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RENORMALIZATION GROUP CONSTRAINTS IN TWO-HIGGS THEORIES*

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ABSTRACT

We use the SU(3) $\times$ SU(2) $\times$ U(1) renormalization group equations to constrain fermion masses and charged-scalar couplings in two-Higgs grand unified theories. We find upper bounds on the sum of all quark (or lepton) masses and show that the Cabibbo mixings of heavy quarks tend to be small. We bound the vacuum expectation values of the Higgs fields, and use these limits to place strong restrictions on the couplings of the charged Higgs scalar.

1. Introduction

Several popular extensions of the standard SU(3) $\times$ SU(2) $\times$ U(1) model have two Higgs doublets at low energies. The scalar fields appear in Glashow-Weinberg-Paschos form [1], where one Higgs doublet couples to up-type quarks, and the other couples to down-type quarks. This mechanism naturally suppresses flavor-changing neutral currents. The Yukawa couplings are as follows*

$$\mathcal{L}_Y = \overline{u}\,\mathcal{U}Q\,\phi_u + \overline{d}\,\mathcal{D}Q\,\phi_d + \overline{\nu}\,\mathcal{N}L\,\phi_u + \overline{e}\,\mathcal{E}L\,\phi_d\,, \tag{1}$$

where $\mathcal{U}$, $\mathcal{D}$, $\mathcal{N}$ and $\mathcal{E}$ are the $N_F \times N_F$ Yukawa matrices of the up-, down-, neutrino- and electron-type fermions, Q and L are the quark and lepton isodoublets, and u, d, ν and e are the corresponding singlet fields.

The standard model does not limit the number of families N_F, nor does it constrain the Yukawa matrices $\mathcal{U}$, $\mathcal{D}$, $\mathcal{N}$ and $\mathcal{E}$. Recent theoretical arguments [2] suggest that $N_F > 3$, and that the new quarks and leptons should live near the weak scale M_W. The masses and mixings of the heavy families are tightly constrained by the infrared fixed-point structure of the SU(3) $\times$ SU(2) $\times$ U(1) renormalization group equations [3,4].

The purpose of this letter is to derive model-independent constraints on the masses and mixings of heavy families in two-Higgs grand unified theories. We make only two assumptions:

1. The desert: We assume that SU(3) $\times$ SU(2) $\times$ U(1) is the effective gauge theory between the weak scale M_W and the unification scale M_X, and that all Yukawa couplings are of Glashow-Weinberg-Paschos form.
2. Perturbative unification: We require all gauge and Yukawa couplings to be small enough for perturbation theory to be valid all the way up to the scale M_X. This assumption is an essential requirement for grand unification.

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* For completeness we consider the possibility of singlet neutrinos in the low-energy theory. If there are no such neutrinos, then $\mathcal{N} = 0$.

Our results rely on perturbation theory throughout the entire SU(3) $\times$ SU(2) $\times$ U(1) desert. They follow from the infrared fixed-point structure of the SU(3) $\times$ SU(2) $\times$ U(1) renormalization group equations. Our results do not depend on the details of unification at the scale M_X.

2. Infrared fixed points

We start by stating our fundamental equations. In the two-Higgs case, the renormalization group equations for the Yukawa couplings take the following form,

$$
\begin{aligned}
\mathcal{U}^{-1}\frac{d\mathcal{U}}{dt} &= G_U - 3T_U - T_N - \frac{1}{2}(3\,\mathcal{U}^\dagger\mathcal{U} + \mathcal{D}^\dagger\mathcal{D})\,,\\
\mathcal{D}^{-1}\frac{d\mathcal{D}}{dt} &= G_D - 3T_D - T_E - \frac{1}{2}(3\,\mathcal{D}^\dagger\mathcal{D} + \mathcal{U}^\dagger\mathcal{U})\,,\\
\mathcal{N}^{-1}\frac{d\mathcal{N}}{dt} &= G_N - T_N - 3T_U - \frac{1}{2}(3\,\mathcal{N}^\dagger\mathcal{N} + \mathcal{E}^\dagger\mathcal{E})\,,\\
\mathcal{E}^{-1}\frac{d\mathcal{E}}{dt} &= G_E - T_E - 3T_D - \frac{1}{2}(3\,\mathcal{E}^\dagger\mathcal{E} + \mathcal{N}^\dagger\mathcal{N})\,.
\end{aligned}
\tag{2}
$$

We have defined

$$
\begin{aligned}
G_U &= 8\,{g_3}^2 + \frac{9}{4}{g_2}^2 + \frac{17}{20}{g_1}^2\,, \qquad & G_N &= \frac{9}{4}{g_2}^2 + \frac{9}{20}{g_1}^2\,,\\
G_D &= 8\,{g_3}^2 + \frac{9}{4}{g_2}^2 + \frac{1}{4}{g_1}^2\,, \qquad & G_E &= \frac{9}{4}{g_2}^2 + \frac{9}{4}{g_1}^2\,,
\end{aligned}
\tag{3}
$$

where g_3, g_2, and g_1 are the SU(3) $\times$ SU(2) $\times$ U(1) gauge couplings, and

$$
\begin{aligned}
T_Y &= \mathrm{Tr}\,\mathcal{Y}^\dagger\mathcal{Y}\,,\\
t &= -\frac{1}{16\pi^2}\log\left(\frac{M}{M_X}\right)\,,
\end{aligned}
\tag{4}
$$

with $\mathcal{Y} = \mathcal{U}, \mathcal{D}, \mathcal{N}$ or $\mathcal{E}$.

As shown in Reference [4], it is useful to consider the fixed point structure in the mathematical limit $t \to \infty$. In this limit, the equations (2) have two distinct fixed points:

1) The quark fixed point, with

$$\begin{aligned} \mathcal{U}^\dagger\mathcal{U} &= \mathcal{D}^\dagger\mathcal{D} = \frac{G_Q}{3N_F+2}\,, \\ \mathcal{E} &= \mathcal{N} = 0\,; \end{aligned} \tag{5}$$

2) The lepton fixed point, with

$$\begin{aligned} \mathcal{E}^\dagger\mathcal{E} &= \mathcal{N}^\dagger\mathcal{N} = \frac{G_L}{N_F+2}\,, \\ \mathcal{U} &= \mathcal{D} = 0\,. \end{aligned} \tag{6}$$

In equation (5), G_Q denotes an appropriate average of G_U and G_D, with $g_1 = 0$. G_L represents a similar average over G_E and G_N.

Equations (5) and (6) are incompatible, so only one of the two fixed points* is reached as $t \to \infty$. For physical gauge couplings, the quark fixed point is strongly preferred. It has a much larger domain of attraction than the lepton fixed point.

The fixed point conditions (5) determine the low-energy Yukawa spectrum of quarks and leptons. All quarks have the same Yukawa coupling at M_W, independent of the initial conditions at M_X. Furthermore, all weak mixings and CP-violating phases vanish. Note that both isospin and family symmetry† are restored in the infrared limit. In the one-doublet case, family symmetry is not restored [4].

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* There are additional fixed points when some masses vanish. These fixed points are unstable, so we do not consider them here.

† The isospin and family symmetry is at the level of Yukawa couplings. Different values of the vacuum expectation values of the Higgs fields, $\langle\phi_u\rangle$ and $\langle\phi_d\rangle$, introduce different masses for up- and down-type quarks.

3. Physical fixed points

The mathematical fixed point in (5) is reached as $t \to \infty$. However, in a realistic grand unified theory, the physical range of t is rather small, $0 \lesssim t \lesssim 1/5$. In this case, the fixed point is reached only if the Yukawa couplings are sufficiently large. In the rest of this paper we restrict our attention to the fixed points that are reached in physical time.

We first consider the renormalization of the overall scale of the heavy quarks, given by T_U and T_D. From (2) it is easy to show

$$\begin{aligned} \frac{dT_U}{dt} &= 2\,(G_U - 3\,T_U - T_N)\,T_U - 3\,\mathrm{Tr}\,(\mathcal{U}^\dagger \mathcal{U})^2 - \mathrm{Tr}\,(\mathcal{U}^\dagger \mathcal{U}\,\mathcal{D}^\dagger \mathcal{D}) \\ \frac{dT_D}{dt} &= 2\,(G_D - 3\,T_D - T_E)\,T_D - 3\,\mathrm{Tr}\,(\mathcal{D}^\dagger \mathcal{D})^2 - \mathrm{Tr}\,(\mathcal{D}^\dagger \mathcal{D}\,\mathcal{U}^\dagger \mathcal{U})\,. \end{aligned} \tag{7}$$

For large initial Yukawa couplings, the fixed point is reached in physical time. At $t \simeq 1/5$, we have

$$T_U \simeq T_D \simeq \frac{N_H\,G_Q}{3\,N_H + 2}\,, \tag{8}$$

where N_H is the number of heavy families. We will see that these values are also approached for moderate values of the Yukawa couplings. Even for relatively small entries in the $\mathcal{U}$ and $\mathcal{D}$ matrices, the initial velocities are large enough for T_U and T_D to approach their fixed point values.

In Figure 1 we show the evolution of T_U and T_D in a theory with two heavy and three light families. We choose several initial conditions for the Yukawa couplings, and renormalize T_U and T_D to M_W using equations (2). For simplicity, we have set $\mathcal{E} = \mathcal{N} = 0$. We plot T_U versus T_D in Figure 1a. We see that the fixed point (8) is reached in physical time. In Figure 1b we show the evolution of T_D with energy. The fixed point is approached very rapidily.

We now discuss the fixed points associated with the restoration of isospin and family symmetry. These fixed points are approached only if the Yukawa couplings are large and the initial breakings are small. To illustrate this, we return to our previous example. Let u_1, d_1 and u_2, d_2 be the Yukawa couplings of the two heavy families. Isospin restoration is associated with $u_i = d_i$, whereas family symmetry corresponds to $u_1 = u_2$ and $d_1 = d_2$. The evolution of d_2/u_2 is shown in Figure 2*a*, and that of d_2/u_1 in Figure 2*b*. We see that the approach to these fixed points is much slower than the approach to (8). However, isospin and family splittings renormalize substantially between the unification scale M_X and the weak scale M_W. In two-Higgs theories, heavy quarks tend to have identical Yukawa couplings.

At the mathematical fixed point (5), all weak angles and phases vanish. For example, the evolution equation for the Cabibbo mixing between two families is given by

$$\begin{aligned}\frac{d}{dt}\sin\theta_C &= \frac{1}{2}\sin\theta_C\cos^2\theta_C\left\{({d_1}^2 + {d_2}^2)\left(\frac{{u_1}^2 - {u_2}^2}{{d_1}^2 - {d_2}^2}\right)\right.\\ &\left.\quad + ({u_1}^2 + {u_2}^2)\left(\frac{{d_1}^2 - {d_2}^2}{{u_1}^2 - {u_2}^2}\right)\right\}.\end{aligned}\tag{9}$$

This equation has two fixed points, $\theta_C = 0$ and $\theta_C = \pi/2$. The sign of the product $({u_1}^2 - {u_2}^2)({d_1}^2 - {d_2}^2)$ determines which of the two fixed points is approached. This can be seen in Figure 3, where we show the evolution of $\sin\theta_C$ with energy. Although the fixed point is not reached in physical time, $\sin\theta_C$ renormalizes substantially. The mixings between heavy and light quarks renormalize in a similar way. Therefore heavy quarks in the models we are considering have relatively long lifetimes.

4. Bounds on heavy fermion masses

The evolution equations (7) can be used to bound the scale of the heavy quark masses. This can be done by dropping positive-definite terms. The resulting equations,

$$\begin{aligned}\frac{dT_U}{dt} &= 2\,(G_U \;-\; 3\,T_U)\,T_U \\ \frac{dT_D}{dt} &= 2\,(G_D \;-\; 3\,T_D)\,T_D\,,\end{aligned} \tag{10}$$

give upper bounds on T_U and T_D at the weak scale M_W. One can perform a similar analysis in the lepton sector. Using the gauge couplings corresponding to $N_F = 8$, we find

$$\begin{aligned}T_U\,,\;\; T_D &\lesssim 4.1 \\ T_N\,,\;\; T_E &\lesssim 3.6\,.\end{aligned} \tag{11}$$

Since theories with more than eight families are not perturbatively unifiable, these are rigorous upper bounds for any number of families.

To convert (11) into bounds on the masses, we introduce vacuum expectation values v_u and v_d for the scalar fields ϕ_u and ϕ_d. By using $\sum M_U{}^2 = (v_u)^2\; T_U$ and $\sum M_D{}^2 = (v_d)^2\; T_D$, we place limits on the heavy quark mass spectrum:

$$\begin{aligned}\sum\, M_U{}^2 &\lesssim (v_u/v)^2\; (355\ \mathrm{GeV})^2 \\ \sum\, M_D{}^2 &\lesssim (v_d/v)^2\; (355\ \mathrm{GeV})^2 \\ \sum\, M_Q{}^2 &\lesssim (355\ \mathrm{GeV})^2\,,\end{aligned} \tag{12}$$

where $v_u{}^2 + v_d{}^2 = v^2 = (175\ \mathrm{GeV})^2$, and all masses are evaluated at the weak scale M_W. In equation (12) the sum over Q runs over both up- and down-type quarks. If there are fewer than eight families, the bounds can be tightened still further. For example, in the four family case we find $\sum M_Q{}^2 \lesssim (290\ \mathrm{GeV})^2$.

The corresponding bounds for leptons are

$$
\begin{aligned}
\sum M_N{}^2 &\lesssim (v_u/v)^2\,(330\ \mathrm{GeV})^2 \\
\sum M_E{}^2 &\lesssim (v_d/v)^2\,(330\ \mathrm{GeV})^2 \\
\sum M_L{}^2 &\lesssim (330\ \mathrm{GeV})^2\,.
\end{aligned}
\tag{13}
$$

In contrast to (12), the inequalities (13) are typically far from being saturated. This follows from the fact that the lepton fixed point is unstable. As a consequence, lepton masses tend to be much smaller than their quark counterparts.

Equations (12) and (13) lead immediately to bounds on the masses of new heavy families. If m_Q and m_L denote the masses of the lightest new quark and lepton, we find

$$
\begin{aligned}
m_Q &\lesssim (250/\sqrt{N_H})\ \ \mathrm{GeV} \\
m_L &\lesssim (235/\sqrt{N_H})\ \ \mathrm{GeV}\,.
\end{aligned}
\tag{14}
$$

5. Bounds on scalar vacuum expectation values

Equation (12) can be used to infer limits on the ratio of the vacuum expectation values v_u/v_d. To see this, suppose that the heaviest up- and down-type quarks have masses m_u and m_d. The corresponding Yukawa couplings are given by $g_u = m_u/v_u$ and $g_d = m_d/v_d$. The fact that g_u and g_d satisfy (11) sets limits on v_u and v_d. For $N_F > 3$, we find*

$$
\begin{aligned}
(v/v_u)^2 &\lesssim 220 \qquad (\text{ or }\ \ v_u \gtrsim 12\ \mathrm{GeV}) \\
(v/v_d)^2 &\lesssim 220 \qquad (\text{ or }\ \ v_d \gtrsim 12\ \mathrm{GeV})\,,
\end{aligned}
\tag{15}
$$

where we have used the fact that new quarks must have masses above 23 GeV. Heavier quarks lead to better limits. For the popular value $m_t = 45$ GeV, we

* See next section for the case $N_F = 3$.

find $(v/v_u)^2 \lesssim 60$. More generally, an up-type (down-type) quark with mass m_T (m_B) implies

$$\left(\frac{m_B}{355\ \mathrm{GeV}}\right)^2 \lesssim \frac{\sum M_D{}^2}{(355\ \mathrm{GeV})^2} \lesssim \left(\frac{v_d}{v_u}\right)^2 \lesssim \frac{(355\ \mathrm{GeV})^2}{\sum M_U{}^2} \lesssim \left(\frac{355\ \mathrm{GeV}}{m_T}\right)^2 . \tag{16}$$

Our limits (15) and (16) give strict upper bounds on the charged-Higgs couplings. They are independent of the charged-Higgs mass M_H and the number of families. In contrast, the phenomenological bounds from $K^o - \overline{K^o}$ and $D^o - \overline{D^o}$ mixing [5] depend strongly on M_H and become weak if $M_H \gtrsim 100$ GeV.

In Figure 4 we plot the bounds on $(v_d/v_u)^2$ as a function of the Higgs mass. Our bounds imply that the charged-Higgs contribution to the $K_L - K_S$ mass difference is smaller than the usual W^+W^- contribution for $M_H \gtrsim 50$ GeV. (The charged-Higgs contribution to $B^o - \overline{B^o}$ mixing is practically unrestricted by this analysis.)

Analyses of $D^o - \overline{D^o}$ mixing [5] lead to $(v/v_d)^2 \lesssim 10^4\ M_H/(20\ \mathrm{GeV})$. The bound (15) is much more stringent. It implies that the charged-Higgs contribution to $D^o - \overline{D^o}$ mixing is smaller than the usual GIM contribution for $M_H \gtrsim 0.5$ GeV. When $N_F = 3$, this bound relaxes to $M_H \gtrsim 6$ GeV.

6. Improved bounds for three families

Some of the bounds that we have discussed are improved if there are only three families. This follows from the fact that equations (2) simplify when the top and bottom quarks do not mix with the lighter families. In this case, the evolution of the top and bottom Yukawas becomes

$$\begin{aligned} \frac{d}{dt}\log u_3 &= G_U - 3\,T_U - T_N - \frac{1}{2}(3\,u_3{}^2 + d_3{}^2)\,, \\ \frac{d}{dt}\log d_3 &= G_D - 3\,T_D - T_E - \frac{1}{2}(3\,d_3{}^2 + u_3{}^2)\,. \end{aligned} \tag{17}$$

Corrections to these equations are of order $\sin^2\theta_2 \lesssim 10^{-2}$. A bound on the value of u_3 can be obtained by setting $T_U = u_3^2$, $d_3 = 0$ and $T_N = 0$ (and similarly for d_3). This gives u_3, $d_3 \lesssim 1.4$, which in turn implies†

$$\begin{aligned} v_u &\gtrsim 17 \text{ GeV} \qquad \text{for } m_t \gtrsim 23 \text{ GeV} \\ v_u &\gtrsim 33 \text{ GeV} \qquad \text{for } m_t \gtrsim 45 \text{ GeV} \\ v_d &\gtrsim 3 \text{ GeV}. \end{aligned} \tag{18}$$

In Figure 4 we compare these bounds with the phenomenological limit of Reference [5]. The renormalization group bounds are stronger for $M_H \gtrsim 10$ GeV.

Recently, phenomenological bounds on v_u/v_d were obtained by considering CP violation in the $K^o - \overline{K^o}$ system [6]. These limits are only valid for three families. In deriving them it is necessary to assume that the CP violation from Higgs exchange is less than that induced by W bosons.

† The limits (18) on v_u are also valid when $N_F = 4$, provided the fourth family barely mixes with the other three. In this case, the bound on v_d can be tightened to $v_d \gtrsim 17$ GeV, where we have used the lower bound of 23 GeV on the mass of a fourth family.

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FIGURE CAPTIONS

1. (a) The evolution of T_U and T_D, for two heavy and three light families. The arrows indicate the flow of increasing t. The initial conditions are $u_1 = d_1 = 1$ for the first family, and various values of u_2 and d_2 for the second. The Cabibbo angle is taken to be zero. (b) The evolution of T_D with energy for the same initial conditions.

2. The evolution of (a) d_2/u_2 and (b) d_2/d_1 as a function of the energy for the initial conditions of Figure 1. We see that splittings of 1000% at M_X get reduced to less than 50% at the weak scale.

3. The evolution of the Cabibbo angle with energy for $N_F = 5$. We have taken $u_1 = 2$, $d_1 = 3$, $u_2 = 1$ and $d_2 = 4$ at the scale M_X. The dashed line corresponds to identical initial conditions, except that $u_2 = 5$.

4. Upper limits for $(v_d/v_u)^2$ as a function of the charged-Higgs mass. The solid line is the renormalization group bound, valid for any number of families. The dashed line corresponds to our limit in the special case when there are only three families (or when there are four families with small Cabibbo mixings). We have used $m_t \simeq 45$ GeV. The dotted line is the phenomenological bound from the $K_L - K_S$ mass difference. We have used the approximations of equation (3.5) of Reference [5].

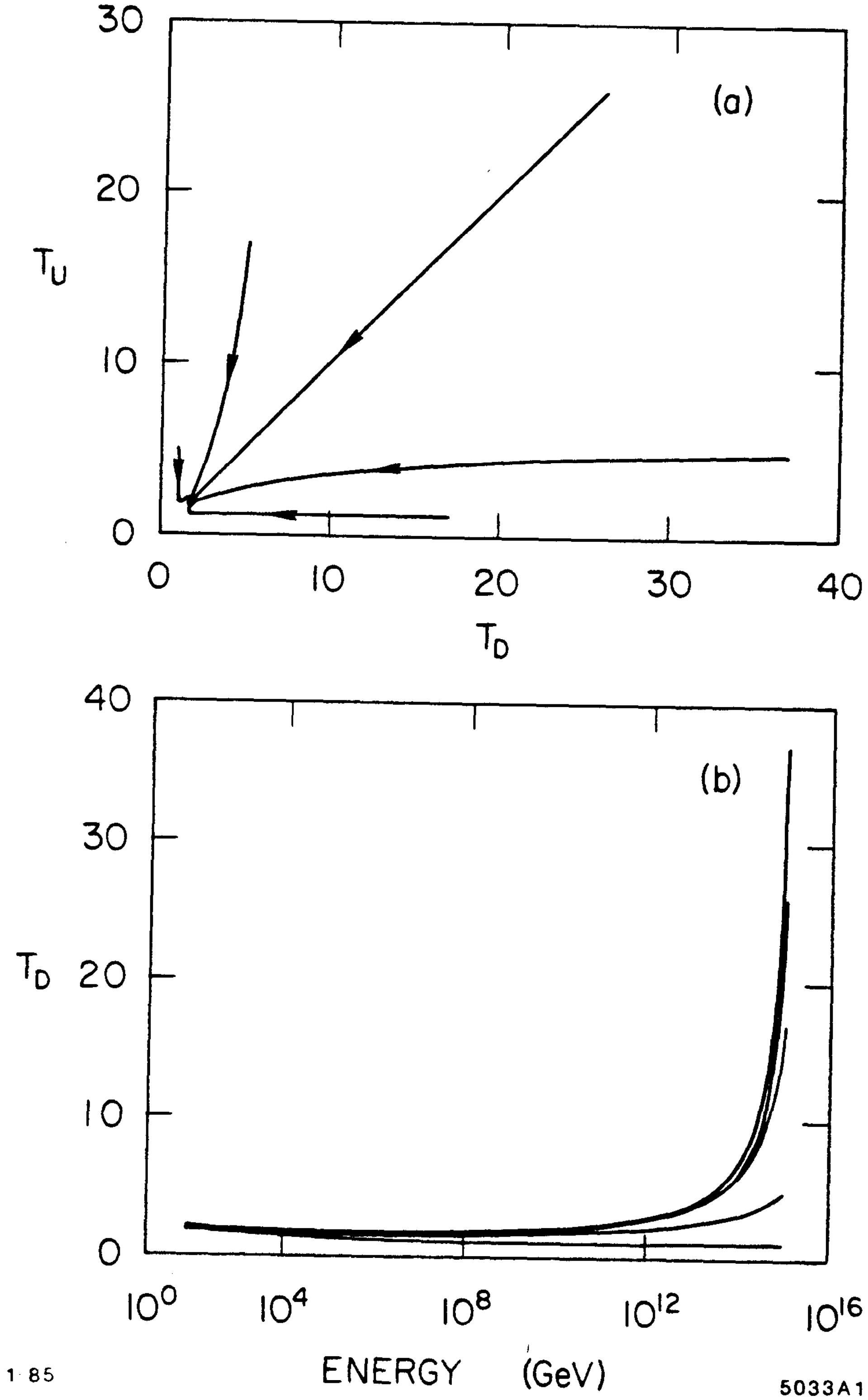
(a)
30
20
10
0
T_U
0
10
20
30
40
T_D
(b)
40
30
20
10
0
T_D
10^0
10^4
10^8
10^{12}
10^{16}
ENERGY (GeV)
1 85
5033A1

Fig. 1

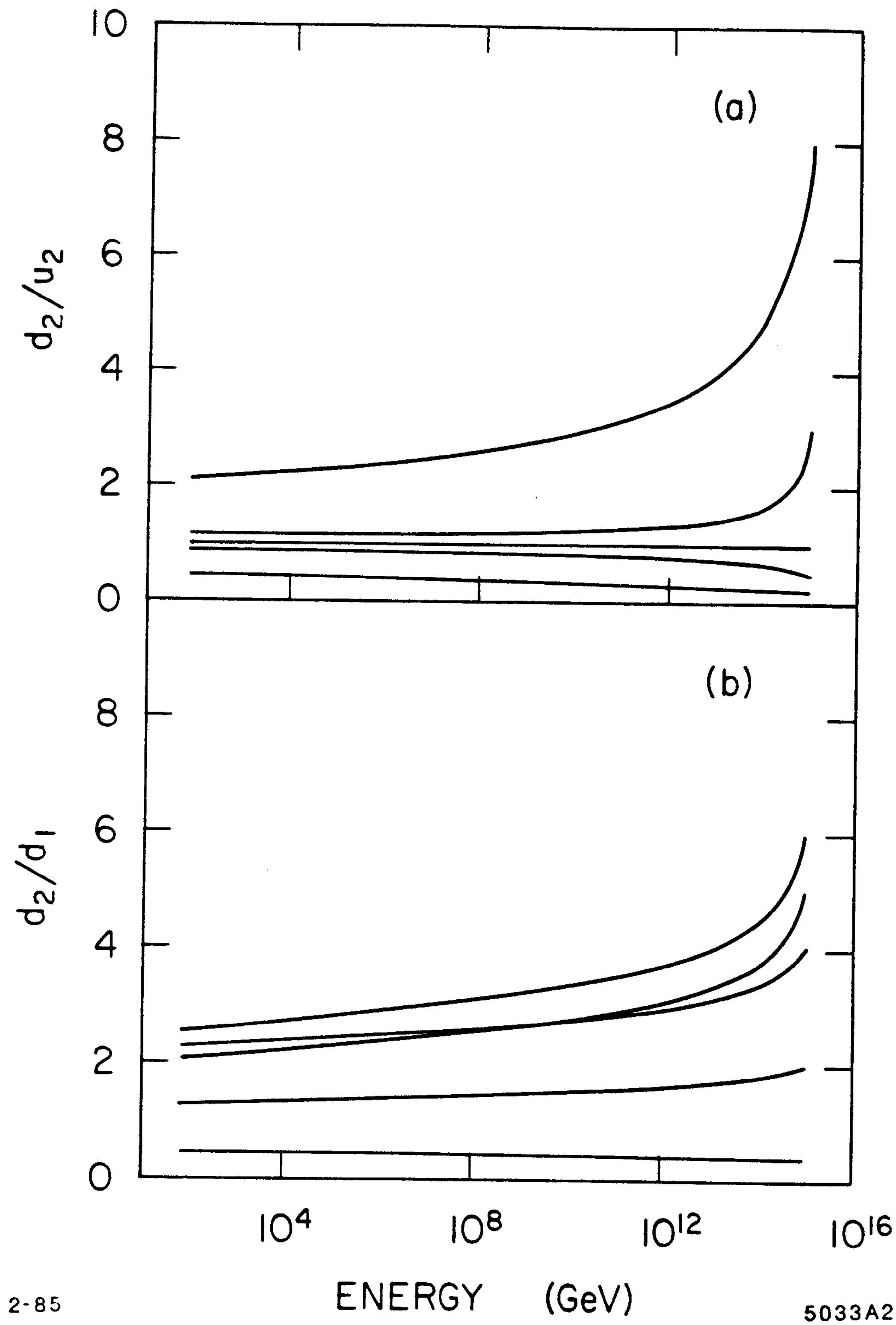

Fig. 2

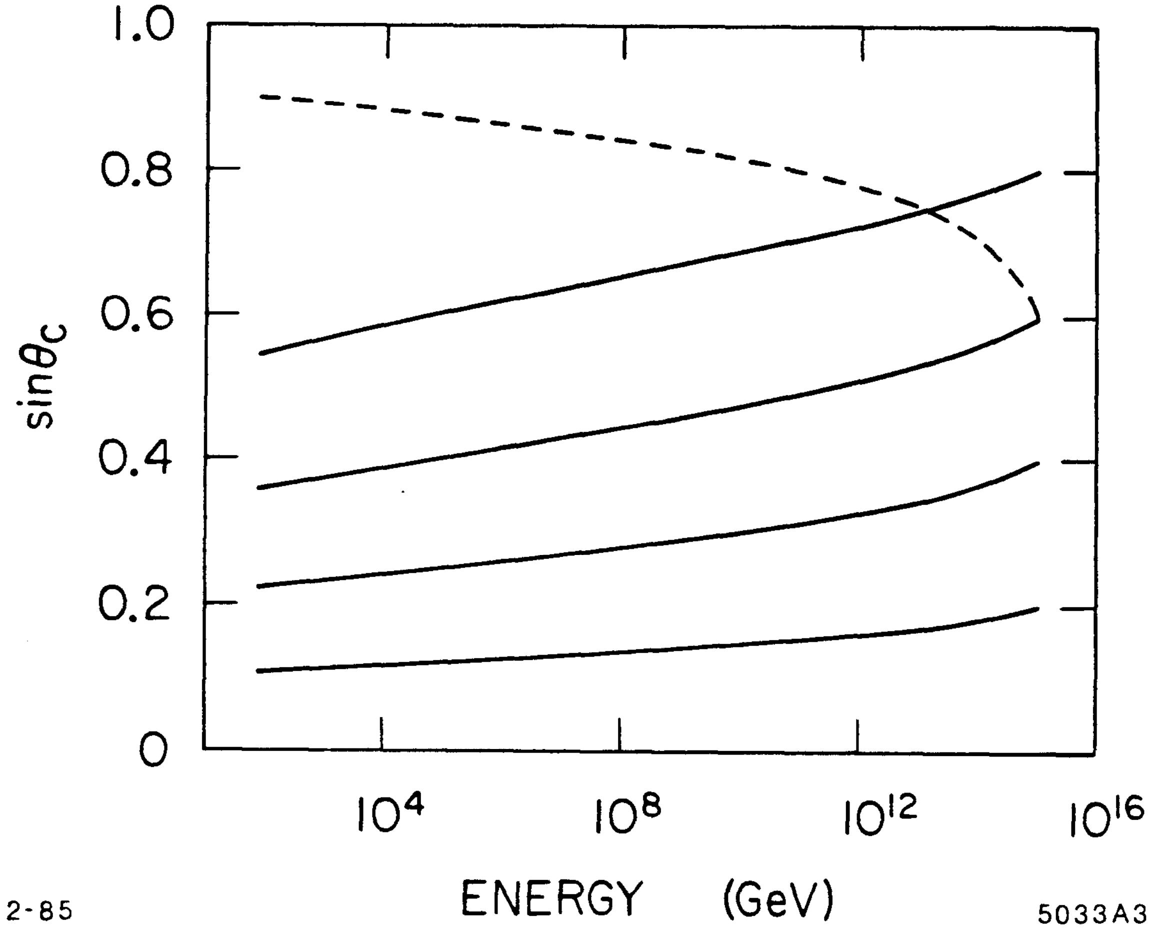
1.0
0.8
0.6
0.4
0.2
0
$\sin\theta_C$
10^4
10^8
10^{12}
10^{16}
2-85
ENERGY (GeV)
5033A3

Fig. 3

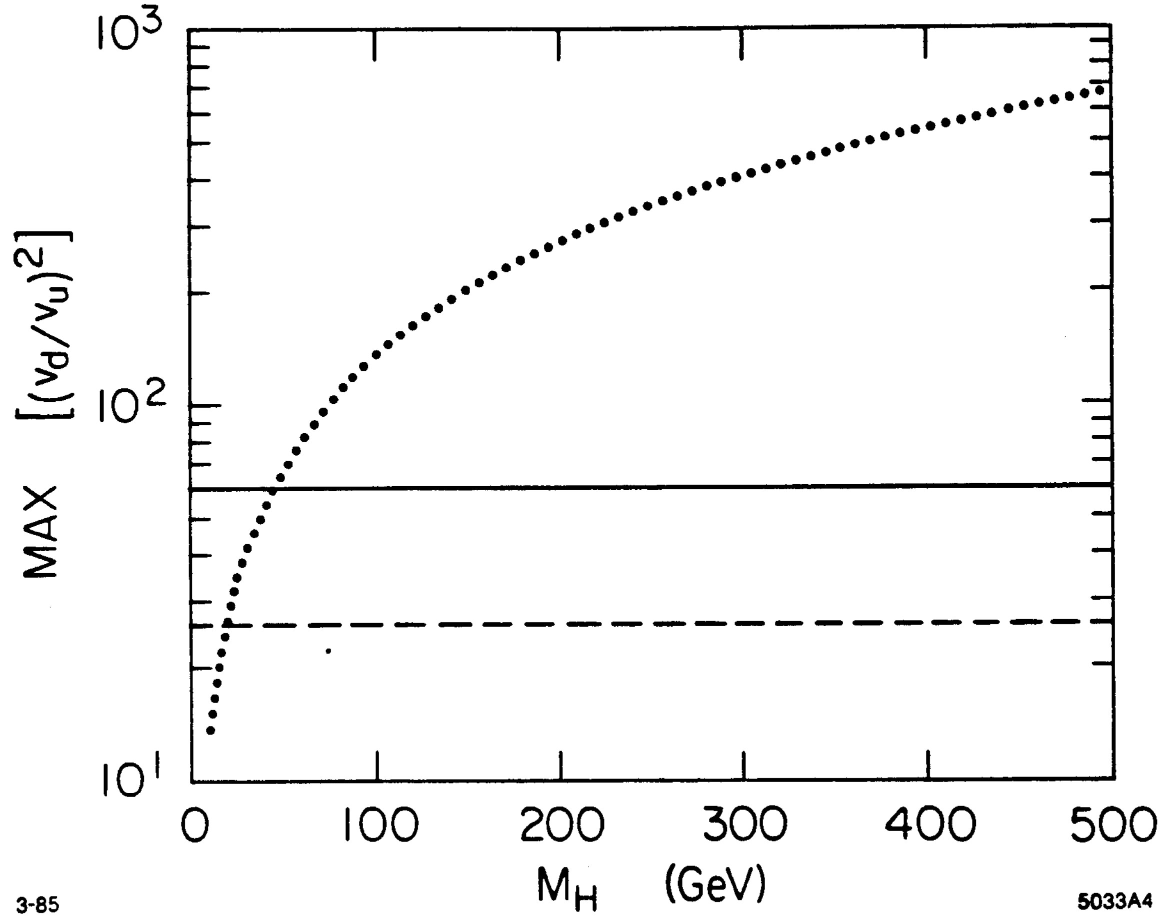

Fig. 4