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MONOPOLE INDUCED BARYON NUMBER VIOLATION IN "REALISTIC" GRAND UNIFIED THEORIES*

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ABSTRACT

We consider the embedding of the SU(2) monopole of 't Hooft and Polyakov into "realistic" grand unified theories and find a complication that may possibly interfere with the Callan-Rubakov effect. The fine tuning that keeps the weak interaction scale much smaller than the GUT scale is necessarily upset in the vicinity of a magnetic monopole, and there is a probability of order one that the Weinberg-Salam Higgs field will have a GUT scale expectation value at the monopole core. This means that the fermions could have a large effective mass at the core which could act as a barrier to exclude the fermions from the baryon number violating interactions inside the monopole core. In this paper we determine whether such a barrier is likely to cause a significant suppression of the Callan-Rubakov effect. Our analysis involves a variational determination of a position dependent mass for the fermions in Callan's soliton formalism. Once the position dependent mass of the solitons has been determined, simple energy arguments allow us to determine if the Callan-Rubakov effect will be suppressed. We find that in ordinary GUTs, the small Yukawa couplings between the Weinberg-Salam Higgs and the light fermions keep the effective soliton masses very small so baryon number violation is not affected, but if the Yukawa couplings are larger than 10^{-3} (instead of the usual 10^{-5}) or with the appropriate tuning of Higgs parameters, the soliton masses can become large at the core, and the Callan-Rubakov effect can be prevented. In the event that the Callan-Rubakov effect is suppressed, we find that baryon number violation is still possible through the weak anomaly induced process discussed by Schellekens and Sen. However, this process is not likely to be important phenomenologically. We also correct an ambiguity with the bosonization procedure that was noticed by Yamagishi.

1. INTRODUCTION

One of the most spectacular predictions of Grand Unified Theories is that magnetic monopoles[1] can catalyze baryon decay at a strong interaction rate.[2-4] This prediction has important implications for the phenomenology of monopoles. If GUT monopoles do indeed exist in a reasonable abundance, this process could lead to some very exciting events should a monopole pass through a proton decay detector. On the other hand, if monopoles catalyze baryon decay, then it would seem unlikely that monopoles exist in a detectable abundance due to monopole-baryon interactions inside neutron stars.[5] Since neutron stars should capture virtually all the monopoles that collide with them, a high cross section for monopole induced baryon decay implies that the monopoles inside a neutron star should be a tremendous heat source. Thus, observations of the x-ray emission from neutron stars (or rather, the *lack* of x-ray emission) put a stringent limit on the galactic monopole abundance.

The initial analysis of this process in monopole-fermion interactions, due to Rubakov and Callan, involved a simple SU(2) model and included only the s-wave degrees of freedom. This analysis was generalized to SU(5) and other "realistic" GUTs, and this generalization seemed to indicate that monopoles will catalyze baryon decay at a strong rate.[6] Because of the numerous approximations used by Rubakov and Callan to reduce the monopole-fermion system to a simple one-dimensional model, there has been much subsequent work attempting to justify and clarify these approximations.[7] There have also been several papers exploring the model dependence of this effect.[8] The general conclusion of most of these papers is that the results of Rubakov and Callan are qualitatively correct and that they apply to most Grand Unified Theories.

In this paper, we discuss a complication which arises in realistic GUTs, as a consequence of the Higgs field configuration in the monopole core. In general, the Higgs field which breaks the GUT symmetry deviates from its vacuum value at the monopole core. (This is partially responsible for the large mass of the monopole.) This GUT scale Higgs field is, in general, coupled to the Higgs which breaks SU(2) × U(1), and a fine tuning is required to keep the weak scale low. At the monopole core, the GUT scale Higgs shifts from its asymptotic vacuum value, and the fine tuning is destroyed. Then, there are two possibilities: (1) The weak scale Higgs acquires zero expectation value at the monopole core and SU(2) × U(1) symmetry is restored; or (2) the weak scale Higgs acquires a vacuum expectation value at the GUTs scale. We will show that this second alternative is not unlikely in realistic GUTs.

In the second case, since the weak scale Higgs is responsible for the fermion masses, the fermions may acquire a large effective mass at the monopole core. This mass can, in principle, act as a barrier which prevents baryon number violation. (This problem was noted by Callan at the end of Ref. 4) The main goal of this paper is to analyze this situation in detail, and to understand under what circumstances this barrier actually suppresses baryon number violation appreciably. Our final conclusion is that this complication is probably not important in practice; the analysis, however, is nontrivial and the conclusion is in doubt until the last moment. Our result finally depends on the presence in this problem of a small dimensionless parameter, namely, the Yukawa coupling ($\sim 10^{-5}$) between the Higgs field and the light fermions. We find that the Callan-Rubakov effect can be suppressed only if one of the Higgs sector coupling constants is tuned to be a factor of $\sim$ 100 greater than another, (or equivalently, if the Yukawa coupling is $\sim 10^{-3}$ instead of $\sim 10^{-5}$).

This paper is organized as follows. In Section 2 we consider the fundamental monopole of the SU(5) GUT and indicate explicitly how the weak scale Higgs field can acquire a large expectation value at the monopole core. In Section 3 we follow the familiar procedure of Callan to reduce the monopole-fermion systems to a $1+1$ dimensional bosonic theory. In contrast to Callan's treatment, however, we are careful to keep track of all the terms depending on the monopole core size since the effective fermion mass term that we wish to study is dependent on the core size as well. We also address a question about Callan's bosonization procedure that was raised by Yamagishi.[9] Section 4 deals with the interpretation of the bosonized Hamiltonian obtained in Section 3. The spatial dependence of the fermion masses is obscured in the bosonized Hamiltonian, so we do a variational calculation to reveal this dependence. This allows us to use simple semiclassical arguments to determine the circumstances in which the Callan-Rubakov effect will be suppressed. We also estimate the probability of tunneling through this mass barrier. In section 5, we remove an approximation that was made in previous sections, and show how weak isospin is conserved at short distances when the baryon number violating reactions are suppressed. The effect of these rotations is to allow a $\Delta B = 3$ process that involves all the fermions present in the s-wave, and we interpret this to be the result of the $SU(2)_W$ anomaly. In Section 6, we briefly discuss some of the phenomenological implications of our results for non-minimal GUTs and find some models for which the Callan-Rubakov effect is likely to be suppressed. Finally, in the appendix, we present some calculations that are used in section 4.

2. MONOPOLES IN GUTs WITH FINE TUNING

The theory we now consider is the minimal SU(5) GUT. The SU(5) gauge symmetry is spontaneously broken down to $SU(3)_C \times SU(2)_W \times U(1)_Y$ at a scale $v \sim 10^{15}$ GeV by the vacuum expectation value of a Higgs field (Φ) in the adjoint ($\underline{24}$) representation. Another Higgs field (h) in the $\underline{5}$ representation is also present with a vacuum expectation value that breaks $SU(3)_C \times SU(2)_W \times U(1)_Y$ down to $SU(3)_C \times U(1)_{EM}$ at a scale $v_w \sim 300$ GeV. The Lagrangian density without fermions is given by

$$\mathcal{L} = -\frac{1}{2g^2}\,\mathrm{Tr}\,(F^{\mu\nu}F_{\mu\nu}) + \mathrm{Tr}\,(\mathcal{D}_\mu\Phi)^2 + |D_\mu h|^2 - V(\Phi,h)\ , \qquad (2.1)$$

where

$$F_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu + i[A_\mu, A_\nu]\ ,$$
$$A_\mu \equiv A^a_\mu \lambda^a\ , \qquad a = 1,\ldots,24\ , \qquad (2.2)$$

and

$$D_\mu h \equiv \partial_\mu h + iA_\mu h\ ,$$
$$\mathcal{D}_\mu\Phi \equiv \partial_\mu\Phi + i[A_\mu,\Phi]\ . \qquad (2.3)$$

The SU(5) generators λ^a are chosen to satisfy $\mathrm{Tr}\,\lambda^a\lambda^b = \frac{1}{2}\,\delta^{ab}$ and $\lambda^{a^\dagger} = \lambda^a$. The Higgs potential[10] is given by

$$V(\Phi,h) = -\frac{1}{2}\,\mu^2\mathrm{Tr}\,(\Phi^2) + \frac{1}{4}\,a\Big(\mathrm{Tr}\,(\Phi^2)\Big)^2 + \frac{1}{2}\,b\mathrm{Tr}\,(\Phi^4)$$
$$-\frac{1}{2}\,m^2 h^\dagger h + \frac{1}{4}\,\lambda(h^\dagger h)^2 + \alpha(h^\dagger h)\,\mathrm{Tr}\,\Phi^2 + \beta h^\dagger\Phi^2 h\ . \qquad (2.4)$$

The values of the coupling constants must be chosen to obtain a Hamiltonian that is bounded from below and that has the correct symmetry breaking pattern.[11]

Hence, we require $b > 0$, $a > -\frac{7}{15}b$, $\lambda > 0$, and $\beta < 0$ to obtain $SU(5) \to SU(3)_C \times SU(2)_W \times U(1)_Y$. (There are additional very complicated constraints on α and β to ensure that the Hamiltonian is bounded from below, but we expect no problems if $-\alpha, -\beta \lesssim a, b$.) The coupling constants α and β can be generated by renormalization and are actually required to give the unobserved colored components of h a large mass. When the potential (2.4) is minimized, we obtain the symmetry breaking scales v and v_w in terms of the coupling constants in the potential,

$$\begin{aligned} \mu^2 &= \left(\frac{15}{2}a + \frac{7}{2}b\right)v^2 + \left(2\alpha + \frac{3}{5}\beta\right)v_w^2 \,, \\ m^2 &= \left(\lambda + \frac{9}{10}\frac{\beta^2}{b}\right)v_w^2 + \left(15\alpha + \frac{9}{2}\beta\right)v^2 \,. \end{aligned} \tag{2.5}$$

Since we require $(v_w/v)^2 \sim \mathcal{O}(10^{-25})$ we obtain the fine tuning equation

$$m^2 = \frac{30\alpha + 9\beta}{15a + 7b}\,\mu^2 + \mathcal{O}(10^{-25}) \,, \tag{2.6}$$

which constrains the coupling constants.

Now, we will examine the behavior of the Higgs fields in the presence of the fundamental monopole. To simplify our calculations, we will assume that our fields have the symmetric form given by Dokos and Tomaras.[12] The fundamental monopole is embedded in the SU(2) subgroup of SU(5) given by

$$\vec{T} = \frac{1}{2}\begin{pmatrix} 0 & & & \\ & 0 & & \\ & & \vec{\tau} & \\ & & & 0 \end{pmatrix} \,, \tag{2.7}$$

where τ^a, $a = 1, 2, 3$, are the Pauli matrices. Thus, the magnetic charge of the fundamental monopole is a sum of ordinary magnetic charge and magnetic color hypercharge.

The general form of the fields given by Dokos and Tomaras is

$$h(\vec{r}) = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ h_5(r) \end{pmatrix} \qquad \Phi(\vec{r}) = \begin{pmatrix} \phi_1(r) & & & \\ & \phi_1(r) & & \\ & & \phi_2(r) + \phi_3(r)\hat{r}\cdot\vec{\tau} & \\ & & & -2(\phi_1+\phi_2) \end{pmatrix} , \tag{2.8}$$

$$A_i = (\vec{T} \times \hat{r})_i \frac{1 - K(r)}{r} . \tag{2.9}$$

As $r \to \infty$ the fields take their vacuum values which are

$$\begin{aligned} \phi_1 &\to v \ , \\ \phi_2 &\to -\frac{1}{4} v \ , \\ \phi_3 &\to \frac{5}{4} v \ , \\ h_5 &\to v_w \ , \\ K &\sim e^{-vr} \ , \end{aligned} \tag{2.10}$$

to leading order in v_w/v. As $r \to 0$ we must have

$$K \to 1 \ , \qquad \phi_3 \to 0 \ , \tag{2.11}$$

in order that the energy be finite. Thus, the fine tuning that worked as $r \to \infty$ to keep $h_5(r) = v_w$ small is upset near $r = 0$ since $\phi_3(0) \neq \phi_3(\infty)$. Naively, there are two possibilities as $r \to 0$. Either h_5 develops a large positive mass squared and no expectation value, or it develops a large negative mass squared and a vacuum expectation value of $\mathcal{O}(v)$.

In order to determine whether $h_5(r)$ obtains a large value near the monopole core, we will study the stability of the configuration $h_5 = 0$. Let us expand (2.4) in powers of $h_5(r)$:

$$V_{\text{eff}}(h_5) = \frac{1}{2}M^2(r)h_5^2 + \frac{\lambda}{4}h_5^4 \ . \tag{2.12}$$

and obtain $M^2(r)$ by plugging into (2.4) the solutions for $\phi_1(r), \phi_2(r), \phi_3(r)$ obtained by solving the equations of motion derived from (2.1). If $M^2(r) > 0$ near the core, then $h_5(r) \to 0$ near $r = 0$. But if $M^2(r) < 0$ near the core, then $h_5(r)$ may attain a large expectation value as $r \to 0$. The solutions for $\phi_1(r), \phi_2(r)$ and $\phi_3(r)$ were found numerically by Eckert *et al.*[13] In Table 1, we have used these numerical results (for selected values of a and b) to obtain the conditions we must impose on the coupling constants α and β to obtain $M^2(0) < 0$. In every case, we can satisfy $M^2(0) < 0$ by picking α large enough. We must be a little bit careful, however, because $M^2(0) < 0$ does not necessarily imply that $h_5(0) \sim \mathcal{O}(v)$. That is, if $M^2(0) < 0$ but small in magnitude, it is possible that the derivative term in the Hamiltonian, $(h_5')^2$, will keep the minimum of the potential at $h_5(0) = 0$. This is exactly analogous to nonrelativistic quantum mechanics where we can have a weakly attractive potential without any bound states. In order to ensure $h_5(0) \sim \mathcal{O}(v)$, we must have a potential $(M^2(r))$ which is deep enough. This depends in detail on all the coupling constants in (2.4), but it should be clear that by choosing α large enough we can insure that $h_5(0) \sim \mathcal{O}(v)$. For an arbitrary choice of coupling constants, we expect that the probability that $h_5(0)$ will attain a large value at the monopole core should be of order one. Later in this paper, we will be interested in configurations which give $h_5(0) \gg v$. Apparently, this can be obtained by setting $\alpha \gg \lambda$ and $\alpha > \beta$.

We should also mention that, due to the fine tuning for $r \gg 1/v$, the form of $h_5(r)$ will be very much different from the forms of the other Higgs fields (ϕ_i). The ϕ_i fields approach their vacuum values as $\frac{1}{r}e^{-vr}$ so they vanish outside the monopole core, but $h_5(r) \sim \frac{1}{r}$ for $1/v_w \gg r \gg 1/v$. Thus, if the Weinberg-Salam Higgs field has a large vacuum expectation value at the monopole core, the large VEV will extend all the way out to the weak breaking scale.

One possibility that we have so far neglected is that the symmetry of the Dokos-Tomaras ansatz will be spontaneously broken. This does not seem to occur with just the $\underline{5}$ of Higgs, but it could occur with a more complicated Higgs sector. Clearly, if this symmetry can be broken, the odds that none of the fields contributing to fermion masses will get a large expectation value at the core will be substantially decreased, but this does not mean that monopole induced baryon number violation would be more likely to be suppressed because some of these fields can violate baryon number themselves. In the rest of this paper, we will ignore this possibility because it does not seem to affect our results qualitatively.

Finally, note that the effective potential (2.12) has no terms linear in h_5. If it did, the weak scale Higgs would acquire a large VEV for any choice of the coupling constants. In the case of SU(5), the lack of linear terms is quite easy to prove. Since the coefficients in the effective potential come from couplings to the adjoint Higgs Φ, a linear term would have to be generated by a term like $h\Phi^3$ in the potential (2.4). But terms like this must always carry quintality because h has nonzero quintality but Φ does not. Thus, such terms cannot be SU(5) invariant. For non-SU(5) GUTs this argument clearly does not hold, but there seems to be a similar argument proving the same result in other GUTs (at least those which are known to the author). So it appears that it is always possible to choose a set of coupling constants such that the fermions do not acquire a large effective mass at the monopole core. The situation in which the Higgs field becomes large at the monopole core is, however a generic case, and in fact, for theories with a very complicated Higgs structure at the weak scale, it is difficult to avoid without a rather precise tuning of Higgs sector coupling constants.

3. REDUCTION TO 1+1 DIMENSIONS AND BOSONIZATION

What is the effect of these large Higgs VEV's in the core region on light fermions? Let us study this question for the fundamental SU(5) monopole. As usual, we will only be considering fermions in the s-wave. This means that we only need consider those fermions that transform under the monopole's SU(2), eq. (2.7). Thus we can reduce the problem to two Dirac SU(2) doublets (per generation),

$$\psi_1 = \begin{pmatrix} e^+ \\ d_3 \end{pmatrix} \qquad \psi_2 = \begin{pmatrix} u_1 \\ u_2^c \end{pmatrix} , \tag{3.1}$$

interacting with an SU(2) monopole. This is just the monopole-fermion system originally considered by Callan[3] with the following modifications. First, we must include the two SU(2) doublets given by (3.1) (or more if we wish to consider more than one generation) rather than the single doublet of Callan's original work. Second, we must include all the relevant Coulomb interactions, not just the ones corresponding to generators in the monopole's SU(2). Finally, we should include the position dependence of the VEV of the weak scale Higgs as a position dependent mass for the fermions.

To keep things simple, we will do the main analysis in the model used by Callan with only one fermion doublet and only one Coulomb term but including a position dependent coefficient for the mass term; we will indicate the how to generalize this model to the SU(5) case whenever necessary. Our procedure is essentially equivalent to that of Callan, but we will not take the limit $r_0 \to 0$ (where r_0 is the radius of the monopole) until after bosonization. The bosonization procedure used here is also slightly simpler and perhaps more transparent than that used by Callan.

The Lagrangian for our system is given by

$$\mathcal{L} = \mathcal{L}_G + \mathcal{L}_{\text{Higgs}} + \mathcal{L}_f \ . \tag{3.2}$$

The magnetic and Higgs fields will be regarded as classical background fields, and we will allow for radial excitations of the unbroken electric field. (Quantum corrections to the Higgs field will be discussed in section 5.) The radial electric field is parametrized by a "gauge rotation" of the monopole field (2.9) (with $\vec{T} = \vec{\tau}/2$)

$$A_i^\lambda = U_\lambda A_i U_\lambda^{-1} + iU_\lambda \nabla_i U_\lambda^{-1} \ , \tag{3.3}$$

where $U_\lambda = e^{i\lambda(r,t)\hat{x}\cdot\vec{\tau}/2}$, and we require that $\lambda(0,t) = 0$. This "gauge rotation" produces an electric field given by

$$E_i^a = -\hat{x}^a\hat{x}^i\lambda' - \frac{K(r)\dot{\lambda}}{r}\left(\delta^{ia} - \hat{x}^i\hat{x}^a\right) \ , \tag{3.4}$$

because we have not rotated A_0. The first term on the right hand side of (3.4) is just an ordinary radial electric field, but the second term is not radial and is orthogonal to the ordinary electric field in SU(2) space. It is of little consequence, however, because it exists only inside the monopole core and will vanish altogether when we take the core size (r_0) to zero. Thus, if we include the effect of a θ (vacuum angle) term, the gauge field Lagrangian is given by

$$\mathcal{L}_G = \frac{1}{2g^2}\left(E_i^a\right)^2 + \frac{\theta}{8\pi} E_i^a B_i^a = \frac{(\dot{\lambda}')^2}{2g^2} + \left(\frac{K(r)\dot{\lambda}}{gr}\right)^2 + \frac{\theta}{8\pi^2}\frac{\dot{\lambda}'}{r^2}\left(1 - K^2(r)\right) . \quad (3.5)$$

We now turn to the fermion doublet. Its Lagrangian is given by

$$\mathcal{L}_f = \bar{\psi}\left(i\gamma^\mu D_\mu - m(r)\right)\psi , \quad (3.6)$$

where

$$D_\mu\psi \equiv \left(\partial_\mu + i\vec{A}_\mu \cdot \frac{\vec{\tau}}{2}\right)\psi .$$

Following Callan's reduction to $1+1$ dimensions we set

$$\psi = \frac{1}{\sqrt{2}}\left[\begin{pmatrix} X_+ \\ X_+ \end{pmatrix} + \begin{pmatrix} X_- \\ -X_- \end{pmatrix}\right]$$
$$X_\pm = \frac{1}{\sqrt{8\pi}\, r}\left(g_\pm + p_\pm \hat{x}\cdot\vec{\tau}\right)\tau_2 , \quad (3.7)$$

in a representation where

$$\gamma^0 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \qquad \gamma^i = -i\begin{pmatrix} \sigma^i & 0 \\ 0 & -\sigma^i \end{pmatrix} .$$

$X_\pm$ is a 2×2 matrix with the first index describing spin and the second describing isospin. Since we are in the s-wave, $g_\pm$ and $p_\pm$ depend only on r and t. We can write down gauge invariant two-component fields describing each helicity,

$$\chi_+ = e^{i\lambda\tau_2/2}\begin{pmatrix} g_+ \\ ip_+ \end{pmatrix} \quad \chi_- = e^{-i\lambda\tau_2/2}\begin{pmatrix} ig_- \\ p_- \end{pmatrix} . \quad (3.8)$$

The gauge invariant form is used in order to simplify the form of the fermion interaction with the monopole core. With this reduction we can now write the action for our system in terms of the $1+1$ dimensional Fermi fields, $\chi_\pm$. (Note that the Higgs field appears only in the fermion mass term in our approximation.)

$$S = \int dt \int_0^\infty dr \Big[\bar{\chi}_+ \Big(i\partial\!\!\!/ - A\!\!\!/ + i\gamma^5 \frac{K(r)}{r} \Big) \chi_+ \\ + \bar{\chi}_- \Big(i\partial\!\!\!/ + A\!\!\!/ + i\gamma^5 \frac{K(r)}{r} \Big) \chi_- + m(r) \Big(\bar{\chi}_+ \chi_- + \bar{\chi}_- \chi_+ \Big) \\ + \frac{2\pi r^2}{g^2} (\dot{\lambda}')^2 + \frac{4\pi}{g^2} \Big(K(r) \dot{\lambda} \Big)^2 + \frac{\theta}{2\pi} \dot{\lambda}' \Big(1 - K^2(r) \Big) \Big] . \tag{3.9}$$

We have now switched to $1+1$ dimensional notation where $\gamma^0 = -\tau_3$, $\gamma^1 = i\tau_1$, $\gamma^5 = \gamma^0\gamma^1 = \tau_2$ and $A_\mu = \frac{1}{2}\,\delta_{\mu 1}\dot{\lambda}$.

Since our fields are defined only on the half line $0 \leq r < \infty$ we must impose a boundary condition at $r = 0$ on our fermi fields $\chi_\pm$ in order that our Hamiltonian be self-adjoint. The correct boundary condition, obtained from the solution of the corresponding Dirac equation, is

$$(1 - \gamma_0)\, \chi_\pm(0, t) = 0 . \tag{3.10}$$

Usually at this stage, the limit of a point monopole is taken and all the terms in the action (3.9) dependent on $K(r)$ vanish. In our case, however, we wish to study the fermion fields when $m(r)$ becomes very large at the monopole core. Thus, to avoid confusion, we will leave these terms in our action when we bosonize the theory.

We will now proceed to bosonize[14,15] the action (3.9). Due to the boundary condition (3.10), we can write the two fermi fields on the half line as one boson field on the full line. In the representation where $\gamma_0 = \tau_2$, $\gamma_1 = i\tau_1$ and $\gamma_5 = \tau_3$, we can write

$$
\begin{aligned}
\chi_{+u} &= \sqrt{\frac{c\mu}{2\pi}}\, N_\mu \; e^{i\sqrt{\pi}\left(\phi(-r)+\int_\infty^{-r}\dot{\phi}\,ds\right)} , \\
\chi_{+\ell} &= i\sqrt{\frac{c\mu}{2\pi}}\, N_\mu \; e^{i\sqrt{\pi}\left(\phi(r)+\int_\infty^{r}\dot{\phi}\,ds\right)} , \\
\chi_{-u} &= i\sqrt{\frac{c\mu}{2\pi}}\, N_\mu \; e^{-i\sqrt{\pi}\left(\phi(r)-\int_\infty^{r}\dot{\phi}\,ds\right)} , \\
\chi_{-\ell} &= -\sqrt{\frac{c\mu}{2\pi}}\, N_\mu \; e^{-i\sqrt{\pi}\left(\phi(-r)-\int_\infty^{-r}\dot{\phi}\,ds\right)} ,
\end{aligned}
\tag{3.11}
$$

where $\chi_\pm = \binom{\chi_{\pm u}}{\chi_{\pm\ell}}$, and $c \simeq e^{.577}$ is the exponential of Euler's constant. Here, N_μ denotes normal ordering with respect to an arbitrary normal ordering mass μ.

Clearly, the boundary condition (3.10) is satisfied. Also note that this boson correspondence gives the correct anticommutation relations between the fermi fields in contrast to Callan's treatment. (This is a minor point because it is easily corrected, and it has no influence on any of Callan's results.) A similar problem is encountered when we try to add more fermion doublets because, if we just copy (3.11) for each doublet, we obtain commuting fermion fields. This problem is easily solved with the use of Klein transformations described by Halpern,[16] and we will not need to concern ourselves with it again.

With the transformations given by (3.11) the action becomes

$$S = \int dt \int_0^\infty dr \left[\frac{1}{2} \left(\partial_\mu \phi(r) \right)^2 + \frac{1}{2} \left(\partial_\mu \phi(-r) \right)^2 \right.$$

$$+ \frac{\dot{\lambda}}{2\sqrt{\pi}} \left(\phi'(r) - \phi'(-r) \right) + m(r) \frac{c\mu}{\pi} N_\mu \left(\cos 2\sqrt{\pi}\, \phi(r) + \cos 2\sqrt{\pi}\, \phi(-r) \right)$$

$$+ \frac{K}{2\pi r^2} N_\mu \cos\sqrt{\pi} \left(\phi(r) - \phi(-r) \right) \times \cos\sqrt{\pi} \textstyle\int_{-r}^{r} ds\, \dot{\phi}$$

$$\left. + \frac{2\pi r^2}{g^2} (\dot{\lambda}')^2 + \frac{4\pi}{g^2} (K\dot{\lambda})^2 + \frac{\theta}{2\pi} \dot{\lambda}'(1 - K^2) \right] . \tag{3.12}$$

Note that by multiplying χ_- by an arbitrary phase ($e^{i\alpha}$) in Eqs. (3.11) we can introduce an arbitrary phase into the mass term in (3.12). This phase can be regarded either as a redefinition of χ_- or as a chiral rotation of the fermion doublet. In the first case, we should expect that this phase will have no physical effect on the system which, as we shall see, is indeed the case. On the other hand, if we interpret α as a chiral rotation, we would expect that α would manifest itself as a contribution to the vacuum angle $\bar{\theta} = \theta + \alpha$. The fact that this does not occur is, apparently, a deficiency of the bosonization technique.

We will now examine the point-like ($r_0 \to 0$) monopole limit. If we recall that $K(r) \sim e^{-r/r_0}$, we can see that the terms proportional to K^2 will vanish in this limit. The remaining core term describes the interaction between the fermions and the monopole core which was responsible for the boundary conditions between Φ and Q in Callan's treatment.[3] (Φ and Q are given by $\Phi(r) = \phi(r)$ and $Q(r) = \phi(-r)$ in our notation.) A classical analysis of this term will indicate that the correct boundary condition is already incorporated into our bosonization scheme. The potential corresponding to this term is

$$V = -\frac{K(r)}{2\pi\, r^2}\, N_\mu \cos\sqrt{\pi}\left(\phi(r) - \phi(-r)\right) \times \cos\sqrt{\pi}\textstyle\int_{-r}^{r} ds\,\dot{\phi}\ . \qquad (3.13)$$

Since this potential is singular at $r = 0$, it should be forced to its minimum at $r = 0$, and this gives us a boundary condition. In our case, this just means $\phi(0) = \phi(-0)$ (or $\Phi(0) = Q(0)$ in Callan's notation).

Now that we have recovered the usual point-like results, we should examine the core dependent terms to see what influence they have. If we assume that $Er_0 \ll 1$ where $E \simeq$ the energy scale of the fermions, then we can still neglect the terms proportional to K^2. The term describing the interaction of the fermions with monopole core, however, is not so simple. A naive classical interpretation of (3.13) would indicate that it has very little influence on low energy physics. For instance, if we expand (3.13) as a power series in ϕ and r, we obtain (to lowest order)

$$V = C + K(r)\left(\dot{\phi}(0)^2 + \phi'(0)^2\right)\ , \qquad (3.14)$$

where C is a (divergent) constant. This term will have a completely negligible effect when we integrate over r, so if we could neglect the quantum mechanical effects associated with (3.13), we could drop it from the action and just keep the boundary condition.

The quantum mechanical effects of (3.13) are considerably more interesting. It turns out that (3.13) is capable of inducing baryon number violation (or charge violation with just one doublet of fermions) nonlocally throughout the monopole core. This would be important if the range of our effective fermion mass term was smaller than r_0, the monopole's size. (As it turns out, this is not the case.) That (3.13) provides for baryon number violation throughout the core is easily seen if we make a canonical transformation to a new set of boson fields,

$$\begin{aligned}
\eta_{\pm}(r) &= \frac{1}{2}\Big(\phi(r) - \phi(-r)\Big) \pm \frac{1}{2}\int\limits_{-r}^{r} \dot{\phi} ds \, , \\
\dot{\eta}_{\pm}(r) &= \frac{1}{2}\Big(\dot{\phi}(r) - \dot{\phi}(-r)\Big) \pm \frac{1}{2}\Big(\phi'(r) - \phi'(-r)\Big) \, ,
\end{aligned} \tag{3.15}$$

so that (3.13) takes the form of an ordinary sine-Gordon mass term with position dependent coefficient,

$$V = -\frac{K(r)}{2\pi r^2}\Big(\cos 2\sqrt{\pi}\,\eta_+(r) + \cos 2\sqrt{\pi}\,\eta_-(r)\Big) \, . \tag{3.16}$$

Then baryon number violation can be seen classically; an incoming soliton will reflect off the large mass term at the core which, with these $\eta_{\pm}$ fields, implies baryon number violation. To see how baryon number violation occurs without transforming the fields, we should note that the fermion operators defined by (3.11) are capable of creating and annihilating solitons and antisolitons at the point r. So it seems reasonable that (3.13) should be able to create a soliton-antisoliton pair extending from r to $-r$. We will show below (eq. (3.26)) that this implies baryon number violation. For now, we will drop (3.13) from the action, but we must remember that, if we are to suppress baryon number violation, our barrier must prevent the solitons from reaching the monopole core.

We now turn our attention to the λ-dependent terms of (3.12) and write

$$S_\lambda = \int dt \int\limits_0^\infty dr \left[\frac{\dot{\lambda}}{2\sqrt{\pi}}\Big(\phi'(r) - \phi'(-r)\Big) + \frac{2\pi r^2}{g^2}(\dot{\lambda}')^2 + \frac{\theta}{2\pi}\dot{\lambda}'\right] , \tag{3.17}$$

for the λ-dependent part of the action. Integrating by parts we obtain

$$S_\lambda = \int dt \, \frac{\dot{\lambda}}{2\sqrt{\pi}} \left(\phi(r) + \phi(-r)\right)\Big|_0^\infty$$
$$+ \int dt \int_0^\infty dr \Big[-\frac{\dot{\lambda}'}{2\sqrt{\pi}} \left(\phi(r) + \phi(-r) - \frac{\theta}{\sqrt{\pi}}\right) + \frac{2\pi r^2}{g^2} (\dot{\lambda}')^2 \Big] \, . \tag{3.18}$$

The surface term merits careful consideration. The boundary condition on λ implies that $\dot{\lambda}(0,t) = 0$, but we cannot also have $\dot{\lambda}(\infty,t) = 0$ because this would make the θ term vanish. Adding a term proportional to $\dot{\lambda}(0,t)$ we obtain

$$S_\lambda = \int dt \int_0^\infty dr \, \Big[\frac{2\pi r^2}{g^2} (\dot{\lambda}')^2 - \frac{\dot{\lambda}'}{2\sqrt{\pi}} \left(\phi(r) + \phi(-r) - \phi(\infty) - \phi(-\infty) - \frac{\theta}{\sqrt{\pi}}\right) \Big] \, . \tag{3.19}$$

Now we can solve for $\dot{\lambda}'$ to get

$$-E_r = \dot{\lambda}' = \frac{g^2}{8\pi^{3/2} r^2} \left(\phi(r) + \phi(-r) - \phi(\infty) - \phi(-\infty) - \frac{\theta}{\sqrt{\pi}}\right) \, ,$$

and

$$S_\lambda = -\int dt \int_0^\infty dr \, \frac{g^2}{32\pi^2 r^2} \left(\phi(r) + \phi(-r) - \phi(\infty) - \phi(-\infty) - \frac{\theta}{\sqrt{\pi}}\right)^2 \, . \tag{3.20}$$

In order to interpret this Coulomb term we will need the electric charge operator. This is given (in 4-dimensional and then bosonic notation) by

$$Q = \int \bar{\psi} \, \hat{x} \cdot \vec{\tau} \gamma_0 \psi \, d^3x = \frac{1}{2\sqrt{\pi}} \int_0^\infty dr \left(\phi'(r) - \phi'(-r)\right)$$
$$= \frac{1}{2\sqrt{\pi}} \left(\phi(\infty) + \phi(-\infty) - 2\phi(0)\right) \, . \tag{3.21}$$

Thus, the Coulomb part of the Hamiltonian is

$$H_\lambda = \int_0^\infty dr\, \frac{g^2}{32\pi^2 r^2} \left(\phi(r) + \phi(-r) - 2\phi(0) + \frac{1}{\sqrt{\pi}}(2\pi Q - \theta)\right)^2 . \qquad (3.22)$$

So, in order to have finite energy we must have $Q = \theta/2\pi$ in agreement with Witten.[17] Also, we notice that the charge on the monopole does not depend on the phase of the mass term. Apparently, this is the answer to one of Yamagishi's objections[9] to Callan's bosonization procedure. Yamagishi had noted that, as we have mentioned above, we can change the phase of the mass term by an arbitrary amount by rotating χ_- by an arbitrary phase. If we interpret this phase as a redefinition of χ_- (and also neglect the surface term), we might conclude that electric charge on the monopole (from (3.21)) depends on $\phi(\pm\infty)$ and hence, on an arbitrary phase in the definition of our bosonization correspondence. The surface term prevents this ambiguity so that the monopole electric charge is uniquely determined by Witten's formula. Unfortunately, when this surface term is included, we can no longer implement chiral rotations correctly. This is not a problem for practical calculations, however, because we can rotate away any chiral phases in the fermion masses before we bosonize. (This has been done by Harvey.[18])

We must also note that the Coulomb term does not seem to allow states with $Q = \theta/2\pi + n/2$ for $n \neq 0$. If we wish to scatter fermions off the monopole, we can evade this problem by placing an appropriate number of stationary solitons at large r so that $Q = \theta/2\pi$. If we are also careful to pick the correct phase for our mass terms (so that $\phi(\pm\infty) = 0$ is a minimum), we can drop all the surface terms and obtain the following Hamiltonian,

$$H = \int_{-\infty}^{\infty} dr \left[\frac{1}{2}\pi^2(r) + \frac{1}{2}\left(\phi'(r)\right)^2 - m(r)\,\frac{c\mu}{\pi}\,N_\mu \cos 2\sqrt{\pi}\,\phi(r)\right] + \frac{g^2}{64\pi^2 r^2}\left(\phi(r) + \phi(-r) - \frac{\theta}{\sqrt{\pi}}\right)^2 . \tag{3.23}$$

This is essentially Callan's Hamiltonian[3] with a position dependent mass term.

In order to study the scattering of fermions off the monopole, it is convenient to find the charges of the soliton states. The electric charge is given by (3.21), and the axial charge (or helicity) and fermion number are given by

$$Q_5 = \int \bar{\psi}\gamma_0\gamma_5\psi\, d^3x = \frac{1}{\sqrt{\pi}}\int_{-\infty}^{\infty} dr\, \dot{\phi}(r) , \tag{3.24}$$

and

$$F = \int \bar{\psi}\gamma_0\psi\, d^3x = \frac{1}{\sqrt{\pi}}\int_{-\infty}^{\infty} dr\, \phi'(r) = \frac{1}{\sqrt{\pi}}\Big(\phi(\infty) - \phi(-\infty)\Big) , \tag{3.25}$$

respectively. The charges of the various soliton states are given in Table 2.

If $m(r) = \text{const.}$ and we neglect the Coulomb term, we just have a free Sine-Gordon theory, but as the solitons propagate through $r = 0$ they reverse their charge. So an incoming d_{3L} would change into a e_L^+ at $r = 0$ while conserving fermion number and helicity. (Recall that either charge or helicity must change in order to conserve angular momentum.) This is exactly the result obtained by Marciano and Muzinich,[19] and D'Hoker and Farhi[20] when they solved the Dirac equation in a non-abelian monopole field. When we turn on the Coulomb term, we obtain $\phi(0) = 0$ as a dynamically enforced boundary condition, and the solitons will reflect from the monopole core and emerge with the same charge and fermion number but with the opposite helicity.

With an SU(5) monopole, we have two Dirac doublets but only one effective boundary condition due to Coulomb terms. (There are three conserved abelian charges, Q_{EM}, Y_c and I_c, but two of the dynamical boundary conditions are degenerate with our boundary condition matching $\phi(r)$ with $\phi(-r)$.) Thus for SU(5), some baryon violating processes remain, such as

$$u_{1R}(\text{state a}) + d_{3L}(\text{state d}) \rightarrow \bar{u}_{2R}(\text{state b}) + e^+_L(\text{state c}) \ , \qquad (3.26)$$

which is shown schematically in Fig. 1 The reactions involving a single incoming particle are more complicated (involving "half soliton" states), and are discussed in some detail elsewhere.[4] The important point is that for baryon number violation to occur, one or more solitons must pass through the origin. Actually, we must be more careful if we take the monopole size to be finite because we must include nonlocal interactions caused by (3.13). A soliton reaching the edge of the monopole core can annihilate with the antisoliton of a soliton-antisoliton pair created by (3.13). The pair produced soliton then propagates away from the opposite "side" of the monopole core without having passed through the origin. Fortunately, we will be able to neglect this fine point because the effective mass terms that we will consider extend far outside the monopole core. In the next section we will see if this type of process is allowed when $m(r)$ takes a large value at $r = 0$.

4. THE EFFECTIVE FERMION MASS AT THE MONOPOLE CORE

A semiclassical analysis of the monopole fermion system described by the Hamiltonian (3.23) is rather subtle when $m(r) \neq$ constant. To simplify the system further, we will drop the Coulomb term so that

$$H = \int_{-\infty}^{\infty} dr \left(\frac{1}{2}\pi^2 + \frac{1}{2}\phi'^2 - m(r)\frac{c\mu}{\pi} N_\mu \cos 2\sqrt{\pi}\,\phi \right) . \tag{4.1}$$

The Coulomb term can be added in later if we wish, but it can only serve to restrict baryon number violation, not enhance it. We will do a semiclassical analysis of (4.1) in order to see whether a large mass near $r = 0$ will lead to suppression of the baryon number violating reactions mentioned in the previous section. If we find that the solitons of (4.1) cannot pass through the barrier at $r = 0$, then we can conclude that the fermion scattering off a GUT monopole cannot violate baryon number.

We now attempt a semiclassical analysis of (4.1). This analysis will center on one important point: The mass term in equation (4.1) formally depends on an arbitrary normal ordering mass μ:

$$H_m = m(r)\frac{c\mu}{\pi} N_\mu cos 2\sqrt{\pi}\,\phi. \tag{4.2}$$

(4.2) is actually independent of μ in the full quantum theory. However, if we are performing a semiclassical analysis, there is some particular value of μ for which this analysis will be the most accurate. The bulk of this analysis will be devoted to finding this preferred value.

First, however, we will consider the consequences of various choices for μ: Let us take $m(r)$ to be of the form discussed in Section 2, that is

$$m(r) \sim \begin{cases} \dfrac{\gamma}{r_0}, & |r| < r_0 \,, \\ m_\infty, & |r| \gg r_0 \,, \end{cases} \tag{4.3}$$

where r_0 is (roughly) the GUT scale $1/v$, m is the ordinary fermion mass and γ is a factor proportional to the Yukawa coupling between h_5 and the fermions. From the discussion in Section 2, we can make a rough estimate of γ,

$$\gamma \sim \frac{\alpha}{\lambda} \times 10^{-5} \,. \tag{4.4}$$

Thus, we expect that $\gamma \sim \mathcal{O}(10^{-5})$, but we can make it larger than this by tuning α to be larger than λ. The long range of the weak scale Higgs ($h_5 \sim \frac{1}{r}\, e^{-v_w r}$) will be neglected for now, but we will consider its consequences later in this section.

Naively, we have two possible choices for μ in a semiclassical analysis:

1. Set $\mu \simeq m_\infty$ (a constant), so

$$H_c = \int_{-\infty}^{\infty} dr \left(\frac{1}{2}\,\pi^2 + \frac{1}{2}\,\phi'^2 - m_\infty m(r)\,\frac{c}{\pi} \cos 2\sqrt{\pi}\,\phi \right) \,. \tag{4.5}$$

2. Set $\mu \simeq m(r)$ (a function of r!)

$$H_c = \int_{-\infty}^{\infty} dr \left(\frac{1}{2}\pi^2 + \frac{1}{2}\,\phi'^2 - \frac{c}{\pi}\, m^2(r) \cos 2\sqrt{\pi}\,\phi \right) \,. \tag{4.6}$$

For each possibility, it is simple to determine if a classical soliton can pass through the barrier at the origin. If it can, then the large mass at the origin presents essentially no barrier for baryon number violating processes. If the soliton cannot penetrate the barrier then we will have a suppression of baryon number violation. (Provided that the tunnelling amplitude is small; we will show later that this is so.) We can answer this question straightforwardly by determining the energy of a soliton at rest at $r = 0$. This energy will be interpreted as the minimum energy that an incoming soliton must have in order to pass through the barrier at $r = 0$. We will take $\gamma \sim 10^{-5}$ and $1/r_0 \sim 10^{16}$GeV as approximate values. For H_c given by (4.5), we have

$$E \simeq m_{\rm soliton} + \frac{c}{\pi} m_\infty \int_{-r_0}^{r_0} dr\, m(r)$$

$$E \simeq m_{\rm soliton}(1+\gamma) \simeq m_{\rm soliton} \tag{4.7}$$

where $m_{\rm soliton} = \text{const.} \times m_\infty$. In this case, the kinetic energy required to penetrate the core is essentially the same as the fermion mass so that relativistic particles will see no barrier. For H_c given by (4.6), however, we obtain

$$E \simeq m_{\rm soliton} + \frac{c}{\pi} \int_{-r_0}^{r_0} dr\, m^2(r)$$

$$\simeq m_{\rm soliton} + \frac{\gamma^2}{r_0} \sim 10^6 \text{ GeV} . \tag{4.8}$$

Here, the kinetic energy required to penetrate the core much greater than the fermion's energy, and any baryon number violation appears to be ruled out. We shall see that (4.8) is substantially correct for $\gamma \gg 10^{-3}$, while (4.7) is correct for $\gamma \ll 10^{-3}$.

Now, let us begin an investigation into what value of μ is the best for a semiclassical analysis. For the case of $m(r) = m_\infty$, a constant, the question was settled some time ago by Coleman;[14] we will reproduce his analysis here so that it will be clear how to generalize his argument to our case where $m(r) \neq$ a constant.

To obtain a relationship between the normal ordered mass term and a bare mass term we will need Wick's theorem. This tells us that, for a free field of mass μ and any space-time function $J(x)$,

$$\exp\Big[i\int J(x)\phi(x)d^2x\Big] = N_\mu \exp\Big[i\int J(x)\phi(x)d^2x\Big] \times \exp\Big[-\frac{1}{2}\int J(x)\Delta(x-y;\mu)J(y)d^2x\,d^2y\Big] , \tag{4.9}$$

where $\Delta(x-y;\mu)$ is the free-field two-point Wightman function. For small separations, it is given by

$$\Delta(x;\mu) = -\frac{1}{4\pi}\log c^2\mu^2\Big[(x^1)^2 - (x^0 - i\epsilon)^2\Big] + O(x_\mu x^\mu) . \tag{4.10}$$

To cut off the theory, we define a cutoff Wightman function by

$$\Delta(x;\mu;\Lambda) \equiv \Delta(x;\mu) - \Delta(x;\Lambda) , \tag{4.11}$$

where Λ is the cutoff. At zero separation we have

$$\Delta(0;\mu;\Lambda) = -\frac{1}{4\pi}\log\frac{\mu^2}{\Lambda^2} . \tag{4.12}$$

Now, if we set J in (4.9) equal to a δ function, we obtain

$$e^{2i\sqrt{\pi}\phi} = \frac{\mu}{\Lambda} N_\mu e^{2i\sqrt{\pi}\phi} . \tag{4.13}$$

From (4.13) we can obtain the normal ordering relationship for our mass term

$$\frac{c}{\pi}\, m_{\infty} m_0 \cos 2\sqrt{\pi}\phi = \frac{c}{\pi}\, m_{\infty} \mu\, N_{\mu} \cos 2\sqrt{\pi}\phi \ , \tag{4.14}$$

where we must set the bare mass $m_0 = \Lambda$. This shows that our mass term is indeed independent of the normal ordering mass μ. We have also developed the formalism we need to choose μ for a semiclassical analysis.

The correct choice of μ is that one associated with the ground state of the system. We will use the Rayleigh-Ritz variational method to minimize the Hamiltonian density,

$$\mathcal{H} = N_{\mu}\Big(\frac{1}{2}\,\pi^2 + \frac{1}{2}\,\phi'^2 - \frac{c}{\pi}\, m_{\infty}\mu \cos\sqrt{\pi}\phi\Big) \ , \tag{4.15}$$

with respect to the vacuum states for a free scalar field of mass ν. These states are defined by

$$a(k,\nu)\,|\,0,\nu\rangle = 0 \ . \tag{4.16}$$

The minimum of $\langle 0,\nu|\mathcal{H}|0,\nu\rangle$ with respect to ν gives the appropriate choice for μ.

To calculate $\langle 0,\nu|\mathcal{H}|0,\nu\rangle$ we will need to reorder the Hamiltonian density with respect to ν. The reordering of the mass term is given by (4.14) to be

$$\mu\, N_{\mu} \cos 2\sqrt{\pi}\phi = \nu\, N_{\nu} \cos 2\sqrt{\pi}\phi \ , \tag{4.17}$$

and the reordering of the first two terms of (4.15) is given by

$$\begin{aligned} N_{\mu}\Big(\frac{1}{2}\,\pi^2 + \frac{1}{2}\,\phi'^2\Big) &= N_{\nu}\Big(\frac{1}{2}\,\pi^2 + \frac{1}{2}\,\phi'^2\Big) + E_0(\nu) - E_0(\mu) \\ &= N_{\nu}\Big(\frac{1}{2}\,\pi^2 + \frac{1}{2}\,\phi'^2\Big) + \frac{\nu^2 - \mu^2}{8\pi} \ . \end{aligned} \tag{4.18}$$

Therefore,

$$\langle 0,\nu\,|\,\mathcal{H}\,|\,0,\nu\rangle = \frac{\nu^2-\mu^2}{8\pi} - \frac{c}{\pi}\,m_\infty\nu \; . \tag{4.19}$$

The minimum of (4.19) is at $\nu = 4cm_\infty$, so it is most sensible to set $\mu = 4cm_\infty$. Hence, (4.1) becomes

$$H = \int_{-\infty}^{\infty} dr\, N_\mu\Big(\frac{1}{2}\,\pi^2 + \frac{1}{2}\,\phi'^2 - \frac{1}{4\pi}\,\mu^2\cos 2\sqrt{\pi}\phi\Big) \; , \tag{4.20}$$

and the ground state of this system is given by $|0,\mu\rangle$. Thus, the choice of the correct normal ordering mass μ really corresponds to choosing the correct vacuum. After we find the vacuum state we normal order with respect to "its mass" in order to minimize radiative corrections.

Now, we would like to repeat this procedure for $m(r)$ of the form given in (4.3). So the Hamiltonian (4.15) is replaced by

$$\mathcal{H} = N_\mu\Big(\frac{1}{2}\,\pi^2 + \frac{1}{2}\phi'^2 - \frac{c}{\pi}\,m(r)\,\mu\cos 2\sqrt{\pi}\phi\Big) \; . \tag{4.21}$$

Our trial states will now be the ground states of a free scalar field with a position dependent mass $\nu(r)$. Initially, we will take both the $m(r)$ and $\nu(r)$ to be given by step functions:

$$\begin{aligned} m(r) &= m\theta(|r| - r_0) + \frac{\gamma}{r_0}\,\theta(r_0 - |r|) \; , \\ \nu(r) &= \nu\theta(|r| - r_t) + M\theta(r_t - |r|) \; . \end{aligned} \tag{4.22}$$

We make the assumption that γ and Mr_t are small because this seems to be the physically relevant case. With the aid of equation (A24) from the appendix, we obtain the reordering equation for the kinetic terms

$$N_\mu\left(\frac{1}{2}\pi^2+\frac{1}{2}\phi'^2\right)=N_\mu\left(\frac{1}{2}\pi^2+\frac{1}{2}\phi'^2\right)+E_0\left(\nu(r)\right)-E_0(\mu)\ ,$$
$$\simeq N_\mu\left(\frac{1}{2}\pi^2+\frac{1}{2}\phi'^2\right)+\frac{\nu^2(r)-\mu^2}{8\pi} \tag{4.23}$$

essentially identical to the constant mass case. The reordering equation for the mass term is

$$\mu\, N_\mu \cos 2\sqrt{\pi}\,\phi=\hat{\nu}(r)\, N_{\nu(r)}\cos 2\sqrt{\pi}\,\phi\ , \tag{4.24}$$

where

$$\hat{\nu}(r)=\Lambda\left[\exp\left(-4\pi\Delta(0;\nu(r);\Lambda)\right)\right]^{1/2}\ . \tag{4.25}$$

The form of $\hat{\nu}(r)$ is obtained from Eq. (A18) of the appendix,

$$\hat{\nu}(r)=\begin{cases}2M^2r_t & r\ll\frac{1}{M^2r_t}\ ,\\ \nu & r\gg\frac{1}{M^2r_t}\ ,\end{cases} \tag{4.26}$$

Note that $\hat{\nu}(r)$ and $\nu(r)$ have very different profiles (since we have assumed that $Mr_t\ll 1$). Naively, we might expect the minimum energy configuration to occur when the reordering mass $\hat{\nu}(r)\simeq m(r)$ as occurs for $m(r)=$ a constant as well as in the qualitative analysis of (4.6) earlier in this section. If $m(r)$ is a very narrow step function (*e. g.* when γ is small), we can make $\nu(r)$ as narrow as we want but $\hat{\nu}(r)$ will always have a range of roughly the inverse of its height. Thus, for γ small it will be impossible to have $\hat{\nu}(r)\simeq m(r)$ as we would need for the baryon number violating case, (4.6). To see this more quantitatively, we will evaluate the vacuum expectation value of the Hamiltonian density (4.21),

$$\langle 0,\nu(r)|\ \mathcal{H}\ |0,\nu(r)\rangle = \frac{\nu^2(r)}{8\pi} - \frac{c}{\pi}\, m(r)\hat{\nu}(r)\ . \tag{4.27}$$

Integrating over r we obtain (roughly)

$$\langle 0,\nu(r)|\ H\ |0,\nu(r)\rangle \simeq \frac{1}{4\pi} M^2 r_t - \frac{4c}{\pi} M^2 r_t \gamma\ , \tag{4.28}$$

where we have set $m, \nu \simeq 0$ and taken $M^2 r_t r_0 < 1$. Clearly, for $\gamma < 1/16c \sim 1/30$ the minimum is at $M = 0$ and the correct semiclassical picture is given by (4.5).

This conclusion is premature, however, because we have assumed that $m(r)$ is given by a step function and neglected its long range. In order to estimate the effect of the long ($\sim 1/r$) tail of $m(r)$, we will take $m(r)$ to have the form,

$$m(r) = \begin{cases} \frac{\gamma}{r_0} & |r| < r_0\ , \\ \\ \frac{\gamma}{r} & |r| > r_0\ . \end{cases} \tag{4.29}$$

We will also need an approximation for $\hat{\nu}(r)$. In the appendix (eq. (A18)), we have calculated the first few powers of r in the expansion of $\hat{\nu}(r)$. These terms indicate that $\hat{\nu}(r)$ begins to vary from $\hat{\nu}(0)$ at a distance of order $1/M^2 r_t$, so we can approximate $\hat{\nu}(r)$ by

$$\hat{\nu}(r) \simeq 2M^2 r_t\, \theta\left(|r| - \frac{1}{M^2 r_t}\right)\ . \tag{4.30}$$

Using (4.29) and (4.30) we can integrate (4.27) over r to obtain

$$\langle 0,\nu(r)|\ H\ |0,\nu(r)\rangle \simeq \frac{1}{4\pi} M^2 r_t - \frac{4c}{\pi} M^2 r_t \gamma \left(1 + \log\left(\frac{1}{M^2 r_t r_0}\right)\right)\ , \tag{4.31}$$

where the $\log(1/M^2 r_t r_0)$ term comes from the $\sim 1/r$ tail of $m(r)$ cut off by the theta function in (4.30). This result is only weakly dependent on the radius at which we cut off $\hat{\nu}(r)$, so the crudeness of our approximation for $\hat{\nu}(r)$ should not have a great influence on our result. Also, note that most of our barrier is located outside the monopole core so that we need not worry about the nonlocal term, (3.13), which is responsible for baryon number violation in the region $0 < r < r_0$.

Minimizing the Hamiltonian, (4.31), we obtain

$$M^2 r_t = \frac{1}{r_0} \exp(-1/16c\gamma) \ , \tag{4.32}$$

and substituting this back into (4.31) gives us an expression for the minimum vacuum energy of (4.21),

$$E_{\text{min}} \equiv \langle 0, \nu(r)| \ H \ |0, \nu(r)\rangle_{\text{min}} = -\frac{\gamma}{r_0} \frac{4c}{\pi} e^{-1/16c\gamma} \ . \tag{4.33}$$

Evaluating this expression for $\gamma = 10^{-5}$, we obtain $M^2 r_t \sim E_{\text{min}} \sim -10^{-150}/r_0$, so in this case any increase in the mass at the core would be negligible (to say the least). For $\gamma \sim 10^{-3}$ the results are considerably more interesting. At $\gamma = 0.001$ we obtain $M^2 r_t \simeq 6 \times 10^{-16}/r_0$ and $E_{\text{min}} \simeq 10^{-18}/r_0$ which would still seem to have little effect on the baryon number violating reactions, but for $\gamma = 0.002$ we have $M^2 r_t \simeq 2 \times 10^{-8}/r_0$ and $E_{\text{min}} \simeq 10^{-10}/r_0$ which would mean a fermion mass of greater than 10^5 GeV if r_0 is the usual GUT scale. So apparently, it seems that there is a rather abrupt transition between heuristic pictures of (4.5) and (4.6) at $\gamma \sim 10^{-3}$. Therefore, it would seem that, in this case, we must fine tune the Higgs sector coupling constants to about 2 decimal places if we wish to suppress baryon number violation.

Before we can be confident that these results are correct, we should check to see that we have made a reasonable choice for our trial states. Perhaps, with a more clever choice for $\nu(r)$, we could obtain a large effective fermion mass for $\gamma \ll 10^{-3}$. This is very unlikely, because we are interested in choices for $\nu(r)$ that will generate values for $\hat{\nu}(r)$ such that $\hat{\nu}(r) \approx m(r)$, and, as we have seen, when $\nu(r)$ is a narrow function (*i. e.* when $Mr_t \ll 1$ in equation (4.22)), $\hat{\nu}(r)$ is only weakly dependent on the shape of $\nu(r)$. Thus, another choice of $\nu(r)$ will have little influence on the form of $\hat{\nu}(r)$ and therefore, little influence on the effective fermion mass. As an independent check, we have done this calculation numerically for several different choices of $\nu(r)$ and the result is essentially the same (within a factor of two) in every case, so we have good reason to expect that this variational calculation gives the correct result.

Since we have done our calculation in the semiclassical limit, we should try to estimate the tunneling probability. That is, we should try to obtain an estimate of the probability for the low energy solitons of (4.1) to tunnel through the barrier at $r = 0$. At a glance, it might seem that the tunneling probability might be significant. If we take the height of the potential barrier to be E_{min} and the range to be r_0, we might guess that the tunneling probability would be of the order

$$e^{E_{\text{min}} r_0} = e^{-10^{-10}} = 1 \ , \tag{4.34}$$

for $\gamma = 0.002$. But we have neglected the fact that the soliton is not a point particle. Since the soliton has a width of order $1/\mathcal{E}$ (where $\mathcal{E}$ is the soliton's energy), the soliton effectively sees a potential of width $1/\mathcal{E}$.

In particular, the classical soliton profile is given by

$$\phi_c = \frac{2}{\sqrt{\pi}} \tan^{-1}(e^{\mathcal{E} r}) \ , \tag{4.35}$$

at a fixed time. With this, we can obtain an effective mass for a classical soliton of energy $\mathcal{E}$ at position r,

$$\begin{aligned} m_{\text{eff}}(r) &= \frac{c}{\pi} \int_{-\infty}^{\infty} ds\, m(s)\hat{\nu}(s)\Big(1 - \cos 2\sqrt{\pi}\phi_c(r-s)\Big) \\ &= \frac{4c}{\pi r_0}\gamma\, e^{-1/16c\gamma} \left(1 + \frac{1}{16c\gamma}\right) \frac{1}{\cosh 2\mathcal{E} r + 1} , \end{aligned} \tag{4.36}$$

where we have assumed that the energy scale associated with the barrier at the core, $\frac{1}{r_0} e^{-1/16c\gamma}$, is much larger than the fermion energy, $\mathcal{E}$. The tunneling amplitude can now be estimated to be

$$P \sim \exp\Big(-\int_{-r}^{r} ds\, m_{\text{eff}}(s)\Big) \sim \exp\Big(-\frac{e^{-1/16c\gamma}}{16c\,\mathcal{E} r_0}\Big) \sim \exp\Big(-10^6\Big) , \tag{4.37}$$

for $\gamma = 0.002$, $1/r_0 \sim 10^{15}$ GeV and $\mathcal{E} \sim 100$ MeV. ($\mathcal{E}$ is a typical quark energy inside the nucleus.) The tunneling amplitude (4.37) is miniscule, so we conclude that tunneling will not be important when γ is large enough so that the fermions acquire a large effective mass at the monopole core. Thus, if γ tuned to be large enough, we can prevent monopole induced baryon number violation.

5. CONSERVATION OF WEAK ISOSPIN WHEN THE CALLAN-RUBAKOV EFFECT IS SUPPRESSED

In the previous section, we have seen that with the appropriate tuning of parameters, the Higgs field configuration will induce large effective fermion mass terms. These mass terms will prevent the fermions from reaching the monopole core where baryon number violation can occur. Instead, all $J = 0$ fermions will simply flip their helicity upon encountering the monopole. The reader may be a bit suspicious of this scenario because these helicity flip reactions (like $u_{1R} + M \rightarrow$

$u_{1L} + M$) clearly violate weak isospin which we should expect to be conserved at distances of order the monopole size. Furthermore, Schellekens[21] and Sen[22] have argued that monopole induced baryon number violation can occur as a result of the weak ('t Hooft[23]) anomaly even in GUT's that conserve baryon number. How can we reconcile our previous results with these arguments?

First, we should discuss how weak isospin is conserved when the Callan-Rubakov effect is not suppressed. In this case, there are two possible outgoing states when a u_{1R} is incident on a monopole: the helicity flip reaction

$$u_{1R} \rightarrow u_{1L} , \tag{5.1}$$

and the baryon number violating reaction,

$$u_{1R} \rightarrow \bar{u}_{2R}\bar{d}_{3L}e^{+}_{L} . \tag{5.2}$$

Each of these reactions alone violates weak isospin, so how would the system behave in the limit of unbroken weak interactions? This question can be answered with the use of Callan's peculiar "fractional solitons[4]" (which carry fractional quantum numbers). If we allow outgoing solitons with less than unit amplitude, the following process is allowed

$$u_{1R} \rightarrow \frac{1}{2}\, u_{1L} + \frac{1}{2}\, \bar{u}_{2R}\bar{d}_{3L}e^{+}_{L} , \tag{5.3}$$

which conserves weak isospin. Classically, this process produces 4 "fractional particles", but quantum mechanically, it has a more sensible interpretation. The quantum mechanical state implied by (5.3) is just a mixture of the outgoing states from (5.1) and (5.2). When the incoming quark reaches the monopole core it begins to excite the other fermion fields while conserving weak isospin as required by (5.3). Then as outgoing "fractional fermions" reach a distance from the monopole of the order of the weak scale, the mass terms become important and the exotic outgoing state of (5.3) must decay into the outgoing states of either (5.1) or (5.2). Presumably, the probabilities of the reactions (5.1) and (5.2) are each 1/2. In the remainder of this section, we will see how to obtain weak isospin conservation at short distances when the Higgs sector coupling constants are adjusted so that the Callan-Rubakov effect is suppressed. As might be expected, we will find another "fractional soliton" process that works at distances smaller than the weak scale to conserve weak isospin.

The source of the weak isospin violation mentioned above is obvious; we have assumed that the Weinberg-Salam Higgs field (h_5) can be treated as a classical background field. Thus, when it gets a large expectation value at the monopole core, weak isospin is strongly broken at the core. This assumption is not necessarily correct because the Higgs field takes an expectation value of order $\gtrsim v$ in a region of space of radius $\sim 1/v$; hence, we expect might quantum fluctuations to be large. In particular, we might guess that the quantum ground state is a superposition of all possible orientations of h in $SU(2)_W$, so that the $SU(2)_W$ symmetry will be restored at the GUT scale. To check this, we should try to include these $SU(2)_W$ rotations of h_5 as dynamical degrees of freedom. These degrees of freedom can be parametrized by performing a time dependent $SU(2)_W$ rotation (at the monopole core) on the classical solution for $h(r)$, (2.8). When we attempt to do this however we encounter some difficulty in constructing the $SU(2)_W$ rotations in the region of the monopole. In fact, as Nelson and Coleman have shown,[24] in the presence of a monopole we can only define those $SU(2)_W$

rotations which commute with the monopole's charge. Thus, we can only include the Abelian component (I_{3W}) of the $\mathrm{SU}(2)_W$ rotations, or rather, more properly, the combination of I_{3W} and Y_W which is orthogonal to the monopoles charge.

Let us describe this system by writing an effective Lagrangian including the Abelian rotation degree of freedom for $h(r)$. Here, for convenience we will represent the fermions in Callan's notation for boson fields on the half line. To connect with our previous notation we must set $\phi_{e^+}(r) = \phi_{\overline{d}_3}(-r)$ and $\phi_{u_1}(r) = \phi_{u_2}(-r)$, and let the $\overline{d}_3$ and u_2 fields be represented by the e^+ and u_1 fields for $r < 0$. The Lagrangian, neglecting the Coulomb terms, is given by

$$\begin{aligned} L = \int_0^\infty dr \Bigg[& 2\pi\, r^2 h_5^2(r) \left(\dot{\alpha}^2(r) + \alpha'^2(r)\right) \\ & + \sum_{i=1}^{3} \Bigg\{ \frac{1}{2}\left(\partial_\mu \phi^i_{e^+}\right)^2 + \frac{1}{2}\left(\partial_\mu \phi^i_{\overline{d}_3}\right)^2 + \frac{1}{2}\left(\partial_\mu \phi^i_{u_1}\right)^2 + \frac{1}{2}\left(\partial_\mu \phi^i_{u_2}\right)^2 \\ & + \frac{c}{\pi} m^i_d(r) \hat{\nu}^i_d(r) \left(\cos(2\sqrt{\pi}\,\phi^i_{e^+} + \alpha) + \cos(2\sqrt{\pi}\,\phi^i_{\overline{d}_3} + \alpha)\right) \\ & + \frac{c}{\pi} m^i_u(r) \hat{\nu}^i_u(r) \left(\cos(2\sqrt{\pi}\,\phi^i_{u_1} - \alpha) + \cos(2\sqrt{\pi}\,\phi^i_{u_2} - \alpha)\right) \Bigg\} \Bigg] \end{aligned} \tag{5.4}$$

where the proper normal ordering (discussed in Section 4) is assumed, and we have summed over three fermion generations. (We have assumed the existence of just three generations, but the generalizations for more generations should be obvious.) Notice that the phase α appears with the opposite sign in the mass terms for the two $\mathrm{SU}(2)_{\mathrm{monopole}}$ doublets. This is because the mass terms for the two doublets come from different couplings to h.

In order to obtain a simple system from (5.4), we will make some rather severe approximations. We drop the $\alpha'^2(r)$ term and replace $\alpha(r)$ by $\alpha\theta(r_0 - r)$

(where α is no longer a function of r). Evidently, these approximations will tend to reduce the correlation between the phases of $h_5(r)$ at $r = 0$ and $r = \infty$. There is a separate motivation for studying the system described by (5.4). Suppose we consider the monopole of a theory which conserves baryon number. If this monopole has the same charge as the fundamental SU(5) monopole, and if we neglect the Coulomb terms and set $\alpha = 0$, its interaction with low energy fermions can be described by (5.4). This works because, for low energy fermions, the large masses at the core serve only to enforce the boundary conditions $\phi_i(0) = 0$, which are the only boundary conditions that conserve baryon number (modulo mixing between generations). The scattering processes implied by these boundary conditions are pure helicity flip ($u_R \rightarrow u_L$) so that weak isospin is not conserved. Thus, if we demand weak isospin conservation at the monopole core we must modify the boundary conditions (or mass terms in (5.4)). This is accomplished by including a nonzero α in (5.4) with the approximations mentioned above.

A further approximation that we will invoke will be to take the bosonized fermi fields as classical objects. This is justified because we are considering only low energy fermions so that the time scale of the variations of the ϕ_i fields is much smaller than the time scale ($\sim 1/r_0$) associated with α. Similarly, we will assume that the ϕ^i fields are constant over the range $(0, r_0)$. In this region we will denote h_5 by

$$h_5(r < r_0) = \frac{\sigma}{r_0} , \tag{5.5}$$

where $\sigma \sim \alpha/\lambda$ measures the tuning of the Higgs sector coupling constants. (σ is just the ratio of γ to the Yukawa coupling.) In order for the ordinary Callan-Rubakov effect to be suppressed, we will take $\sigma > 100$.

With these approximations, and the assumption that the fermion fields are in their ground state, (5.4) can be replaced by a simple effective Hamiltonian for the α dependent terms,

$$H_\alpha = \frac{3}{8\pi\, r_0 \sigma^2}\, J_\alpha^2 - \frac{c}{\pi} \cos\alpha \sum_i \int_0^{r_0} dr\, m^i(r) \hat{\nu}^i(r) \ , \tag{5.6}$$

where $J_\alpha = \frac{4\pi}{3} r_0 \sigma^2 \alpha'$, and the sum is now over all fermions. Since the top quark is much heavier than any of the other fermions, we can neglect the contribution to this sum from all the other fermions. The effective mass for the top quark is given by

$$m_t(r) = \begin{cases} \sigma\Gamma_t / r_0 & r < r_0 \ , \\ \sigma\Gamma_t / r & r > r_0 \ \ , \end{cases} \tag{5.7}$$

where $\Gamma_t \sim 0.1$ is the Yukawa coupling between the top quark and Weinberg-Salam Higgs. Since we have assumed that $\sigma > 100$, the assumption ($\sigma\Gamma = \gamma \ll 1$), we used previously in our calculation of $\hat{\nu}(r)$ does not hold. For $\sigma\Gamma_t > 1$ we obtain $\hat{\nu}_t(r) \approx 4cm_t(r)$ as one might expect from (4.19). Thus, (5.6) becomes

$$H_\alpha = \frac{3}{8\pi\, r_0 \sigma^2}\, J_\alpha^2 - \frac{(\sigma\Gamma^t)^2}{2\pi\, r_0} \cos\alpha \ . \tag{5.8}$$

For $\sigma \sim 100$ and $\Gamma_t \sim 0.1$, the potential term is a factor of 10^6 larger than the kinetic term, so we can neglect the latter. Thus, α need only appear in the mass term of our effective Lagrangian for the solitons,

$$\begin{aligned} L = \int_0^\infty dr \sum_{i=1}^{3} \Big[& \frac{1}{2}\left(\partial_\mu \phi^i_{e+}\right)^2 + \frac{1}{2}\left(\partial_\mu \phi^i_{\bar{d}_3}\right)^2 + \frac{1}{2}\left(\partial_\mu \phi^i_{u_1}\right)^2 + \frac{1}{2}\left(\partial_\mu \phi^i_{u_2}\right)^2 \\ & + \frac{c}{\pi} m^i_d(r) \hat{\nu}^i_d(r) \left(\cos(2\sqrt{\pi}\, \phi^i_{e+} + \alpha) + \cos(2\sqrt{\pi}\, \phi^i_{\bar{d}_3} + \alpha)\right) \\ & + \frac{c}{\pi} m^i_u(r) \hat{\nu}^i_u(r) \left(\cos(2\sqrt{\pi}\, \phi^i_{u_1} - \alpha) + \cos(2\sqrt{\pi}\, \phi^i_{u_2} - \alpha)\right) \Big] \ . \end{aligned} \tag{5.9}$$

This is equivalent to the original weak isospin violating Lagrangian with one additional degree of freedom which will allow baryon number violation. Previously, without the requirement of weak isospin conservation, we could summarize the effect of the large mass terms with the fermion boundary conditions $\phi_i(0) = 0$. Weak isospin conservation requires that we change these boundary conditions to $\phi_i(0) = \pm\alpha$ where the (+) sign is for the charge 2/3 quarks and the (−) sign is for the charge 1/3 quarks and leptons. (Note that the Coulomb terms play no role here; they only serve to duplicate the boundary conditions due to the mass terms.)

Qualitatively, it is easy to see what types of processes are allowed by (5.9). In addition to the helicity flip reaction, (5.1), which leaves α unchanged from its initial state ($\alpha = 0$), we can also allow α to change continuously from 0 to 2π. For one generation of fermions, this is just the usual type of baryon number violating reaction, (5.2), but for the phenomenologically relevant case of three fermion generations, we can see that every fermion must be produced. Thus, (5.2) is replaced by

$$u_{1R} \rightarrow \bar{u}_{2R}\bar{d}_{3L}e_L^+ \ \bar{c}_{1R}\bar{c}_{2R}\bar{s}_{3L}\mu_L^+ \ \bar{t}_{1R}\bar{t}_{2R}\bar{b}_{3L}\tau_L^+ \ . \tag{5.10}$$

As we could have anticipated for a weak anomaly induced effect, this is a $\Delta B = 3$ process involving all three generations of fermions.

Now, (5.10) violates weak isospin by an even greater amount that the helicity flip process (5.1) does, so we will have to construct a "fractional soliton" process in order to conserve weak isospin. The reaction that conserves weak isospin is

$$u_{1R} \rightarrow \frac{5}{6}\, u_{1L} + \frac{1}{6}\, \bar{u}_{2R}\bar{d}_{3L}e_L^+ \ \bar{c}_{1R}\bar{c}_{2R}\bar{s}_{3L}\mu_L^+ \ \bar{t}_{1R}\bar{t}_{2R}\bar{b}_{3L}\tau_L^+ \ . \tag{5.11}$$

We interpret this outgoing state just as we did in (5.3), as a quantum mechanical superposition of the outgoing states from (5.1) and (5.10). This scattering process is shown schematically in Fig. 2 For one fermion generation, the weak isospin conserving reaction is just the "ordinary" Callan-Rubakov process, (5.3), discovered by Callan[4] without postulating conservation of weak isospin. If we generalize our arguments to an arbitrary number of fermion generations, we can see that $\langle \Delta B \rangle = 1/2$ for a single incoming fermion.

We should note that this situation changes slightly (in the case where α represents the phase of the Higgs field), if we relax our approximation neglecting $\alpha(r)$ for $r > r_0$. Since the baryon number violating process (5.10) changes $\alpha(0)$ from 0 to 2π, we are left with a twist in the Higgs field. If the Hamiltonian (5.8) were not so dominated by the potential energy term, this twist would quickly decay via a quantum fluctuation (in which $h_5(0) \to 0$) or by tunneling through the potential barrier at $\alpha \sim \pi$. Instead, this twist would probably propagate out to a distance of the order of the weak breaking scale where it could manifest itself as a physical Higgs boson or decay through a quantum fluctuation or tunneling. While this possibility of Higgs field excitation does not change the fermion content of the possible final states (5.1) and (5.10) produced in fermion-monopole scattering, it could change the intermediate state (5.11). This is because we can now include a Higgs particle among our outgoing intermediate states, so that conservation of weak isospin does not imply a unique intermediate state. In the case where α does not represent the phase of the Higgs field, Sen[22] has shown that the Higgs field excitations are not important so that (5.11) should be valid.

Finally, we should make a few remarks about the real cross section for this weak anomaly induced baryon number violating reaction (5.10). If we wish to calculate this, we must, of course, include the effects of Cabibbo mixing between the generations. Unfortunately, our bosonization approach is poorly suited for this calculation, and we will not attempt it. Sen has done this calculation for massless fermions and has shown that, with Cabibbo mixing, processes involving only first and second generations of fermions are allowed. If fermion masses are included, it may be possible to obtain a process involving only light particles (no charmed quarks). In any event, this type of process would seem to have little effect on present day attempts at monopole detection through catalysis of baryon decay (either in proton decay experiments or in neutron stars) both because of the high powers of mixing angles involved and because only $\Delta B = 3$ processes are allowed.

6. CONCLUSIONS

Thus far, we have only considered fermions interacting with the fundamental monopole of the minimal SU(5) model, but since this model has many problems (such as being excluded experimentally), we should like to generalize our results to other, presumably more realistic, models. As we have seen in Section 2, our conclusions depend critically on coupling constants in the Higgs sector which are unlikely to be observed in the near future. We expect the situation to be essentially the same for non-SU(5) GUTs, as well, because the fine tuning that keeps the weak scale small is present for any GUT, and as in the SU(5) case, this fine tuning can be destroyed by the GUT monopole. The analysis should be essentially the same for any GUT with a monopole of the same charge as the SU(5) monopole. Thus, we expect that our conclusions are essentially independent of any details of the theory that manifest themselves only at the GUT scale.

The situation is quite different, however, when we consider the structure of the $SU(2)_W$ breaking sector of the theory. Since this sector of the standard model is not well understood, we are free to consider many extensions of the single Higgs doublet considered above. For example, several authors have imagined models with two Higgs doublets and an enhanced Higgs-fermion coupling.[25] In such a model, the Higgs boson which gives mass to the fermions has a vacuum expectation value of only about 25 GeV; the Yukawa couplings are increased by a factor of 10. Then, the Yukawa coupling to the down quark might be as large as a few times 10^{-4}, so that we could achieve a significant suppression of the Callan-Rubakov effect by tuning some of our Higgs sector parameters to less than one decimal place.

An even more interesting variation of the weak scale Higgs sector is the attempt by Bagger *et. al.*[26] to explain the fermion mass spectrum. We need not explore their ideas in detail here, but we will just note that in their model, the small Yukawa couplings to the light fermions are replaced by $\langle\phi\rangle / \langle\chi\rangle$ raised to a power that is roughly the number of fermion generations. $\langle\phi\rangle$ and $\langle\chi\rangle$ are the vacuum expectation values of additional Higgs fields, and in general, we can expect that these expectation values will change in the vicinity of a monopole. In fact, in the explicit example given by Bagger *et. al.* (for an SO(10) theory), both ϕ and χ get their vacuum expectation values in either the Y (weak hypercharge) or B−L directions. Since neither Y or B−L commutes with $SU(2)_{\text{monopole}}$, both $\langle\phi\rangle$ and $\langle\chi\rangle$ must change from their vacuum values at the monopole core. So, if $\langle\phi\rangle / \langle\chi\rangle$ is increased at the monopole core, then the effective Yukawa couplings to the light fermions would be increased as well, and we could have suppression of the Callan-Rubakov effect without any extra fine tuning.

Thus, if we consider alternatives to the minimal Grand Unified Theories, it seems quite possible that, under special circumstances, we can obtain a very large suppression of monopole induced baryon number violation. However, we are always at the mercy of the coupling constants in the Higgs sector. Finally, we must note that the process we have discussed here depends entirely on the disruption of the fine tuning which enforces the gauge hierarchy between the GUT scale and the weak scale. This fine tuning is usually regarded as the most unnatural aspect of Grand Unified Theories, so we might expect that this fine tuning will be explained by some other mechanism. In supersymmetric GUTs, this fine tuning is stable under radiative corrections, but since the relevant parameters must still be tuned at the tree level, the possibilities described herein still remain. In technicolor theories, however, there is no fine tuning at all, so monopole induced baryon number violation could not be suppressed by our mechanism. Each of these approaches to the solution of the fine tuning problem has its drawbacks, so we might expect that the real solution has yet to be proposed. In any event, it is difficult to say whether the suppression of the Callan-Rubakov effect described here will survive the solution to gauge hierarchy puzzle or not.

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APPENDIX

In this appendix, we will calculate some expressions necessary for the variational calculations of Section 4. Our trial functions will be the ground states of a free massive scalar field with a spatially varying mass term. The Hamiltonian density for this trial system is

$$\mathcal{H} = \frac{1}{2}\pi^2 + \frac{1}{2}\phi'^2 - \frac{1}{2}\nu^2(r)\phi^2 \ , \tag{A1}$$

and the ground state of this system is denoted by $|0, \nu(r)\rangle$. In order to perform the variational calculation we will need to calculate

$$E_0(\nu(r)) \equiv \langle 0, \nu(r)| \left(\frac{1}{2}\pi^2 + \frac{1}{2}\phi'^2\right) |0, \nu(r)\rangle \ ,$$

and the Wightman function at zero separation.

Initially, for simplicity, we will take $\nu(r)$ to be a step function:

$$\nu(r) = \nu + M\theta(r_t - |r|) \ , \tag{A2}$$

where $\nu \ll M, 1/r_t$. The Wightman function at zero spacetime separation is now given by

$$\Delta(0; \nu(r)) = \langle 0| \phi(t, r)\phi(t, r) |0\rangle$$

$$= \langle 0| \left[\iint_0^\infty \frac{dp\, dk}{2\pi\sqrt{2w_p}\sqrt{2w_k}} e^{-iw_p t}\Big(\varphi_+(k, r)a_+(k) + \varphi_-(k, r)a_-(k)\Big) \right.$$

$$\left. + e^{iw_k t}\Big(\varphi_+^*(k, r)a_+^\dagger(k) + \varphi_-^*(k, r)a_-^\dagger(k)\Big)\right] |0\rangle \ , \tag{A3}$$

where we have expanded $\phi(t,r)$ in terms of creation and annihilation operators, $a_i^\dagger(k)$ and $a_i(k)$, and dropped terms where $a_i(k)$ acts on $|0\rangle$. The functions, $\varphi_\pm(k,r)$, are just the orthogonal solutions of the field equations obtained from the Hamiltonian (A1). These are given by

$$\begin{aligned} \varphi_+(k,r) &= \begin{cases} Ae^{ikr} & r > r_t , \\ Be^{\alpha r} + Ce^{-\alpha r} & |r| < r_t , \\ De^{ikr} + Ee^{-ikr} & r < -r_t , \end{cases} \\ \varphi_-(k,r) &= \varphi_+(k,-r) , \end{aligned} \tag{A4}$$

where

$$M^2 = \alpha^2 + k^2 , \tag{A5}$$

and

$$\begin{aligned} A &= \frac{1}{1 + \frac{i}{2}\frac{M^2}{k\alpha}\sinh 2\alpha r_t} , \\ B &= e^{-\alpha r_t} e^{ikr_t} \frac{\alpha + ik}{2\alpha} A , \\ C &= e^{\alpha r_t} e^{ikr_t} \frac{\alpha - ik}{2\alpha} A , \\ D &= e^{2ikr_t} \left(\cosh 2\alpha r_t + \frac{i}{2}\left(\frac{\alpha}{k} - \frac{k}{\alpha}\right) \sinh 2\alpha r_t \right) A , \\ E &= -\frac{i}{2}\frac{M^2}{k\alpha} \sinh 2\alpha r_t \, A . \end{aligned} \tag{A6}$$

These solutions are normalized so that

$$\int_{-\infty}^{\infty} dr\, \varphi_i(k,r)\varphi_j^*(p,r) = 2\pi\, \delta_{ij}\delta(k-p) . \tag{A7}$$

Using the commutator

$$\left[a_i(k), a_j^\dagger(p)\right] = \delta_{ij}\delta(k-p) ,$$

we can obtain an integral expression for our Wightman function, (A3),

$$\Delta(0;\nu(r)) = \int_0^\infty \frac{dk}{4\pi\sqrt{k^2+m^2}}\Big(\varphi_+(k,r)\varphi_+^*(k,r) + \varphi_-(k,r)\varphi_-^*(k,r)\Big)\ . \tag{A8}$$

We will now evaluate (A8) for $r > r_t$ in the limit where $Mr_t \ll 1$ and $M^2 r_t \gg \nu$ which, as we have mentioned in Section 4, seems to be the most interesting case. Substituting for $\varphi_\pm(k,r)$ in (A8), we obtain

$$\Delta(0;\nu(r)) = \Delta(0;\nu) + I\ , \tag{A9}$$

where

$$\begin{aligned} I = \frac{1}{4\pi}\int_0^\infty \frac{dq}{\sqrt{q^2+\epsilon^2}} &\left(\frac{1}{q^2(1-q^2) + \frac{1}{4}\sinh^2 2\sqrt{1-q^2}\,z_t}\right) \\ \times &\Big(q\sqrt{1-q^2}\sinh 2\sqrt{1-q^2}\,z_t \cosh 2\sqrt{1-q^2}\,z_t \sin 2q(z-z_t) \\ &- \frac{1}{2}(1-2q^2)\sinh^2 2\sqrt{1-q^2}\,z_t \cos 2q(z-z_t)\Big)\ . \end{aligned} \tag{A10}$$

We have switched to the dimensionless variables $q = k/M$, $z_t = Mr_t$, $z = Mr$, and $\epsilon = \nu/M$. It is useful to break this integral up into two parts

$$I = I_0^1 + I_1^\infty\ , \tag{A11}$$

according to the sign of (q^2-1). In the interval $0 < q < 1$, we can use our assumption, $Mr_t = z_t \ll 1$, to simplify the integral

$$I_0^1 = \frac{1}{4\pi}\int_0^1 \frac{dq}{\sqrt{q^2+\epsilon}}\left(\frac{1}{q^2+z_t^2}\right)\left(2q\,z_t\sin 2qz + 2q^2z_t^2\cos 2qz - 2z_t^2\cos 2qz\right)\ . \tag{A12}$$

We will denote each term in the integrand by $I_0^1(i)$, $i = 1,2,3$. The last term,

$I_0^1(3)$, will give the largest contribution because it is infrared divergent for $\epsilon = 0$. (Recall that we have assumed $\epsilon \ll z_t$.) The integral of this term has the upper limit

$$I_0^1(3) \leq -\frac{2z_t^2}{4\pi} \int\limits_0^1 \frac{dq}{\sqrt{q^2+\epsilon^2}} \frac{1-2q^2z^2}{q^2+z_t^2} = \frac{1}{4\pi} \left(\log \frac{\epsilon^2}{4z_t^2} - 2z_t^2 z^2 \log z_t^2 \right) . \quad \text{(A13)}$$

The upper limit of the other two terms is given by

$$I_0^1(1) + I_0^1(2) \leq \frac{1}{4\pi} 4(z_t^2 + zz_t) \int\limits_0^1 \frac{qdq}{q^2+z_t^2} = -\frac{1}{4\pi} 2zz_t \log z_t^2 \, , \quad \text{(A14)}$$

where we have dropped a term proportional to $z_t^2 \log z_t^2$. Note that the inequalities in (A13) and (A14) become equalities for $z \ll 1$.

Finally, we must consider the integral I_1^∞. Changing variables again to $u = \sqrt{q^2-1}$, we obtain

$$\begin{aligned} I_1^\infty = \frac{1}{4\pi} \int\limits_0^\infty \frac{udu}{u^2+1} & \frac{\sin 2uz_t}{u^2(u^2+1) + \frac{1}{4}\sin^2 2uz_t} \\ & \times \left[u\sqrt{u^2+1} \cos 2uz_t \sin 2\sqrt{u^2+1}\, z \right. \\ & \left. + \left(u^2 + \frac{1}{2}\right) \sin 2uz_t \cos 2\sqrt{u^2+1}\, z \right] . \end{aligned} \quad \text{(A15)}$$

The upper bound on I_1^∞ is easily found to be

$$I_1^\infty \leq \frac{1}{4\pi} 2z_t \int_0^\infty du \; \frac{u\sqrt{u^2+1} + \left(u^2 + \frac{1}{2}\right)}{\left(u^2+1\right)^2}$$

$$\leq \frac{1}{4\pi} 2z_t \int_0^\infty du \, \frac{2}{u^2+1} \tag{A16}$$

$$= \frac{1}{4\pi} \left(2\pi z_t\right) .$$

So I_1^∞ makes essentially no contribution for $z_t \ll 1$.

Now, we can combine our results to obtain our expression for the zero separation Wightman function,

$$\Delta(0;\nu(r)) \leq \Delta(0;\nu) + \frac{1}{4\pi} \left[\log \left(\frac{\nu}{2M^2 r_t} \right)^2 - 2\left(M^2 r_t r + M^4 {r_t}^2 r^2\right) \log M^2 {r_t}^2 \right] . \tag{A17}$$

Using (4.11) and (4.12) we can obtain the cutoff Wightman function,

$$\Delta(0;\nu(r);\Lambda) \leq -\frac{1}{4\pi} \left[\log \left(\frac{2M^2 r_t}{\Lambda} \right)^2 + 2(M^2 r_t r + M^4 {r_t}^2 r^2) \log M^2 {r_t}^2 \right] . \tag{A18}$$

Although we have derived this formula only for $|r| > r_t$, it can be easily shown to hold for $|r| < r_t$ as well.

Next, we will calculate E_0. Expanding in terms of creation and annihilation operators for $|r| > r_t$, we obtain

$$E_0\left(\nu(r);\; |r| > r_t\right) = \int \frac{kdk}{4\pi} + \nu^2 \times \text{finite terms} . \tag{A19}$$

Since we are considering the case where ν is small we will keep only the leading term. The quadratic divergence will require a little bit of care to regulate (because an infinitesimal change in the cutoff, $\Lambda \to \Lambda + \epsilon$, can lead to a finite change in $\Lambda^2 \to \Lambda^2 + 2\Lambda\epsilon$). The correct procedure is to use point splitting. Strictly speaking, point splitting doesn't work for the quadratic divergence of (A19), but in a rigorous treatment this term would be replaced by space and time derivatives of a logarithmically divergent integral that can be regulated by point splitting. Another, simpler procedure that will give the same result is to cut off the integral at the same momenta that point splitting would cut it off. Thus, the quadratically divergent part of (A19) is given by

$$\int_0^\infty \frac{kdk}{4\pi} \to \frac{1}{4\pi} \int_0^\infty \cos(k/\Lambda)\, kdk \; .$$

So, we cut this off at $k/\Lambda = 1$, and we obtain

$$\int_0^\infty \frac{kdk}{4\pi} \to \frac{1}{4\pi} \int_0^\infty kdk = \frac{1}{8\pi} \Lambda^2 \; . \tag{A20}$$

Now for $|r| < r_t$, the cutoff will be different because the functions $\varphi_\pm$ will oscillate with a different wave number $p = \sqrt{k^2 + M^2}$. Thus, to use (A20), we must change variables from k to p, and the leading term is given by

$$\int_0^\infty \frac{kdk}{4\pi} = \int_{iM}^\infty \frac{pdp}{4\pi} = \frac{1}{8\pi} \left(\Lambda^2 + M^2\right) \; . \tag{A21}$$

Here the non-leading terms may be as large as M^2 so we should calculate them, too. The creation and annihilation operator expansion gives

$$E_0\left(\nu(r);|r|>r_t\right)=\int\frac{kdk}{4\pi}+M^2J\ ,$$

$$J\equiv\int\limits_i^\infty\frac{udu}{8\pi}\ \frac{\cos 2uz_t(\cos 2uz_t-\cos 2uz)}{u^4+u^2+\frac{1}{4}\sin^2 2uz_t}\ , \tag{A22}$$

where, as before, $u=\sqrt{k^2/M^2-1}$, $z=Mr$ and $z_t=Mr_t$. It is convenient to break J up into intervals,

$$J=J_i^{1/z_t}+J_{1/z_t}^\infty\ .$$

These integrals are easily evaluated (for small z_t):

$$J_i^{1/z_t}\approx(z^2-z_t^2)\int\limits_i^{1/z_t}\frac{udu}{4\pi}\ \frac{1}{u^2+(1+z_t^2)}=\frac{1}{4\pi}\left(z^2-z_t^2\right)\log\frac{1}{z_t^2}\ ,$$

$$J_{1/z_t}^\infty\approx\int\limits_{1/z_t}^\infty\frac{1}{16\pi}\ \frac{du}{u^2}=\frac{1}{32\pi}z_t^2\ . \tag{A23}$$

Combining Eqs. (A19) through (A23) we obtain

$$E_0\left(\nu(r)\right)=\frac{1}{8\pi}\left(\Lambda^2+\nu^2(r)\right)+\theta(r_t-|r|)\,\mathcal{O}\left(M^4{r_t}^2\log\left(Mr_t\right)\right)+\mathcal{O}\left(\nu^2\right)\ , \tag{A24}$$

for $\nu(r)$ given by (A2)).

REFERENCES

1. G. 't Hooft, *Nucl.Phys.* **B79** (1974) 276;

   A. M. Polyakov, *Pis'ma Zh. Eksp. Teor. Fiz.* **20** (1974) 430[*JETP Lett.* **20** (74) 194].

2. V. A. Rubakov, Inst. for Nucl. Res. preprint P-0211 (1981); *Nucl. Phys.* **B203** (82) 311.

3. C. G. Callan, Jr., *Phys. Rev.* **D26** (1982) 2058.

4. C. G. Callan, Jr., in *Problems in Unification and Supergravity,* (La Jolla Institute 1983) edited by G. Farrar and F. Henyey (AIP, New York, 1984).

5. E. W. Kolb, S. A. Colgate and J. A. Harvey, *Phys. Rev. Lett.* **49** (1982) 1373;

   S. Dimopoulos, J. Preskill and F. Wilczek, *Phys. Lett.* **119B** (1982) 320;

   V. A. Kuzmin and V. A. Rubakov, *Phys. Lett.* **125B** (1983) 372;

   E. W. Kolb and M. S. Turner, Fermilab-Pub-84/30-A (1984).

6. C. G. Callan, Jr., *Nucl. Phys.* **B212** (1983) 391.

7. M. A. Virasoro, *Phys. Lett.* **125B** (1983) 161;

   V. A. Rubakov and M. S. Serebryakov, *Nucl. Phys.* **B237** (1984) 329;

   N. S. Craigie, W. Nahm and V. A. Rubakov, *Nucl. Phys.* **B241** (1984) 274;

   W. Goldstein, SLAC-PUB-3272 (1983).

8. F. A. Bais, J. Ellis, D. V. Nanopoulos and K. A. Olive, *Nucl. Phys.* **B219** (1983) 189;

   V. A. Rubakov and M. S. Serebryakov, *Nucl. Phys.* **B218** (1983) 240;

S. Dawson and A. N. Schellekens, *Phys. Rev.* **D27** (1983) 2119;
B. Sathiapalan and T. Tomaras, *Nucl. Phys.* **B224** (1983) 491;
S. Rao, *Phys. Rev.* **D29** (1984) 2387.

9. H. Yamagishi, MIT preprint CTP-1139 (1984).
10. A. J. Buras, J. Ellis, M. K. Gaillard and D. V. Nanopoulos, *Nucl. Phys.* **B135** (1977) 66.
11. F. Buccella, H. Ruegg and C. A. Savoy, *Nucl. Phys.* **B169** (1980) 68.
12. C. P. Dokos and T. T. Tomaras, *Phys. Rev.* **D21** (1980) 2940.
13. P. Eckert, D. Altschüler and T. Schücker, *Phys. Lett.* **132B** (1983) 360; *Nucl. Phys.* **B231** (1984) 40;
P. Eckert, D. Altschüler, T. Schücker and G. Wanner, *Nucl. Phys.* **B226** (1983) 387.
14. S. Coleman, *Phys. Rev.* **D11** (1975) 2088.
15. S. Mandelstam, *Phys. Rev.* **D11** (1975) 3026.
16. M. B. Halpern, *Phys. Rev.* **D12** (1975) 1684.
17. E. Witten, *Phys. Lett.* **86B** (1979) 283.
18. J. A. Harvey, *Phys. Lett.* **131B** (1983) 104.
19. W. J. Marciano and I. J. Muzinich, *Phys. Rev. Lett.* **50** (1983) 1035.
20. E. D'Hoker and E. Farhi, *Phys. Lett.* **127B** (1983) 360.
21. A. N. Schellekens, *Phys. Rev.* **D29** (1984) 2378.
22. A. Sen, Fermilab-Pub-84/42-T and Fermilab-Pub-84/42-T-Add. (1984).
23. G. 't Hooft, *Phys. Rev. Lett.* **37** (1976) 8.
24. P. Nelson and S. Coleman, *Nucl. Phys.* **B237** (1984) 1.

25. F. Wilczek, *Phys. Rev. Lett.* **39** (1977) 1304;

H. E. Haber, G. L. Kane and T. Sterling *Nucl. Phys.* **B161** (1979) 493

26. J. Bagger, S. Dimopoulos, H. Georgi, and S. Raby, SLAC-PUB-3342 (1984).

TABLE 1

24 Self Couplings		Minimum Value of α
a	b	such that $M^2(0) < 0$
0	0	0
0	.1	$-.02\beta$
0	1	$-.05\beta$
0	100	$-.23\beta$
$-.4$	1	$-.14\beta$
-2	5	$-.18\beta$
1	1	$-.09\beta$
1	10	$-.11\beta$
5	1	$.56\beta$
10	1	1.02β
10	10	$.05\beta$

Note that $b > 0$, $a > -\frac{7}{15}b$ and $\beta < 0$ are required for the correct symmetry breaking pattern.

TABLE 2

	Boson type	r	Motion	Electric Charge	Helicity	Fermion Number
(a)	soliton	> 0	left	+	+	+
(b)	soliton	< 0	left	−	+	+
(c)	soliton	> 0	right	+	−	+
(d)	soliton	< 0	right	−	−	+
(e)	antisoliton	> 0	left	−	−	−
(f)	antisoliton	< 0	left	+	−	−
(g)	antisoliton	> 0	right	−	+	−
(h)	antisoliton	< 0	right	+	+	−

FIGURE CAPTIONS

1. This is the baryon number violating process $u_{1R}+d_{3L} \to \bar{u}_{2R}+e^+_L$ as described by soliton scattering on the full line. Fig. (a) shows the incoming solitons. In (b), the solitons are passing through the origin and begin to change their identity, and (c) shows the outgoing particles.

2. This figure shows the scattering of a u_{1R} quark, (a), off the monopole while imposing weak isospin conservation on the monopole core. (b) shows the weak isospin conserving intermediate state,

$$u_{1R} \to \frac{5}{6}u_{1L} + \frac{1}{6}\sum_i \sum_j \bar{u}^j_{1R}\bar{u}^i_{1R}\bar{d}^i_{3L}e^{+i}_L$$

where i runs over all generations and j runs over only heavy ones. (c) shows one possible final state, $u_{1L} \to u_{1R}$, with $\alpha = 0$ which is the only possible final state when the incoming quark has low energy. Finally, (d) shows the only possible baryon number violating process,

$$u_{1R} \to \bar{u}_{2R}\bar{d}_{3L}e^+_L\ \bar{c}_{1R}\bar{c}_{2R}\bar{s}_{3L}\mu^+_L\ \bar{t}_{1R}\bar{t}_{2R}\bar{b}_{3L}\tau^+_L\ ,$$

while α goes from 0 to 2π.

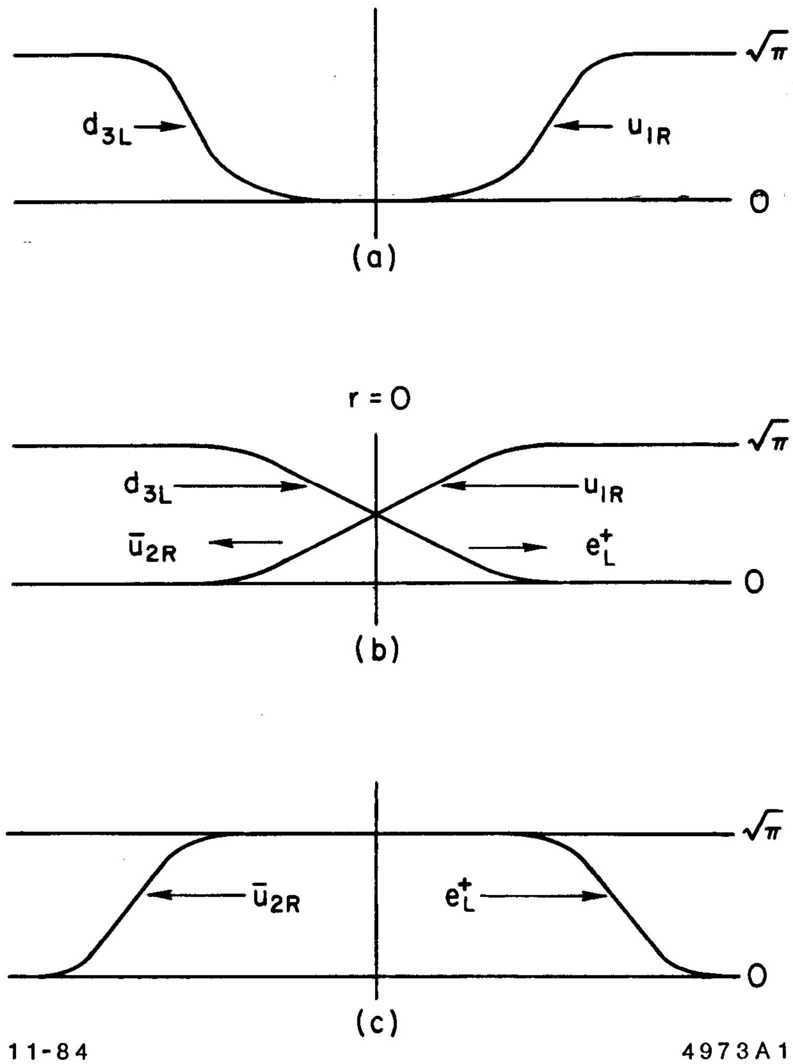
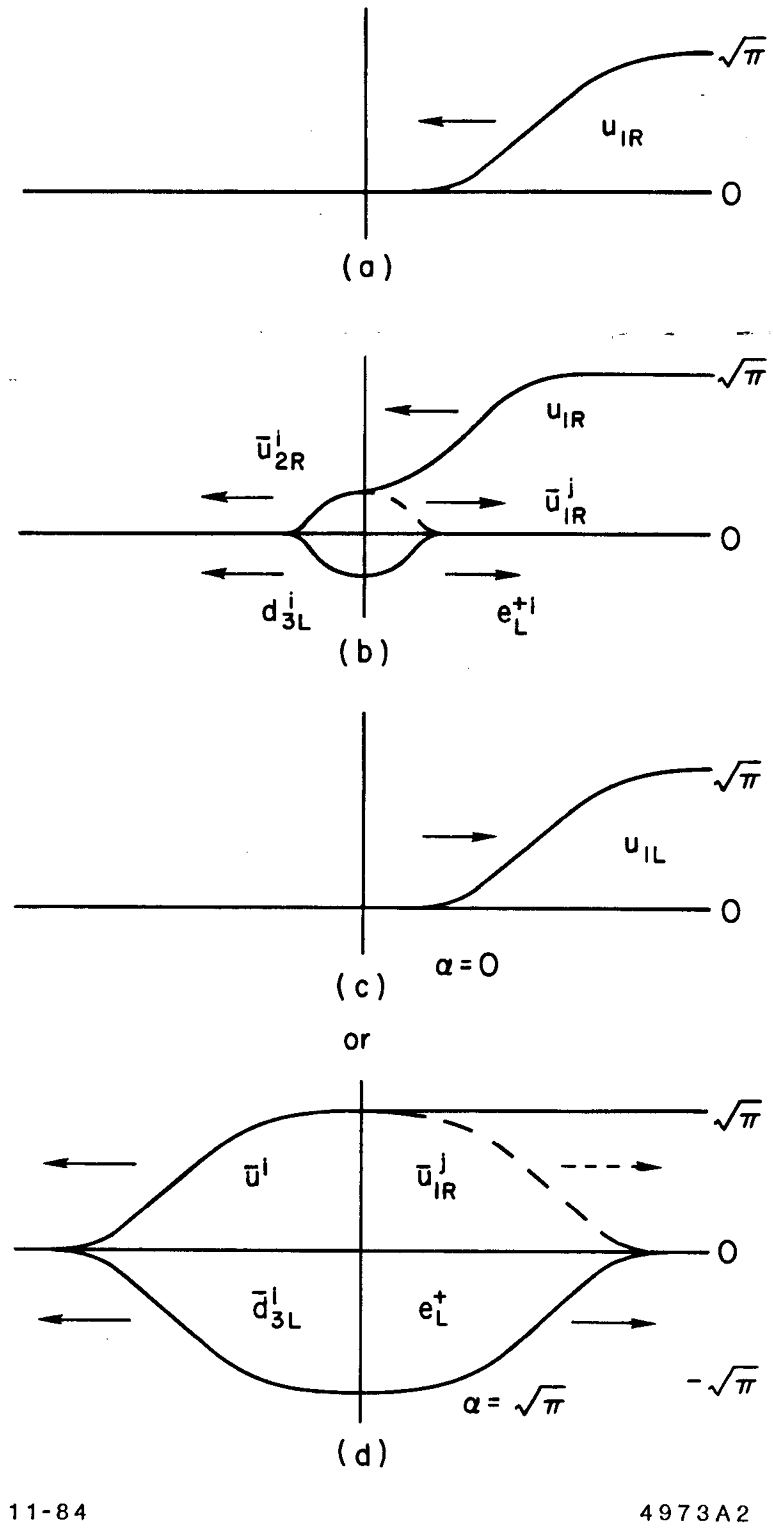


FIGURE 1



$\sqrt{\pi}$
u_{1R}
0
(a)
$\sqrt{\pi}$
u_{1R}
$\bar{u}^i_{2R}$
$\bar{u}^j_{1R}$
0
d^i_{3L}
e^{+i}_L
(b)
$\sqrt{\pi}$
u_{1L}
0
$\alpha = 0$
(c)
or
$\sqrt{\pi}$
$\bar{u}^i$
$\bar{u}^j_{1R}$
0
$\bar{d}^i_{3L}$
e^+_L
$-\sqrt{\pi}$
$\alpha = \sqrt{\pi}$
(d)
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FIGURE 2