SLAC – PUB – 3529
December 1984
(M)

ELECTROSTATIC LEVITATION OF A DIPOLE*

DAVID J. GRIFFITHS†

Stanford Linear Accelerator Center

Stanford University, Stanford, California, 94305

Submitted to *American Journal of Physics*

* Work supported by the Department of Energy, contract DE-AC03-76SF00515.

† On leave from the Physics Department, Reed College, Portland, OR 97202.

In a recent paper[1] F. H. J. Cornish has shown that an electric dipole can undergo self–sustaining accelerated motion in a direction perpendicular to its axis, with

$$a = \left(\frac{2c^2}{d}\right) \sqrt{(e^2/2mc^2d)^{2/3} - 1} \; , \tag{1}$$

where a is the acceleration, $\pm e$ are the charges, d is the separation distance, $2m$ is the dipole mass, and c is the speed of light.[2] According to the Equivalence Principle, it should therefore be possible for a dipole to "float" in a uniform gravitational field of strength

$$g = a \; . \tag{2}$$

The purpose of this note is to confirm that such levitation does indeed occur, and to elucidate the mechanism responsible.

Viewed from a freely falling reference system, there is no gravity, and the dipole accerates upward in accordance with Cornish's formula (1). Viewed from a stationary reference frame, the dipole is subject to a gravitational force downward

$$F_{\text{grav}} = 2\, mg \tag{3}$$

which must be balanced – if the dipole is to be at rest in this system – by an electrical force upward. But why should there be an upward electrical force on the dipole? The answer goes back to an observation of Boyer[3] and others that the electric field lines "droop" in the presence of gravity (see Figure). At the location of $-e$ the field of $+e$ has a downward component, and hence there is an upward force on the charge. Boyer calculated the electric field to first order in g, and showed that the vertical force precisely accounts for the electrostatic contribution to the gravitational mass of the object. For present purposes, however, we require the exact electric field, to all orders in g. This is to be found, for instance, in a classic paper of Rohrlich:[4] at a horizontal distance d from a point charge e, the vertical component of the electric field is

$$E = -\frac{eg}{2c^2 d[1 + (dg/2c^2)^2]^{3/2}} \; . \tag{4}$$

Thus, the net upward force on the dipole, due to drooping of the field lines, is

$$F_{\text{elec}} = \frac{e^2 g}{c^2 d[1 + (dg/2c^2)^2]^{3/2}} \tag{5}$$

For perfect levitation, the electrical force upward (5) must balance the gravitational force downward (3):

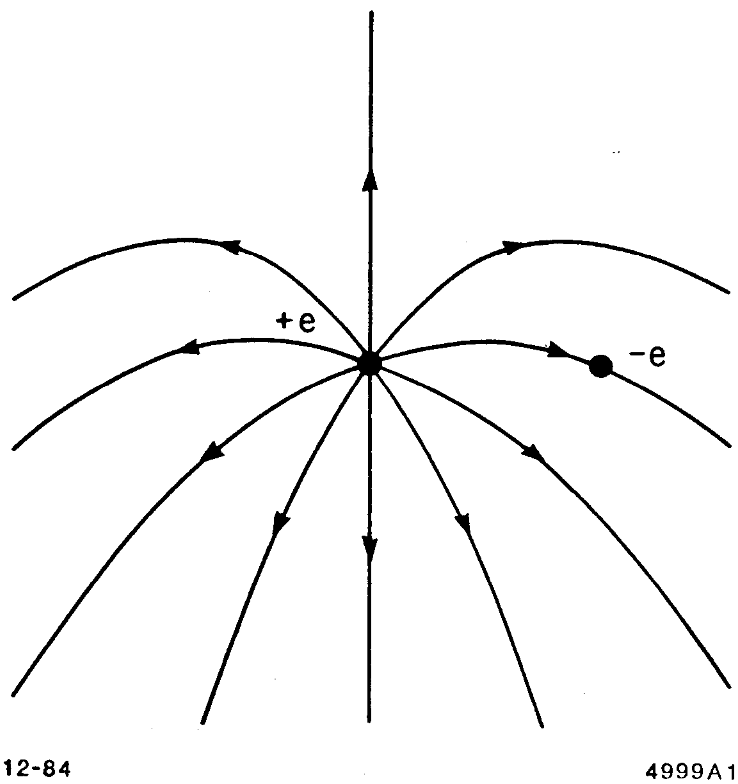
$$2\, mg = \frac{e^2 g}{c^2 d[1 + (dg/2c^2)^2]^{3/2}} \; . \tag{6}$$

Solving for g, we recover Cornish's formula (1) — with $a = g$ (2).

Unfortunately, one is unlikely to witness this levitation in the laboratory. For the electron, $e^2/mc^2 = 2.8 \times 10^{-15}$ meter (the classical electron radius), and the dipole separation d would have to be almost exactly half of this: 1.4×10^{-15} meter.

Footnotes

1. F. H. J. Cornish, An Electric Dipole in Self–Accelerated Transverse Motion, (submitted to Am. J. Phys.).
2. Gaussian *cgs* units are used.
3. T. H. Boyer, Am. J. Phys. 47, 129 (1979).
4. F. Rohrlich, Ann. Phys. 22, 169 (1963), equation (7.10).



"Drooping" electric field lines in a uniform gravitational field.