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SURVEY COMPUTATION PROBLEMS ASSOCIATED WITH MULTI–PLANAR ELECTRON–POSITRON COLLIDERS*

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ABSTRACT

At the Stanford Linear Accelerator Center (SLAC) a new type of electron-positron collider (the SLC) is being built. In contrast to colliding beam storage rings, this machine will be built in multiple inclined planes. This makes the placement of magnets difficult to control with conventional surveying techniques. The geometry of the beam line is explained to provide the background necessary to understand the transformations between the various coordinate systems implied. These calculations are used to provide layout information to the surveyor. The steps from the magnet support to the final smoothing surveys are explained along with the associated calculations.

INTRODUCTION

The Stanford Linear Accelerator Center (SLAC) is a national research laboratory operated by Stanford University for the Department of Energy. The major objective of this facility is to conduct basic research in elementary particle physics. To continue this research, it has become desirable to increase the interaction energies of the accelerated electrons and positrons produced by the Linear Accelerator (Linac). For this reason a $113 million project called the Stanford Linear Collider (SLC) has been funded by DOE. This machine will provide center-of-mass energies in the range of 100 GeV. To reach this goal while utilizing SLAC's existing Linac, a new type of machine which is radically different from traditional colliding beam storage rings was designed (SLC Group 1981). The 1000 SLC bending magnets will steer accelerated electron and positron bunches around two arcs so that they face one another. The bunches are then focused and allowed to interact at a single interaction region. The beam size after it has traveled through approximately 1.4 km of bending and focusing magnets will be on the order of 2 square microns at the collision point.

Two of the major restrictions in the design phase of the SLC were the topography and the boundaries of the site. The area available for the tunnels of the collider is about 1.2 km by 1.2 km and contains several large hills. These hills make it impossible to build a set of collider arcs which lie in a common plane. Instead a system of bending magnets which lie in multiple inclined planes was designed to steer the beam not only around the two 1.4 km arcs, but also up and down grades of up to 10%. That is, groups of 20 bending magnets lying in a pitched and rolled plane are used to guide the beam. This group of magnets

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makes up what is called an achromat (Fischer 1984). Having pitched and rolled magnets makes it very difficult to separate the six degrees of freedom of each magnet into horizontal and vertical components. In addition, traditional surveying equipment measures with respect to gravity and not in inclined planes. This makes the computations for placement and adjustment of the 1000 arc magnets complex.

One must first study the geometry of the SLC magnets and beam line to understand the difficulties in surveying it. The alignment tolerances which must be meet for proper machine performance are a prime consideration when planning for the survey. Manufacturing inconsistencies in magnets and supports make it necessary to perform transformations between ideal beam line coordinates and actual coordinates in providing layout data to the surveyor. Finally, all the computations and specifications are coupled with a knowledge of the available instrumentation to design a survey which meets the requirements of the machine. This paper will explain some of these calculations and preparations necessary to perform a survey on a machine like the SLC.

BEAM COORDINATE SYSTEMS

The geometry of the SLC arcs is defined by a beam transport simulation program named TRANSPORT (Brown 1973). This program utilizes two coordinate systems; the absolute reference system and the beam-following system. The three rotations and three shifts to transform the absolute system to the beam-following system provide layout information for the beam line. This information is used with the geometry of an achromat and the bend magnets to compute coordinates for the magnets.

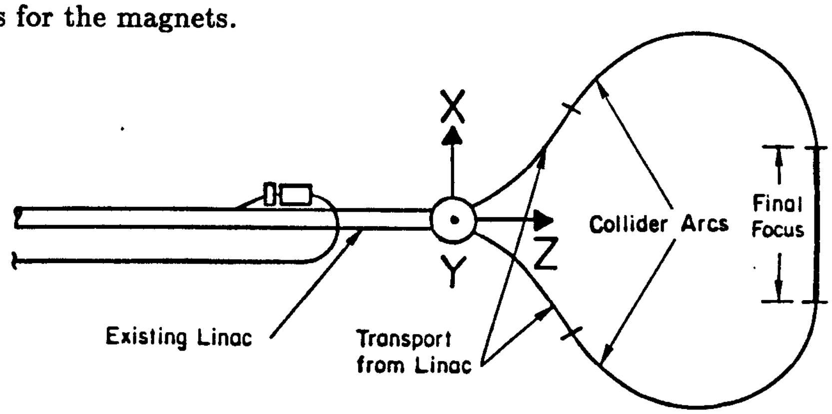

Figure 1.

The absolute reference system is a right hand coordinate system with its origin at Linac station 100+00 (see Figure 1). All survey coordinates for the arcs are expressed in this reference frame while any local system like the beam-following system has a known relationship to it. The Y-axis of the absolute coordinate system is defined as the direction of gravity at station 100+00 with positive up. The Z-axis is in the vertical plane of the Linac and perpendicular to gravity. Positive Z is in the direction of the beam line. The X-axis which completes the right hand system, is perpendicular to the Y-axis and the beam line.

The beam-following coordinate system is used to describe the orientation of the beam at any point along the arcs. This system remains tangent to the beam line with its positive z-axis pointing downstream. The system is rotated so that the positive x-axis always points outwards of the bending arc and lies in the plane of the current achromat. The positive y-axis is oriented to complete the right handed system of the local coordinate system.

To bring the absolute coordinate system into coincidence with the local system, three shifts (Z_i, X_i, Y_i) are first executed. This moves the origin to the point of interest along the beam line. Three sequential rotations are then applied which make the axes of the shifted absolute system parallel to those of the beam following system. These three rotation angles are defined as follows:

yaw (θ)	a rotation around the Y-axis of the shifted absolute coordinate system
pitch (ϕ)	a rotation around the once rotated X-axis
roll (ψ)	a rotation angle around the twice rotated Z-axis.

These are sequential rotation angles which must be applied in the order specified. Their signs are determined using the right hand rule. With these six transformation parameters the beam-following system is defined and will be called the z_i, x_i, y_i coordinate system.

The complete orientation matrix corresponding to the above rotations is formed from three single rotation matrices (Moffitt 1980):

$$R = R_{(\psi)}\, R_{(\phi)}\, R_{(\theta)}$$

with

$$R_{(\psi)} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\psi & \sin\psi \\ 0 & -\sin\psi & \cos\psi \end{bmatrix}$$

$$R_{(\phi)} = \begin{bmatrix} \cos\phi & 0 & -\sin\phi \\ 0 & 1 & 0 \\ \sin\phi & 0 & \cos\phi \end{bmatrix}$$

$$R_{(\theta)} = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad .$$

The product matrix is

$$R = \begin{bmatrix} \cos\phi\cos\theta & \cos\phi\sin\theta & -\sin\phi \\ \sin\psi\sin\phi\cos\theta - \cos\psi\sin\theta & \sin\psi\sin\phi\sin\theta + \cos\psi\cos\theta & \sin\psi\cos\phi \\ \cos\psi\sin\phi\cos\theta + \sin\psi\sin\theta & \cos\psi\sin\phi\sin\theta - \sin\psi\cos\theta & \cos\psi\cos\phi \end{bmatrix} .$$

The total transformation equation is as follows:

$$\begin{bmatrix} z \\ x \\ y \end{bmatrix}_i = [R] \begin{bmatrix} Z - k_1 \\ X - k_2 \\ Y - k_3 \end{bmatrix}_i \tag{1}$$
$$\underline{x}_i = \underline{R}\,[\underline{X}_i - \underline{k}_i] \quad .$$

Since $\underline{R}$ is orthogonal, the inverse transformation can be written as follows:

$$\underline{X}_i = \underline{R}^t \underline{x}_i + k_i \quad . \tag{2}$$

The six parameters to perform this transformation are given by TRANSPORT at the beginning and end of drift spaces and magnets along the beam. The constant shift vector $\underline{k}_i$ contains the actual beam coordinates at these points because the origin of the beam-following system lies on the beam line. However, this is not true for points on the arc magnets which are offset from the beam.

MAGNET AND ACHROMAT GEOMETRY

The bending magnets of the SLC are combined function magnets. They have magnetic properties of dipoles, quadrupoles and sextupoles. They are made by stacking and welding together 1,560 *e*-shaped laminations to form magnets approximately 2.5 meters long. The finished magnet is actually stacked in an arc with a sagitta of 2.75 mm. This makes it possible for the magnet to bend the beam through a radius which is the same as that of the magnet.

The magnets are connected in a sausage-link fashion to form the arcs of the SLC. Figure 2 shows a typical two magnet section made up of one focus and one defocus magnet. It should be pointed out that the beam path is not a simple arc. It is made up of a series of curves connected by straight lines. The curves are the result of the effective bending length of a magnet. The magnetic fields actually extend beyond the physical limits of the magnet, therefore the magnetic length is longer than the length of the magnet iron. These bends are connected with straight lines where no magnetic fields affect the beam's path. The straight sections, therefore are tangent to both the preceding and following bending arcs. This pattern of bend and drift sections is repeated 20 times to form an achromat.

An achromat is a section in the arc where the outgoing beam has the same shape as the incoming one. The distortions caused by the combined function magnets are cancelled by the time a particle bunch has traversed a complete achromat. Therefore, achromats are stacked one after the other down the beam line until the electron and positrons have been turned to face one another. Each of the 20 bending magnets lie in a common plane and seem to trace out an arc on this surface. When this achromat plane is rolled the effect is to steer the beam up and over a slope as well as along an arc. Twenty-three achromats per arc are strung together to guide the beam up and down the grades while maintaining a coherent particle bunch. Each one of these achromat planes is rolled and pitched differently to achieve this. Therefore, the roll and pitch of the magnets with

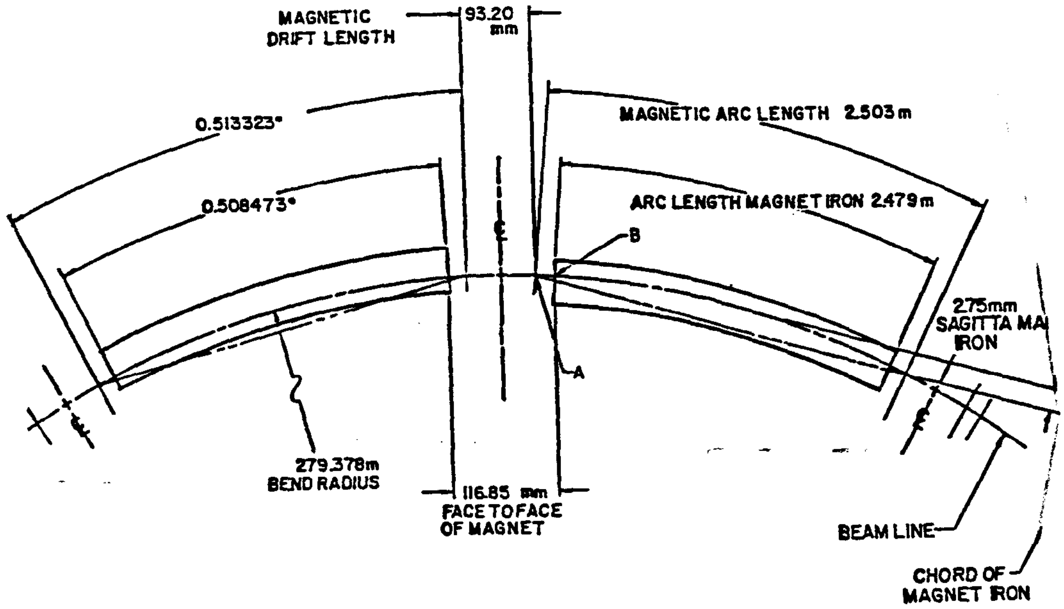

Figure 2.

respect to the absolute coordinate system change continuously as one proceeds down the beam. These angles vary regularly within an achromat as well as between them. This can be seen by looking at the sequential rotations needed to move the absolute system to the beam-following system. The only common section of beam line between differentially rolled achromats is the linear drift section between the end magnet of the preceding achromat and the beginning magnet of the current achromat.

This is an appropriate place to point out that TRANSPORT provides layout coordinates and rotation angles at the beginning and end of the drift sections. These points are also the beginning and end of the magnetic arcs which have a known bending radius. However, the magnetic arcs do not have a common radius point due to the drift section between them. This makes it somewhat difficult to make computations between magnets. Therefore, additional coordinates are given at the midpoints of bending arcs and drift sections. In this paper the center of the drift section will be referred to as the vertex point.

ALIGNMENT TOLERANCES

The most important point in aligning a machine like the SLC is smoothness within an achromat. No kinks or sudden changes of direction are permissible if the arcs are to function properly. The alignment tolerances reflect this need by placing tight restrictions on magnet to magnet alignment accuracies (Friedsam 1984). These tolerances are presented below.

1. Two magnets within an achromat must point at each other with an angular accuracy of .04 mrad.
2. Two magnets must point toward each other in the $z_i x_i$ and $z_i y_i$ planes to within 0.1 mm.

3. The distance between two magnets must be adjusted with an accuracy of 0.5 mm.

4. Roll must be set to within 1.0 mrad of its ideal value.

It can be seen that it is necessary to take into account any small manufacturing imperfections while calculating coordinates to place magnets to these tolerances.

PREPOSITIONING OF SUPPORT SYSTEMS

The alignment of the magnets in the tunnel will involve three steps. In the first step, the bolt locations to mount support pedestals are surveyed and drilled. To compute the position of the vertex point projected onto the floor, TRANSPORT coordinates and rotations for the beam-following system at the vertex are used. The pedestals will be set so that they are perpendicular to the pitched floor, but plumbed in the transverse direction. That is, their pitch will be equal to that of the beam line at that point while roll will be adjusted to zero. Since the pedestal is pitched the location of the vertex point cannot be directly plumbed to the floor. A vector which represents the pedestal must be intersected with the floor to find the exact location of the center of the base.

To obtain an exact set of coordinates of the projected vertex point one would have to measure the actual 3-dimensional location of the tunnel floor. This, however, is impractical so calculations are based on ideal floor locations. This seems to be a reasonable assumption because the tolerance for bolt placement is ± 1.0 cm while floor uncertainties will amount to approximately 0.5 cm errors in the Z, X location of the points.

The computations are made by taking each pedestal as a rigid body with its own coordinate system. This system is equivalent to the beam-following system at the vertex point except that it is not rolled. By representing the pedestal with the y_i-axis it is possible to give the ideal floor point the coordinates of $z_i = 0$, $x_i = 0$ and $y_i =$ beam height above the floor at the vertex point. Then Equation (2) can be applied with just yaw and pitch values inserted into the rotation matrix. When this is done, projected coordinates of the vertex are obtained in the absolute coordinate system. These coordinates can then be laid out from control points measured in a tunnel traverse.

After the pedestals are bolted down, step 2 of the alignment procedure can begin. In this step the pedestal position is refined to the 3 mm level and then it is grouted in place. The magnet adjustment system is then positioned so that the magnets can be mounted to within .5 mm of their ideal position. For both parts of step 2, the vertex point is used as the control point for positioning. A jig which sets yaw by registering on a previously positioned pedestal and has a target representing the vertex point will be used.

For part one of step 2, the vertex point will be represented by intersecting laser beams projected through KERN E2 theodolite telescopes. These instruments will occupy control points with coordinates known in the absolute system. By knowing instrument heights and backsighting other control points, both horizontal and vertical angles to the vertex point can be calculated and set on two instruments. Since the vertex point has known coordinates it is easy to calculate these angles. The accuracy of locating the point at this time will be ± 3 mm.

Part two of step 2 will involve the same calculations as part one, except that now the procedure is changed slightly. Here the actual position of the vertex is measured through leveling and intersection. Its true coordinates are then compared with the ideal position and offsets calculated. The adjustment system on top of the pedestal is then used to move the vertex target to its ideal location. These motions will be controlled with dial gages connected into a computer feedback loop. This prevents mistakes in making the adjustments. The new position of the vertex is then measured and the procedure is repeated if necessary to achieve the desired .5 mm level. This method can only be used as long as the vertex is visible. As soon as the magnets are mounted the vertex point and beam line are obscured.

FINAL POSITIONING OF THE MAGNETS

The final positioning of the magnets is step 3 of the alignment process. At this point the magnets have been mounted on the support pedestal and the beam line is no longer accessible. The two major goals here are to smooth the magnets into an arc and make them face one another to the final tolerances. However, the alignment process is much more difficult in this case than in the previous two steps, because coordinates on the magnets are not provided by TRANSPORT. They must be calculated by the surveyor according to the locations of the magnet fiducial points in relation to the beam line. The three rotational elements yaw, pitch and roll must also be controlled. The yaw and pitch are set by moving fiducial points at each end of the magnet to their proper 3-dimensional positions. This unfortunately does not set the roll of the midplane of symmetry of the magnet. Therefore, it is necessary to calculate a roll about the beam line for one fiducial point on the magnet. Finally, corrections for the actual magnet lengths and longitudinal twists caused by welding and handling must be taken into account.

To start these calculations, one must first look at a simple case. Assume that a magnet fiducial point is located directly above point A in Figure 2. This of course is impossible because the magnet iron ends at point B. However, this is a convenient place to start because TRANSPORT coordinates are provided for the point on the beam line below the fiducial mark. The orientation angles of the local coordinate system are also given. This makes the computation of the needed coordinates obvious. One must only know the coordinates of the fiducial point in the beam-following system. This can be done by building fixtures which locate the mark in a known position with respect to the magnet's magnetic center line. Then Equation (2) can be applied directly to obtain TRANSPORT coordinates of the desired point. However, this point only exists in space because it is located at the end of the magnetic bend arc but not on the magnet iron.

To find coordinates on the actual magnet iron, the local coordinate system must be translated along the beam line to a point beneath the fiducial mark. By referring to Figure 3 one can see how this would be done. First the local system $z_i'x_i'y_i'$ at point B is rotated through a yaw angle α to orient it to the

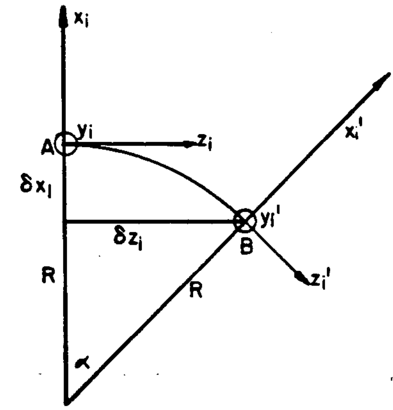

Figure 3.

$z_ix_iy_i$ system. The rotated system is then shifted by δz_i and δx_i so that it origin coincides with the origin of the beam-following system at point A. In doing this the $z_i'x_i'y_i'$ coordinates of the fiducial mark which are set through fixturing are transformed into beam-following coordinates. The controlling equation is as follows:

$$\begin{bmatrix} z \\ x \\ y \end{bmatrix}_i = \begin{bmatrix} \cos\alpha & \sin\alpha & 0 \\ -\sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} z' \\ x' \\ y' \end{bmatrix}_i + \begin{bmatrix} R\sin\alpha \\ -R(1-\cos\alpha) \\ 0 \end{bmatrix} \quad . \tag{3}$$

The angle α can be calculated by using the radius of the curve provided by TRANSPORT and either the measured length of the arc or the chord. After the $z_ix_iy_i$ coordinates are computed, Equation (2) is again applied to find TRANSPORT coordinates of the fiducial mark above point B. This calculation can be done for any point along the bending arc of one magnet if a chord or arc length is measured from a point with known TRANSPORT coordinates. In the case of the SLC magnets, one end will be held fixed and all calculations will begin at this point. The actual chord length between the fixed end and the fiducial mark on the floating end will be measured in the laboratory.

These, however, will not be the final coordinates of the fiducial marks since the magnet maybe manufactured with a twist around its magnetic axis. If this is true the $z_i'x_i'y_i'$ system must undergo an additional rotation to compensate for the twist. This is necessary because the coordinates of the fiducial mark are known only in a system which is rolled with respect to the $z_i'x_i'y_i'$ system. To do this Equation (3) must be modified to include a roll (γ) about the tangent to the beam line. This would result in the following equation:

$$\begin{bmatrix} z \\ x \\ y \end{bmatrix}_i = \begin{bmatrix} \cos\alpha & \sin\alpha\cos\gamma & \sin\alpha\sin\gamma \\ -\sin\alpha & \cos\alpha\cos\gamma & \cos\alpha\sin\gamma \\ 0 & -\sin\gamma & \cos\gamma \end{bmatrix} \begin{bmatrix} z' \\ x' \\ y' \end{bmatrix}_i + \begin{bmatrix} R\sin\alpha \\ -R(1-\cos\alpha) \\ 0 \end{bmatrix} \tag{4}$$

$$\underline{x}_i = \underline{Bx}_i' + \underline{c} \quad .$$

Equation (2) is then applied to these results to find the needed coordinates. It should be pointed out that a total of five sequential rotations are needed to translate the position of the fiducial point in a magnet coordinate system to the absolute system of TRANSPORT.

Now that these coordinates are found, a roll value (ψ') with respect to gravity must be calculated. However, this is not a simple matter because of the five sequential rotations needed to transform the above coordinate systems. For this reason it is easiest to go back to point A where reference coordinates and rotations are provided by TRANSPORT. One may think that the problem is easy at this point because a roll value is provided. This is not the case though because the roll given is a sequential roll. It is not measured on a plane which is parallel to gravity, but about a twice rotated Z-axis. Therefore, this number cannot accurately be used to set roll with an inclinometer which uses gravity as a reference.

One must understand how the inclinometer works, to solve this problem. In the case of the SLC, a type of inclinometer (Schaevitz 1978) which is not affected by tilts transverse to the direction of measurement is utilized. This means that it can be used on a magnet which is both pitched and rolled, to measure a roll angle with respect to gravity. It will not measure the compound angle formed by the pitch and roll angles. To do this accurately and quickly it must be easy to orient the inclinometer in a convenient direction which is repeatable for every setup. In this case, it is easiest to orient in the direction of the x_i-axis of the beam-following coordinate system; i.e. perpendicular to the beam line.

As was said above, the TRANSPORT roll angle is an angle measured about a twice rotated axis and is not the roll to be set with respect to gravity. The measured roll is a projection of the TRANSPORT roll on to the plane formed by the gravity vector and the x_i-axis. The formula to calculate the correct roll at point A can be found by using the fact that the rotation matrix for a given orientation of the beam-following coordinate system is unique, but the combination of sequential rotations is not. In other words, the values of the nine elements of the rotation matrix are fixed but these nine numbers can be calculated from several different sequential rotations. This makes it possible to equate corresponding elements of different sequential rotation matrices which define a given orientation. In this case the roll must be calculated in a system which has been yawed but not pitched. To do this, the sequence of rotations is changed to yaw, roll then pitch. This gives the following rotation matrix $\underline{M}$:

$$\begin{bmatrix} \cos\phi'\cos\theta - \sin\phi'\sin\psi'\sin\theta & \cos\phi'\sin\theta + \sin\phi'\sin\psi'\cos\theta & -\sin\phi'\cos\psi' \\ -\cos\psi'\sin\theta & \cos\psi'\cos\theta & \sin\psi' \\ \sin\phi'\cos\theta + \cos\phi'\sin\psi'\sin\theta & \sin\phi'\sin\theta - \cos\phi'\sin\psi'\cos\theta & \cos\phi'\cos\psi' \end{bmatrix} .$$

It should be noted that ϕ' is not equal to the TRANSPORT pitch for the same reason that ψ' is not equal to ψ. Now the m_{23} element can be equated to the r_{23} element of the TRANSPORT rotation matrix $\underline{R}$. This gives the following formula for ψ':

$$\psi' = \sin^{-1}(\sin\psi\cos\phi) \quad . \tag{5}$$

In the worst case, the difference between the TRANSPORT roll and ψ' is 0.9 mrad. This is a significant amount and must be taken into account. A correction for the earth's curvature could also be applied to ψ', but it was found to be insignificant in this case.

This is fine for any point along the beam line that has given TRANSPORT coordinates, but the case where there are five rotations must still be calculated. The same procedure as above can be applied to the t_{23} element of the total rotation matrix $\underline{T}$ for the twisted magnet at point B. This $\underline{T}$ matrix is formed by multiplying $\underline{R}$ by $\underline{B}^t$ of Equation (4):

$$\underline{T} = \underline{B}^t \, \underline{R} \quad . \tag{6}$$

The resulting formula for ψ' is:

$$\psi' = \sin^{-1}\left(-\sin\phi\sin\alpha\cos\gamma + \sin\psi\cos\phi\cos\alpha\cos\gamma - \cos\psi\cos\phi\sin\gamma\right) \quad . \tag{7}$$

This completes the calculations necessary to find ideal coordinates of fiducial marks and roll values to be set on the magnet.

CONCLUSION

The coordinate systems of the beam design program, TRANSPORT, were explained along with the associated geometry. This made it possible to understand the tolerances for final magnet placement. The steps involved in the surveys of the magnet support systems and the mounted magnets were also explained. It was shown that the calculations necessary to prepare these surveys involved coordinate transformations with up to five sequential rotations. These types of computations are not usually encountered in conventional surveys but are necessary here because of the complex 3-D geometry of the SLC beam line.

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