# ARE $D^{*}$-DECAYS ALL RIGHT?* 

R. L. Thews<br>Department of Physics<br>University of Arizona, Tucson, Arizona 85721<br>and<br>A. N. KAMAL ${ }^{\dagger}$<br>Stanford Linear Accelerator Center<br>Stanford University, Stanford, California, 94305

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## ABSTRACT

We study the hadronic and radiative decays of $D^{*+}$ and $D^{* 0}$. Using $S U(4)$ symmetry for the hadronic decays, we convert the branching ratios into total rates for $D^{*+}$ and $D^{* 0}$. We point out a potential problem with the ratio $\Gamma\left(D^{*+} \rightarrow D^{+} \pi^{0}\right) / \Gamma\left(D^{*+} \rightarrow D^{0} \pi^{+}\right)$. Finally using the measured branching ratios for radiative decays, we extract the "experimental" radiative rates. We find these radiative rates puzzling as they are difficult to understand in an $S U(4)$ or in broken $S U(4)$ schemes.

The mass splitting between $D^{*}$ and $D$ mesons is such that the only hadronic decay modes available to $D^{*}$ are $D \pi$ states. $D^{*+}$ mass is such that it can decay to $D^{+} \pi^{0}$ and $D^{0} \pi^{+}$; however, $D^{* 0}$ can decay hadronically only into $D^{0} \pi^{0}, D^{+} \pi^{-}$ channel is not allowed kinematically. In the following we summarize what is known ${ }^{1}$ about the branching ratios and the total rates for the $D^{*}$ 's:

$$
\begin{align*}
D^{*+}: \quad B R\left(D^{0} \pi^{+}\right) & =49 \pm 8 \% \\
B R\left(D^{+} \pi^{0}\right) & =34 \pm 7 \% \\
B R\left(D^{+} \gamma\right) & =17 \pm 11 \% \\
\Gamma_{T}\left(D^{*+}\right) & <2.0 \mathrm{MeV}  \tag{1}\\
D^{* 0}: \quad B R\left(D^{0} \pi^{0}\right) & =54 \pm 9 \% \\
B R\left(D^{0} \gamma\right) & =46 \pm 9 \% \\
\Gamma_{T}\left(D^{* 0}\right) & <5.0 \mathrm{MeV} . \tag{2}
\end{align*}
$$

We believe that the total rates for $D^{*+}$ and $D^{* 0}$ decays can be calculated with reasonable accuracy. In this paper we calculate $D^{*} \rightarrow D \pi$ rates and then using the branching ratios given in Eq. (1) and (2) we estimate the total rates for $D^{*+}$ and $D^{* 0}$. We then use the branching ratios for the radiative modes to estimate $\Gamma\left(D^{*+} \rightarrow D^{+} \gamma\right)$ and $\Gamma\left(D^{* 0} \rightarrow D^{0} \gamma\right)$. Finally we discuss the theoretical implications of our results.

We compute $D^{*} \rightarrow D \pi$ rates from an $S U(4)$ invariant interaction, ${ }^{2}$

$$
\begin{equation*}
\Gamma\left(V_{i} \rightarrow P_{j} P_{k}\right)=\frac{2}{3} \frac{g_{i j k}^{2}}{4 \pi} \frac{|\vec{p}|^{3}}{M_{V}^{2}} \tag{3}
\end{equation*}
$$

where $i, j, k$ are $S U(4)$ indices and

$$
\begin{equation*}
g_{i j k}=i f_{i j k} g_{V P P} \tag{4}
\end{equation*}
$$

Applying formula (3) to $\rho \rightarrow \pi \pi$, with $\Gamma(\rho \rightarrow \pi \pi)=154 \pm 5 \mathrm{MeV},{ }^{1}$ we obtain

$$
\begin{equation*}
\frac{g_{V P P}^{2}}{4 \pi}=2.98 \pm 0.10 \tag{5}
\end{equation*}
$$

From $\Gamma\left(K^{*} \rightarrow K \pi\right)=51.3 \pm 1.0 \mathrm{MeV},{ }^{1}$ we obtain

$$
\begin{equation*}
\frac{g_{V P P}^{2}}{4 \pi}=3.42 \pm 0.07 \tag{6}
\end{equation*}
$$

The ratio of the last two numbers is $1.15 \pm 0.05$ representing about $15 \% S U(3)$ breaking effect. As the individual errors in (5) and (6) are small, we choose to work with a mean value ${ }^{3}$

$$
\begin{equation*}
\frac{g_{V P P}^{2}}{4 \pi}=3.20 \pm 0.22 \tag{7}
\end{equation*}
$$

where the errors connect the two central values of (5) and (6).
In applying Eq. (3) to $D^{*} \rightarrow D \pi$ one has to be very precise in computing the phase space as it depends very sensitively on the masses. We used the mass difference measurements for $m_{D^{+}}-m_{D^{0}}, m_{D^{*+}}-m_{D^{0}}$ and $m_{D^{\bullet 0}}-m_{D^{0}}$ quoted in Ref. 1 to compute $|\vec{p}|$ for the various decay modes. We find,

$$
\begin{align*}
& \Gamma\left(D^{*+} \rightarrow D^{+} \pi^{0}\right)=(2.32 \pm 0.28) \frac{g_{V P P}^{2}}{4 \pi} \mathrm{KeV}  \tag{8}\\
& \Gamma\left(D^{*+} \rightarrow D^{0} \pi^{+}\right)=(5.0 \pm 0.19) \frac{g_{V P P}^{2}}{4 \pi} \mathrm{KeV}  \tag{9}\\
& \Gamma\left(D^{* 0} \rightarrow D^{0} \pi^{o}\right)=(3.5 \pm 0.96) \frac{g_{V P P}^{2}}{4 \pi} \mathrm{KeV} \tag{10}
\end{align*}
$$

If we use $g_{V P P}^{2} / 4 \pi$ from Eq. (7) we obtain,

$$
\begin{equation*}
\Gamma\left(D^{*+} \rightarrow D^{+} \pi^{0}\right)=7.4 \pm 1.0 \mathrm{KeV} \tag{11}
\end{equation*}
$$

$$
\begin{gather*}
\Gamma\left(D^{*+} \rightarrow D^{0} \pi^{+}\right)=16.0 \pm 1.3 \mathrm{KeV}  \tag{12}\\
\Gamma\left(D^{* 0} \rightarrow D^{0} \pi^{0}\right)=11.2 \pm 3.1 \mathrm{KeV} \tag{13}
\end{gather*}
$$

The experimental branching ratios shown in Eq. (1) and (2) can then be used to estimate the following total rates

$$
\begin{align*}
& B R\left(D^{*+} \rightarrow D^{+} \pi^{0}\right)=34 \pm 7 \% \text { yields } \Gamma_{T}\left(D^{*+}\right)=22 \pm 6 \mathrm{KeV}  \tag{14}\\
& B R\left(D^{*+} \rightarrow D^{0} \pi^{+}\right)=49 \pm 8 \% \text { yields } \Gamma_{T}\left(D^{*+}\right)=32 \pm 6 \mathrm{KeV}  \tag{15}\\
& B R\left(D^{* 0} \rightarrow D^{0} \pi^{0}\right)=54 \pm 9 \% \text { yields } \Gamma_{T}\left(D^{* 0}\right)=21 \pm 7 \mathrm{KeV} . \tag{16}
\end{align*}
$$

Again the individual errors on the two values of $\Gamma_{T}\left(D^{*+}\right)$ in Eq. (14) and (15) are small and the central values are separated by almost 2 standard deviations. We choose to work with an average for some of the following calculations,

$$
\begin{equation*}
\Gamma_{T}\left(D^{*+}\right)=27 \pm 5 \mathrm{KeV} \tag{17}
\end{equation*}
$$

where the error is chosen to connect the two central values.
We are now in a position to estimate the radiative rates. Using the branching ratios for the radiative modes from Eq. (1) and (2) and the average total rate $\Gamma_{T}\left(D^{*+}\right)$ from Eq. (17) we find that

$$
\begin{equation*}
B R\left(D^{*+} \rightarrow D^{+} \gamma\right)=17 \pm 11 \% \text { yields } \Gamma\left(D^{*+} \rightarrow D^{+} \gamma\right)=4.6 \pm 3.0 \mathrm{KeV} \tag{18}
\end{equation*}
$$

and

$$
\begin{equation*}
B R\left(D^{* 0} \rightarrow D^{0} \gamma\right)=46 \pm 9 \% \text { yields } \Gamma\left(D^{* 0} \rightarrow D^{0} \gamma\right)=9.7 \pm 3.7 \mathrm{KeV} \tag{19}
\end{equation*}
$$

We now discuss the theoretical implications of the experimental data and our
calculations.
Application of the symmetry in the $S U(2)$ sector of the charm subspace results in

$$
\begin{equation*}
\frac{\Gamma\left(D^{*+} \rightarrow D^{+} \pi^{0}\right)}{\Gamma\left(D^{*+} \rightarrow D^{0} \pi^{+}\right)}=0.466 \pm 0.057 \tag{20}
\end{equation*}
$$

This is to be compared with the experimental ratio, ${ }^{1}$

$$
\begin{equation*}
\frac{\Gamma\left(D^{*+} \rightarrow D^{+} \pi^{0}\right)}{\Gamma\left(D^{*+} \rightarrow D^{0} \pi^{+}\right)}=\frac{34 \pm 7}{49 \pm 8}=0.694 \pm 0.178 \tag{21}
\end{equation*}
$$

We have propagated the errors as if the data sample were independent. The actual errors could well be smaller. Clearly theory and experiment are 1 standard deviation apart. Since this limited $S U(2)$ charm sector symmetry should not be broken (more than in the corresponding strangeness sector) $\Gamma\left(D^{*+} \rightarrow D^{+} \pi^{0}\right)$ would be expected to come down to its lowest value, $\approx 27 \%$, and $\Gamma\left(D^{*+} \rightarrow D^{0} \pi^{+}\right)$ would have to rise to its highest value, $\approx 57 \%$. We may remark that the branching ratios in Ref. 1 before the revision were in better agreement with the theory.

We now turn our attention to the radiative decay. The ratio of the radiative widths is,

$$
\begin{equation*}
\frac{\Gamma\left(D^{*+} \rightarrow D^{+} \gamma\right)}{\Gamma\left(D^{* 0} \rightarrow D^{0} \gamma\right)}=\frac{(17 \pm 11)}{(46 \pm 9)} \frac{\Gamma_{T}\left(D^{*+}\right)}{\Gamma_{T}\left(D^{* 0}\right)} \tag{22}
\end{equation*}
$$

We can obtain three different values for this ratio depending upon what we use for $\Gamma_{T}\left(D^{*+}\right)$ and $\Gamma_{T}\left(D^{* 0}\right)$.

$$
\begin{equation*}
(a): \frac{\Gamma_{T}\left(D^{*+}\right)}{\Gamma_{T}\left(D^{* 0}\right)}=\frac{\Gamma\left(D^{*+} \rightarrow D^{+} \pi^{0}\right)}{\Gamma\left(D^{* 0} \rightarrow D^{0} \pi^{0}\right)} \frac{B R\left(D^{* 0} \rightarrow D^{0} \pi^{0}\right)}{B R\left(D^{*+} \rightarrow D^{+} \pi^{0}\right)}=1.05 \pm 0.28 \tag{23}
\end{equation*}
$$

using Eq. (8) and (10) for the hadronic rates. Using Eq. (23) in Eq. (22) we
obtain

$$
\begin{equation*}
\frac{\Gamma\left(D^{*+} \rightarrow D^{+} \gamma\right)}{\Gamma\left(D^{* 0} \rightarrow D^{0} \gamma\right)}=0.370 \pm 0.272 \tag{24}
\end{equation*}
$$

$$
\begin{equation*}
\text { (b) : } \frac{\Gamma_{T}\left(D^{*+}\right)}{\Gamma_{T}\left(D^{* 0}\right)}=\frac{\Gamma\left(D^{*+} \rightarrow D^{0} \pi^{+}\right)}{\Gamma\left(D^{* 0} \rightarrow D^{0} \pi^{0}\right)} \frac{B R\left(D^{* 0} \rightarrow D^{0} \pi^{0}\right)}{B R\left(D^{*+} \rightarrow D^{0} \pi^{+}\right)}=1.56 \pm 0.4 \tag{25}
\end{equation*}
$$

using Eq. (9) and (10) for the hadronic rates, and results in

$$
\begin{equation*}
\frac{\Gamma\left(D^{*+} \rightarrow D^{+} \gamma\right)}{\Gamma\left(D^{* 0} \rightarrow D^{0} \gamma\right)}=0.576 \pm 0.415 \tag{26}
\end{equation*}
$$

(c): If we use the average value for $\Gamma\left(D^{*+}\right)$ given in Eq. (17) and $\Gamma\left(D^{* 0}\right)$ from Eq. (16), we obtain,

$$
\begin{equation*}
\frac{\Gamma\left(D^{*+} \rightarrow D^{+} \gamma\right)}{\Gamma\left(D^{* 0} \rightarrow D^{0} \gamma\right)}=0.480 \pm 0.336 \tag{27}
\end{equation*}
$$

All the three ways of obtaining $\Gamma\left(D^{*+} \rightarrow D \gamma\right) / \Gamma\left(D^{* 0} \rightarrow D^{0} \gamma\right)$ yield a central value $\approx 0.5$. Given the large errors this ratio could be as low as $\approx 0.1$. Does this pose a problem?

If we assume that the radiative decays $V \rightarrow P \gamma$ can be described by an $S U(4)$-symmetric interaction ${ }^{2}$ then,

$$
\begin{equation*}
\Gamma\left(V_{i} \rightarrow P_{j} \gamma_{k}\right)=\frac{1}{3} \frac{g_{i j k}^{2}}{4 \pi}|\vec{p}|^{3} \tag{28}
\end{equation*}
$$

where $i, j$ and $k$ are $S U(4)$-labels and

$$
\begin{equation*}
g_{i j k}=d_{i j k} g_{V P P} \tag{29}
\end{equation*}
$$

Using the tabulation ${ }^{4}$ of the symmetric symbol $d_{i j k}$ and the fact that the $S U(4)$
label, $k$, of the photon is

$$
\begin{equation*}
k=\frac{\sqrt{2}}{3}(0)+(3)+\frac{1}{\sqrt{3}}(8)-\sqrt{\frac{2}{3}}(15) \tag{30}
\end{equation*}
$$

we find that ${ }^{2}$

$$
\begin{equation*}
\frac{\Gamma\left(D^{*+} \rightarrow D^{+} \gamma\right)}{\Gamma\left(D^{* 0} \rightarrow D^{0} \gamma\right)}=\frac{1}{16} \tag{31}
\end{equation*}
$$

which is certainly not consistent with the estimates from the experimental branching ratios (27).

A simple way of breaking $S U(4)$ symmetry is to write the M1-transition operator for $1^{-} \rightarrow 0^{-} \gamma$ transition in the non-relativistic form

$$
\begin{equation*}
H(M 1)=\sum_{q} \frac{e_{q}}{2 m_{q}} \vec{\sigma} \cdot(\vec{\epsilon} \times \vec{k}) \tag{32}
\end{equation*}
$$

where the sum is over the quark flavors, $e_{q}$ and $m_{q}$ are the quark charge and the mass respectively and $\vec{\epsilon}$ and $\vec{k}$ the photon polarization vector and momentum respectively. Use of Eq. (32) in $D^{*}$-radiative decays leads to

$$
\begin{equation*}
\frac{\Gamma\left(D^{*+} \rightarrow D^{+} \gamma\right)}{\Gamma\left(D^{* 0} \rightarrow D^{0} \gamma\right)}=\frac{1}{4}\left(\frac{m_{c}-2 m}{m_{c}+m}\right)^{2} \tag{33}
\end{equation*}
$$

where $m_{c}=$ charmed quark mass and $m=u p$ (or down) quark mass. Constituent quarks are used in this naive quark model. If we set $m_{c}=m$ we recover the suppression factor of $1 / 16$. If we use $m_{c}=1500 \mathrm{MeV}$ and $m=340 \mathrm{MeV}$ we obtain,

$$
\begin{equation*}
\frac{\Gamma\left(D^{*+} \rightarrow D^{+} \gamma\right)}{\Gamma\left(D^{* 0} \rightarrow D^{0} \gamma\right)} \simeq \frac{1}{20} \tag{34}
\end{equation*}
$$

Thus symmetry breaking through quark masses suppresses this ratio even further. Yet we expect that in $D^{*} \rightarrow D \gamma$ the naive quark model should work well since the photon momentum is small compared to the $D$ or $D^{*}$ mass.

Another possibility is to relate these widths directly to non-charmed decays via pole-dominated duality sum rules. Due to large symmetry-breaking effects in $\phi \rightarrow \eta \gamma$ and $J / \psi \rightarrow \eta_{c} \gamma$, one can predict large deviations from symmetry values for the ratio (31) and the corresponding quantity in the strangeness sector. ${ }^{5}$ We note that the symmetry-breaking mechanisms which deal with nonrelativistic corrections ( $p / M_{V}$ ) or overall mass-dependent factor (e.g. $M_{V}^{2} g_{V P \gamma}^{2}$ obeys $S U(4)$ ) all primarily effect absolute rates only, and do not alter conclusions on the ratios within a heavy quark sector.

We now turn to the absolute radiative decay widths for the $D^{*}$ 's. We take as "experimental" values (18) and (19), based on $S U(4)$ symmetry for strong decays plus the experimental branching ratios. In Table I we list the theoretical predictions. The $S U(4)$ symmetry predictions are obtained from (28) and (29), using $\Gamma(\omega \rightarrow \pi \gamma)$ for normalization. Broken $S U(4)$ symmetry rates from the interaction (32) are also normalized to $\Gamma(\omega \rightarrow \pi \gamma)$. Finally, values based on $S U(4)$ symmetry for the dimensionless coupling $M_{V}^{2} g_{V P \gamma}^{2}$ are also tabulated.

It is clear from Table I that the problem of explaining the ratio (27) remains in all cases, so that a consistent set of rates cannot be obtained in any of these schemes.

In summary, it should be emphasized that we regard the new data on ratios of $D^{*}$ decays as possibly indicative of some theoretical puzzles. If the ratio of strong decays continues to diverge from $S U(2)$ symmetry in the charm sector as in (20) and (21), this would indicate a qualitatively new behavior for heavyquark systems. Theoretical prejudice, however, would be strongly in favor of $S U(2)$ symmetry and one would suspect the experimental data. That the ratio of $D^{*}$ radiative widths (22) does not obey $S U(4)$ symmetry (again in a restricted
sense) may not be totally due to problems in the heavy quark sector. However, straightforward ways of breaking the symmetry do not improve agreement with present experimental indications.

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## Table 1

Radiative rates for $D^{*+}$ and $D^{* 0}$.

| $\frac{\Gamma\left(D^{*+} \rightarrow D^{+} \gamma\right)(\mathrm{KeV})}{4.6 \pm 3.0}$ | $\frac{\Gamma\left(D^{* 0} \rightarrow D^{0} \gamma\right)(\mathrm{KeV})}{19.7 \pm 3.7}$ | Source <br> Experimental B.R. <br> $4.4 \pm 0.3$ |
| :---: | :---: | :--- |
|  | $70 \pm 5$ | $S U(4)$ for $g_{V P \gamma}$ |
| $1.3 \pm 0.1$ | $27.0 \pm 1.8$ | Broken $S U(4)$ by M1 |
|  |  | quark transition |
|  | $10.6 \pm 0.8$ | Broken $S U(4)$ by |
| $0.67 \pm 0.05$ |  | $1 / M_{V}^{2}$ |

## REFERENCES

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    $\dagger$ On leave from Department of Physics, University of Alberta, Edmonton, Alberta, Canada T6G 2 J 1.

