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EFFECTS OF THE PARITY-ODD SINGLET ON THE $N = 1$ SUPERGRAVITY THEORY WITHIN THE LEFT-RIGHT SYMMETRIC MODEL*

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ABSTRACT

We study the $N = 1$ supergravity theory within the left-right symmetric model, based on the gauge symmetry $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$, when the parity-odd singlet field is added, in addition to the minimal set of Higgs fields. This model allows for a vacuum solution with the hierarchy ratio $\eta = (m_{W_R}/m_{W_L}) \gg 1$. Also, the gravitino mass is likely to set the scale of m_{W_L} rather than the one of m_{W_R}. These features of the presented model should be contrasted with the results of the left-right symmetric model with the minimal set of Higgs fields, where $\eta < O(10)$ and the gravitino mass naturally sets the scale of m_{W_R}.

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1. INTRODUCTION

Models with $N = 1$ supergravity (SG) coupled to the Maxwell-Yang-Mills gauge interactions[1,2] provide an attractive linkage, though not a true unification of gravity with the other forces of nature. These models are attractive, because they provide an elegant way of breaking of local supersymmetry (SS) spontaneously.[3,4] That can lead to the realistic low energy models[5] with new interesting physics at an energy scale much smaller than the Planck mass ($M_{P\ell}$). Realistic models based on the "left-handed" electro-weak symmetry $SU(2)_L \times U(1)$ have been presented. The striking aspect of these models is a scenario[5,6] where the "left-handed" electro-weak symmetry ($SU(2)_L \times U(1)_Y$) breaking is driven by the soft SS breaking terms. These terms arise when local SS is spontaneously broken and are proportional to the gravitino mass, $m_{3/2}$, so that the gravitino mass sets the mass scale of $W_L(m_{W_L})$.

In the previous publication[7] we examined the consequences which arise when constraints of $N = 1$ SG are imposed on the minimal left-right (L-R) symmetric model based on the gauge group $G = SU(2)_L \times SU(2)_R \times U(1)_{B-L}$[8] and the *minimal* set of Higgs fields which were necessary for exhibiting the fermionic masses and the realistic spontaneous symmetry breaking (SSB) pattern. We chose the set of Higgs superfields with the following transformation properties:

$$\chi = \begin{bmatrix} \chi_1^0 & \chi_1^+ \\ \chi_2^- & \chi_2^0 \end{bmatrix} \sim (\underset{\sim}{2}, \underset{\sim}{2}, 0) \tag{1a}$$

$$\Delta_{1L} = (L_1^0, L_1^-, L_1^{--}) \sim (\underset{\sim}{3}, \underset{\sim}{1}, -2) \quad \Delta_{2R} = (R_2^0, R_2^-, R_2^{--}) \sim (\underset{\sim}{1}, \underset{\sim}{3}, -2) \tag{1b}$$

$$\Delta_{2L} = (L_2^{++}, L_2^+, L_2^0) \sim (\underset{\sim}{3}, \underset{\sim}{1}, 2) \quad \Delta_{1R} = (R_1^{++}, R_1^+, R_1^0) \sim (\underset{\sim}{1}, \underset{\sim}{3}, 2)\ . \tag{1c}$$

It turned out that in this case constraints of $N = 1$ SG shed a new light on the origin of m_{W_L} and m_{W_R} masses. We were forced to derive m_{W_L} radiatively as a consequence of quantum corrections to the tree level Higgs potential (TLHP) while the magnitude of m_{W_R} was determined already at the TLHP. Thus $m_{3/2}$ sets the scale of m_{W_R} rather than that of m_{W_L}. In addition, constraints of $N = 1$ SG do not permit, barring unnatural fine tuning of parameters, a maxi-hierarchy, i.e., $\eta \equiv (m_{W_R}/m_{W_L}) \to \infty$. In contrast, they impose an upper bound: $\eta < (2 \text{ to } 6)\,(\pi^2/g^2)^{1/4} = \mathcal{O}(10)$. Here g is the gauge coupling constant for $SU(2)_{L,R}$. Thus, the realistic minimal L-R symmetric model within $N = 1$ SG necessarily calls for a mini-hierarchy.

Motivated by those results we would like to extend the minimal model to the case where a maxi-hierarchy, i.e., $\eta \gg 1$, would be a permissible solution. The most obvious extension of the minimal model would be the replacement of triplet fields $\Delta_{(1,2)(L,R)}$ with doublet fields $H_{(1,2)L}$ which transform under G as $(\underset{\sim}{2}, \underset{\sim}{1}, -1)$ and $(\underset{\sim}{2}, \underset{\sim}{1}, 1)$, respectively and with $H_{(1,2)R}$ defined likewise. The superpotential would have permitted an *additional* term $H^T_{1L}\tau_1\phi H_{1R} + H^T_{2R}\phi^T\tau_2 H_{2L}$ which was not permitted in the case of triplet fields only. This new term breaks a rotational invariance in the space of the L-type and R-type doublet fields, i.e., it is not of the form $H^2_L + H^2_R$. It turns out that in that case the desired vacuum solution with a maxi-hierarchy would have been permitted if in addition the inequality $M^2_1 \neq M^2_2$ for the mass parameters M^2_1 and M^2_2 of the type 1 and type 2 Higgs doublets would have been satisfied. Such an inequality however does not appear to emerge at the renormalization mass $\mu = \mathcal{O}(m_{W_R})$, because in a model without triplets there are no quantum corrections which would lead to a distinction between the parameters of the type 1 and type 2 Higgs fields via the renormalization group equations (RGE's). However, a model where triplets $\Delta_{(1,2)(L,R)}$ as well as doublets $H_{(1,2)(L,R)}$ are introduced might permit a maxi-hierarchy. In such a model the Higgs content would then be proliferated. The quantitative features of this model were not investigated closely.

Another possibility would be to add an ordinary singlet field Y. In the superpotential, Y can couple, due to the L-R symmetry, only to the combination $\Delta_{1L}^T\Delta_{2L} + \Delta_{1R}^T\Delta_{2R}$ of $\Delta_{(1,2)(L,R)}$ fields which has a rotational invariance in the space of the L-type and R-type triplet fields. As it turns out, such a restrictive coupling does not permit a maxi-hierarchy and quantum corrections are again essential for obtaining a stable vacuum solution.

An appealing possibility is an introduction of the so-called parity-odd singlet field σ:

$$\sigma = (\sigma^0) \sim (\underset{\sim}{1}, \underset{\sim}{1}, 0) \ . \tag{1d}$$

This field transforms as a singlet under the gauge group but *changes sign* under the $L \leftrightarrow R$ transformation. This in turn implies that σ couples in the superpotential only to the linear combination $(\Delta_{1L}^T\Delta_{2L} - \Delta_{1R}^T\Delta_{2R})$ of $\Delta_{(1,2)(1,R)}$ fields. Such a coupling breaks a rotational invariance in the space of the L-type and the R-type triplet fields an thus has a potential to yield new qualitative features of the model.

2. THE MODEL

In the following we shall investigate the SSB pattern of the model with the parity odd singlet (1d) added to the minimal set of Higgs fields (1a, 1b, 1c).

One seeks the following pattern of vacuum expectation values (VEV's) for the scalar components of the Higgs chiral superfields:

$$\langle \chi \rangle = \begin{bmatrix} \chi_1 & 0 \\ 0 & \chi_2 \end{bmatrix} \tag{2a}$$

$$\langle \Delta_{1L} \rangle = (L_1, 0, 0) \ , \quad \langle \Delta_{2R} \rangle = (R_2, 0, 0) \tag{2b}$$

$$\langle \Delta_{2L} \rangle = (0, 0, L_2) \ , \quad \langle \Delta_{1R} \rangle = (0, 0, R_1) \tag{2c}$$

$$\langle \sigma \rangle = \sigma \tag{2d}$$

with the following hierarchy between VEV's:

$$\{L_1, L_2\} \ll \{\chi_1, \chi_2\} \ll \{R_1, R_2\} < \sigma \ . \tag{3}$$

For simplicity we consider only the case with real positive VEV's. The VEV pattern (2) with the hierarchy requirement (3) provides a desired breaking of the gauge symmetry consistent with the known phenomenology. In addition, VEV's of the multiplet χ give Dirac masses to fermions. We choose the fields Δ_{iL} and Δ_{iR} ($i = 1, 2$) to be the triplet fields under the symmetries $SU(2)_L$ and $SU(2)_R$, respectively. Thus, $\Delta_{2(L,R)}$ fields give Majorana masses to neutrinos. For convenience of algebraic manipulation, we impose $\chi_2 \ll \chi_1$. This incidentally also restricts the $W_L - W_R$ mixings. Constraint $\sigma > \{R_1, R_2\}$ is only technical, so that we can obtain explicit expressions for the vacuum solution.

We assume the by-now familiar $N = 1$ SG coupled to the Maxwell-Yang-Mills interactions in which SS is spontaneously broken when the fields, which are singlets under the gauge group and belong to the so-called hidden sector,[3,4] acquire VEV's of order M_{Pl}. This mechanism yields the following restricted form[4] for the effective low energy TLHP which contains only the light scalar fields Z_i:

$$V = \sum_i |g_i|^2 + \frac{1}{2}\sum_\alpha D_\alpha^2 + m_1^2 \sum_i |Z_i|^2 + \left(m_2 \sum_i Z_i g_i + m_3 g + \text{h.c.} \right) . \quad (4)$$

Here $g_i = \partial g / \partial Z_i$, $D_\alpha = g_\alpha Z^{*i} (T_\alpha)_i^j Z_j$, g is the superpotential, T_α is the α-th generator of the gauge group and g_α is the associated couplings constant. Parameters $m_{1,2,3}$ determine the soft SS breaking terms, and they depend on the VEV's of the fields in the hidden sector. Mass m_1 is the gravitino mass $m_{3/2}$ and masses $m_{2,3}$ are of order m_1.

In our case the superpotential g consistent with the L-R gauge symmetry, renormalizability and SS has the following form:[9]

$$g = g_0 + g_1 \quad (5a)$$

where

$$g_0 = m_\Delta \left(\Delta_{1L}^T \Delta_{2L} + \Delta_{1R}^T \Delta_{2R} \right) + m_\chi \text{tr} \left(\tau_2^T \chi^T \tau_2 \chi \right) \quad (5b)$$

$$g_1 = m_\sigma \sigma^2 + \lambda \sigma \left(\Delta_{1L}^T \Delta_{2L} - \Delta_{1R}^T \Delta_{2R} \right) . \quad (5c)$$

Here g_1 contains the new terms in the superpotential, which arise when σ field is added.

We use the form (4) of the low energy $N = 1$ SG potential V. For the sake of future discussion we shall give V in the form which is obtained by substituting the Higgs field (1) by their VEV's (2). Then V is of the following form:

$$V = V_0 + V_1 \tag{6a}$$

with

$$\begin{aligned} V_0 = & M_1^2(L_1^2 + R_1^2) + M_2^2(L_2^2 + R_2^2) - 2M_3^2(L_1L_2 + R_1R_2) \\ & + \mu_1^2(\chi_1^2 + \chi_2^2) - 2\mu_3^2\chi_1\chi_2 + D \end{aligned} \tag{6b}$$

$$\begin{aligned} V_1 = & \mathcal{M}\sigma^2 + \mathcal{M}_4\sigma(L_1L_2 - R_1R_2) + \mathcal{M}_5\sigma(L_1^2 + L_2^2 - R_1^2 - R_2^2) \\ & + \lambda^2\left[(L_1L_2 - R_1R_2)^2 + \sigma^2(L_1^2 + L_2^2 + R_1^2 + R_2^2)\right] \ . \end{aligned} \tag{6c}$$

Here:

$$\begin{aligned} D = \frac{g^2}{8} & \Big[(2L_1^2 - 2L_2^2 + \chi_1^2 - \chi_2^2)^2 + (2R_1^2 - 2R_2^2 - \chi_1^2 + \chi_2^2)^2 \\ & + q^2(-2L_1^2 + 2L_2^2 + 2R_1^2 - 2R_2^2)^2\Big] \ . \end{aligned} \tag{7}$$

Here $q = g'/g$ where g and g' are the gauge coupling constants for the $SU(2)_{L,R}$ and $U(1)_{B-L}$ groups, respectively.

Mass parameters are defined as follows:

$$\begin{aligned} M_1^2 = M_2^2 = m_\Delta^2 + m_1^2 \ , \qquad & M_3^2 = -(2m_2 + m_3)\, m_\Delta \\ \mu_1^2 = m_1^2 + 4m_\chi^2 \ , \qquad & \mu_3^2 = -2(2m_2 + m_3)\, m_\chi \end{aligned} \tag{8a}$$

and

$$\begin{aligned} & \mathcal{M}^2 = 4m_\sigma^2 + m_1^2 + 2(2m_2 + m_3)m_\sigma \ , \\ & \mathcal{M}_4 = 2\lambda(m_\sigma + 3m_2 + m_3) \ , \quad \mathcal{M}_5 = 2\lambda m_\Delta \ . \end{aligned} \tag{8b}$$

The above parameters are determined at the renormalization mass $\mu = \mathcal{O}(M_{pl})$

where SG is spontaneously broken. They evolve according to the RGE's and need not be the same at $\mu = O(m_{W_R})$ where we are actually looking for our vacuum solution. In fact, the value of these parameters may change significantly because the renormalization mass μ is changed by the large value from $O(M_{P\ell})$ to $O(m_{W_R})$.

From (6) we see that V possesses new features compared to potential V_0 of the minimal model; V is at most quadratic in σ field, and it contains new trilinear terms for $\Delta_{(1,2)(L,R)}$ and σ fields which break the rotational invariance in the space of the L-type and R-type triplet fields. This is again suggesting that one is dealing with a physically different system.

We obtain the extremum solution consistent with hierarchy (3) already at the TLHP by solving the extremum equations in the following two steps:

(i) First we assume $M_1 \equiv M_2$ which is actually the case at $\mu = O(M_{P\ell})$. This implies a solution $R_1 = R_2 \equiv R \neq 0$, $\sigma = \sigma_0 \neq 0$, and $L_{1,2} = 0$, $\chi_{1,2} = 0$. At this stage $SU(2)_L \times U(1)_Y$ remains unbroken.

(ii) Then we evaluate small derivations from solution (i) in terms of small nonzero parameters $(M_1^2 - M_2^2)/M_1^2$ and $\mu_1^2/M_1^2 \ll 1$ which induce $0 \neq \left|R_1^2 - R_2\right|^2 / R_1^2 \ll 1$ and in turn $0 \neq \chi_{1,2}^2/R_1^2 \ll 1$.

In the leading order, the explicit expessions for the extremum solution are of the following form:

$$\sigma = \sigma_0 + \delta\sigma \ , \quad R_{1,2} = R + \delta R_{1,2} \tag{9a}$$

$$\chi_1^2 = R^2 \times \frac{M_1^2 - M_2^2 - \mu_1^2\left[4(1+q^2) + \frac{4\lambda^2}{g^2} + \frac{2(2M_3^2 + \mathcal{M}_4\sigma)}{g^2R^2}\right]}{2M_3^2 + \mathcal{M}_4\sigma + 2\left[(1+2q^2)g^2 + \lambda^2\right]R^2} \ , \quad \chi_2 = \frac{\mu_3^2}{2\mu_1^2}\chi_1 \tag{9b}$$

$$L_1 = L_2 = 0 \tag{9c}$$

with

$$\sigma_0 = \frac{(2\mathcal{M}_5 + \mathcal{M}_4) - \left[(2\mathcal{M}_5 + \mathcal{M}_4)^2 - 16\lambda^2(M_1^2 - M_3^2)\right]^{1/2}}{4\lambda^2}$$

$$\times \left[1 + \frac{4\lambda^2 \mathcal{M}^2}{(2\mathcal{M}_5 + \mathcal{M}_4)^2 - 16\lambda^2(M_1^2 - M_3^2)}\right] \tag{10a}$$

$$R^2 = \frac{\mathcal{M}^2\{(2\mathcal{M}_5 + \mathcal{M}_4) - \left[(2\mathcal{M}_5 + \mathcal{M}_4)^2 - 16\lambda^2(M_1^2 - M_3^2)\right]^{1/2}\}}{2\lambda^2[(2\mathcal{M}_5 + \mathcal{M}_4)^2 - 16\lambda^2(M_1^2 - M_3^2)]^{1/2}} \tag{10b}$$

$$\frac{\delta\sigma}{\sigma_0} = \frac{R^2(M_2^2 - M_1^2)}{2\sigma_0^2 \mathcal{M}^2} \tag{10c}$$

$$\frac{\delta R_1 + \delta R_2}{R} = \frac{(\mathcal{M}^2 + 2\lambda^2 R^2)R^2(M_2^2 - M_1^2)}{2\sigma_0^2 \mathcal{M}^4} \tag{10d}$$

$$\frac{\delta R_1 - \delta R_2}{R} = \frac{M_2^2 - M_1^2 + 2\mu_1^2}{2M_3^2 + \mathcal{M}_4\sigma_0 + 2[(1 + 2q^2)g^2 + \lambda^2]R^2} \, . \tag{10e}$$

The obtained extremum solution satisfied hierarchy (3) which is determined in terms of the small parameters $(M_1^2 - M_2^2)/M_1^2$ and μ_1^2/M_1^2. The requirement $0 \neq \left|M_1^2 - M_2^2\right|/M_1^2 \ll 1$ calls for nonzero Majorana coupling $0 \neq h_M \ll 1$. This comes about as follows. At $\mu = \mathcal{O}(M_{P\ell})$ one has $M_1^2 \equiv M_2^2$ (see Eq. (8a)). However, due to the RGE's these two masses *do* differ from each other at m_{W_R}, where in fact we are looking for a desired vacuum solution. This is because only $\Delta_{2(L,R)}$, but not $\Delta_{1,(L,R)}$ can have Majorana Yukawa type interaction with chiral matter superfields $\psi_L^c \sim (\underset{\sim}{2}, \underset{\sim}{1}, -1)$ and $\psi_L^c \sim (\underset{\sim}{1}, \underset{\sim}{2}, 1)$ whose fermionic components are leptons. Then the evolution of $(M_1^2 - M_2^2)/M_1^2$ is governed in the leading approximation only by the coupling h_M and one obtains the above estimate for h_M.

One the other hand, in contrast to the minimal model,[7] no large Dirac coupling h_D is needed which would assure $\mu_1^2 \ll M_1^2$ at m_{W_R}. This requirement can be satisfied already at the TLHP by the initial choice of parameters in the superpotential, i.e., $m_\chi \ll m_\Delta$. Parameters $\mathcal{M}$, $\mathcal{M}_4$, $\mathcal{M}_5$ and λ must also satisfy hierarchical constraints, e.g., one can choose $\mathcal{M} < \mathcal{O}(\mathcal{M}_4) = \mathcal{O}(\mathcal{M}_5) = \mathcal{O}(\lambda M_1^2)$ and $\lambda = \mathcal{O}(g)$.

Additional constraints on the parameters come from the minimization of the potential. It is interesting to notice that the constraints for a local minimum of the Higgs potential can be satisfied already at the TLHP without introducing quantum corrections of the Coleman-Weinberg type. This is again different form the minimal model. Namely, in this model the parity odd nature of the singlet field σ assures that in the superpotential the couplings of the fields $R_{1,2}$ with one power of the field σ have the opposite sign from the couplings of the fields $L_{1,2}$ fields with one power of the field σ. This in turn favors the situation where the solution with $R_{1,2} \neq 0$ and $L_{1,2} = 0$ and therefore $\eta \gg 0$ is the preferred solution already at the TLHP.

For the sake of completeness, we shall state the relevant constraints for the local minimum of the potential:

$$\mu_1^2,\ \mu_3^2,\ M_1^2,\ (M_1^2 - M_3^2),\ (2\mathcal{M}_5\sigma_0 + 2M_3^2),\ (2\mathcal{M}_5 + \mathcal{M}_4) > 0 \tag{11a}$$

$$\mathcal{M}_5\sigma_0 + M_1^2 + \lambda^2\sigma_0^2 + \left[2g^2(1+q^2) + \lambda^2\right] R^2 > 0 \tag{11b}$$

$$(\mathcal{M} + 2\lambda^2 R^2)(2\lambda^2 - g^2)\, R^2 - \frac{2\mathcal{M}^4\sigma^2}{R^2} > 0 \tag{11c}$$

$$M_1^2 - M_2^2 - \mu_1^2 \left[4(1+q^2) + \frac{4\lambda^2}{g^2} + \frac{2(2M_3^2 + \mathcal{M}_4\sigma)}{g^2R^2}\right] > 0\,. \tag{11d}$$

Constraints (11) can be all satisfied for an appropriate choice of the TLHP parameter. Thus, one has a vacuum solution satisfying maxi-hierarchy (3) already at the TLHP, but the algebraic expressions for the VEV's (see Eqs. (9,10)) and constraints on the parameters of the TLHP (see Eq. (11)) are complicated. However, if one imposes the additional hierarchical requirement $\sigma \gg R$, the following simplified constraints among the mass parameters of the superpotential g (see Eq. (5)) and the mass parameters of the soft SS breaking terms is obtained:

$$2\left(\frac{m_\Delta}{m_\sigma}\right)^{3/2} \gg \left(\frac{m_1}{m_\sigma}\right)^2 \gtrsim 4\left(\frac{m_\Delta}{m_\sigma}\right)^{1/2} \tag{12a}$$

which in turn leads to the following inequalities:

$$m_\Delta^2 \gg m_\Delta m_\sigma \gg m_1^2 \gg m_\sigma^2 \,, \qquad m_1 = O(m_1, m_2) \,. \tag{12b}$$

We see that when $\sigma \gg R$ one has a constraint $m_\Delta \gg m_\sigma$, i.e., the hierarchy between these two mass parameters is reversed compared to the hierarchy between the corresponding VEV's. This is another peculiar feature of the TLHP (6).

The hierarchical constraint $\sigma \gg R$ also enables us to express the vacuum solution in the following, more transparent way:

$$\sigma = \frac{m_\Delta}{\lambda} \,, \qquad R = \left(\frac{m_\Delta}{m_\sigma}\right)^{1/4} \frac{m_1}{\sqrt{2}\lambda} \tag{13a}$$

$$\chi_1 = R \times O\left(\sqrt{\frac{m_1}{m_\Delta}}\right) \,, \qquad \chi_2 = \chi_1 O\left(\frac{\mu_3^2}{m_1^2}\right) \tag{13b}$$

$$L_1 = L_2 = 0 \tag{13c}$$

and consequently the hierarchy ratio η becomes:

$$\eta \sim \frac{\sqrt{8}R}{\chi_1} = O\left(\sqrt{\frac{m_1}{m_\Delta}}\right) \gg 1 \,. \tag{14}$$

The obtained vacuum solution therefore leads to a maxi-hierarchy which depends only on the hierarchy (12) between the *free* mass parameters of the TLHP.

In addition, the gravitino mass ***need not*** set the scale for the mass of W_R. From Eqs. (12) and (13) we see that it is likely that the gravitino mass sets the scale for the mass of W_L, because $m_{W_L} \gtrsim O(g/\lambda m_1)$.

3. CONCLUSIONS

The introduction of the parity-odd singlet fields in the $N = 1$ SG theory within the L-R symmetric model allows for a scenario which is qualitatively different from the one[7] with the minimal set of Higgs fields. The nature of the TLHP is now drastically changed due to the new couplings of the singlet with the L-type and R-type triplet superfields. This amounts to the following new features:

(i) There are more than one free parameter in the theory. These parameters set the scale for the VEV's of the different fields. The gravitino mass is likely to set the scale for the mass of W_L rather than the mass of W_R.

(ii) The hierarchical vacuum solution can be insured already at the TLHP and no substantial quantum corrections are needed, except the ones which induce $M_1^2 \neq M_2^2$ via the RGE's.

(iii) The hierarchy ratio η does not have an upper bound and depends on the free parameters of the TLHP. In this case the maxi-hierarchy can emerge but an underlying understanding of its origin cannot be gained.

Both scenarios proposed with the parity-odd singlet as well as without it are realistic in the sense that none of them contradicts observation. However, nature has to decide whether the existence of the parity odd singlet is relevant for the real world.

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10. Note that the parity-odd singlet has been introduced[11] in the non-supersymmetric $SU(10)$ grand unified theory. The RGE's permit in this case an interesting SSB pattern with a low mass for W_R.

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