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A NOTE ON CP-NONCONSERVATION IN $K \to \pi\pi^{\star}$

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ABSTRACT

A relation between absolute values of η_{00} and η_{+-} and their phases is studied. An inconsistency in the measured values is demonstrated and discussed.

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The CP-nonconservation in two-body decays of kaons still attracts a lot of attention. It is expected that once the theoretical and experimental uncertainties are put under control, an important test of a nonleptonic part of the standard model will be available. In this note relations between absolute values and phase angles of some characteristic parameters are studied. The analysis is done within the standard model with K-M mixing matrix, and by using the so-called "diagrammatic" approach[1]. (A similar analysis can be repeated in other models, or by using the more established approaches.)

With a notation for diagrams similar to the one used in Ref. 1, one can write the amplitudes of K decays in a form

$$\begin{aligned} \mathcal{A}(K^+) &= \frac{1}{\sqrt{2}}(A+B) \quad ; & \mathcal{A}(K^-) &= -\mathcal{A}(K^+) \\ \mathcal{A}(K^0_{+-}) &= (A+C+E+F)+i(P+Q) \quad ; & \mathcal{A}(\overline{K}^0_{+-}) &= -\mathcal{A}^*(K^0_{+-}) \\ \mathcal{A}(K^0_{00}) &= (-B+C+E+F)+i(P+Q) \quad ; & \mathcal{A}(\overline{K}^0_{00}) &= -\mathcal{A}^*(K^0_{00}) \end{aligned} \tag{1}$$

Seven real parameters $A, B, \ldots, Q$ correspond to amplitudes of generic diagrams in Fig. 1. P and Q are related to CP-nonconservation. The term "generic" denotes that only a representative skeleton graph (and not all possible gluonic and quark—see corrections that actually contribute to $A, B, \ldots$) is drawn. In addition to parameters in (1), which basically are on-shell amplitudes of corresponding diagrams, one must introduce a few more parameters "by hand". (These cannot be represented by on-shell diagrams.) One is a complex number α, appearing in

$$K^{L,S} = \frac{1}{\sqrt{2(1+|\alpha|^2)}}\left[(1+\alpha)K^0 \pm (1-\alpha)\overline{K}^0\right] \ , \tag{2}$$

while the other two are the phases of a final state interaction, δ_0 and δ_2. By identifying projections of amplitudes (1) to $\Delta I = 0, 2$ final states, one can find the probability amplitudes for decays of $K^{\pm}$, K^S and K^L. In a condensed notation they are given by

$$
\begin{aligned}
\mathcal{P}^{\pm} &= \pm\,\frac{3}{4}\,Y\left\{\rho\, e^{i\delta_2}\right\} \\
\mathcal{P}^{L}_{+-} &= NY\left\{\alpha\left[(1+\rho)+\frac{1}{2}\,\rho\, e^{i\Delta}\right]+i\tau\right\} \\
\mathcal{P}^{S}_{+-} &= NY\left\{\left[(1+\rho)+\frac{1}{2}\,\rho\, e^{i\Delta}\right]+i\alpha\tau\right\} \\
\mathcal{P}^{L}_{00} &= NY\left\{\alpha\left[(1+\rho)-\rho\, e^{i\Delta}\right]+i\tau\right\} \\
\mathcal{P}^{S}_{00} &= NY\left\{\left[(1+\rho)-\rho\, e^{i\Delta}\right]+i\alpha\tau\right\}\ .
\end{aligned}
\tag{3}
$$

Here

$$
\begin{aligned}
& Y=\sqrt{2}\,(C-B+E+F)\ ,\quad W=\frac{\sqrt{8}}{3}\,(A+B)\ ,\quad Z=\sqrt{2}\,(P+Q) \\
& N=e^{i\delta_0}\Big/\sqrt{1+|\alpha|^2}\ ,\quad \rho=\frac{W}{Y}\ ,\quad \tau=\frac{Z}{Y}\ ,\quad \Delta=\delta_2-\delta_0\ .
\end{aligned}
\tag{4}
$$

Clearly, W denotes the $\Delta I=2$ projection, while Y and Z are $\Delta I=0$ projections. The ratio τ is a measure of CP-violation. All parameters but α are real. From (3) one can now calculate partial decay rates. However, it is instructive to find first the rough magnitudes of parameters. From experimental values,[2] $\Gamma(K^{\pm})\approx 0.1\times10^{-16}\ GeV$, $\Gamma(K^S_{+-})\approx 2\Gamma(K^S_{00})\approx 50\times10^{-16}\ GeV$, $\Gamma(K^L_{+-})\approx 2\Gamma(K^L_{00})\approx 0.00025\times10^{-16}\ GeV$, one easily obtains

$$
|\rho|\sim 0.05\ ,\qquad |\alpha|\ ,\ |\tau|\lesssim 0.002\ . \tag{5}
$$

Note that relations (5) are based on the "order of magnitude" analysis and do not depend on precise values of measured decay rates. Therefore we can conclude that the choice of a nature is

$$1 \gg |\rho| \gg \rho^2 \ , \ |\alpha| \ , \ |\tau| \ . \tag{6}$$

Relations (3) and (6) are the key steps in the following analysis. Expression (3) contains the most general parameterization of probability amplitudes, while (6) is a phenomenological constraint on parameters. Since due to (6) one can safely neglect α, τ and ρ^2 when appearing by quantities of the order one, the amplitudes (3) and the expressions for related observables can be significantly simplified. Let me illustrate that in the evaluation of η_{00} and η_{+-}. Using the definition $\eta = \mathcal{P}(K^L)/\mathcal{P}(K^S) \equiv |\eta| \exp(i\vartheta)$, one can easily calculate from (3) (and neglecting double suppressed terms) the absolute values and phases of η. It comes out that the results can be expressed most suitably in terms of a new angle Φ, and a "typical length", L, defined by

$$tg\,\Phi = \frac{\mathrm{Im}\,\alpha + \tau}{\mathrm{Re}\,\alpha} \ ; \qquad L = \sqrt{(\mathrm{Im}\,\alpha + \tau)^2 + (\mathrm{Re}\,\alpha)^2} \ . \tag{7}$$

With (7), it follows

$$\begin{aligned}
|\eta_{+-}| &= L\left\{1 - \frac{\rho\tau}{L}\left[\sin\Phi - \frac{1}{2}\sin(\Delta - \Phi)\right]\right\} \\
|\eta_{00}| &= L\left\{1 - \frac{\rho\tau}{L}\left[\sin\Phi + \sin(\Delta - \Phi)\right]\right\} \\
tg\,\vartheta_{+-} &= tg\,\Phi\left\{1 - \frac{\rho\tau}{L\cos\Phi\sin\Phi}\left[\cos\Phi + \frac{1}{2}\cos(\Delta - \Phi)\right]\right\} \\
tg\,\vartheta_{00} &= tg\,\Phi\left\{1 - \frac{\rho\tau}{L\cos\Phi\sin\Phi}\left[\cos\Phi - \cos(\Delta - \Phi)\right]\right\} \ ,
\end{aligned} \tag{8}$$

and also

$$S = \frac{1}{3} tg\,(\vartheta_{00} - \vartheta_{+-}) = \frac{1}{2}\frac{\rho\tau}{L}\cos(\Delta - \Phi)$$
$$R = \frac{1}{6}\left(1 - |\eta_{00}/\eta_{+-}|^2\right) = \frac{1}{2}\frac{\rho\tau}{L}\sin(\Delta - \Phi) = S\,tg\,(\Delta - \Phi)\ . \tag{9}$$

In (8) and (9) the terms of the order ρ^2 were neglected. R is an observable usually denoted as $\mathrm{Re}\,(\epsilon'/\epsilon)$. Note that four observables in (8) are described in terms of four parameters, L, $(\rho\tau/L)$, Φ and Δ. However, the parameter $\Delta = \delta_2 - \delta_0$ can be determined from experiments not directly measuring the CP-nonconservation. (One can e.g. use decay rates of $K^{\pm}$ and K^S.[3] Alternatively, Δ can be obtained from a measurement of some scattering amplitudes; see Ref. 4.) Therefore ϑ_{+-} and ϑ_{00} in (8) really depend on only two unknown parameters, $(\rho\tau/L)$ and Φ. Conversely, the measurement of ϑ_{+-} and ϑ_{00} determines uniquely these two parameters, and thus also the quantity R in (9). In other words, if the phase difference is large enough, one can indirectly learn the value of $\mathrm{Re}(\epsilon'/\epsilon)$, which is quite interesting property.

Experimental results are as follows: $\vartheta_{+-} = (44.7 \pm 1.2)^\circ$, $\vartheta_{00} - \vartheta_{+-} = (10 \pm 6)^\circ$ (from Ref. 5); $\Delta = (-45 \pm 10)^\circ$ (Ref. 4); $|R| \leq 0.015$ (Ref. 6). So, the phase difference is really large. However, it will be shown now that there is an inconsistency in the above results which prevents one from reaching firm conclusions. Let us assume first that $\vartheta_{00} - \vartheta_{+-}$ difference is at least 4° (as suggested by Ref. 5), and $\vartheta_{+-} = 44.7^\circ$. Then, from Eqs. (8) and (9) one can deduce that only for $-165^\circ < \Delta < -105^\circ$ and $+15^\circ < \Delta < +85^\circ$ the present bound on $|R|$ can be satisfied. If $\vartheta_{00} - \vartheta_{+-}$ difference comes out to be larger, and/or the value for $|R|$ smaller, the set of allowed value for Δ is even more restricted. It is clear that the measured value for Δ (Ref. 4) is not in the "allowed" interval.

One can conversely use $\Delta = (-45 \pm 10)^\circ$ and determine the bounds on $|\vartheta_{00} - \vartheta_{+-}|$. From (8) and (9) then follows that $|R| \leq 0.015$ is satisfied only if $|\vartheta_{00} - \vartheta_{+-}| \lesssim 0.5^\circ$. Therefore the difference $\vartheta_{00} - \vartheta_{+-}$ should be at least an order of magnitude smaller than the value presently quoted. Consequently, although in principle the ratio $|\eta_{00}/\eta_{+-}|$ can be close to one, and the phase difference still large, it is clear from (8) that Δ in the fourth quadrant doesn't allow such a situation.

In conclusions, an analysis which shows a close relationship between $\mathrm{Re}(\epsilon'/\epsilon)$ and the phase difference $\vartheta_{00} - \vartheta_{+-}$ is presented. The obtained relationship can have practical consequences only if the phase difference is significant. Experimentally, this difference is indeed large, but from Eqs. (8) and (9) it follows that the large $\vartheta_{00} - \vartheta_{+-}$ is not consistent with the measured value of Δ. There is no doubt that future experiments can clarify the situations. However, if the phase difference $\vartheta_{00} - \vartheta_{+-}$ repeatedly comes out to be larger than a degree or two, a new interesting problem will arise in physics of CP-nonconservation.

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FIGURE CAPTIONS

1. Generic diagrams in $K \to \pi\pi$ decays. For simplicity, only valence quark lines are drawn. Implicitly, however, amplitudes $A, B, \ldots,$ represent sums of all possible diagrams with the same skeleton.

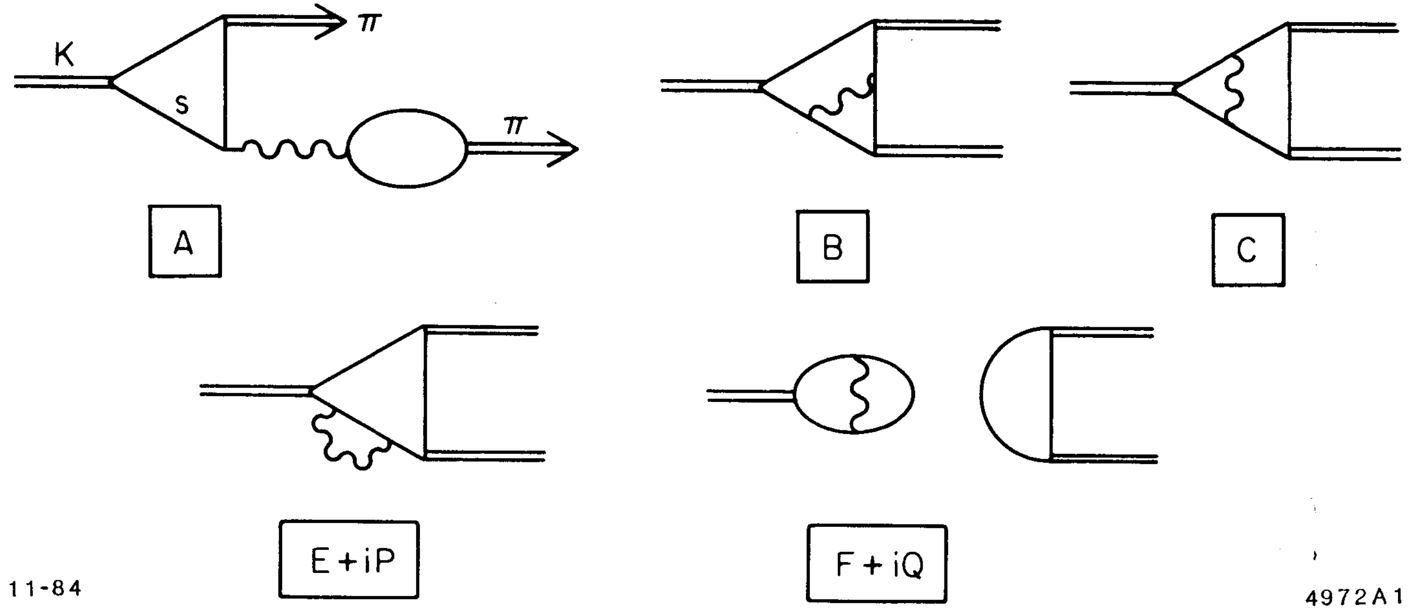

Fig. 1