SLAC – PUB – 3566
January 1985
(T/E)

A DISCRETE FOUNDATION FOR PHYSICS AND EXPERIENCE*

H. PIERRE NOYES

Stanford Linear Accelerator Center

Stanford University, Stanford, California, 94305

ABSTRACT

We derive a discrete quantum scattering theory using bit strings generated and labeled by a simple recursive algorithm. We believe that our results could lead to a discrete reconstruction of quantum mechanics compatible with current experience. Implications for the mind-physics problem are briefly discussed.

Submitted to *Quantum Mechanics and the Nature of Reality*

The Esalen Mind-Physics Conference

Monterey, California, 4-8 February, 1985

⋆ Work supported by the Department of Energy, contract DE – AC03 – 76SF00515.

1. INTRODUCTION

1.1 CONTEXT

Quantum mechanics has been highly successful in practice; tension between the continuum classical mechanics used to interpret the discrete phenomena which provide its empirical roots and the uncompromising indivisibility of quantum effects which provide the core of the new theory remains. *Granted* quantum mechanics, the fact that space-like separated events can change probabilities without transmitting causally effective information is consistent with both theory and experiment[1] ; in a sense this only deepens the mystery. Chew has gone a long ways toward grounding quantum mechanics in a topological bootstrap theory[2] ; however, his basic treatment requires *continuous* momenta, an assumption which he admits[3] is inconsistent with the fully discrete theory quantum mechanics requires. Chew's theory contains necessarily massless photons and neutrinos; and both he and Stapp agree that the soft photons provide the route for passing from the micro to the macro world[4–6] without any logical hiatus. Yet both approaches are still infected by continuum assumptions. In this paper we present a consistent discrete approach which removes this fundamental inconsistency. Implications for the mind-physics problem are briefly discussed in the concluding section.

1.2 DISCRIMINATION AND TICK

The approach presented here has a long history[7–17] , most of which we must ignore. The critical ingredient we focus on is a growing universe of bit strings generated by two operations: *discrimination* and *tick*. A bit string $S(N)$ is an ordered sequence of length N composed of the symbols $0,1$:

$$S(N) = (....,b_n,...)_N;\ b \in 0,1;\ n \in [1,2,...,N]$$

For strings of the same length, discrimination ($\oplus$) between two *labeled* strings S_1, S_2 has the two equivalent definitions

$$S_1 \oplus S_2 \equiv (...,{}^1b_n +_2 {}^2b_n,...)_N \equiv (...,({}^1b_n - {}^2b_n)^2,...)_N)$$

where "$+_2$" is addition, mod 2 or "exclusive or" and "$-$" has the usual significance for integers. Note that if we call the *null string* $(0,0,...,0)_N$ "0_N" and the *anti-null string* $(1,1,...,1)$ "1_N", $S_1 \oplus S_1 = 0_N$ and we can define $\bar{S} = S \oplus 1_N$. Tick invokes the random operator R which yields the symbol "0" or "1" with equal probability. For any assemblage of strings with SU elements called $U(SU,N)$, tick is defined by $U := U||R, N := N+1$; in words, tick adjoins a single random bit (separately chosen for each string) to the growing end and hence also increments the bit length by one.

1.3 Program Universe

To generate the strings, we start by taking $U(1,1) = R$, $U(2,1) = \bar{R}$ and go to $PICK$. This operation selects any one the SU strings in $U(SU,N)$ with probability $1/SU$, which we call S_1, picks a second string S_2 in the same way, and forms $S_{12} = S_1 \oplus S_2$. If $S_{12} = 0_N$, we have picked the same string twice and must keep on picking S_2 again and again until we generate a non-null S_{12}. We now ask whether or not S_{12} is already in $U(SU,N)$. If it is *not*, we adjoin it ($U := UuS_{12}, SU := SU+1$) and return to $PICK$. If S_{12} *is* already in the universe we go to $TICK$ ($U := U\|R, N := N+1$) and also return to $PICK$. This simple algorithm has been explicitly coded by M.Manthey[15]. In words, we increase the content of the bit string universe either by generating a new string of the current length N by discriminating between two different randomly chosen strings without changing N, or increase the length of each string by a separately chosen random bit without changing SU.

Since, after we go from TICK to PICK, we go on picking pairs of strings at random, guaranteeing that they are not the same, and discriminating them to produce new strings which are adjoined to the universe until we obtain a string that *already in* the universe, a number of things can happen*between* ticks – that is while the length of the strings is fixed but unique strings of that length are being adjoined to the universe. One interesting question is how we get from this situation to the next tick. We focus on the active element in this process, which is discrimination. The simplest way to get out of the boxis when one discrimination, which may or may not be the first one, produces a string which was *not* created since the last tick, but simply happens to coincide with a string produced by that tick. We will call this situation a *vertex event*, but defer further consideration until we understand the structure better. The only other way we can get out of the box is when the string which is already in the universe is there because of a previous discrimination since the last tick. A little thought should convince the reader that this route to $TICK$ can lead through only six different possibilities, namely

$$S_1 \oplus S_2 = S_{12} = S_3 \oplus S_4 \; or \; S_1 \oplus S_2$$

$$S_2 \oplus S_3 = S_{23} = S_1 \oplus S_4 \; or \; S_2 \oplus S_3$$

$$S_3 \oplus S_1 = S_{31} = S_2 \oplus S_4 \ or \ S_3 \oplus S_1$$

involving the seven non-null strings $S_1, S_2, S_3, S_4, S_{12} = S_{34}, S_{23} = S_{14}, S_{31} = S_{24}, S_{123} = S_1 \oplus S_2 \oplus S_3$ and the null string 0_N, or three non-null strings $S_i \oplus S_j \oplus S_{ij} = 0_N$. Note that in each case the non-null strings *close* under discrimination. We call this occurance a *4-event* (or usually simply an *event*), and note that all cases are contained in the *symmetric* equation

$$S_1 \oplus S_2 \oplus S_3 \oplus S_4 = 0_N$$

2. THE STRUCTURE OF EVENTS

2.1 A Simple Label Paradigm

As the program proceeds, the initial bitscan change only due to the random discriminations. If we call the first $N_L$bits in any string the *label*, and we ticked before all possible $(2^{N_L} - 1)$ non-null strings of that length have been generated, these subsequent discriminations will not necessarily produce all possibilities. In fact, as we discuss in more detail below, if there are $n_L < N_L$ strings which are linearly independent (which we call a *basis*), the number of non-null strings ultimately produced by the random discriminations will be $2^{n_L} - 1$. Subsequent discriminations can never take us outside of this set, so the labels ultimately become *fixed*. For the initial development of our discrete S-matrix theory we use a simple model with only three strings in the basis. We label the strings by the first four bits, using $L_1 = (1110), L_2 = (1100), L_3 = (1101)$ as the basis. Then the remaining labels which close under discrimination are $L_4 = (0011) = \bar{L}_2 = L_{13}$,

$L_5 = (0001) = \bar{L}_1 = L_{23}$, $L_6 = (1111) = L_{123} = L_{24} = 1_4$, $L_7 = (0010) = \bar{L}_3 = L_{23}$; $L_8 = (0000) = 0_4 = \bar{1}_4 = \bar{L}_6$ can also occur as a label. We note first that these divide into two systems whose labels contain an even or an odd number of bits. We will show later that the operation $\bar{L} = L \oplus 1_4$ takes particles into antiparticles and that we can associate a mass with each label. For this particular paradigm we associate a single mass m with the odd strings, and a single mass μ with the even strings. Calling the bits in the string (b_1, b_2, b_3, b_4), we then define two quantum numbers by $x = b_1 - b_2 + b_3 - b_4$ and $2y = b_1 + b_2 - b_3 - b_4$. For m we have the four states $x_m = \pm 1, y_m = \pm 1/2$ and for μ $x_\mu = 0, y_\mu = 0, \pm 1$. We note that L_6 and L_8 are degenerate, so we have in fact a system with only seven particles distinguishable by their quantum numbers and masses.

By examining the scattering equations developed later, we find that, thanks to our basic symmetries, these quantum numbers are conserved. Further, when we examine the probability of scattering compared to the probability of no scattering, since all cases have equal prior probability, the relative "coupling constant" for our basic amplitudes is $g^2 = 1/7$. If we call x baryon number and y the "z-component" of isospin, this system therefore has the quantum numbers for a familiar model of scalar nucleons and antinucleons with isospin 1/2 and scalar mesons with isospin 1, and the model entails a lowest order coupling constant in the range required for an approximate description of nuclear forces. If we choose to insert empirical masses and coupling constants, we can adjust the coupling constant by a density matrix argument, as has been discussed elsewhere[21], and construct a covariant phenomenology. But we are playing for higher stakes.

2.2 The Free Particle Propagators

In this simple environment, any string of length $N+4$ must carry one of these eight labels, only three of which are linearly independent under $\oplus$, and a string of length N which we call the *address* $A_w(N), w \in [1,2,..,8]$. Then our definition of event splits into the two equations

$$L_a \oplus L_b \oplus L_c \oplus L_d = 0_4; \;\; A_a \oplus A_b \oplus A_c \oplus A_d = 0_N$$

If the number of ones in an address string is called k, the number of zeros in the string is $N-k$, and lacking further information the probability of encountering such an address string for any label is

$$P(N,k) = 2^{-N} N!/k!(N-k)!$$

We now define a rational fraction β in the interval $[-1,+1]$ by

$$2k = N(1+\beta); \;\; 2(N-k) = N(1-\beta)$$

It is then easy to show that

$$P(N,k-1)P(N,k+1) = \frac{1-\beta^2}{1-(\beta+2/N)^2} P(N,k)$$

We can now proceed to physical interpretation by assuming that to each label we can assign a parameter m, which we prove below is *conserved* in events, and define $E = m/\sqrt{1-\beta^2}$. If N is very large, proceed for $n \ll N$ ticks, and define $E_1 = m/\sqrt{1-(\beta+2/N)^2}$; the probability to order n/N that E and E_1 will retain this connection for each tick, normalized to the probability that we encounter β in the first place, is

$$P(E,E_1) = \Sigma_{j=0}^{n}(E/E_1)^j,\ E < E_1; P(E,E_1) = \Sigma_{j=0}^{n}(E_1/E)^j,\ E > E_1$$

and hence

$$P(E,E_1) \to 1/|E-E_1|$$

in the large number limit appropriate to scattering boundary conditions. (Scattering boundary conditions require a large number of ticks before we enter the scattering region in order that β_i be confined to some small interval $\Delta\beta$ and that, for the same reason, there be a large number of ticks after we leave the scattering region; hence $n \ll N_i, n \ll N_f$.) By the same reasoning we can extend the spread in values of E and E_1 to larger intervals appropriate to these boundary conditions. Finally, the absolute value we derived shows that we must consider both values for the spread in β which we showed above is $\pm 2/N$ (with a corresponding spread in E) and hence can use the two expressions

$$P^{\pm}(E_1,E_2) = 1/[(E_1-E_2) \pm i0^+]$$

in the appropriate limit. We therefore claim to have derived the *free particle propagators* from our discrete theory.

2.3 ENERGY-MOMENTUM CONSERVATION IN EVENTS

We now introduce *four-vectors* by the definitions $\vec{F} = (f^0, \mathbf{f})$, $\vec{F}_a \cdot \vec{F}_b = f_a^0 f_b^0 - \mathbf{f}_a \cdot \mathbf{f}_b$, $\vec{F}_a + \vec{F}_b = (f_a^0 + f_b^0, \mathbf{f}_a + \mathbf{f}_b)$, $\mathbf{f}_a \cdot \mathbf{f}_b = f_a f_b cos\theta_{ab}$. By defining $\gamma^2\beta^2 = \gamma^2 - 1$ we also have the unit four-vectors $\vec{u} = (\gamma, \gamma\beta)$ with $\vec{u} \cdot \vec{u} = 1$ provided we can find algebraic relations specifying the associated angles θ_{ab}, ... etc.. We can now give the relation between the parameter m for each label, which our derivation of the propagators connected to E and the β for any address string carrying that label, a formally Lorentz invariant significance by first defining four-momenta $\vec{P} = m\vec{u} = M(\gamma, \gamma\beta)$ with the associated Lorentz invariant m^2; clearly $\vec{P} \cdot \vec{P} = m^2$ for *any* $\beta^2 = \beta \cdot \beta$ in any address string carrying any label associated with m.

It was realized by Mach long ago that the most useful way to *define* scalar mass ratios is to use the conservation of 3-momentum in collisions between massive objects (Newton's Third Law) involving some standard mass object. This definition survives the transition to special relativity, and has no known experimental counter-examples when extended to four-momentum conservation with Lorentz invariant masses.

For the basic events generated by our algorithm we therefore *define* mass ratios by requiring that

$$\vec{P}_a + \vec{P}_b + \vec{P}_c + \vec{P}_d = 0$$

which is the usual definition in S-Matrix theory when discussing the Mandelstam representation. In the laboratory, the conventional starting point is to impose *scattering boundary conditions* for the case when, for example, identified particles a and b enter the scattering region and c and d leave it and are also identified. The simplest experimental setup uses counter telescopes which measure the velocities $\beta_a, \beta_b, \beta_c, \beta_d$ (in units of the limiting velocity c) with the associated invariants $m_a^2, m_b^2, m_c^2, m_d^2$. The geometrical arrangement then specifies four four-vectors, and the system has 16 degrees of freedom. Compared to our theoretical description, we now define

$$\vec{P}_a + \vec{P}_b - \vec{P}_c - \vec{P}_d = \vec{P}_{abcd}$$

imposing four constraints on our basic events. Four constants are fixed by the invariant M_{abcd}^2 and the three components of $\beta = \beta_{abcd}$, or equivalently β^2 and two angles (a *direction*) defined by $\beta = (\beta sin\theta cos\phi, \beta sin\theta sin\phi, \beta cos\theta)$. Since these four parameters simply represent the constraints due to energy-momentum conservation, four of the sixteen experimental numbers are redundant, and provide a useful experimental check on the particle identification and other systematic errors (or on special relativity if we accept the particle identifications). To complete the invariant description we define $\vec{P}_a + \vec{P}_b = -(\vec{P}_c + \vec{P}_d) = \vec{P}_{ab} = \vec{P}_{cd}$, $\vec{P}_b + \vec{P}_c = -(\vec{P}_a + \vec{P}_d) = \vec{P}_{bc} = \vec{P}_{ad}$ and $\vec{P}_c + \vec{P}_a = -(\vec{P}_b + \vec{P}_d) = \vec{P}_{ca} = \vec{p}_{bd}$, with $\vec{P}_{ab} = M_{ab}\vec{u}_{ab}$, $\vec{P}_{bc} = M_{bc}\vec{u}_{bc}$, $\vec{P}_{ca} = M_{ca}\vec{u}_{bc}$; hence we have the three invariants $M_{ab}^2, M_{bc}^2, M_{ca}^2$ or equivalently the three angles

$$cos\theta_{ab} = \frac{M_{ab}^2\gamma_{ab}^2\beta_{ab}^2 - m_a^2\gamma_a^2\beta_a^2 - m_b^2\gamma_b^2\beta_b^2}{2m_a\gamma_a\beta_a m_b\gamma_b\beta_b}$$

with the obvious cyclic extensions. In our bit string event context, the "particle identification" is replaced by our association of masses with labels, the "counter telescopes" by specifying $\beta_a, \beta_b, \beta_c, \beta_d$; we conclude that we will be able to relate our bit string events to laboratory measurements with standard scattering boundary conditions if we can derive the four invariants $M_{ab}^2, M_{bc}^2, M_{ca}^2, M_{abcd}^2$ or their equivalents. Note that the "angles" can be defined algebraically and need not be given *direct* geometrical interpretation in terms of the bit strings.

We now return to our basic definition of event and note that any address string involved at a specific value of N referred to any three linearly independent labels a, b, c can be specified by eight integers: $n_a, n_b, n_c, n_{ab}, n_{bc}, n_{ca}, n_{abc}, n_0$ where $n_a(n_b, n_c)$ is the number of ones which do *not* coincide (in any position n of the ordered bits b_n) with the ones in the other two strings, $n_{ab}(n_{bc}, n_{ca})$ *do* coincide between the indicated pairs, n_{abc} is the number of ones which coincide for all three strings, while n_0 is the number of zeros which coincide for all three strings. Clearly $N = n_a + n_b + n_c + n_{ab} + n_{bc} + n_{ca} + n_{abc} + n_0$. Further, we can replace these eight integers by the eight integers $k_a = n_a + n_{ab} + n_{ca} + n_{abc}$, $k_b = n_b + n_{ab} + n_{bc} + n_{abc}$ and $k_c = n_c + n_{bc} + n_{ca} + n_{abc}$ referring to the strings S_a, S_b, S_c, $k_{ab} = n_a + n_b + n_{bc} + n_{ca}$, $k_{bc} = n_b + n_c + n_{ab} + n_{ca}$ and $k_{ca} = n_c + n_a + n_{ab} + n_{bc}$ referring to the strings $S_a \oplus S_b = S_{ab} = S_{c,abc} = S_c \oplus S_{abc}$, $S_b \oplus S_c = S_{bc} = S_{a,abc} = S_a \oplus S_{abc}$ and $S_c \oplus S_a = S_{ca} = S_{b,abc} = S_b \oplus S_{abc}$, and $k_{abc} = n_a + n_b + n_c + n_{abc}$ referring to the string $S_{abc} = S_a \oplus S_b \oplus S_c$, and finally $k_0 = n_a + n_b + n_c + n_{ab} + n_{bc} + n_{ca} + n_{abc} + n_0$ referring to a string with k_0 ones and $N - k_0$ zeros. These in turn can be replaced by the eight rational fractions $\beta_a, \beta_b, \beta_c, \beta_{ab}, \beta_{bc}, \beta_{ca}, \beta_{abc}, \beta_0$.

This is not quite the general experimental situation we described above, since the arbitrary mass m_d has been replaced by m_{abc}, which is fixed by the label $L_{abc} = L_a \oplus L_b \oplus L_c$. When we have succeeded below in associating conserved quantum numbers with labels, in addition to masses, this will be extremely important as it puts constraints on the relationships between "elementary particles". Further, β_0 and the related invariant M_0^2 are not related in any obvious way to the experimental invariant M^2_{abcd}. This turns out to have an interesting significance. Since the basic algorithm creates address strings in which the number of ones is most likely to be equal to the number of zeros, the most probable value for β_0 is zero. Refered to this universal coordinate system, experimentally defined by motion relative to the 2.7 ^oK universal background radiation, we see that the parameter β_0 specifies for our arbitrary event the propagators $P_0^{\pm} = 1/(M^2_{abcd} - M_0^2 \pm i0^+)$. Hence our basic theory carries with it a unique momentum space coordinate system, with an implied connection to any terrestrial laboratory. We have thus proved that the basic events generated by our discrete algorithm provide a manifestly covariant description of scattering with the conventional invariance properties. Further, we have the bonus that they necessarily can be interpreted as tied to motion of the solar system through the background radiation.

2.4 SCATTERING EQUATIONS

So far we have only discussed events for a single tick at bit string length N. However, our algorithm also provides strings in which a discrimination involving one pair is separated by n ticks from a discrimination involving the other pair while satisfying the same asymptotic constraints. By our basic interpretive postulate, this intermediate string is associated with the invariant mass of the pair. Hence we must extend our description of scattering to include these cases by including the three *elementary scattering amplitudes* referred to these masses which have to be proportional to $A^{\pm}_{ab}(M^2_{ab}) = 1/(M^2_{ab} - m^2_{ab} \pm i0^+)$, $A^{\pm}_{bc}(M^2_{bc}) = 1/(M^2_{bc} - m^2_{bc} \pm i0^+)$ and $A^{\pm}_{ca}(M^2_{ca}) = 1/(M^2_{ca} - m^2_{ca} \pm i0^+)$. Restricting ourselves for the moment to a single pair ab in a case where the other possibilities can be neglected, the probability of starting and ending with this pair going once through this intermediate mass must be proportional to the product of the initial and final probability amplitudes:

$$P^{Born}(M_{ab}^2, m_{ab}^2) = A_{ab}^{\pm} A_{ab}^{\mp} = [M_{ab}^2 - m_{ab}^2]^{-2}$$

where we have had to take the product of complex conjugates before taking the limit because probabilities are positive numbers. We have thus derived the familiar "Born approximation" in an unfamiliar way.

But this is only the start of our problem. Even though we can start and end with the same invariant M_{ab}^2, there are many ways consistent with these boundary conditions to conserve 3-momentum but have energies M_n^2 different from M_{ab}^2, and we must sum over all these possibilities. For this purpose we first define *half off shell* elementary amplitudes by

$$A^{\pm}(M^2, M_n^2) = (1/[M^2 - M_n^2 \pm i0^+])A^{\pm}(M_n^2)$$

$$A^{\pm}(M_n^2, M^2) = A^{\pm}(M_n^2)(1/[M_n^2 - M^2 \pm i0^+])$$

and then form the sums

$$t^{\pm}(M_i^2, M_f^2) - A^{\pm}(M_i^2, M_f^2)$$

$$= \Sigma_n A^{\pm}(M_i^2, M_n^2) P^{\pm}(E_i, E_n) t^{\pm}(M_n^2, M_f^2)$$

$$= \Sigma_n t^{\pm}(M_i^2, M_n^2) P^{\pm}(E_n, E_f) A^{\pm}(M_n^2, M_f^2)$$

where the basic symmetries of our algorithm require both forms to define the same function t. Thus we have derived the relativistic Lippmann-Schwinger equation from our discrete algorithm. By paying due attention to momentum conservation and the correct normalizations to unit probability, J.V.Lindesay has proved[18] that the solutions of these integral equations define unitary scattering amplitudes, and even contain such bizarre, but well understood, examples of the extreme non-locality of quantum mechanics as the Efimov effect and the "eternal triangle" effect. These examples of the extreme non-locality of quantum mechanics have relevance to the mind-physics problem which we will explore in the concluding section.

Now that we have unitary two-particle amplitudes, we can extend the treatment to include the *bc* and *ca* channels by deriving relativistic Faddeev equations, and presumably to general N-particle equations by the Faddeev-Yakubovsky combinatorics, starting from arbitrary masses $m_a, m_b, m_c, m_d$with *kinematically defined* "bound states" of mass $m_{ab} = m_{cd}, m_{bc} = m_{ad}, m_{ca} = m_{bd}$. We can also take m^2 to $-m^2$ in the equations and thus include "s-channel resonances" or "virtual states". We have thus proved that our discrete algorithm contains a rich phenomenological model for relativistic N-particle scattering theory.

3. ELEMENTARY PARTICLES

3.1 ANTI-PARTICLES; "SOFT PHOTONS"

We now return to the *elementary particle* problem. In the context of what we are calling here the labels, J.Amson[19] has shown that the entire algebraic content is also contained in a dual theory in which the null string is replaced by the anti-null string. Thus under our basic interpretive assumption, the mass corresponding to any label L_w must be the same as the mass corresponding to $\bar{L}_w = L_w \oplus 1_{N_L}$

where N_L is the (fixed) length of the label bit strings. We therefore interpret this operation as CP, noting that this invariance only applies to the basic strings generated by PROGRAM UNIVERSE, and need not hold for the much more complicated "particles" encountered in the laboratory. For the address strings this symmetry is even simpler, since $\bar{A}(N,\beta) = A(N,\beta) \oplus 1_N = A(N,-\beta)$ corresponding to the velocity reversing operation T, sometimes incorrectly called the "time reversal" operation. We have thus proved that our theory is CPT invariant without any fuss. Further, since reversing the velocities, or changing particles to antiparticles, does not change our invariant amplitudes, our theory has the usual "crossing symmetry" in an equally transparent manner. Whether the dynamic equations then produce manifestly covariant, crossing invariant, unitary amplitudes with the proper cluster decomposition is not so obvious, and is under investigation[20] . Preliminary results are encouraging.[21–23]

The next step is to show that our theory contains "soft photons". The invariance properties already established make this easy. We have already established that the amplitudes are necessarily invariant under the operations $\bar{L} = L \oplus 1_{N_L}$ and/or $\bar{A} = A \oplus 1_N$, and trivially invariant when we discriminate with the null label string or the null address string. But by our basic definition 1_N corresponds to $\beta = +1$ and 0_N to $\beta = -1$. Thus all we need do is to require that the labels 1_{N_L} and 0_{N_L} are associated with $m = 0$. If our normalization of the single quantum exchange amplitudes associated with these strings requires the factor $e^2/\hbar c$, a point we establish below, we then have directly from our scattering equations the description of Rutherford scattering and coulomb bound states. For this to be convincing we must, of course, take the final step of connecting our formal scattering theory in momentum space to laboratory experiments, which again we discuss below.

3.2 THE COMBINATORIAL HIERARCHY

We now show how the *combinatorial hierarchy*[9,10,13,15–17] can be used to label the strings generated by our algorithm, which we call PROGRAM UNIVERSE. Since $a \oplus a = 0$, and a,b linearly independent (l.i.) iff $a \oplus b \neq 0$, there are sets of strings which close under discrimination called discriminately closed subsets (DCsS). For example, if a and b are l.i., the set $\{a, b, a \oplus b\}$ closes, since any two when discriminated yield the third. Similarly if c is l.i. of both a and b, we have the DCsS $\{a, b, c, a \oplus b, b \oplus c, c \oplus a, a \oplus b \oplus c\}$. Provided we call singletons such as $\{a\}$ DCsS's as well, it is clear that from n l.i. non-null strings we can form $2^n - 1$ DCsS, since this is simply the number of ways we can choose n distinct thing one, two,... up to n at a time.

The first construction of the hierarchy[9] started from *discrimination* using ordered bit strings as already defined. Starting from strings with two bits (N=2) we can form $2^2 - 1 = 3$ DCsS's, for example $\{(10)\}, \{(01)\}, \{(10), (01), (11)\}$. To preserve this information about discriminate closure we map these three sets by non-singular, linearly independent 2×2 matrices which have only the members of these sets as eigenvectors, and which are linearly independent. The non-singularity is required so that the matrices do not map onto zero. The linear independence is required so that these matrices, rearranged as strings, can form the basis for the next level. Defining the mapping by (ACDB)(xy)=(Ax+Cy,Dx+By) where A,B,C,D,x,y $\in 0, 1$, using standard binary *multiplication*, and writing the corresponding strings as (ABCD), three strings mapping the discriminate closure at level 1 are (1110), (1101), and (1100) respectively, which is the basis used for our introductory paradigm. Clearly this rule provides us with a linearly independent set of three basis strings. Consequently these strings form a basis for $2^3 - 1 = 7$ DCsS's. Mapping these by 4x4 matrices we get 7 strings of 16 bits which form a basis for $2^7 - 1 = 127$ DCsS's. We have now organized the information content of 137 strings into 3 levels of complexity. We can repeat the process once more to obtain $2^{127} - 1 \simeq 1.7 \times 10^{38}$ DCsS's composed of strings with 256 bits, but cannot go further because there are only 256×256 linearly independent matrices available to map them, which is many to few. We have in this way generated the critical numbers $137 \simeq hc/2\pi e^2$ and$2^{127} + 136 \simeq 1.7 \times 10^{38} \simeq hc/2\pi G m_p^2$ and a hierarchical structure which terminates at four levels of complexity: $(2,3),(3,7),(7,127),(127,2^{127}-1)$. It should be clear that the hierarchy defined by these rules is *unique*, a result achieved in a different way by John Amson[10].

3.3 Labeling the Strings

In the context of program universe, since the running of the program provides us with the strings and also an intervention point (adjoin the novel string produced by discrimination from two randomly chosen strings) where we can organize them conceptually without interfering with the running of the program, we can achieve the construction of a representation of the hierarchy in a simpler way, invented by Manthey and Noyes[15–16]. The procedure is to construct first the basis vectors for the four levels by requiring linear independence both within the levels and between levels. Since adding random bits at the head of the string will not change the linear independence, we can do this at the time the string is created, and make a pointer to that $U[i], i \in [1,2,...,SU]$ which is simply i, and which does not change as the string grows.

Once this is understood, the coding is straightforward, and has been carried through by Manthey[15–16]. Each time a novel string is produced by discrimination, it is a candidate for a basis vector for some level. All we need do is find out whether or not it is l.i. of the current (incomplete) basis array, and fill the levels successively. Calling the basis strings $B_\ell[m]$ where $\ell \in 1,2,3,4$ and $m \in 1,..,B[\ell]$ with $B[1]..B[4] = 2,3,7,127$, we see that the basis array will be complete once we have generated 139 l.i. strings. Since the program fills the levels successively, it is easy to prove that if we discriminate two basis strings from *different* levels we must obtain one of the basis strings in the highest level available during the construction, or level 4 when the construction is complete, i.e. $if\ i \neq j\ and\ both\ < \ell_{last}\ then\ B_i + B_j =\ some\ B_{last}$.

Once we have 139 l.i. basis strings, which will happen when the bit string length N_{139} is greater than or equal to 139, we can guarantee that we will generation *some* representation of the combinatorial hierarchy in these initial bits as we go on ticking. The only alteration of these N_{139} initial bits that can occur from then on will be the filling up, by discriminate closure, of any of the remaining elements of the hierarchy in this representation as a consequence of the continuing random discriminations. Since we keep on choosing strings at random and discriminating them, discriminate closure insures that we will eventually generate all $2^{127} + 136$ elements of the hierarchy [BUT NO MORE]. Of course there will eventually come to be many different strings with the same initial bits, N_{139}. We fix this number, and from now on call the first N_{139} bits in a string the *label*, and the remaining bits the *address*. Finally we note that when the label array is complete we know that among the labels L_i at any one level we can find exactly B(i) l.i. strings and no more; it becomes arbitrary which of the many possible choices we make, so the "basis" becomes a structural fact and does not single out any particular strings. It follows immediately that *if* $i \neq j$ *and both* $< \ell_{last}$ *then* $L_i + L_j =$ *some* L_{last}.

We have seen above that, given four labeled strings $L_w(N_L), A_w(N)$ where $w \in a, b, c, d$ we can construct the full formalism of a relativistic finite particle number S-matrix theory with all the usual invariances. Our next step is to investigate how the combinatorial hierarchy labeling scheme allows us to assign physically significant quantum numbers and masses to these labeled strings generated by PROGRAM UNIVERSE.

3.4 Conserved Quantum Numbers: Leptons

PROGRAM UNIVERSE "starts up" in such a way that we reach the the situation with $SU = 3, N_U = 2$ composed of the three strings (10),(01),(11), or their equivalent, which is the first level of the hierarchy. Since

$$(10) \oplus (01) \oplus (11) = (00)$$

the universe must then "tick". This tick adds either a one or a zero at the end of each string; we can now interpret the first two bits as labels with $N_L = 2$, and the third bit as an address. Since this corresponds to $\beta = \pm 1$, these level 1 labels must be assigned ***exactly*** zero mass, as already noted.

As the program chooses and discriminates between these strings, we can generate eight labeled strings corresponding to $m = 0$ which are

$$(00)(0),\ (00)(1),\ (10)(0),\ (10)(1),\ (01)(0),\ (01)(1),\ (11)(0),\ (11)(1)$$

It can also happen that the universe "ticks" for a while in such a way that 0 becomes 0_N and 1 becomes 1_N, which obviously does not change this structure. It is important to realize that once we have introduced the label-address dichotomy, the string 0_N, which is excluded in the hierarchy construction itself, can have interpretable significance.

We now turn to physical interpretation by taking the critical step of defining a "quantum number" $2h^1_w$ for the level 1 labels, $({}^wb_1, {}^wb_2)$, where w takes on the values $a = (10), b = (01), c = (11), d = (00)$, as $2h^1_w = {}^w b_1 - {}^w b_2$, with the consequence that $h_a = +1/2, h_b = -1/2, h_c = 0 = h_d$. It is easy to show, in the current context, that this quantum number is *conserved* in all events.

The next critical fact to note is that the string (11)(1) reverses both the sign of this quantum number and the sign of the velocity parameter β when any string is discriminated with it. Thus labels fall into two classes, L and $\bar{L} = L \oplus 1_{N_L}$ which we call *particles* and *antiparticles* respectively. Further, the reversal of the sign of the velocity caused by discrimination with the address string 1_N applies just as well to strings with $|\beta| < 1$ as to the case we are considering at the moment. We are now in a position to identify the quantum numbers h^1_a and h^1_b as the two *helicity* states of some massless particle-antiparticle pair. Since the helicity does not reverse when we reverse the velocity (but not the overall time sense, which is defined by the irreversible growth of N), these are"pseudovectors", and if we take the dimensional unit of this quantum number as $\hbar$, we can identify them as strictly massless *chiral* two-component *neutrinos*. From now on we will refer to labels with $|h_L| = 1/2$ in terms of the unit $\hbar$ as particles and (when we encounter them later on) with $|h| = 0, 1$ as *quanta*. We also see that if we think of the reversal of the velocity as the reversal of the time sense instead we have the usual Feynman rule that a particle "moving forward in time" will be equivalent to an antiparticle "moving backward in time". Therefore we have established the CPT theorem in our context.

Two of the remaining four strings, namely (00)(0) and (11)(1) are of particular interest, since the former leaves any string untouched on discrimination, while the second, thanks to the CPT theorem, has the same effect. If we articulate our basic event structure further in the case of neutrino-antineutrino "scattering" ($w, w' \in a, b$) by writing

$$^{w}S \oplus^{w'} S =^{e} S =^{w} S \oplus^{w'} S$$

we see that $e \in c, d$ and that these two strings can be "exchanged" without altering the system. They are therefore our candidates for "soft" quanta, which are necessarily massless - a point which Stapp and Chew emphasize. As we have proved, our scattering theory allows us to sum any number of such processes and then lead to the kinematics of Rutherford scattering in an appropriate large number approximation.

Before we leave this primitive universe of massless neutrinos and quanta, it is interesting to note that they will remain constituents of the universe as it evolves and provide an ultimate (but ever increasing) boundary. Since we do not as yet have enough structure to define directions, this boundary is *isotropic*. Once we have developed enough structure for them to scatter from massive constituents, the first scatterings will define an "event horizon" whose isotropy or lack of it will depend on the details of the way PROGRAM UNIVERSE generates these scatterings. About this we will only be able to make statistical statements. Strings which engage in scatterings after these first "horizon" events will then define, statistically, the energy and particle density of the universe. We will not discuss cosmology further in this paper.

The level 1 structure we have discussed will persist until we encounter an address string with the structure (1_N0) or (0_N1). Then the program will start to construct level 2. The basis will close off when we have three l.i. basis strings, which are also l.i. of the level 1 strings, and their discriminate closure in a total of seven strings. The simplest representation of this situation is to use level two label strings with the structure $(00)(b_3b_4b_5)$ with basis strings $(00)(100), (00)(010), (00)(001)$. The mapping matrix construction can give the equivalent set $(1100), (1110), (1101)$, which is more convenient to use when defining quantum numbers. After the labels close off, we can again encounter the situation in which, for a while, the only address strings will be 1_N and 0_N, so we continue our discussion in terms of the structure for the first level

$$level\ 1:\ ({}^ib_1\ {}^ib_2)(0000)(1_N\ or 0_N)$$

where $i \in 1,2,3,4$ and, to be specific, $1:(10),\ 2:(01),\ 3:(11),\ 4:(00)$. Note that $1=\bar{2}$ and $3=\bar{4}$. The corresponding structure for the second level is

$$level\ 2:\ (00)({}^jb_3\ {}^jb_4\ {}^jb_5\ {}^jb_6)(1_N\ or\ 0_N);\ b_3=b_4$$

where $j \in 1,2,3,4,5,6,7,8$ and, again to be specific, $1:(1110),\ 2:(1100),\ 3:(1101),\ 4:(0011),\ 5:(0001),\ 6:(1111), 7:(0010),\ 8:(0000)$. Again note that $1=\bar{5},\ 3=\bar{7},\ 2=\bar{4},\ 7=\bar{8}$.

Within level 2, we now define helicity by $2h_j = {}^jb_3 + {}^jb_4 - {}^jb_5 - {}^jb_6$ and find that $h_1=h_3=+1/2;\ h_5=h_7=-1/2;\ h_2=+1;\ h_4=-1;\ h_6=h_8=0$. We now have enough structure to define a second quantum number within this level, $\ell_j =^j b_3 - {}^j b_4 + {}^j b_5 \; -^j b_6$ with the consequence that $\ell_1 = +1,\ \ell_3 = -1,\ \ell_5 = -1,\ \ell_7 = +1,\ \ell_2 = \ell_4 = \ell_6 = \ell_8 = 0$. By appropriate invocation of the Feynman rules, we again can show that these quantum numbers are conserved in events, that the elementary scattering diagrams have crossing symmetry, and that the CPT theorem is satisfied. Clearly ℓ can now be identified as lepton number. Thus, with both level 1 and level 2 before us, we claim to have, still massless, *chiral* (two component) neutrinos, *achiral* (four component) leptons and massless vector and scalar quanta with zero lepton number. We do not explore here the coupling between level 1 and level 2, since by our constructive algorithm for the hierarchy this necessarily involves level 3 labels. We note that, in contrast with the conventional theory, and in agreement with the topological bootstrap theory, our basic neutrinos and scalar and vector quanta are massless. When we go on to the next two levels, we will see how the *achiral* leptons acquire mass.

3.5 Conserved Quantum Numbers: Hadrons

Once again, when we encounter an address string of the form 1_N0 or 0_N1, PROGRAM UNIVERSE requires us to start constructing level 3. In analogy with our previous step, we now use for the third level structure

$$level\ 3 : (00)(0000)({}^kb_7\ {}^kb_8\ {}^kb_9\ {}^kb_{10}\ {}^kb_{11}\ {}^kb_{12}\ {}^kb_{13}\ {}^kb_{14})(1_N\ or 0_N)$$

with $k \in [1, 2, 3, ..., 128]$. We also add 0_8 at the end of the level 1 and level 2 labels, before starting the new address labels. For the moment we will restrict ourselves to the situation in which $b_{11} = b_{12} = b_{13} = b_{14} = 0$, and consider only the 16 strings generated from some l.i. choice of four basis vectors of length 4.

Consider first the strings (1110), (0001), (1101), (0010) we encountered before, and the four new ones now available (1011), (0100), (0111), (1000). We define the quantum numbers $2h_k =^k b_7 +^k b_8 -^k b_9 -^k b_{10}$, $B =^k b_7 -^k b_8 +^k b_9 -^k b_{10}$and $2i_z =^k b_7 -^k b_8 -^k b_9 +^k b_{10}$. Using the usual Gell-Mann Nishijima relation $Q = i_z + B/2$, we have precisely the quantum numbers for protons and antiprotons with baryon number and charge $B = \pm 1 = Q$ and neutrons and anti-neutrons with $B = \pm 1,\ Q = 0$; the two helicity states $\pm 1/2$ also occur in the correct way. As before, all the usual rules of S-matrix theory work out.

What about $b_{11} - b_{14}$? Since we already have four l.i. basis vectors, only 3 of these are allowed to be l.i to complete the basis for level 3. We take the basis to be the familiar (1100), (1110), (1101), but now with the interpretation given in Table I.

Although for brevity in the caption we have called this the "SU3 octet", speaking with more precision what we have is just the discrete quantum numbers which are conventionally discussed in terms of that octet. From our point of view, all we have is a transparent rule for defining two sets of eight quantum numbers for eight bit strings we have derived from the combinatorial hierarchy. We believe that it is a conceptual advantage in our approach that discrete quantum numbers are just that, and need never be referred to "continuous groups". All we encounter in high energy experimental physics are discrete quantum numbers and their connections. These are all we need to, or intend to, construct.

We now have a ready interpretation for level 3. We identify this octet with the "color octet" of QCD. We started our discussion of baryons by taking these four bits to be (0000). Since, as we can see from Table I, either this string or (1111) represent a "color singlet" our initial discussion of nucleons and anti-nucleons,

Table I

The SU3 octet for "I,U,V spin"

	$(b_{11}b_{12}b_{13}b_{14})$	$2I_z$	$2U_z$	$2V_z = 2(I_z + U_z)]$
STRING:	1110	+1	+1	+2
	0010	-1	+2	+1
	1100	+2	-1	+1
	1111	0	0	0
	0000	0	0	0
	0011	-2	+1	-1
	1101	+1	-2	-1
	0001	-1	-1	-2

$$2I_z = b_{11} + b_{12} - b_{13} - b_{14}$$

$$2b_{11} + b_{12} + 2b_{13} - b_{14}$$

$$2V_z = -b_{11} + 2b_{12} + b_{13} - 2b_{14}$$

with associated mesons generated by discrimination, remains valid. But with color added, these two-particle, two-antiparticle spin states can become "up" and "down" quarks and antiquarks. All that remains is to show that the only states we can form as particles correspond to (qqq) and $(q\bar{q})$, and that the quarks and associated gluons remain in the picture as "partons" . That any particle has three "partons" is clear from the fact that for any particle d, our events allow it to make the sequential transitions $d \rightarrow a, b, c \rightarrow d$ as it propagates. These (finite) "virtual transitions" are unobservable until we intervene with some additional high energy particle and study "deep inelastic" processes. Our S-matrix theory then insures that we will have the usual kinematics of the parton model. All we need do is to show that we can calculate the correct masses and coupling constants.

To go on to level 4, we see that we can have two basis vectors with the structure of $(B)_2 0_{12}$ at level 1, three basis vectors with structure $0_2(B)_4 0_8$ at level 2 and seven basis vectors with structure $0_6(B)_8$ at level 3. According to our constructive algorithm, we can immediately put together $2 \times 3 \times 7 = 42$ of these to form 42 of the basis vectors for level 4, without changing the massless address strings 0_N and 1_N. But this does not complete the 127 basis strings needed for constructing the level. Hence, for the last time, we argue that PROGRAM UNIVERSE will eventually produce an address string with the structure $0_N 1$ or $1_N 0$ and from then on will have to continue adding to the label string ensemble until at some label length $N_L + N_{139} \geq 139$ the basis is complete and the label length fixed from then till doomsday. If we are content to stick with the first three level labels as an approximation and interpret these added bits as addresses, we see that they correspond to systems with $|\beta| < 1$, and hence to massive particles. In this way our hardons are shown to have to be massive but the first generation leptons and electromagnetic quanta remain exactly massless. We will discuss below how the electrons and positrons acquire mass. Further discriminations will eventually produce all $2^{127} + 136$ non-null labels at this label length, while the addresses continue to grow both in bit length and in number as long as the program continues.

Clearly the eventual structure, with $2^{127} + 136$ distinct quantum number states, is immensely complicated in detail, but we can already make some useful comments about some of the connections which will have to emerge. One is that there are three simple structures of the form $(B)_{14}0_{28}$, $0_{14}(B)_{14}0_{14}$, $0_{28}(B)_{14}$ where (B) are the 42 basis vectors already discussed. This gives $3 \times 42 = 126$ of the 127 basis vectors needed to close the hierarchy. Yet each of them will also close on itself, so we anticipate that the coupling between these three structures will be weak. The first one looks like it still has a massless address label, but if we use instead simply three identical repetitions, i.e. $(B)_{14}(B)_{14}(B)_{14}$, the properties will be the same, and we trust can be discussed ignoring, in first approximation, the anticipated weak coupling to the rest of the scheme. If we now consider only the label 1_{14} or its equivalent 0_{14} which couples "softly" to all of the first three levels, this will occur with probability 1/137 and we can now, with confidence, accept this as our first approximate evaluation of the strength of the coulomb interaction.

With this in hand, we can then expect that the structures we first encounter in particle experiments at low energy will be the familiar $\nu_e, \bar{\nu}_e; e^{\pm}, \gamma; p, \bar{p}, n, \bar{n}$ with the weak vector bosons, up and down quarks, and gluons coming along in due course. At least we have the right quantum numbers for the first generation of the standard model, and believe we have made it look worth while to see if the couplings can be worked out and compared with experiment. Further, the structure we discussed above suggests that the next two generations will also be there. Finally, when we ask about the 127^{th} basis vector, 1_{42} with the associated 0_{42} which occurs with probability $1/(2^{127} + 136)$ and couples to everything, we can also with confidence assume that this is the "soft" Newtonian gravitational interaction with this number as a first approximation to the coupling constant $Gm_p^2/\hbar c$, and choose our final dimensional constant to be either m_p *or* G according to our taste.

4. LABORATORY EXPERIENCE

4.1 SPIN AND SCATERING

Our next step is to note that since our theory contains the dichotomous spin labels (01) and (10), which when augmented by an address string with $|\beta|$ less than unity must carry mass, we can scatter this particle elastically from any other system and emerge with a *coherent* amplitude composed of both states. The Lorentz invariance of our theory then requires us to introduce the full formalism for dichotomous relativistic spinors. Because we insist on unitarity, it is most convenient to use the Wigner two-component unitary representations of the Lorentz group. The details have been worked out and will be published[24]. We are now finally ready to move from our abstract momentum space S-matrix theory to the interpretation of that theory in terms of laboratory experiments.

We consider first a *counter* activated by some ionization process, for example the ionization of a hydrogen atom whose levels are predicted by the poles in the appropriate S-matrix (Franck-Hertz experiment). This counter will have some length Δx which we can measure with "rods" and will fire during a time interval Δt which we can measure with "clocks", but we cannot localize the firing directly with any greater precision. Following our interpretation of the random walk model pioneered by Stein[11], we interpret the address string associated with the particle label as a random walk connecting two tick-separated discriminations or events with step length $\ell = hc/E$ in which a one represents one step in the positive direction and a zero a step in the opposite direction. All we know from the firing of a single counter is that we have some ensemble of strings for which, lacking further information, the probability of finding any velocity $2\beta = 2k - N$

is $2^{-N}N!/k!(N-k)!$.

We now consider a situation in which two counters are separated by some distance $D >> \Delta x$, and the second counter fires at some time $T >> \Delta t$ after the first. Assuming a source of particles, and selecting only those laboratory events for which D/T has some constant value βc with a precision $\pm\Delta x/\Delta t = \Delta\beta$, we have thus prepared a *beam* of particles of specified velocity. The number N_i used in our abstract discussion is now obviously given approximately by $D = N_i hc/E, E = m\gamma c^2 = mc^2/\sqrt{1-\beta^2}$. In general the uncertainty $\Delta\beta$ will be much greater than the intrinsic digital uncertainty $2/N_i$, which uncertainty our discrete theory will never allow us to reduce.

We can now obviously form two initial beams which intersect in a scattering region and two detectors each containing two counters, and have connected the boundary conditions of our abstract theory to actual laboratory experiment. We must of course supplement these counter telescopes by particle identification devices and impose on the data selective criteria which satisfy the energy-momentum conservation laws, agonizing details all too familiar to the high energy experimentalist; we leave them in his competent hands. We have thus proved that our theory has explicit connection to laboratory *practice*, a necessary part of any constructive theory, as has been emphasized by Gefwert[25].

4.2 THE WAVE-PARTICLE DUALISM

We now investigate the single particle beam in more detail. Knowing β and N_i, we see that we have specified a random walk ensemble with step length $\ell = hc/E$. For any particular string, we assume that each step takes a time $\delta t = \ell/c$. We further assume that after n steps, the most probable position of the random walk distribution will move one step length. We identify the velocity at which this most probable position moves, $v = \ell/n\delta t$, with the velocity of the particle $\beta c = c/n$. Our previous discussion insures that we can always find such an integer n within the range allowed by our boundary conditions with a correction of order n/N_i, which we ignore. Thus in our distribution there will be a *coherence length* $\lambda = n\ell = \ell/\beta c = h/p$ at which any pattern with this velocity repeats. From our scattering theory it now follows that this will be the most probable point at which a scattering will occur.

If we divide our beam into two coherent beams (eg. by a double slit) and bring them together again the maximum detectable intensity will occur at positions where the two path lengths are an integral number of coherence lengths apart. Further, as we have seen above, if our particle carries spin, we can prepare a beam with any specified spin *direction* by a suitable sequence of scatterings, a process familiar to particle physicists who measure spin polarization and correlation in scattering. But our *directions* approximate *classical* vectors for sufficiently large N; hence this internal direction (which acts like the polarization vector in classical optical bench experiments) can add to zero when we bring two coherent beams together and produce interference nulls. Thus our interpretive postulate guarantees that for spinning particles our theory exhibits the usual interference phenomena of quantum mechanics with the relativistic deBroglie group velocity wave length. According to our discussion of partons given above, any particle, even one of zero spin, will carry these coherent internal degrees of freedom, so this conclusion is quite general. To get the "wave theory", all we need do is construct the appropriate interpolation between the basic digital phenomena given by our bit strings using Fourier analysis and paying due attention to the necessary uncertainties which go with our measurement paradigm.

5. THE MASS-RATIO SCALE

What is still missing in our fundamental theory are the mass ratios of the particles relative to our standard m_p identified by $\hbar c/Gm_p^2 = 2^{127} + 136$. Here we adapt a calculation of Parker-Rhodes[26] based on his alternative, but closely related, approach to the problem of constructing a fundamental theory. He confronts the problem of *indistinguishability*, which in modern science goes back at least to Gibbs, but poses the problem in the logical (static) framework of how we can make sense of the idea that there are *two* (or more) things which are indistinguishable other than by the *cardinal* number for the assemblage *without* introducing either "space" or "time" as primitive notions. Clearly his starting point is distinct from the constructive program, and the "fixed past - uncertain future" implicit in our growing universe with randomly selected bit strings.

We have seen above that, for a system at rest in the coordinate system defined internally by $< \beta >= 0$ or externally by zero velocity with respect to the background radiation, the minimal fundamental length is h/m_pc, inside which length we have no way of giving experimental meaning to the concept of length without external coupling[27] . We have also seen that our scattering theory has, for zero mass coulomb photons, a macroscopic limit in Rutherford scattering, a non-relativistic limit in Bohr's theory of the Hydrogen atom, a continuum approximation in deBroglie's wave theory provided by continuum interpolation using Fourier analysis, and hence the usual formalism for the macroscopic e^2/r "potential" up to $O(1/137)$ spin-dependent corrections or relativistic corrections of the same order (either of which corrections — relativistic spin(Dirac) or relativistic motion (Sommerfeld) — account quantitatively for the empirical hydrogen fine-structure to that order). We have also seen that our momentum-space S-matrix theory has (within our digital restrictions) the usual properties of rotational and Lorentz invariance in $3+1$ momentum-energy space, and hence by our interpretive paradigms in 3-space.

We therefore can assert that outside a radius of $h/2m_pc$, the energy associated with the (minimally three) partons connected to an electron, the electrostatic energy of an electron can be calculated statistically from $< e^2/r >$ with three degrees of freedom and $r \geq (h/2m_pc)y, y \geq 1$. Since the conservation laws we have already established require charge conservation, the electrostatic energy must be calculated from the charge separation outside this radius with charges ex and $e(1-x)$, so $< e^2 >= e^2 < x(1-x) >$. At first glance x can have any value, but in any statistical calculation the charge conservation we have already established requires that these cancel outside of the interval $0 \leq x \leq 1$. We have seen that the leptons are massless until they are coupled to hadrons at level 3 of the hierarchy (with, as the first approximation, $e^2/\hbar c = 1/137$). Hence, in this approximation, we can equate m_ec^2 with $< e^2/r >$, and arrive at the first Parker-Rhodes formula

$$m_p/m_e = \frac{137\pi}{< x(1-x) >< 1/y >};\ 0 \leq x \leq 1;\ 0 \leq (1/y) \leq 1$$

From here on in, the only point to discuss is the weighting factors used in calculating the expectation values, since we now have from our S-matrix theory the same number of degrees of freedom (three) as Parker-Rhodes arrives at by a different argument based on the *Theory of Indistinguishables.* For the $(1/y)$ weighting factor this is almost trivial; our carefully constructed derivation of the Coulomb law and the symmetries of 3-space imply that $P(1/y) = 1/y$. For $x(1-x)$ the two-vertex structure of our S-matrix theory requires one such factor at each vertex in any statistical calculation: $P(x(1-x)) = x^2(1-x)^2$. The calculation for three degrees of freedom is then straightforward, and has been published several times[10,12–13,15–17]. The result is $< 1/y >= 4/5$, $< x(1-x) >= (3/14)[1 + (2/7) + (2/7)^2]$, leading immediately to the second Parker-Rhodes formula

$$m_p/m_e = 137\pi/[(3/14)[1 + (2/7) + (2/7)^2](4/5)] = 1836.151497...$$

in comparison with the experimental value of 1836.1515 ± 0.0005. Although this result has been published and presented many times, we know of no published challenge to the calculation.

The success of this calculation encourages us to believe that the seven basis vectors of level 3 will lead to a first approximation for $m_p/m_\pi \approx 7$ with corrections of order $1/7$, but this has yet to be demonstrated.

6. CONCLUSIONS FOR PHYSICS

We believe that the structure developed in this paper, including the connection between the mathematical structure and laboratory practice is not in contradiction with currently non-controversial "facts" accepted by most elementary particle experimentalists or theorists. Our theory has many features in common with Bell's[28] interpretive schema that introduces "beables", which are conceptually close to our bit strings; we have given a more detailed construction than his, and one that eventually will be experimentally refutable. Although the theory is finite and discrete, it cannot support any direct physical interpretation which would allow the supraluminal transmission of information; of course the randomness and and non-locality which have been carefully incorporated in the construction can provide for the supraluminal correlations experimentally demonstrated in EPR experiments. We are eager to hear any careful argument purporting to show that our theory is necessarily constrained by Bell's inequality, or of any point where we are in conflict with either accepted experimental results or theories within the framework of constructive mathematics. We believe that we can have our cake in the sense of successful contact with experiment, and eat it too in the sense that we have an underlying digital algorithm which can be directly grounded in constructive mathematics and which never need invoke *completed* infinities. Thus we claim to have arrived at an *objective* quantum mechanics with all the needed properties.

In this paper we have proved that by starting from bit strings generated by *program universe* and labeled by the $2^{127} + 136$ strings provided by any representation of the four-level *combinatorial hierarchy* leads to an S-matrix theory with the usual C, P, T properties, CPT and crossing invariance, manifest covariance and a candidate to replace quantum field theory by an N-particle scattering theory which will not be in conflict with practice for some sufficiently large *finite* N.

Many points along the way should, and will[29] , receive much more careful discussion from a number of points of view. We believe, however, that the essential features of the new theory have been outlined here. This theory is corrigable by laboratory experiment or logical analysis. If an apparent conflict arises, we must (a) question the accuracy or relevance of the experiment, or (b) locate a logical flaw (within the confines of constructive mathematics) or an ambiguity among the paradigms by means of which we relate our mathematics to laboratory practice, or (c) modify the theory at one or more of these points to meet the problem posed. Failing any successful way of meeting objections (a), (b) or of succeeding with (c) this author would be constrained to (d) abandon the theory. We spell out these obvious criteria — which are close to what Popper holds to be necessary (but not sufficient) for a theory to be *scientific* — because our approach has sometimes been misinterpreted as "Pythagorean" or "*a priori*".

7. IMPICATIONS FOR THE PHYSICS-MIND PROBLEM

My approach to the broader implications of quantum mechanics has not as yet led to much new insight on the mind-physics problem. I have assumed for a long time that the creation of an *objective* quantum mechanics would eventually prove to be possible. I believe that in different ways Stapp and Chew and Bell have already come close to, or perhaps have, reached that objective; of course I hope that the approach presented in this paper will also be considered a viable alternative. However, what follows will be compatible with any of these differing foundational theories.

Although there are problems in detail as to how to extend cosmology back before the first three minutes, from then on I believe that we already have in hand a pretty good, and reasonably stable, account of how the universe as a whole, or at least the portion of it within our event horizon, has evolved. I think that *program universe* provides an interesting way of understanding how the event horizon and the special zero velocity reference frame may have come about. It may also have interesting technical implications about cosmological details that may be testable, but we will not pursue those here.

In the same sense I believe that we have a good handle on the 4.5×10^9 year history of the solar system and of biological evolution on the surface of our own planet. There was, in Manfred Eigen's phrase, "once for all" selection for the three codon DNA organ of heredity. There too many, rather than too few, plausible scenarios for biopoesis and the origin of biomolecular chirality. Once biological complexity got off the ground Eigen has also provided a useful statistical model for species defined by a gaussian spread of genotypes, each of which can have different phenotypic expressions in different environments. Within appropriate limits this model explains how reversable adaptation to a wide variety of environments can work for sufficiently large populations. Speciation occurs due to geographical isolation of small populations, genetic drift, and subsequent interactions which either extend the range of variation or fix a non-interbreeding new species. The length of time it took to go from monocellular life to multicellular organisms, and the origin of the largest taxonomic groupings may still be something of a mystery, but work on correlating this problem with secular changes in the surface environment, particular temperature, looks promising.

Again, the origin of both social structure and intelligence fits easily, for me, into the overall pattern of biological evolution, with only problems of detail rather than principle to be met. Cultural evolution brings in new principles which are not as yet as well understood, and which the simplifications used in sociobiology are to naive to be the the whole story. Consciousness (in the sense of *self*-consciousness might be of quite recent origin; at least Julian Jaynes has made a provocative argument along that line. Clearly more work is needed in these areas.

However, one can also get cluse from the extreme non-locality of quantum mechanics which leap across the intervening complexity, as I realized when I first encountered the *eternal triangle* effect[30,31]. It is easy to show from the conventional Faddeev description of the quantum mechanical three-body problem in configuration space that the introduction of the third body into a two-body systems results in a change in the interaction between an "isolated" pair even when the forces are of strictly finite range. As already noted, this effect also occurs in a relativistic scattering theory even when the "range of forces" is zero (i.e. in a scattering length model).

The name for the effect comes from considering the behavioral analogy of two persons in a closed room whose behaviors change when they come to think that there is a third person outside the door. I see this as a consequence of their past history which has trained them to use different behaviors when there are three rather than two persons present. This is connected to the quantum mechanical model by the fact that quantum mechanics also requires that the entire past history of the system be known before one can start to compute current probabilities. This effect also has analogical connection to the phenomenon called "synchronicity" discussed by Pauli and Jung, and other experiential phenomena, as I have discussed elsewhere[32].

For me the most important aspect of this theoretical structure which we are discussing at this conference is the implication that we can make, at the present, only calculations of the probabilities that various alternatives will take place; hence our predictions must be *a-deterministic.* I have called this "fixed past - uncertain future" and idea which goes back at least as far as Aristotle, but which has now been made a part of science by the creation of an *objective* quantum mechanics. Therefore we can never be absolved of the moral responsibility for our actions or inactions and should guide our behavior accordingly. My thoughts on this are somewhat expanded in the Appendix, which first appeared in the SLAC *Beam Line*.

ACKNOWLEDGEMENTS

This synthesis would not have been possible without the work of Bastin and Kilmister[7–8] which started from an examination by Bastin of Eddington's "fundamental theory" and led (in collaboration with Amson, Pask and Parker-Rhodes) to the combinatorial hierarchy[9]. T.E.Phipps' work on the connection between quantum and classical mechanics, and time irreversability[33–35] started this author on a serious search for a finite scattering theory incorporating the "fixed past — uncertain future" philosophy. Work by Stein, Aerts, Manthey, McGoveran and Gefwert was brought into this focus thanks to the founding of the Alternative Natural Philosophy Association in 1979. This paper owes much to critical discussions with M.Peskin, H.Partovi, H.P.Stapp and G.F.Chew, which closes the recursive loop for this author because he was Chew's first graduate student.

REFERENCES

1. H.P.Stapp, "Bell's Theorem and the Foundations of Quantum Mechanics", *Amer. J. Phys.* (in press) and LBL-16482 preprint.

2. G.F.Chew, *Found. of Phys.* **13**, 273 (1983)

3. G.F.Chew, talk at the First Annual Regional Meeting of the Alternative Natural Philosophy Association, Stanford, Nov. 23-25, 1984, available from HPN.

4. H.P.Stapp, *Phys. Rev.* **D 28**, 1386 (1983)

5. –, "Time and Quantum Process", presented at Conference *Physics and the Ultimate Significance of Time*, March 16, 1983, Center for Process Studies, Claremont, CA 91711, and LBL-17576 preprint.

6. G.F.Chew, "Gentle Quantum Events as the Source of Explicate Order", LBL-5811 preprint.

7. E.W.Bastin and C.W.Kilmister, *Proc. Roy. Soc.*, **A 212**, 559 (1952).

8. –*Proc. Camb. Phil. Soc.*, **50**, 254, 278 (1954), **51**, 454 (1955); **53**, 462 (1957); **55**, 66 (1959).

9. T.Bastin, *Studia Philosophica Gandensia*, **4**, 77 (1966).

10. T.Bastin, H.P.Noyes, J.Amson and C.W.Kilmister, *Int'l J.Theor.Phys.*, **18**, 455-488 (1979).

11. I.Stein, Seminar at Stanford, 1978, and papers at the 2nd and 3rd annual meetings of the Alternative Natural Philosophy Association, Cambridge 1980, 1981.

12. A.F.Parker-Rhodes, *The Theory of Indistinguishables*, Synthese Library, *150*, Reidel, Dordrecht, 1981.

13. H.P.Noyes, in *The Wave-Particle Dualism* S.Diner, et al., (eds), Reidel, Dordrecht, 1984, pp. 537-556.

14. M.J.Manthey and B.M.E.Moret, Communications of the ACM, **26**, 137-145, 1983.

15. H.P.Noyes, C.Gefwert and M.J.Manthey, in *Proc. 7th Int'l Cong. on Logic, Methodology and Philosophy of Science.* Salzburg, 1983 (in press) for a much more detailed version see – "Toward a Constructive Physics", SLAC-PUB-3116 (rev. September 1983).

16. H.P.Noyes, M.J.Manthey and C.Gefwert "Constructing a Bit String Universe: a Progress Report", *Proc. of the Oxford Quantum Gravity Discussion Conference* (in press) and SLAC-PUB-3290.

17. H.P.Noyes and C.Gefwert, "The Measurement Problem in Program Universe", SLAC-PUB-3537, submitted to the *Symposium on the Foundations of Modern Physics*, Joensuu, Finland, June, 1985.

18. J.V.Lindesay, PhD Thesis, Stanford, 1981, available as SLAC Report No. 243.

19. J.Amson, papers at the 5^{th} and 6^{th} Annual International Meetings of the Alternative Natural Philosophy Association, 1983 and 1984.

20. J.V.Lindesay, H.P.Noyes, A.Markevich and G.Pastrana (in preparation).

21. H.P.Noyes and J.V.Lindesay, *Australian J. Phys.*, **36**, 601 (1983).

22. H.P.Noyes and G.Pastrana, *Proc. Few Body X*, Karlsruhe, 1983.

23. J.V.Lindesay and A.Markevich, *Proc. Few Body X*, Karlsruhe, 1983.

24. A.Markevich, Ph.D Thesis, Stanford, 1985 (in preparation).

25. C.Gefwert, *A Participator: the Metaphysical Subject*, SLAC-PUB-3277 (December, 1983) and *The Proposition-as-Rules Idea*, SLAC-PUB-3303 (March 1944), to be submitted to *Synthese; On The Logical Form of Primitive Recursive Functions*, SLAC-PUB -3334 (May 1984) and *On The Logical Form of Mathematical Language*, SLAC-PUB-3344 (May 1984), to be submitted to the *Journal of Philosophical Logic.*

26. Ref. 12, pp 182-185.

27. G.C.Wick, *Nature*, **142**, 993 (1938).

28. J.S.Bell, "Beables for Quantum Field Theory", CERN-TH.4035/84 preprint.

29. D.Aerts, J.Amson, T.Bastin, C.Gefwert, C.W.Kilmister, M.J.Manthey, D.McGoveran, H.P.Noyes, A.F.Parker-Rhodes, I.Stein and J.C.van den Berg, "A Discrete Reconstruction of Quantum Mechanics" (in preparation).

30. H.P.Noyes, in *The Three Body Problem in Nuclear and Particle Physics*, J.S.C.McKee and P.M.Rolph, eds., North Holland, Amsterdam, 1970, p.494.

31. -, *Bull. Am. Phys. Soc.*, **16**, 20 (1971).

32. H.P.Noyes, "The Eternal Triangle Effect", in *Non-Verbal Communication Today*, Mary Ritchie Key, ed., Moulton, New York, 1982, pp. 279-284.

33. T.E.Phipps, Jr., *Phys. Rev.* **118**,1653 (1960).

34. T.E.Phipps, Jr., *Dialectica* **23**, 189 (1969).

35. T.E.Phipps, Jr., *Found. Phys.* **3**, 435 (1973

APPENDIX: Fixed Past – Uncertain Future[♯1]

For over two thousand years Western philosophers have struggled with two conflicting descriptions of time. One view is that both past and future are completely determined. Complete knowledge of what has and will happen is denied to finite human minds, yet some theistic philosophies hold that God has this complete prescience, while some atheistic philosophies maintain that determinism is the unalterable consequence of immutable scientific laws. Either belief says that, in principle, complete knowledge of the present implies the possibility of predicting the future and reconstructing the past. Competing views are that the unconstrained actions of the gods, or the unpredictable choices of free human beings, or some intrinsic randomness built into the structure of the universe, makes both past and future increasingly chaotic as one looks either forward or backward in time.

Physics employs both models, and like philosophies or religions which try to find a path between the two extreme views cited above, tries to reconcile them. "Classical" physics, which burgeoned out of the Scientific Revolution of the seventeenth century, was primarily deterministic, while quantum mechanics in our own century has usually been taken to imply increasing chaos as one looks farther into either past or future. Since the basic laws of both classical and quantum physics are reversible in time, neither finds it easy to account for many everyday experiences. For example, if a hot and a cold body are put in contact and insulated from their surroundings, we find that, following the usual human sense of the direction of time, they will come closer and closer to a common temperature.

♯1 From the *Beam Line*, Vol. 3, No. 3, published by the Stanford Linear Accelerator Center (SLAC), December 18, 1972.

Thus a physicist (unless he is struggling with a philosophical problem) presented with a sequence of observations of their temperatures will unhesitatingly assert that time was moving forward if he reads the record in the direction of decreasing temperature difference, or backward if the hotter body is growing hotter and the colder body growing colder. But he has great difficulty in "proving" (using either classical or quantum physics) whether he has (or has not) in fact misread the time direction of the record as it would have been given by a clock in touch with the rest of the universe. To put the case more dramatically, contemporary physicists cannot guarantee that even if your refrigerator is working properly, an ice tray (by a very unlikely chance) may not start to boil.

Thus physics as taught and used in our society seems to contradict everyday experience. Few of us believe that we can actually alter the past, yet most of us act as if we believed that our acts can have some effect on the future. Symmetry under time reversal is only one of three symmetries that physicists used to believe were absolute, at least at the level of the elementary particle interactions. They also held that any experiment viewed in a mirror was also an experiment which could conceivably be carried out, and that the same would be true if particles were exchanged with their anti-particles. One of the most significant results achieved by the high energy accelerator laboratories such as the Stanford Linear Accelerator Center has been to prove that none of these "obvious" assumptions are true. The first breakthrough came in the fifties when Lee and Yang suggested that the mirror image of certain experiments might not picture experiments which it is possible to carry out on the surface of the earth. Experimental proof of this hypotheses that "parity" is not conserved was soon forthcoming, but it was still possible to assume that the mirrored experiment could be performed on a planet composed of anti-matter (i.e. in which the atoms of the chemical elements are made up of electrons with positive electric charge and the nuclei of the atoms have negative electric charge). But detailed study of K-meson decay here and elsewhere eventually proved that the decay of anti-K-mesons does not mirror the decay of K-mesons. Current theory requires, and it has since been shown experimentally, that K-meson decay occurs because of an interaction that is not reversible in time. But K-mesons are, so far, the only elementary particles with this peculiar property; it has not been able to connect up this effect with any of the other known facts about other particles. This unique example of the failure of the usual symmetry under time reversal is so weak that ways to connect it up with the obvious lack of time reversal invariance in everyday life have remained completely obscure.

Fortunately, the accumulation of experimental information and theoretical speculation over the last half centry has finally led to a reinterpretation of the laws of quantum mechanics which might be able to remove this paradox. The theory is still highly controversial. The basic idea it contains is that the past is indeed fixed and unique, but can only partially be reconstructed from present evidence. In contrast, the future can be predicted only to the extent that the relative likelihood of different events which are allowed by the basic laws of energy and momentum conservation can be calculated. Since, for most processes, the predictions of the new approach coincide with earlier results, it will be difficult to devise crucial experimental tests. But the conceptual gain is already of great philosophical significance.

In the historical past the philosophical and scientific controversies over determinism and free will have alllowed both sides in religious struggles to call on physics for support of their particular theologies. For example, Calvin held so strictly to the deterministic model that he taught that God decided before he created the world who would be damned and who saved. In contrast, Counter-Reformation Catholics emphasized the importance of the free choice of the individual between salvation and damnation, although they found this difficult to reconcile with the omnipotence of God. In human terms, this conflict was, for a time, quite literally a burning issue. More recently, some thinkers have tried to invoke the uncertainties inherent in quantum mechanics to justify a belief in free will, but many find their arguments unconvincing.

It may be that if this new interpretation of quantum mechanics bears fruit, a conjecture which only the uncertain future can decide, physics will once again be able to reclaim its old title of "Natural Philosophy" in a profoundly significant way. If the past is indeed fixed, but determines the probabilities of future events, study of the past can provide a significant guide to present action. The increasing precision which historical, evolutionary, and cosmological study has given to our understanding of how we have arrived at the current planetary crisis lends hope to this view. Yet if all we can predict are probabilities, we are not forced to choose courses which are likely to lead to disaster. We can always, with some finite hope for success, choose a more humane course of action. It is a tribute to the inherent wisdom of the peoples of this world that they have mainly taken this attitude of responsible moral choice, in spite of the erudite teachings of their theologians, philosophers, and scientists.