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ON THE OBSERVABILITY OF $B^0 - \bar{B}^0$ MIXING IN THE $e^+e^- \to B\bar{B}X$ REACTION AT $\sqrt{s} \approx$ 15 - 30 GEV*

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ABSTRACT

We evaluate the possibility of measuring $B^0 - \bar{B}^0$ mixing in the $e^+e^- \to B\bar{B}X$ reaction (X meaning anything) by detecting like sign dileptons in the final state. We consider the 15-30 GeV c.m. energy region in which any type of B-meson pairs can be produced. The influence of a non-spectator component in the B-decay on the observability of mixing effects is investigated. In addition a method for tagging B's is discussed. Numbers for signal and background have been calculated for an accumulated luminosity of 1000 pb^{-1}.

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1. Introduction

The possibility of observing mixing in the $B^0\bar{B}^0$ system has been rather widely discussed[1–11] . In particular theoretical estimates have been made for the case of $B^0\bar{B}^0$ production in the $e^+e^- \to \Upsilon(4S) \to B^0\bar{B}^0$ reaction. Mixing in this specific channel is expected to appear as a small effect and might thus be hardly measurable. Here we will therefore consider the $e^+e^- \to B\bar{B}X$ reaction (X meaning anything) well above the $B\bar{B}$ threshold where any combination of $B\bar{B}$ mesons[12] ($B_u\bar{B}_u$, $B_u\bar{B}_d$, $B_u\bar{B}_s$, etc.) can be produced. This reaction might be advantageous as mixing is expected to be almost complete for the B_s mesons[5,9] . In this respect the study of mixing effects in the $e^+e^- \to B\bar{B}X$ reaction is complementary to that in the $e^+e^- \to \Upsilon(4S) \to B_d^0\bar{B}_d^0$ process.

In Section 2 we recall briefly the parameters used to describe the $B\bar{B}$ mixing and in Section 3 we give estimates for the mixing effects which could be observed in the $e^+e^- \to B\bar{B}X$ reactions as well as in the $\Upsilon(4S)$ decays. We have also evaluated the influence of a non-spectator component in the B-decay on the observability of the mixing phenomenon. Finally, in Section 4 we give estimates for the number of signal and background events which could be observed in an e^+e^- experiment with an accumulated luminosity of 1000 pb^{-1}.

2. Mixing parameters

As suggested previously[13] the mixing can be measured by either of the following ratios:

$$R = \frac{N^{++} + N^{--}}{N^{+-} + N^{++} + N^{--}} \qquad \text{or} \qquad R' = \frac{N^{++} + N^{--}}{N^{+-}}$$

Here $N^{++} + N^{--}$ denotes the number of events having two leptons of the same charge in the final state, arising from the mixing process and the subsequent semileptonic decays of B^0B^0 or $\bar{B}^0\bar{B}^0$ pairs. The number of events with two leptons of opposite charge is given by N^{+-}. The latter is either due to the semileptonic decays of the B and $\bar{B}$ mesons, $B \to l^- \nu X$ and $\bar{B} \to l^+ \nu X$, or to a double mixing process $B^0 \to \bar{B}^0 \to l^+ \nu X$ and $\bar{B}^0 \to B^0 \to l^- \nu X$. Usually the R and R' ratios are given in terms of the Pais and Treiman parameters[14]

$$r = \frac{N(B^0 \to \bar{B}^0 \to l^+ \nu X)}{N(B^0 \to l^- \nu X)}$$

$$\bar{r} = \frac{N(\bar{B}^0 \to B^0 \to l^- \nu X)}{N(\bar{B}^0 \to l^+ \nu X)}$$

where as before N denotes a number of events. For our present discussion we will ignore eventual CP violation effects, hence $r = \bar{r}$. One has then[1,6,9]

$$r = \bar{r} = \frac{x^2 + y^2}{2 + x^2 - y^2}$$

with $x = \Delta M/\Gamma$ and $y = \Delta\Gamma/\Gamma$. Here ΔM ($\Delta\Gamma$) is the mass (width) difference between the physical B_H and B_L ($H \equiv$ heavy, $L \equiv$ light) state and Γ is the total width of the B meson. One obtains[15]

$$R = \frac{r + \bar{r}}{1 + \bar{r} + r + r\bar{r}} \qquad \to \qquad \frac{2r}{(1+r)^2}$$

$$R' = \frac{r + \bar{r}}{1 + r\bar{r}} \qquad \to \qquad \frac{2r}{1+r^2} \qquad .$$

In Section 3 we will also consider the more experimental oriented parameter

$$R_m = \frac{N^{++} + N^{--}}{N_t}$$

where N_t is here the total number of events containing B's in the final state. This ratio is very convenient as it is straightforward to obtain from it the statistical significance with which a signal can be observed in a given experiment.

It has been shown[4,10] that the above formula giving R and R' cannot be used in the case of the $e^+e^- \to \Upsilon(4S) \to B_d^0 B_d^0, \bar{B}_d^0 \bar{B}_d^0$ reactions as the relative orbital momentum L between the B's has allways the odd value L=1. Therefore the influence of the Bose-Einstein statistics has to be taken into account. This leads to a reduction of the mixing effect[16] and gives[4]

$$R \to R_{odd} = \frac{r}{1+r} \qquad \text{and} \qquad R' \to R'_{odd} = r.$$

In order to appreciate the importance of the mixing, one needs to know the values of x and y which may be estimated within the box diagram approximation[17] . The numerical results depend on several parameters such as the mass m_t of the top quark, the value of the CP violating phase δ in the Kobayashi Maskawa matrix, and the product $f^2 B_b$. Here f is the B meson decay constant and B_b is the so called bag parameter which usually are taken in the range of $f \approx$ 150 - 500 MeV and $B_b \approx$ 0.5 - 1.5 (Ref. 18). Despite the uncertainties in the above parameters it has been shown that for the $B_{s,d}^0$ case the mixing depends essentially on the x parameter[8,9] as $x^2 \ll y^2$ (or $\mid \Delta\Gamma \mid \ll \Delta M$). Taking $y = 0$ and the formula given above we present in Fig. 1 the variation of R' and R'_{odd} as a function of x. One sees from this plot the suppression introduced by the constraint due to the Bose-Einstein statistics. As in the current estimates[6,9] x is small ($x \lesssim 0.4$), one can see that in this range the mixing parameter R'_{odd} for the $e^+e^- \to \Upsilon(4S) \to B_d^0 \bar{B}_d^0$ process is predicted to be rather small. In contrast the mixing for B_s^0 is expected[6,9] to be nearly complete. Moreover the reaction $e^+e^- \to B\bar{B}X$ at high c.m. energy has the advantage that no suppression due to Bose-Einstein statistics occurs since many relative orbital momenta should be present[4,10,16] . In the following we will therefore study the observability of mixing in the 15-30 GeV region where $B_s^0(\bar{B}_s^0)$ can be produced. The results will be compared with the mixing effects which are expected to be observed on the $\Upsilon(4S)$ resonance.

3. The $e^+e^- \rightarrow B\bar{B}X$ process

This reaction well above the $B\bar{B}$ threshold should lead to events having two nearly back-to-back jets due to the decays of the B mesons. These jets will be accompanied by particles due to the fragmentation of the b quarks believed to be produced via the elementary process $e^+e^- \rightarrow b\bar{b}$. In this c.m. energy region the decay products of the two B-mesons are well separated and the $e^+e^- \rightarrow b\bar{b}$ cross section is still sizeable (128 to 32 pb). The mixing can be estimated by using the ratio $R_m = (N^{++} + N^{--})/N_t$. Denoting by P_i the probability to produce a B_i (or a $\bar{B}_i$) in the final state, namely

$$P_i = \frac{b \rightarrow B_i}{b \rightarrow all} = \frac{\bar{b} \rightarrow \bar{B}_i}{\bar{b} \rightarrow all}$$

one has

$$\begin{aligned} R_m \equiv \frac{N^{++} + N^{--}}{N_t} &= 2P_uP_dBr(B_u \rightarrow l)Br(B_d \rightarrow l)\frac{r}{1+r} \\ &+ 2{P_d}^2Br(B_d \rightarrow l)Br(B_d \rightarrow l)\frac{r}{(1+r)^2} \\ &+ 2{P_s}^2Br(B_s \rightarrow l)Br(B_s \rightarrow l)\frac{r_s}{(1+r_s)^2} \\ &+ 2P_sP_uBr(B_u \rightarrow l)Br(B_s \rightarrow l)\frac{r_s}{1+r_s} \\ &+ 2P_dP_sBr(B_d \rightarrow l)Br(B_s \rightarrow l)\frac{r_s}{(1+r)(1+r_s)} \\ &+ 2P_dP_sBr(B_d \rightarrow l)Br(B_s \rightarrow l)\frac{r}{(1+r)(1+r_s)} \end{aligned}$$

neglecting B_c - production[19] . Here r_s represents the Pais and Treiman parameter for the B_s^0 meson and $Br(B_i \rightarrow l)$ is the B_i semileptonic branching ratio into an electron or a muon ($l \equiv e,\ \mu$). For simplicity we neglect in the present discussion the $B \rightarrow \tau\nu X$ decay which is small because of phase space suppression[20] . Note that for the $e^+e^- \rightarrow \Upsilon(4S) \rightarrow B_d^0\bar{B}_d^0$ reaction one has

$$R_m \equiv \frac{N^{++} + N^{--}}{N_t} = f_r \left[Br(B_d^0 \rightarrow l\nu X\right]^2 \frac{r}{1+r}$$

where f_r is the fraction of $B_d^0\bar{B}_d^0$ produced at the $\Upsilon(4S)$.

The possibility to pick up an $s\bar{s}$ pair from the sea is usually assumed to be around 0.1 - 0.2 (Ref. 19) , a value which is supported by a recent evaluation of the data taken at $\sqrt{s}$ = 34 GeV by the TASSO collaboration[21] . Taking P_s = 0.17 and $P_u = P_d$ = (1 - 0.17) / 2, we estimate $R_m = (N^{++} + N^{--})/N_t$ as a function of the phase δ using m_t = 45 GeV, $f^2 B_b = (0.20\text{GeV})^2$ and $\Gamma(b \to l\nu u)/\Gamma(b \to l\nu c)$=0.01 ($\Gamma$ is here the $b \to l\nu q$ decay width). The r and r_s values for our parameters were obtained from Ref. 9 and Ref. 6, respectively, which incorporate in their analyses the latest results on the B meson lifetime. The B meson was assumed to decay according to the spectator model, the semileptonic branching ratio being $Br(B \to l\nu X)$ = 2 x 0.116 as obtained at the $\Upsilon(4S)$ (Ref. 22). As a recent analysis of experimental data has shown that δ should be confined to the $0° - 180°$ range[23] , we have only considered this interval. For the values of the parameters chosen above, R_m (not shown) varies by about 20% when δ changes from 0° to 180°. As an example we give in Table 1 the R_m value for δ = 90° which corresponds to $r \simeq 2.8 \times 10^{-2}$ and $r_s \simeq 0.89$. In the same table we also present the values of R_m calculated with various P_s (P_s=0.17 and the limit $P_u = P_d = P_s = 1/3$), $f^2 B_b$ and m_t values. We utilize the fact that $y \approx 0$ and that x scales approximately[24] with $f^2 B_b m_t^2$. One sees from this table that the mixing parameter is expected to be around 10^{-2} (first two rows in Table 1). It does not depend very much on the value chosen for $\Gamma(b \to l\nu u)/\Gamma(b \to l\nu c)$. An increase by a factor of 3 changes R_m by only $\simeq$ 2%.

Recently an investigation of the available data on B decays and on lifetime measurements lead to the hypothesis that non spectator contributions may be important in the B decay mechanism[25] . An important difference between the lifetime of the B_d^0 (τ_0) and the $B^\pm$ ($\tau_\pm$) was thus predicted, namely $\tau_\pm/\tau_0$ = 1.4 - 1.8. This would correspond to a non-spectator contribution α_{ns} of 29 to 44 % to the total width of the B_d^0 (Ref. 26,27). The increase of the total $B_d^0(\bar{B}_d^0)$ width will lead to a decrease of the $B_d^0(\bar{B}_d^0) \to l\nu X$ branching ratio as semileptonic decays proceed essentially via the spectator mechanism[25,27] . It is usually assumed that it is the diagram shown in Fig. 2a with $B_d^0 \to gc\bar{u}$ which is responsible for the bulk of the non spectator contribution to the B_d^0-decay[27] . For the B_s^0 the process $B_s^0 \to gc\bar{u}$ (Fig. 2b) is Cabibbo-suppressed while the $B_s^0 \to gc\bar{c}$ is expected to be small due to phase space considerations. We will, therefore, first assume that the total B_s^0 decay width, $\Gamma(B_s^0)$, will not be affected by the non spectator contribution ($\Gamma(B_s^0) = \Gamma(B^\pm)$). Then in order to be conservative we will also consider the case where both, $\Gamma(B_s^0)$ and $\Gamma(B_d^0)$, are increased by the same amount.

As examples we will take for α_{ns} the values of 20% and 40%. One then has to extract from the $Br(B \to l\nu X)$ value (measured on the $\Upsilon(4S)$) the semileptonic $B^0 \to l\nu X$ and $B^\pm \to l\nu X$ branching ratios. To this end, we assume that $B_d^0\bar{B}_d^0$ and B^+B^- are produced in the $e^+e^- \to \Upsilon(4S) \to B\bar{B}$ reaction in the ratio of

$$N(B_d^0\bar{B}_d^0) \; : \; N(B^+B^-) \; = \; 40\,\% : \; 60\%$$

as a consequence of the $B_d^0, B^\pm$ mass difference[28−30] $M(B_d^0) - M(B^\pm) \approx 4$ MeV.

Using the above ratio and the assumed α_{ns}, one obtains the $Br(B_d^0 \to l\nu X)$ and $Br(B^\pm \to l\nu X)$ as given in Table 1. Correcting then the B_d^0 (and eventually the B_s^0) total width in order to take into account the non spectator contribution, we recalculate the $x = \Delta M/\Gamma$ values for $\alpha_{ns} = 20,\ 40\%$ (taking $y = \Delta\Gamma/\Gamma = 0$). We obtain thus the new values for R_m also given in Table 1. As can be seen from this table the values obtained for R_m do not depend dramatically on the chosen values of the parameters and have the same order of magnitude.

For comparison we give in Table 2 the R_m values expected for the the $e^+e^- \to \Upsilon(4S) \to B_d^0\bar{B}_d^0$ reaction. One notices that they are an order of magnitude smaller than the corresponding values obtained from the $e^+e^- \to B\bar{B}X$ process. However, the cross section for producing $B_d^0\bar{B}_d^0$ at the $\Upsilon(4S)$ is much larger ($\approx 1nb$, see Ref. 31) than the $e^+e^- \to B\bar{B}X$ cross section in the $\sqrt{s} =$ 15 - 30 GeV region. (Note also, that if δ reaches a value near 180°, R_m for the $\Upsilon(4S)$ could be increased by a factor of $\lesssim 3$). Not accounting for any detection efficiency or background contributions, the product $\sigma(b\bar{b})R_m$ can be used to compare both cases. For $\alpha_{ns} = 0.20$, $B_b f^2 = (0.20\ \text{Gev})^2$ and $m_t = 45$ GeV one has $\sigma(b\bar{b})R_m \approx 0.6$ pb for the $\Upsilon(4S)$ whereas in the $\sqrt{s}$ = 15 - 30 GeV region one obtains $\sigma(b\bar{b})R_m \approx 1.1 - 0.3$ pb (using P_s=0.17). In this respect the two methods are nearly equivalent. On the other hand they are complementary since in one case one measures the mixing properties of the $B_d^0 - \bar{B}_d^0$ system and in the second case primarily those of the $B_s^0 - \bar{B}_s^0$ system. Indeed, in the $\sqrt{s}$ = 15 - 30 GeV region, $\sim 84\%$ of the mixing effect comes from the B_s^0 and $\bar{B}_s^0$ production (as obtained from the formula giving R_m for the $e^+e^- \to B\bar{B}X$ reaction with P_s=0.17).

4. B-Tagging and mixing

In the following we estimate the signal and background obtained by measuring the mixing with like sign dileptons. Apart from mixing, same sign dileptons can also arise from the following decays :

$$B(\bar{B}) \to l^{\mp}\nu X$$

and

$$\bar{B}(B) \to \bar{D}(D)X \qquad \text{where} \qquad \bar{\mathrm{D}}(\mathrm{D}) \to \mathrm{l}^{\mp}\nu\mathrm{X}.$$

This is illustrated by Fig. 3 which presents at the quark level the various ways of producing leptons. In practice one counts only those events having two leptons of the same charge in opposite hemispheres (the latter being defined with respect to the thrust axis) where both leptons exceed a certain momentum cut (we will take successively 1.0 GeV/c and 1.5 GeV/c). This momentum cut will lead to an additional source of background due to asymmetric π^0 Dalitz decays where one of the $e^{\pm}$ has a momentum below the chosen threshold or escapes detection.

Note that if one parameterizes the mixing by R or R' instead of R_m (see Section [2]) additional backgrounds will be generated. This is because one has to determine also N^{+-}, which is contaminated by:

$$e^+e^- \to B\bar{B}X \qquad \begin{bmatrix} B \to DX, & D \to l^+\nu X \\ \bar{B} \to \bar{D}X, & \bar{D} \to l^-\nu X \end{bmatrix}$$

and

$$e^+e^- \to D\bar{D}X \qquad \begin{bmatrix} D \to l^+\nu X \\ \bar{D} \to l^-\nu X \end{bmatrix}$$

(The background due to τ decays and two photon interactions can be easily eliminated by a multiplicity and total energy cut, respectively).

In order to estimate the number of events for signal and background we used the Lund Monte Carlo program[19,32] . We generated $e^+e^- \to b\bar{b}$ events at three different c.m. energies, $\sqrt{s} = 15$, 22, 30 GeV and at the c.m. energy corresponding to the $\Upsilon(4S)$. We also estimated the possible additional background induced by the $e^+e^- \to q\bar{q}$ processes (where $q = u, d, s, c$) by applying the same cuts as those used for selecting the B's.

To appreciate the importance of the detector properties we assumed that the identification of electrons and muons poses no severe problems and we take a lepton detection efficiency of $\epsilon_{lep} = 90\%$ for lepton momenta $p_l > 1.0$ GeV/c. In addition we require that the leptons should have $\mid \cos\theta_l \mid < 0.90$ (where θ_l is the emission angle of the lepton with respect to the beam direction). The results are given in Table 3 for an accumulated luminosity of 1000 pb^{-1} and $R_m = 8.9 \times 10^{-3}$ (see Table 1). The $e^+e^- \to b\bar{b} \to B\bar{B}X$ cross sections at 15, 22 and 30 GeV were taken as 128, 60 and 32 pb, respectively. Also listed in this table are the signal to background ratio as well as the corresponding significance of the signal. One notices that the background coming from the $e^+e^- \to b\bar{b}$ process ($B(\bar{B}) \to l\nu X$ and $\bar{B}(B) \to \bar{D}(D)X \to l\nu X$, asymmetric π^0 Dalitz decays) are much more important than those induced by $e^+e^- \to q\bar{q}$ $(q = u, d, s, c)$. For comparison we give in the same table the results expected from the $\Upsilon(4S)$, using $\sigma(b\bar{b}) = 1nb$ and $R_m = 5.5 \times 10^{-4}$ (see Table 2). Clearly in this case we do not require the leptons to be in opposite hemispheres as the B's are almost emitted at rest.

In the present study the signal to background ratio is the largest at $\sqrt{s} \simeq 15$ GeV. The number of standard deviations (s.d.) quoted in the table might however be somewhat misleading. This is because in the actual experiment one has to evaluate an unknown background which comprises an important part of the observed event sample. Unless the background can be measured somehow, one has to rely on Monte Carlo calculations thus introducing systematic uncertainties.

Therefore, we would like to discuss here an alternative method for tagging the $e^+e^- \to B\bar{B}X$ process. In addition to the cut in the momentum p_l we propose to detect in each jet a kaon of the same charge as the lepton. This is because in the Cabibbo allowed $b \to c \to s$ ($\bar{b} \to \bar{c} \to \bar{s}$) quark decay chain, the charged kaon has the same charge as the lepton if the latter originates from the b decay as can be seen from Fig. 3 and 4. Note that the diagram in Fig. 4b gives a contribution to the l^-K^- production which is about 5 times smaller than that of Fig. 4a because of phase space suppression[33] . We have also presented in Fig. 4c a background process due to the decay of a $\bar{b}$ quark leading to an l^-K^- pair among the $\bar{B}$ decay particles.

The $B\bar{B}$ tagging efficiency is obtained by counting the number of events having their l^+K^+ and l^-K^- emitted in opposite hemispheres. We also count the number of events having l^+K^+ (l^-K^-) in one hemisphere and another l^+K^+ (l^-K^-) pair in the opposite one. These events represent part of the expected background to the mixing signal as no mixing effects have been implemented in the used Monte Carlo program. In fact this background is coming mainly from the processes of the type shown in Fig. 4c. We assume that the identification of kaons can be obtained with a combination of dE/dx, time of flight, and Čerenkov counters. We consider that $0.2 \leq p_K \leq 2.4$ GeV/c is the $K^{\pm}$ momentum window in which identification can be achieved. Assuming that the contamination of $\pi(K)$ in the $K(\pi)$ sample is $\sim 1\%$ between 0.9 and 1.0 GeV/c (a good π/K

separation is assumed below 0.9 GeV/c with the dE/dx and time of flight devices) and that $\sim 4\%$ of the produced $\pi^{\pm}$ can fake $K^{\pm}$ in the 1.0 - 2.5 GeV/c region (a typical value for threshold Čerenkov counters) we obtain the results given in Table 3. The signal to background ratio did not improve with respect to the method that required only the identification of leptons. The efficiency however drops drastically in the $l^{\pm}K^{\pm}/l^{\pm}K^{\pm}$ tagging.

In order to decrease the background due to the diagram of Fig. 4c we refined the above tagging method by requiring that in one hemisphere at least one $l^{\pm}K^{\pm}$ pair should not be accompagnied by an additional $K^{\mp}$. The tagging efficiency is decreased somewhat (Table 3) but one obtains almost in all cases a significantly larger signal to background ratio with a still acceptable statistical significance. The $\sqrt{s} \approx 15 - 22$ GeV c.m. energy range appears thus to be a promising region in which $B^0\bar{B}^0$ mixing might be experimentally accessible.

5. Conclusions

We studied the possibility of measuring $B^0\bar{B}^0$ mixing in the $e^+e^- \rightarrow B\bar{B}X$ reaction by detecting like sign dileptons belonging to opposite jets. We considered the $\sqrt{s}$ = 15 - 30 GeV region where any type of B meson can be produced.

Using current values for the unknown parameters governing the mixing phenomenon (such as the mass of the top quark, the bag parameter, the B meson decay constant) we investigated the observability of mixing in e^+e^- experiments with an accumulated luminosity of 1000 pb^{-1}. The performed Monte Carlo calculations have shown that by selecting leptons with momenta $p_l > 1.0, 1.5$ GeV one can expect signal to background ratios ranging from 0.3 to 1.6. We have also carried out a brief comparison of our predictions with the mixing effects which could be observed at the $\Upsilon(4S)$. The numbers obtained for the ratio of signal to background appear to be comparable. On the other hand both measurements are complementary since in one case one measures primarily the mixing of the $B_s^0 - \bar{B}_s^0$ and in the other case that of the $B_d^0 - \bar{B}_d^0$ system.

As the subtraction of a large background from the observed signal may introduce further uncertainties we have also investigated additional methods for tagging the $e^+e^- \to B\bar{B}X$ process. They are based on the fact that in the Cabibbo allowed $b \to c \to s$ ($\bar{b} \to \bar{c} \to \bar{s}$) quark decay chain, $K^\pm$ and $l^\pm$ have the same charge if the lepton comes from the $B \to l^\pm \nu X$ decay. The proposed tagging methods lead to a drastic decrease in the detection efficiency but tend to increase the signal to background ratio. In this respect these tagging methods are useful and could be applied for a further measurement of mixing phenomena.

It appears that the observable effects of mixing phenomena at the $\Upsilon(4S)$ and in the $e^+e^- \to B\bar{B}X$ reaction are of comparable size within the discussed framework. The $e^+e^- \to B\bar{B}X$ reaction has however the additional advantage that all the current theoretical models predict the mixing of the $B_s^0(\bar{B}_s^0)$ to be almost complete. This means that the observable effects will not depend crucially on the exact values of unknown parameters such as the top quark mass, the bag parameter, etc.. In conclusion the $\sqrt{s} \simeq 15$ GeV region appears in this study as the most promising one for detecting mixing.

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TABLE 1: For the phase $\delta \simeq 90°$ estimates of the mixing parameter $R_m = (N^{++} + N^{--})/N_t$ obtained with various values of α_{ns}, P_i, m_t and $f^2 B_b$. Here α_{ns} is the fraction of the non-spectator contribution to the B^0 decay, P_i is the probability $P_i = (b \to B_i)/(b \to \text{all})$ and m_t is the top quark mass. In the product $f^2 B_b$, f represents the B meson decay constant and B_b the bag parameter (see text). For $\alpha_{ns} \neq 0$ we recalculated the $B^\pm$ and B_d^0 semileptonic branching ratios using as input the experimental $\mathrm{Br}(B \to l\nu X) = 2 \times 0.116$ value. In these cases, R_m was calculated assuming successively that $\mathrm{Br}(B_s^0 \to l\nu X) = \mathrm{Br}(B^\pm \to l\nu X)$ (next to the last column) and $\mathrm{Br}(B_s^0 \to l\nu X) = \mathrm{Br}(B_d^0 \to l\nu X)$ (last column).

$\alpha_{ns}(\%)$	P_s $P_u = P_d$ $P_c = 0$	$\mathrm{Br}(B^\pm \to l\nu X)$	$\mathrm{Br}(B_d^0 \to l\nu X)$	$B_b f^2$ $(\mathrm{GeV})^2$	m_t (GeV)	$\frac{(N^{++}+N^{--})}{N_t}$ $\Gamma(B_s^0) = \Gamma(B^\pm)$	$\frac{(N^{++}+N^{--})}{N_t}$ $\Gamma(B_s^0) = \Gamma(B_d^0)$
0.0				$(0.20)^2$	45	$\approx 8.9\ 10^{-3}$	—
	0.170	2×0.116	2×0.116	$(0.15)^2$	45	$\approx 7.4\ 10^{-3}$	—
				$(0.20)^2$	35	$\approx 7.7\ 10^{-3}$	—
				$(0.20)^2$	45	$\approx 1.5\ 10^{-2}$	—
	0.333	2×0.116	2×0.116	$(0.15)^2$	45	$\approx 1.3\ 10^{-2}$	—
				$(0.20)^2$	35	$\approx 1.3\ 10^{-2}$	—
20.0				$(0.20)^2$	45	$\approx 9.0\ 10^{-3}$	$\approx 7.0\ 10^{-3}$
	0.170	2×0.126	2×0.101	$(0.15)^2$	45	$\approx 7.8\ 10^{-3}$	$\approx 5.6\ 10^{-3}$
				$(0.20)^2$	35	$\approx 8.1\ 10^{-3}$	$\approx 5.9\ 10^{-3}$
				$(0.20)^2$	45	$\approx 1.6\ 10^{-2}$	$\approx 1.2\ 10^{-2}$
	0.333	2×0.126	2×0.101	$(0.15)^2$	45	$\approx 1.4\ 10^{-2}$	$\approx 1.0\ 10^{-2}$
				$(0.20)^2$	35	$\approx 1.5\ 10^{-2}$	$\approx 1.0\ 10^{-2}$
40.0				$(0.20)^2$	45	$\approx 9.4\ 10^{-3}$	$\approx 5.0\ 10^{-3}$
	0.170	2×0.138	2×0.083	$(0.15)^2$	45	$\approx 8.4\ 10^{-3}$	$\approx 3.8\ 10^{-3}$
				$(0.20)^2$	35	$\approx 8.6\ 10^{-3}$	$\approx 4.0\ 10^{-3}$
				$(0.20)^2$	45	$\approx 1.7\ 10^{-2}$	$\approx 8.6\ 10^{-3}$
	0.333	2×0.138	2×0.083	$(0.15)^2$	45	$\approx 1.6\ 10^{-2}$	$\approx 6.7\ 10^{-3}$
				$(0.20)^2$	35	$\approx 1.6\ 10^{-2}$	$\approx 7.1\ 10^{-3}$

Table 2: For the phase $\delta \simeq 90°$ estimates of $R_m = (N^{++} + N^{--})/N_t$ for the $e^+e^- \to \Upsilon(4S) \to B_d^0 \bar{B}_d^0$ reaction using $f_r = 0.40$ for the fraction of $B_d^0 \bar{B}_d^0$ produced at the $\Upsilon(4S)$. Note that if $\delta \simeq 180°$, R_m could be increased by a factor of $\lesssim 3$.

α_{ns}(%)	Br($B^\pm \to l\nu X$)	Br($B_d^0 \to l\nu X$)	$B_b f^2$ (GeV)^2	m_t (GeV)	$\frac{(N^{++}+N^{--})}{N_t}$
0.0	2×0.116	2×0.116	$(0.20)^2$	45	$\approx 5.5\ 10^{-4}$
			$(0.15)^2$	45	$\approx 1.8\ 10^{-4}$
			$(0.20)^2$	35	$\approx 2.1\ 10^{-4}$
20.0	2×0.126	2×0.101	$(0.20)^2$	45	$\approx 2.7\ 10^{-4}$
			$(0.15)^2$	45	$\approx 8.7\ 10^{-5}$
			$(0.20)^2$	35	$\approx 1.0\ 10^{-4}$
40.0	2×0.138	2×0.083	$(0.20)^2$	45	$\approx 6.7\ 10^{-5}$
			$(0.15)^2$	45	$\approx 2.1\ 10^{-5}$
			$(0.20)^2$	35	$\approx 2.6\ 10^{-5}$

Table 3: Estimates of the number of events due to mixing and to background using the tagging procedure described in the text. The numbers were obtained at $\sqrt{s}$= 15, 22 and 30 GeV for an integrated luminosity of 1000 pb^{-1} and $R_m = 8.9\times10^{-3}$. For the $\Upsilon(4S)$ we took $R_m = 5.5\times10^{-4}$. Here N_{mix} represents the number of events due to mixing before applying any tagging method.

	15GeV N_{mix}=1139 events lepton momentum (GeV/c)		22GeV N_{mix}= 534 events lepton momentum (GeV/c)		30GeV N_{mix}= 285 events lepton momentum (GeV/c)		$\Upsilon(4S)$ N_{mix}= 550 events lepton momentum (GeV/c)	
	≥ 1.0	≥ 1.5	≥ 1.0	≥ 1.5	≥ 1.0	≥ 1.5	≥ 1.0	≥ 1.5
$l^{\pm}/l^{\pm}$ tagging	423	214	267	170	161	120	262	99
background ($b\bar{b}$)	529	120	710	280	494	261	2346	230
background ($c\bar{c}$)	47	14	52	14	34	13	<63	<27
signal/ background	0.7	1.6	0.4	0.6	0.3	0.4	>0.1	>0.4
# of s.d.	13	12	8	8	6	6	>5	>5
$l^{\pm}K^{\pm}/l^{\pm}K^{\pm}$ tagging	34	25	27	22	18	16		
background ($b\bar{b}$)	<40	<19	50	30	41	27		
background ($c\bar{c}$)	<11	<5	<6	<6	<5	< 4		
signal/ background	>0.7	>1.0	>0.5	>0.6	>0.4	>0.5		
# of s.d.	>4	>4	>3	>3	> 2	>2		
$l^{\pm}K^{\pm}/l^{\pm}K^{\pm}$, no $K^{\mp}$(tagging)	29	22	18	14	12	10		
background ($b\bar{b}$)	<11	<11	< 13	<11	<13	<13		
background ($c\bar{c}$)	< 7	< 5	< 2	< 2	< 2	< 2		
signal/ background	>1.6	>1.4	>1.2	>1.1	>0.8	>0.7		
# of s.d.	>4	>4	>3	> 3	> 2	>2		

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18. See for instance (5), (8) and (9).
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FIGURE CAPTIONS

Fig. 1. The distributions of R' and R'_{odd} as a function of $x = \Delta M/\Gamma$.

Fig. 2. Non spectator processes which may contribute to the B^0 decays (see Ref. 27). Note that among the combinations of "outgoing" quarks only c-quarks have been considered because of the dominance of the $b \to c$ over $b \to u$ transitions.

Fig. 3. Schematic representation of quark decays which may lead to lepton production in the case of the $e^+e^- \to b\bar{b}$ process.

Fig. 4. Some b and $\bar{b}$ quark decay processes leading to l^-K^- pairs in the final state.

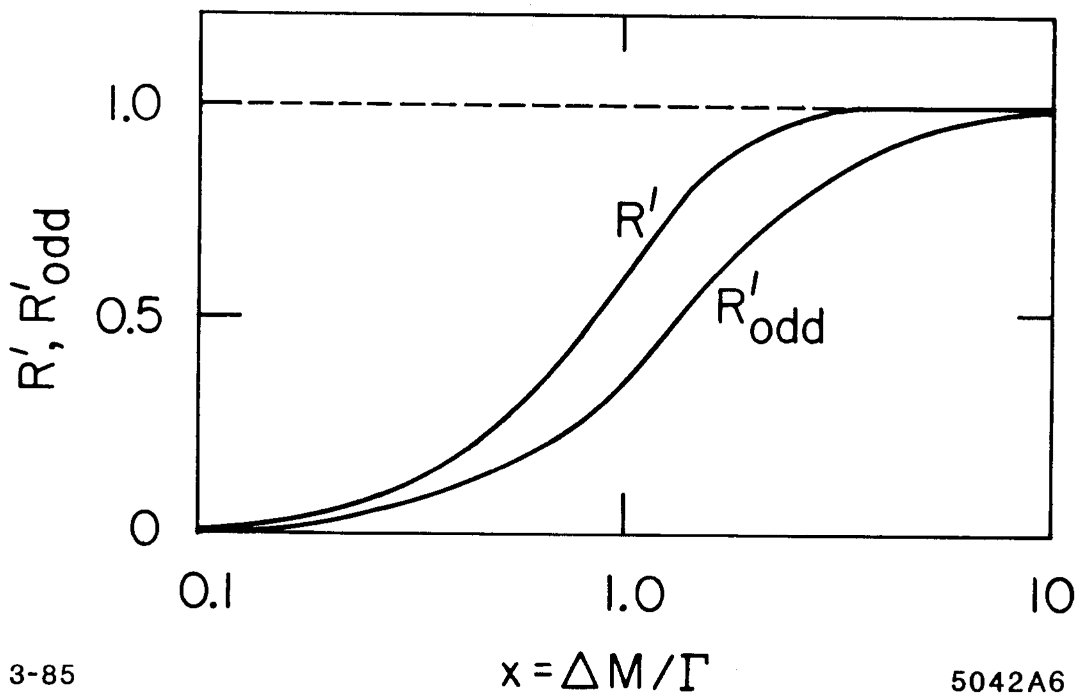


Fig. 1

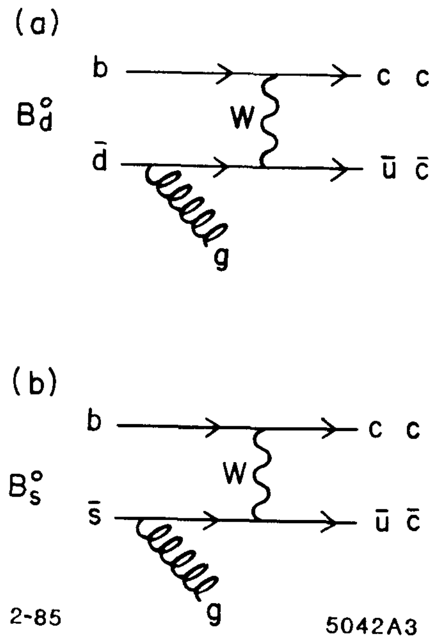

(a)
b
c c
B_d^o
W
$\bar{d}$
$\bar{u}$ $\bar{c}$
g
(b)
b
c c
B_s^o
W
$\bar{s}$
$\bar{u}$ $\bar{c}$
g
2-85
5042A3

Fig. 2

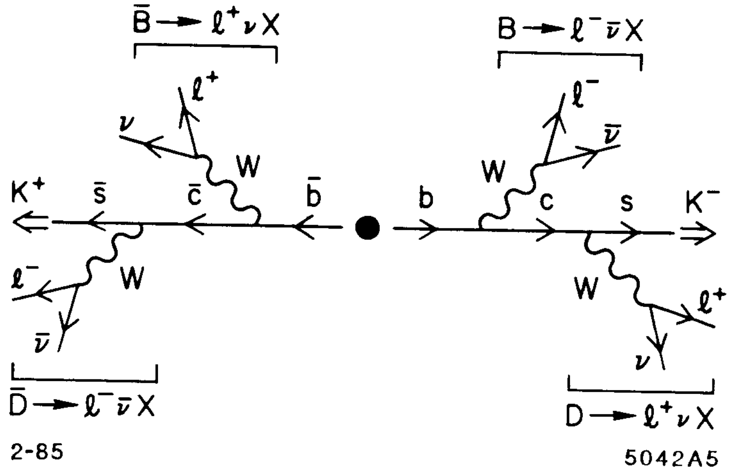
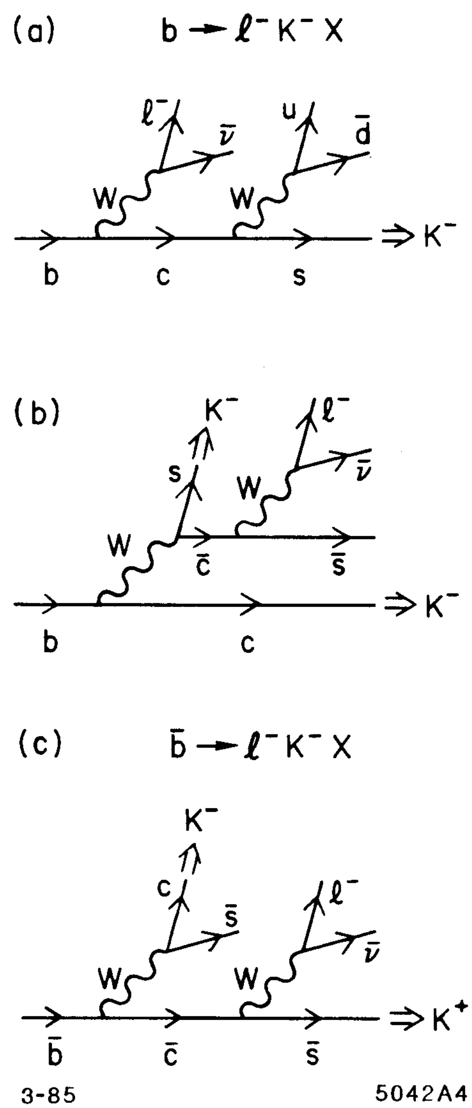


Fig. 3



(a)
b → ℓ⁻ K⁻ X
ℓ⁻
ν̄
u
d̄
W
W
⇒K⁻
b
c
s
(b)
K⁻
ℓ⁻
s
W
ν̄
W
c̄
s̄
⇒K⁻
b
c
(c)
b̄ → ℓ⁻ K⁻ X
K⁻
c
s̄
ℓ⁻
ν̄
W
W
⇒K⁺
b̄
c̄
s̄
3-85
5042A4

Fig. 4