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PHYSICS OF (VERY) HIGH ENERGY e^+-e^- COLLIDERS*

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ABSTRACT

I review the physics capabilities of e^+e^- colliders of hundred GeV to TeV center-of-mass energies, emphasizing issues relevant the physics of symmetry breaking in the weak interactions.

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This lecture concerns the physics possibilities of TeV-energy e^+-e^- colliding beam accelerators. At the moment, the construction of such devices seems to be beyond our technical capabilities. However, refinement of the methods for the production and collision of intense bunches of electrons and positrons being developed for the SLC project, combined with a continuation of the recent progress in the technology of high-gradient linacs, may well produce a feasible design within the next few years. It is then appropriate to ask whether the physics interest in such a device would justify its construction. One could, on the one hand, point to the glorious history of e^+-e^- colliders in uncovering new quarks and leptons, exploring families of narrow resonances, and testing the detailed structure of 3-jet final states. I will approach the question, however, from a somewhat different point of view: In the past decade, the mysteries of elementary particle physics have narrowed to a small number of essential questions whose solution will probably require new experimental information. As I will discuss at some length, the most important experiments are also those best done in the environment of e^+e^- annihilation, since the crucial effects appear there as a large fraction of the total cross section and with a clarity which makes visible their telling details.

What is the question we must answer, and how can we confront it experimentally? Experiments over the past ten years have made clear that the strong, weak, and electromagnetic interactions are described, at least at the energies we can now access, by a gauge theory of the group $SU(3) \times SU(2) \times U(1)$. This structure explains the couplings of quarks and leptons to the gluons and to the weak interactions. However, this synthesis leaves two matters unexplained. The first is the question of the mass spectrum of quarks and leptons. The second is the question of the origin of the weak boson masses, the question, that is, of the mechanism by which $SU(2) \times U(1)$ is spontaneously broken. In the unified gauge theory, the mass terms for quarks and leptons violate $SU(2) \times U(1)$, so that these masses can only be produced by the intervention of the field which breaks this symmetry. Because of this, the solution to the first of these problems itself depends on the resolution of the second. It is crucial, then, that we learn something of the nature of the forces which cause the breaking of $SU(2) \times U(1)$. At the moment, we know nothing about these forces from direct observation. The main piece of information that we have comes simply from examining the mass formula for the W bosons and removing a power of the $SU(2)$ gauge coupling g^2. Since the value of this coupling is known, we can infer the mass scale of the symmetry-breaking forces:

$$\langle \phi \rangle = 240 \text{ GeV} . \tag{1}$$

In the simplest model of this symmetry breaking, $\langle \phi \rangle$ is the vacuum expectation value of a single elementary scalar Higgs field; this quantity may have a more complicated interpretation in other models. But eq. (1) serves to set the scale for our future investigations: when we reach center-of-mass energies (for parton-parton collisions) of some fraction of a TeV, we should be able to create the entities which make up this new sector of forces and thus resolve this central puzzle.

What processes will give us information about this new sector? First of all, if this sector has a structure more complex than the simplest scheme with only one Higgs doublet field, we should expect to see new particles. Most of these will have masses of order 1 TeV. However, many explicit models of this new sector contain new particles with interesting quantum numbers and masses of only a few hundred GeV, smaller than the TeV scale by one or two powers of small gauge couplings. Any of these particles which carry electromagnetic or weak charge should be visible in e^+e^- annihilation experiments. Secondly, one should expect to find information about the new sector from the reactions of the particles which couple to it most directly—the W and Z bosons. One can then learn about this sector from detailed studies of the W and Z production cross-sections. Thirdly, if the new sector involves nonperturbative physics, one should expect these new strong interactions to produce resonances and other dramatic effects as one enters the region of TeV energies. In this lecture, I will discuss these three types of experiments in turn; I will describe, for each case, what new effects might be observed in e^+-e^- collisions.

Many of the effects I will discuss may be accessed also, though often in a more limited way, in p-p collisions. I do not have space to give a detailed comparison of the strengths of e^+-e^- and p-p colliders as tools for exploring the TeV region of energies. Such a comparision is hardly appropriate anyway, because the technology for producing p-p collisions with TeV energies in elementary subprocesses is already available and is, in fact, already the basis for the design of a major new accelerator project. A detailed study of p-p reactions which probe TeV physics has recently been made by Eichten, Hinchliffe, Lane, and Quigg.[1] For most of the processes I will deal with, a discussion of the analogous p-p processes may be found there. I would like to say something, however, about how to compare e^+-e^- and p-p center-of-mass energies in terms of the power of these colliders to search for new particles. I have included an analysis of this question, structured along lines suggested by Ellis,[2] as an appendix to this article.

My discussion will contain one quirky feature which I should warn you of at the outset. The general scale of the rates of all interesting processes in e^+e^- annihilation is set by the size of the cross-section for the electromagnetic annihilation of e^+e^- to $\mu^+\mu^-$:

$$1 \text{ R unit} \equiv \frac{4\pi\alpha^2}{3s} = \sigma(e^+e^- \to \gamma^* \to \mu^+\mu^-) \tag{2}$$

This unit is not large even at present energies and, as eq. (2) already indicates, it drops precipitously as the center-of-mass energy is increased. In convenient units, its value as a function of energy is:

$$1 \text{ R unit} = \frac{86.8\,\text{fb}}{(E(\text{TeV}))^2} \,. \tag{3}$$

Most cross-sections in high-energy e^+e^- annihilation remain fixed in R units as s is increased. As an illustration of this, I have plotted in Fig. 1 the total cross section for e^+e^- annihilation into quark pairs as a function of energy (assuming only the conventional six flavors), computed in the standard model. This cross-section is predicted to be essentially flat in units of R, except for the enormous enhancement in the immediate vicinity of 90 GeV caused by the Z^0 resonance. Throughout this lecture, I will quote all cross-sections in R units. This method of presentation has the advantage that one sees immediately the magnitude of the process being considered relative to the total cross-section for e^+e^- annihilation. It also emphasizes the fact that appropriate event sample sizes are given, not as a fixed number of nanobarns, but, rather, as a fixed number of R units. e^+-e^- luminosities must increase as the energy increases. (How, technically, one does this is a problem which I encourage you to worry about.) In Fig. 2, I have plotted the luminosity required to produce 1 unit of R per day, as a function of energy. Our present e^+-e^- colliders lie well above this curve; future colliders must also, if they are to have sufficient luminosity to do the physics.

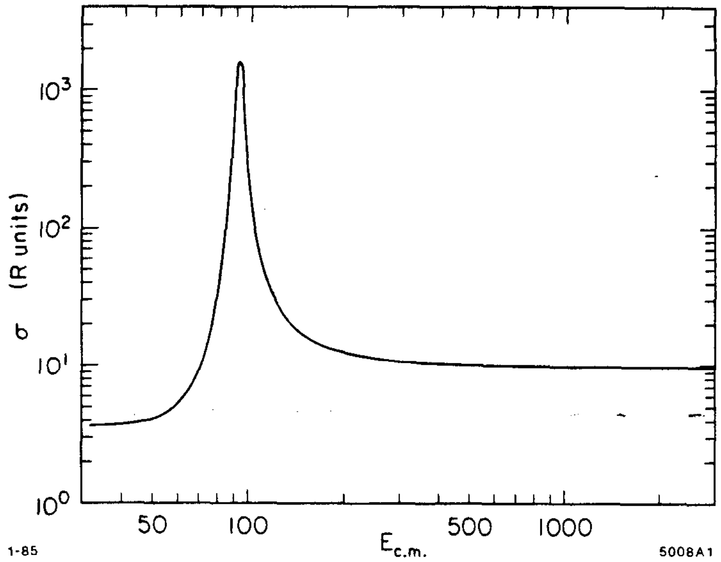

Fig. 1. Total cross-section for $e^+e^- \rightarrow$ quark pairs, in R units, as a function of energy

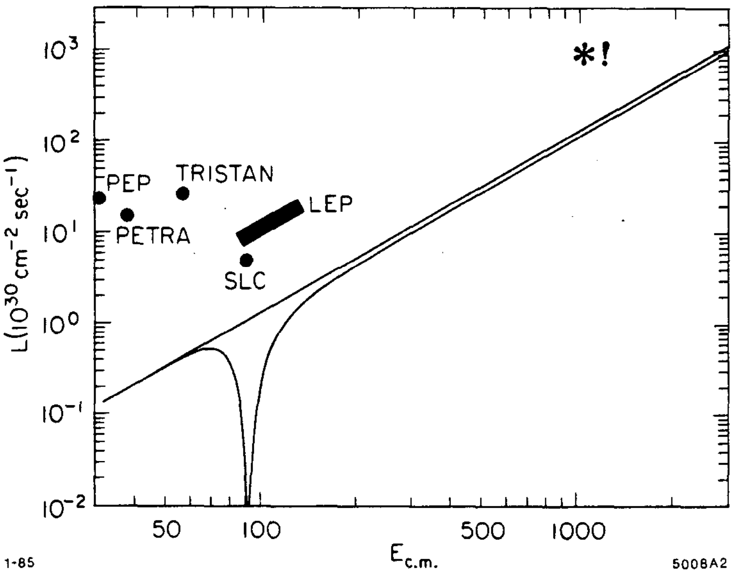

Fig. 2. e^+-e^- luminosity in $cm^{-2}sec^{-1}$ equivalent to 1 R unit/day. The luminosities of various present and proposed colliders are compared to this standard.

A Hint at the Z^0

My first topic will be the ability of high-energy e^+-e^- colliders to search for the novel heavy particles predicted by theories of the TeV-energy sector. Before beginning this discussion, however, I would like to say a few words about e^+-e^- collisions at a modest energy that we will reach very soon—the energy of the Z^0 resonance. The physics of the Z^0 itself has been discussed in great detail by physicists looking forward to experimentation at the SLC and LEP; a particularly fine review has been presented by Dorfan.[3] The ability of experiments at the Z^0 to explore for new physics in the mass region up to $M_Z/2$ is by now well known. What is not so well appreciated is that we will also find at the Z^0 our first hint of new physics in the mass region of several hundred GeV. Let me, then, digress briefly to explain how this can be done.

The production of lepton pairs at the Z^0 is a purely electroweak process; it involves quarks and gluons only a marginal way, in radiative corrections of the form of vacuum polarization diagrams. Thus, the production cross-sections for lepton pairs may be calculated with great precision. Certain parameters of these production cross-sections may also be measured very precisely. By comparing theory and experiment, one might hope to detect experimentally the effects on these parameters of weak-interaction radiative corrections.[4] When one reaches this level, one is sensitive not only to the effects of loops containing W and Z bosons but also to the effects of loops containing new heavy particles. I will outline the situation for the quantity most sensitive to these new effects, the polarization asymmetry in lepton pair production on the Z^0 resonance. (A more detailed analysis of this parameter may be found in ref. 5.)

The polarization asymmetry in lepton pair production is defined to be:

$$A_{\rm pol} = \left.\frac{\sigma(e^-(L)e^+ \to \ell^-\ell^+) - \sigma(e^-(R)e^+ \to \ell^-\ell^+)}{\sigma(e^-(L)e^+ \to \ell^-\ell^+) + \sigma(e^-(R)e^+ \to \ell^-\ell^+)}\right|_{s=m_Z^2} \tag{4}$$

Alternatively, it may be measured as the polarization of τ's produced from unpolarized electrons. In terms of the parameters of the electroweak theory, $A_{\rm pol}$ is given by:

$$A_{\rm pol} = \frac{-8(sin^2\theta_w - \frac{1}{4})}{(1 + (4sin^2\theta_w - 1)^2)^2} + \delta A_{\rm pol}, \tag{5}$$

where the first term is the zeroth-order result. The radiative corrections to $A_{\rm pol}$ in the standard model have been calculated by Lynn and Stuart.[6] In order to evaluate this prediction, one of course needs to know the value of $sin^2\theta_w$ to high accuracy. That value may be obtained from the measurement of the Z^0 mass. To leading order, $sin^2\theta_w$ is given by:

$$sin2\theta_w = \left(\frac{4\pi\alpha}{\sqrt{2}G_\mu M_Z^2}\right)^{\frac{1}{2}}, \tag{6}$$

where G_μ is the muon decay constant. From this equation, we can see that measurement of the Z^0 mass to 100 MeV allows one to determine $sin^2\theta_w$ to $\pm 4 \times 10^{-4}$. A figure of merit for the measurement of $A_{\rm pol}$ is that it gives $sin^2\theta_w$ to comparable accuracy, assuming that $A_{\rm pol}$ can be determined to 1% of its value, ± 0.003 in absolute terms. Both the factor 8 in the tree-level expression and the fact that $sin^2\theta_w$ happens to be close to $\frac{1}{4}$ are helpful in achieving this accuracy.

A measurement of $A_{\rm pol}$ to ± 0.003 looks quite interesting in terms of its sensitivity to new physics. As an example, I show in Fig. 3 the extra contribution to $\delta A_{\rm pol}$ which would arise from the existence of a single additional generation of quarks or of leptons; for this plot I have, pessimistically, chosen the masses of the two members of the isospin doublet to be equal. It is remarkable that the effect of heavy fermions on $A_{\rm pol}$ does not decrease as the fermions are taken to be heavier. Other types of new phyiscs give comparable contributions to $\delta A_{\rm pol}$; many of these are displayed in ref. 5.

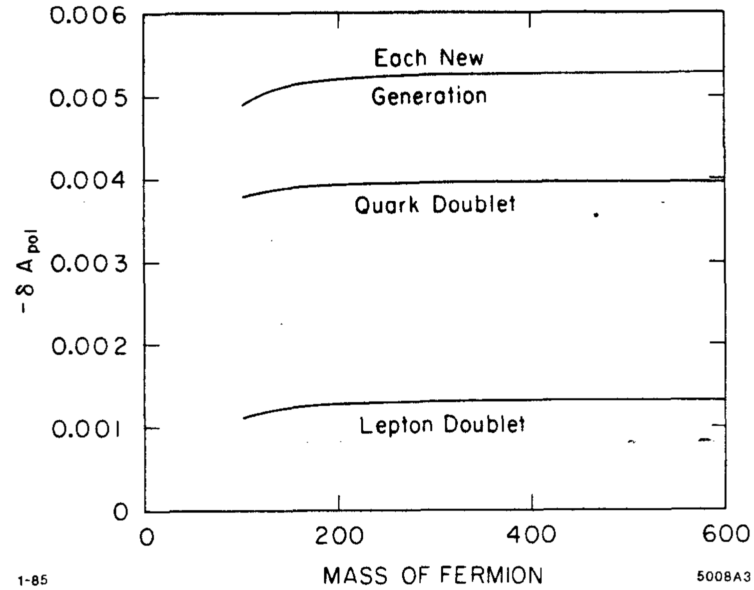

Fig. 3. Correction to the polarization asymmetry at the Z^0 resulting from each new heavy generation of quarks or leptons. These curves assume that the two members of each isodoublet have the same mass.

New Particle Production

Now let us jump in energy up to the region of many hundreds of GeV in the center of mass. Our first concern in this region will be the serach for new types of elementary particles. It is by now widely accepted that e^+e^- annihilation provides an ideal environment for the search for new particles of mass almost up to $\sqrt{s}/2$. The point is such an important one, however, that, despite their familiarity, I should present the basic arguments again here.

The most important reason to use e^+e^- annihilation as a tool in searching for new particles is that the cross-sections for production of new and old species in this reaction are roughly comparable. I have pointed out already that the production rates for the various quark and lepton species are all given as some finite number of R units. This is also true for new types of particles. Up to the effect of phase space, a suppression by β^3 for production of bosons and by $\frac{1}{2}\beta(3-\beta^2)$ for fermions, the production rates are determined entirely by the electromagnetic and weak quantum numbers. To give an idea of the democracy of rates, let me tabulate the production cross-sections, in R units, for a variety of particles, assuming $s \gg M_Z^2$ and $sin^2\theta_w = 0.22$:

Species	Rate (R units)
ℓ	1.15
ν	0.53
u	1.63
ℓ	1.59
. . .	
charged fermion	$\sim Q^2 \cdot N_c$
charged boson	$\sim \frac{1}{4}Q^2 \cdot N_c$
pseudo-Goldstone boson	$\frac{1}{8}[Q + 0.81(I^3 - 2Qsin^2\theta_w)]^2 + \frac{1}{8}[Q - 0.64(I^3 - 2Qsin^2\theta_w)]^2$
. . .	

One should also keep in mind that new particles will not necessarily appear only one at a time. In most models of supersymmetry, the scalar supersymmetric partners of all the leptons and (separately) all the quarks (except t) should appear within a GeV of one another. In models of technicolor with rich spectra of technipions (for example,the model of ref. 7), the various ordinary and exotic new bosons appear according to a predicted pattern, shown in Fig. 4.

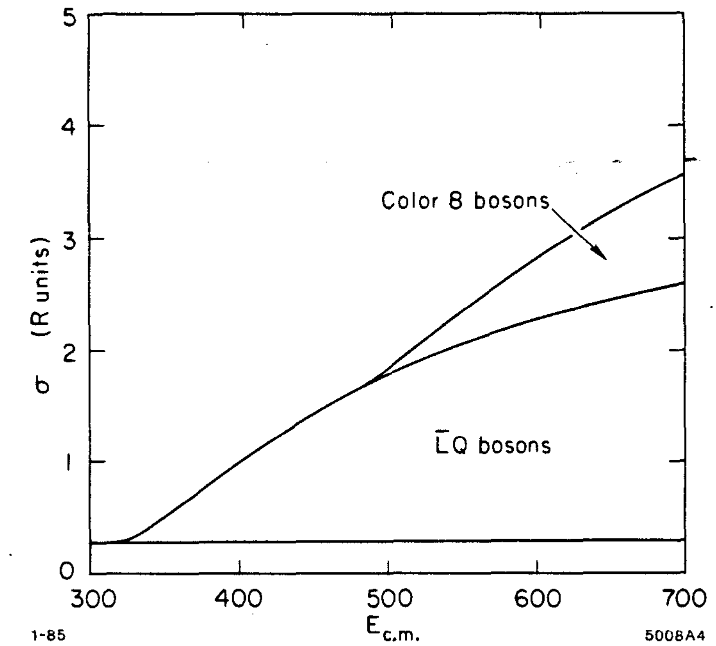

Fig. 4. Pair-production rates for various types of technipions, as a function of $\sqrt{s}$, in the model of ref. 7.

In addition to giving large rates for new particle production (at least, as a proportion of the total cross-section), e^+e^- annihilation offers a very clean environment in which events of novel topology are readily detectable. Particle searches at e^+-e^- colliders of the current generation, PEP and PETRA, have shown that relatively simple cuts against two-lepton and two-jet final states can produce data samples virtually free of background, in which new particle signatures, if they existed in this region, would stand out clearly. Particularly effective in eliminating conventional backgrounds are acoplanarity cuts, cuts which favor the topology of a single μ or τ recoiling against a hadron jet, and cuts which favor pure lepton and photon final states. In addition, since jets produced in e^+e^- annihilation tend to be narrow and relatively isolated, one can recover some information by measuring the jet invariant mass. Figure 5 shows the distribution of invariant masses measured by the MAC experiment at PEP, using the rather crude definition that a jet consisted of all hadrons in a given hemisphere defined relative to the thrust axis. This invariant mass distribution peaks below 5 GeV and is, in fact, sufficiently narrow to allow one to select for bottom quarks on the basis of this invariant mass alone.

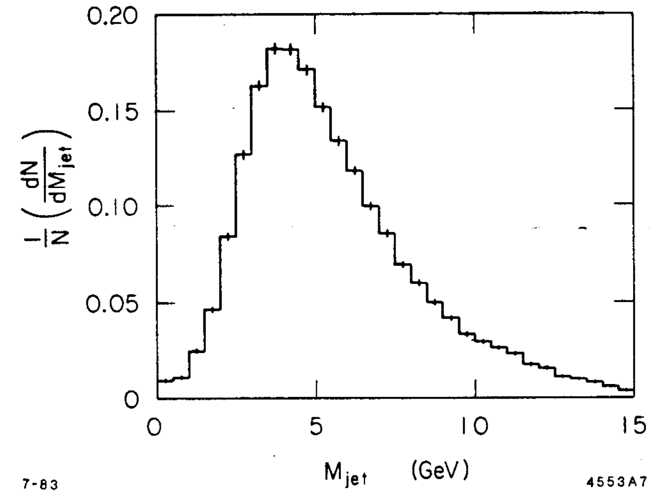

Fig. 5. Jet invariant mass distribution in e^+e^- annihilation at $\sqrt{s} = 29$ GeV, as measured by the MAC experiment; from ref. 8.

A Peripheral Process

In considering the backgrounds to new particle searches in e^+e^- annihilation, I have so far considered only those coming from other reactions in which the electron and positron annihilate. Such backgrounds are associated with processes whose cross-sections decrease with energy in the same fashion as the new production cross-sections; thus, their effects can safely be estimated from our present experience. There is another process present in e^+-e^- collisions, however, which does not fall off with energy—the 2-photon process. It is worth a digression to ask whether, as s increases, this can ever be a significant background to new particle searches. In brief, the answer is no. Figure 6 shows the total cross-section for the production of quark pairs by the 2-photon process under the constraint that the transverse momentum of each quark jet is greater than a given fraction f of the available energy $\frac{1}{2}\sqrt{s}$. Apparently, such a transverse momentum cut does not have to be especially stringent to reduce the background from 2-photon processes to the level of the background from annihilation processes. Remember that in neither case does this background, which is basically composed of 2-jet events, at all resemble the signatures one is trying to find. The 2-photon process could, in principle, also interfere with experiments in another way, by producing a high rate of irrelevant events in a detector with large angular coverage. The analogous effect in p-p reactions is very much a worry, since there the peripheral cross-sections are of strong-interaction magnitude: at TeV energies, they can be tens of millibarns even when a transverse momentum cut is applied. The 2-gamma total cross-section, defined by some weak transverse momentum cuts, is shown in Fig. 7; it is at the level of nanobarns.

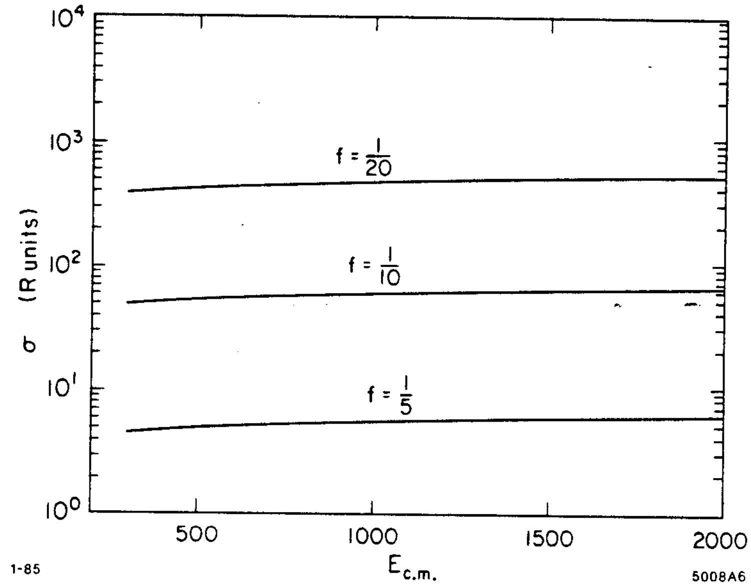

Fig. 6. Cross-sections for production of quark pairs from e^+e^- via the 2-photon process, under the constraint that the transverse momentum of each quark is greater than a given fraction f of its maximum value $\frac{1}{2}\sqrt{s}$. The cross-sections are quoted in R units.

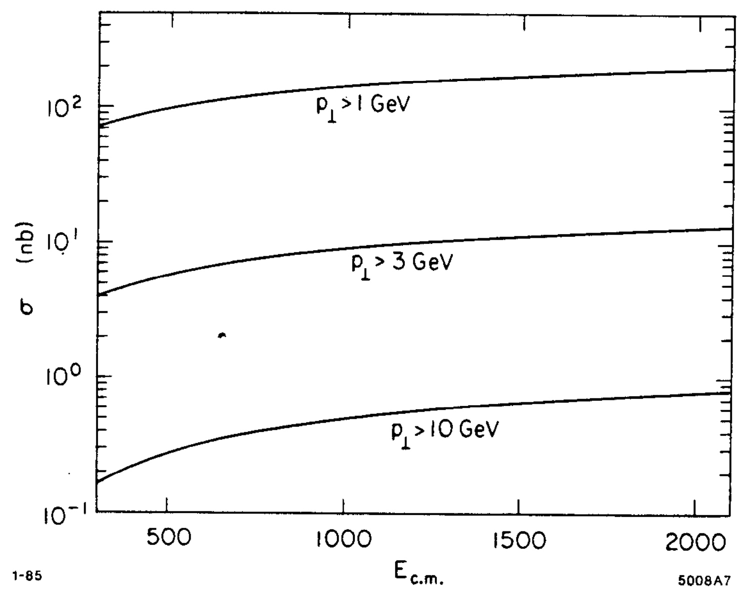

Fig. 7. Cross-sections for production of quark pairs from e^+e^- via the 2-photon process, under the constraint that the transverse momentum of each quark is greater than a given fixed value. The cross-sections are quoted in nanobarns.

W, Z, and Higgs Reactions

Up to this point, we have focussed on the general question of searches for new particles; we have seen that the conventional wisdom that e^+-e^- colliders are excellent devices for particle searches should still hold true at TeV center-of-mass energies. I would like to move now to a detailed consideration of a more specific set of reactions which bear directly on the central questions of weak-interaction symmetry-breaking which I emphasized at the beginning of this lecture. These are the reactions which involve real W and Z production; these reactions are interesting because they probe directly the couplings of the W and Z with the Higgs boson.

I should begin by noting that, simply in terms of absolute rate, the process $e^+e^- \to W^+W^-$ is the most important process in e^+e^- annihilation at very high energies. The total rate is shown in Fig. 8; it rises rapidly from threshold to a value of 10 units of R and continues to rise (in R units), comprising 30 units of R at $\sqrt{s} = 1$ TeV. The form of

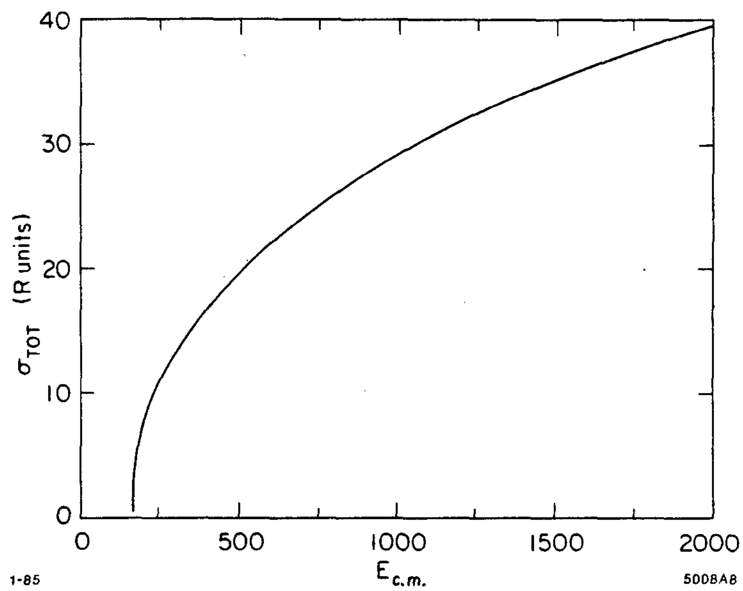

Fig. 8. Total cross-section for the reaction $e^+e^- \to W^+W^-$, in units of R.

this cross-section as a function of energy and angle is precisely predicted by the standard model[9] and, in fact, gives quite a stringent test of the vector boson couplings predicted by the $SU(2) \times U(1)$ gauge theory. The W pair production cross-section is computed from the three diagrams shown in Fig. 9. Any individual term which arises in summing and squaring these diagrams leads to a differential cross-section proportional to s^2; it is only by noting delicate cancellations among the complete set of terms that one finds the dependence s^0 required by unitarity. The predicted angular distribution for $e^+e^- \to W^+W^-$, and the angular distributions for the related processes $e^+e^- \to Z^0Z^0$ and $e^+e^- \to Z^0\gamma$[10], are shown in Fig. 10. It is amusing to note that these differential cross-sections, expressed in R units, are almost independent of s for $\sqrt{s} > 250$ GeV, except at small angles. The whole increase of the total cross-section for W pair production above this energy results from the peripheral process of ν exchange (the third graph of Fig. 9).

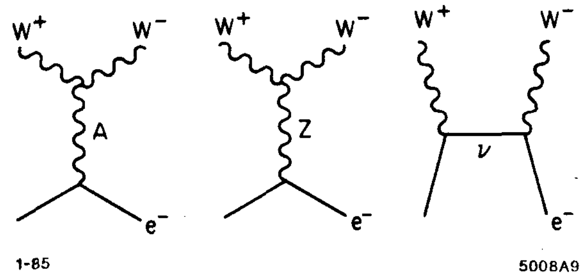

Fig. 9. Feynman diagrams contributing to $e^+e^- \to W^+W^-$.

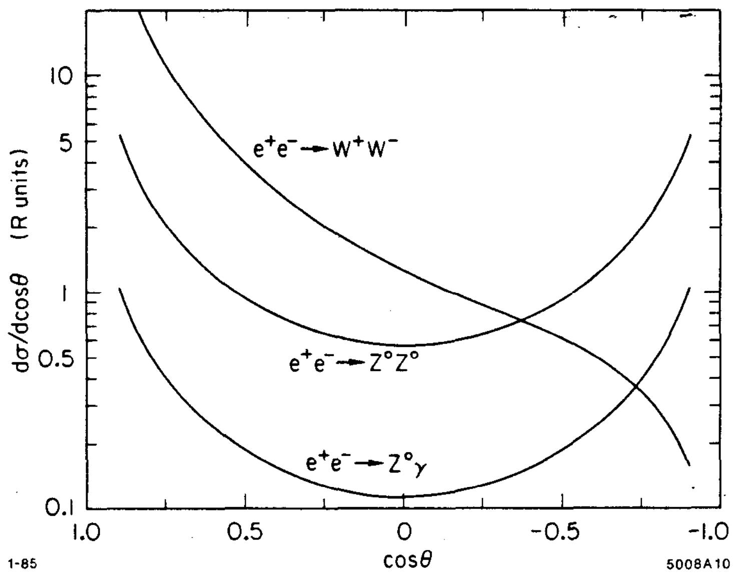

Fig. 10. Angular distributions for the processes $e^+e^- \to W^+W^-$, $e^+e^- \to Z^0Z^0$, and $e^+e^- \to Z^0\gamma$, at $\sqrt{s} = 1$ TeV.

Together with this set of processes in which pairs of vector bosons are produced, one also may find a process (shown in Fig. 11) in which one Z^0 boson is produced in association with a Higgs boson. This process, for which the cross-section was computed in detail in refs. 11 and 12, is the cleanest of all processes for isolating a Higgs boson of mass greater than about 50 GeV. The rate is also quite sizeable; one can understand this by observing that the Higgs-Z^0-Z^0 vertex used comes directly from the term in the gauge theory Lagrangian

$$\frac{1}{4}(g^2 + g'^2)^2 Z_\mu Z^\mu |\phi|^2 \tag{7}$$

which produces the Z^0 mass when the Higgs field ϕ acquires its vacuum expectation value. The rate of this Higgs production process is shown for several values of the Higgs boson mass in Fig. 12 (for the cases of $\sqrt{s} = 250$ GeV, the highest LEP energy) and in Fig. 13 (for the case of $\sqrt{s} = 1$ TeV). Note that the cross-sections are still significant for values of the Higgs mass quite close to the kinematic boundary. The presence of a real Z^0 should allow one to separate this process from background independently of the decay products of the Higgs. However, one should note that, if the mass of the Higgs boson is greater than twice the mass of the W, the decay H $\to W^+W^-$ (along with its partner H $\to Z^0Z^0$) becomes the major decay mode of the Higgs boson. The partial widths for these decays are given by:

$$\Gamma(\mathrm{H} \rightarrow W^+W^-) \approx 2\Gamma(\mathrm{H} \rightarrow Z^0Z^0) \approx 40\mathrm{GeV} \cdot \left(\frac{m_{\mathrm{H}}}{500\mathrm{GeV}}\right)^3. \tag{8}$$

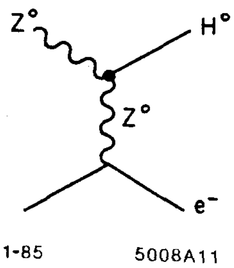

Fig. 11. Feynman diagram contributing to $e^+e^- \rightarrow Z^0 +$ Higgs.

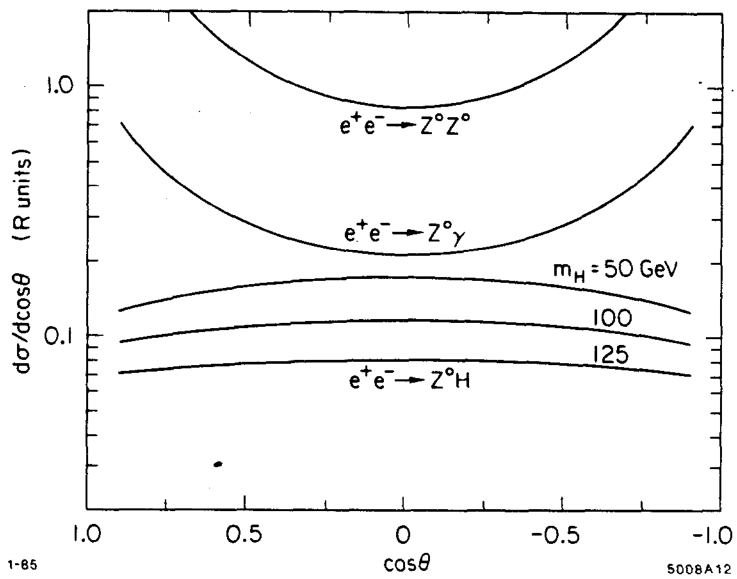

Fig. 12. Differential cross sections for the process $e^+e^- \rightarrow Z^0 +$ Higgs, for several values of the Higgs boson mass, compared to the cross-sections for other Z^0 production processes, at $\sqrt{s} = 1$ TeV.

Thus, a Higgs of several hundred GeV in mass produces a dramatic signature. For a Higgs mas-of 200 – 400 GeV, the Higgs should be visible even unaccompanied by a Z^0; Dawson and Rosner[13] have found that the cross-section for single production of the Higgs is above 1 unit of R in this mass range. If the mass of the Higgs is larger than 1 TeV, however, its width is so large that it should not be directly observable as a resonance in any reaction.

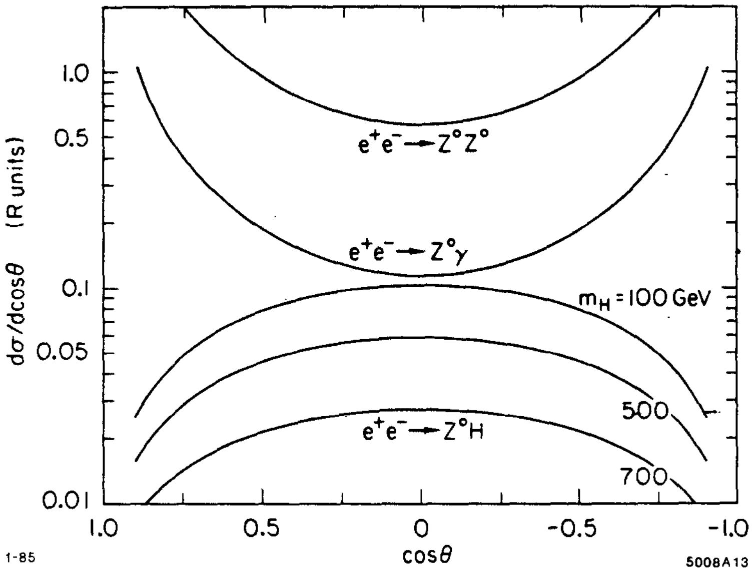

Fig. 13. Differential cross sections for the process $e^+e^- \to Z^0$ + Higgs, for several values of the Higgs boson mass, compared to the cross-sections for other Z^0 production processes, at $\sqrt{s} = 1$ TeV.

Effects of a Heavy Higgs

We have thus seen that e^+e^- annihilation provides a beautiful way of finding the Higgs if the mass of the Higgs boson is below 1 TeV. However, the Higgs boson might well be heavier than this value. This can happen, according to the adjustment of parameters, in models in which the Higgs boson is elementary; it is forced to happen in schemes such as technicolor models[14] in which the Higgs boson has a dynamical origin.[♯1] Can we find evidence of such a heavy Higgs? Though it is very difficult to observe directly, a very heavy Higgs should be observable through its indirect effects, in particular, its influence on the W pair production cross-section at TeV energies. Let me now discuss in some detail why this process is sensitive to the effects of the Higgs boson and what one might expect to observe.

The presence of interesting effects follows from two theoretical observations. The first is that, if the Higgs is heavy, it is also strongly interacting. In the simplest model, the mass of the Higgs is related to the vacuum expectation value of the Higgs field (1) through:

$$m_{\text{Higgs}} = \sqrt{\lambda} \cdot \langle \phi \rangle \,, \tag{9}$$

where λ is the scalar field self-coupling. The same qualitative relation holds in more general schemes. The second is a wonderful theorem on W boson couplings at high energy.[11,16,17] To introduce this theorem, let me remind you of a bit of the structure of the simplest model of $SU(2) \times U(1)$ breaking. This model contains one complex doublet of scalar Higgs

♯1 Except in a particular scenario presented recently by Kaplan, Georgi, and Dimopoulos.[15]

fields; let us write it in the form:

$$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 + i\phi^3 \end{pmatrix}, \qquad \phi^\dagger = \begin{pmatrix} \phi^- \\ \phi^0 - i\phi^3 \end{pmatrix}. \tag{10}$$

We may choose ϕ^0 as the field which acquires a vacuum expectation value; then ϕ^+, ϕ^-, ϕ^0 are the Goldstone bosons which become incorporated as the longitudinal polarization states of the massive W^+, W^-, Z^0. In more complex schemes, the form of the Goldstone bosons may be more involved, but the structure is basically the same. Now I may state the theorem: For $s \gg m_W^2$, the cross-section for producing a longitudinally polarized W^+ is equal to the cross-section, computed in the pure Higgs theory, for producing a ϕ^+, up to corrections of order α. The result is illustrated in Fig. 14. The import of this theorem is that, if the Higgs is heavy, not only the Higgs but also the W bosons are strongly interacting.

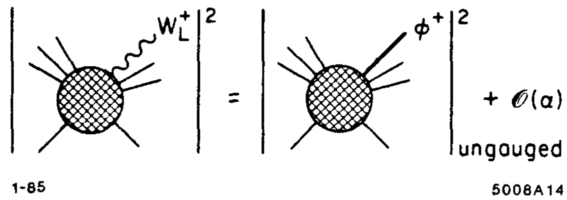

Fig. 14. A theorem on W production cross-sections at very high energies.

To find evidence of a heavy Higgs, then, we need only look for a sign of strong interactions in the production of longitudinal W's.[♯2] Let us, then, consider the dynamics of this process more carefully. Lee, Quigg, and Thacker[11] emphasize that the strongest W-W binding effects should appear in the spin 0, isospin 0 channel. Unfortunately, this channel is not directly accessible in e^+e^- annihilation, or, indeed, in any annihilation process of light fermions. This is because, if we are permitted to ignore the fermion masses, the interactions which allow W production conserve helicity; then, necessarily, the annihilating system has at least spin 1. (See. Fig. 15.) Perturbation theory does indicate attractive interactions in the $J = 1$, $I = 1$ partial wave. ($I = 1$ follows from $J = 1$ by Bose statistics).

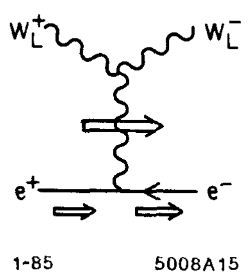

Fig. 15. Constraint of helicity conservation on partial waves accessible in $e^+e^- \to W^+(L)W^-(L)$.

♯2 The production of longitudinal W bosons in p-p collisions has been studied in some detail by Chanowitz and Gaillard.[18]

Figure 15 makes clear that this channel is readily accessed in e^+e^- annihilation. At $\sqrt{s} =$ 1 TeV and above, this partial wave contributes some reasonable part of the full W pair production cross-section in the backwards direction; this is shown in Fig. 16.

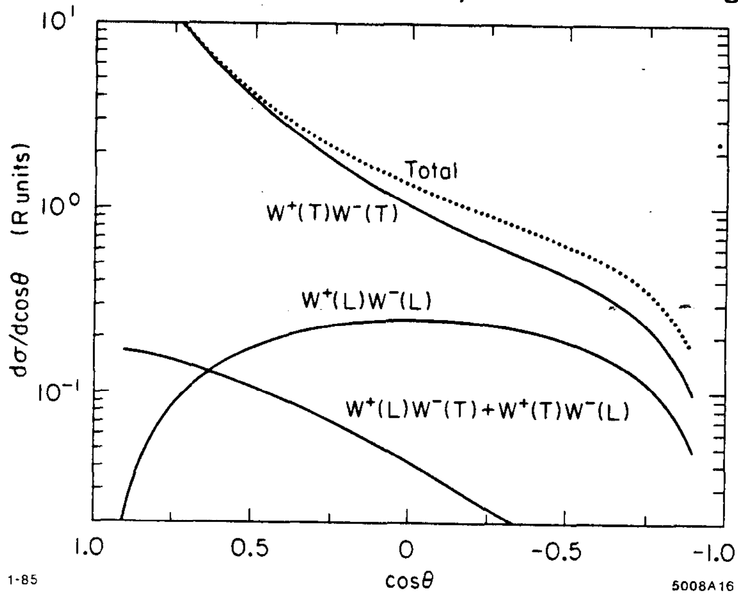

Fig. 16. Differential cross-sections for pair production of W bosons according to their polarization states: longitudinal (L) or transverse (T). $\sqrt{s} = 1$ TeV. The curves make use of formulae given in refs. 19, 20.

Now that we have seen that longitudinal W's are accessible, we must ask what effects we might expect them to show. A quantity which conveniently illustrates the size of the effects is the value of the complete cross-section for $e^+e^- \to W^+W^-$, evaluated at some backwards angle, for example, 120°. Let us first consider the effect of a simple modification of the lowest-order formula by rescattering. Using the theorem quoted above, we may approximate the scattering amplitude for longitudinal W's by that for the ϕ bosons of eq. (10). In a one-Higgs-boson model, the $I = 1$ Born amplitude for ϕ bosons is:

$$= -i\sqrt{2}G_F m_H^2 \cdot \left(\frac{t}{t - m_H^2} - \frac{u}{u - m_H^2}\right) . \tag{11}$$

This amplitude vanishes at threshold, but becomes large when $\sqrt{s}$ becomes of order m_H. To extrapolate to very high energies, let us recast (11) as a formulae for the $I = 1$, $J = 1$ phase shift of charged Goldstone bosons:

$$\delta_1(s) \approx \frac{1}{32\pi} \int_{-1}^{1} d\cos\theta \, \cos\theta \cdot \tag{12}$$

We may then represent the final-state interaction in a simple way[21] by multiplying the longitudinal W pair production amplitude in this partial wave by

$$exp \left[\frac{1}{\pi} \int_0^\infty ds' \, \delta_1(s') \left\{ \frac{1}{s' - s - i\epsilon} - \frac{1}{s'} \right\} \right] . \tag{13}$$

This produces the effect indicated in Fig. 17. The rescattering increases the production cross-section for longitudinal W's by about 10%. The effect is somewhat diluted but still significant in the total W cross-section. A second possibility is that the Goldstone bosons interact so strongly as to produce a resonance in this channel. In the technicolor models of ref. 14, the interactions of the Goldstone bosons are (with a change of mass scale) precisely those of pions in the usual strong interactions. We expect, then, a technicolor analogue of the ρ resonance to appear in precisely this partial wave. Scaling up the parameters of the familiar ρ resonance, we find the effect shown in Fig. 18: The W pair cross-section at 120° is increased by a factor of 10 as a result of the resonance.

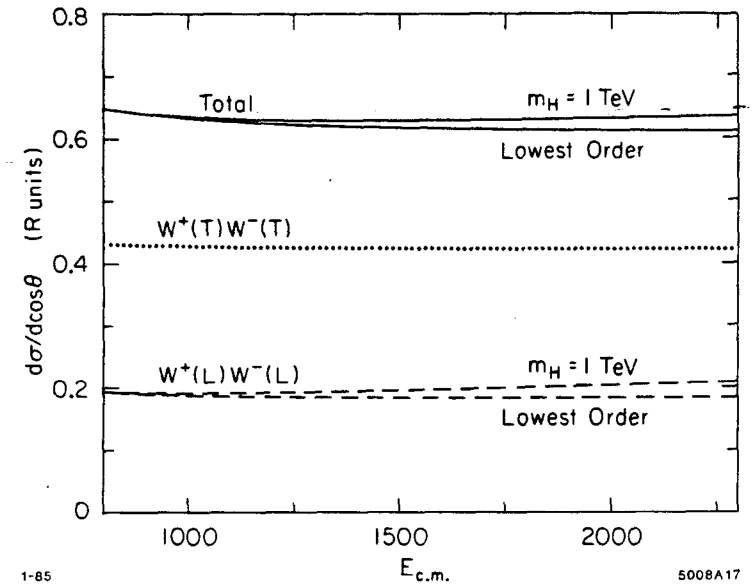

Fig. 17. Effect of rescattering of longitudinal W's, computed in the approximation of eq. (13), on the W pair production cross-section at $\cos\theta = -0.5$. The effect is shown both for the longitudinal W and the total W production cross-sections.

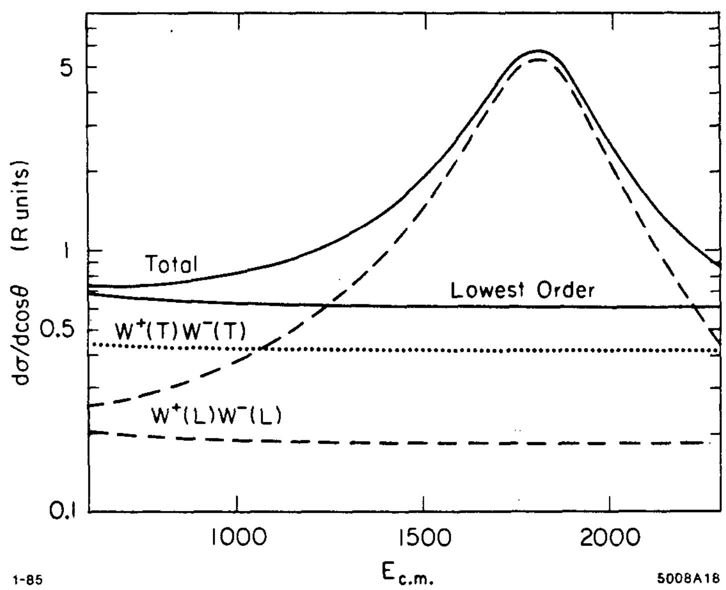

Fig. 18. Effect of a techni-ρ resonance in the Higgs sector, on the W pair production cross-section at $\cos\theta = -0.5$. The notation is as in Fig. 17.

Finally, I should note that even more exotic behavior is possible in specific models. At the end of my discussion of particle production, I referred to a class of technicolor models in which the new technicolor sector contained fermions carrying quark and lepton quantum numbers.[7] In these models, the techni-ρ resonance enhances not only the production of W pairs but also the production of new scalar bosons with exotic quantum numbers. Figure 4 displayed the total cross-section for production of such bosons in e^+e^- annihilation in the region near their thresholds. Over a larger energy region, we must include the effect of the techni-ρ; this gives the cross-sections shown in Fig. 19. Thus, a search in e^+e^- annihilation for the effects of the the strong interactions of Higgs bosons might well turn up, in addition, dramatic resonances which decay into completely new species.

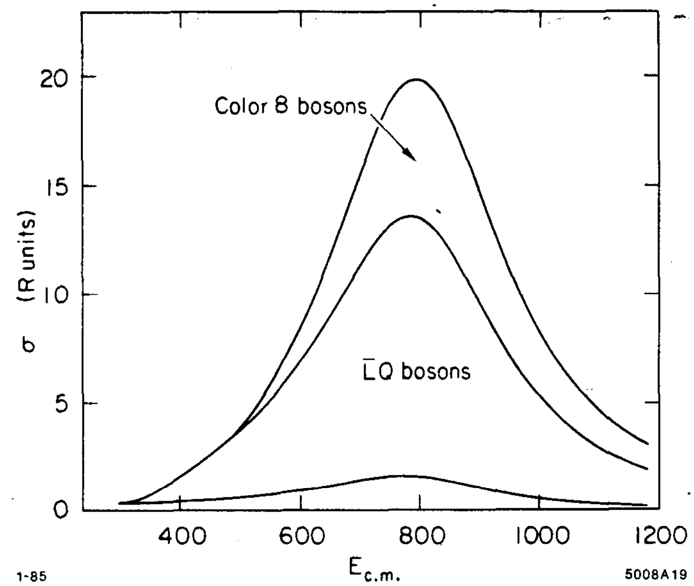

Fig. 19. Effect of a techni-ρ resonance in the Higgs sector on the total cross-section for e^+e^- annihilation into boson pairs, in the model of ref. 7. The figure shows the contributions of W bosons and "ordinary" charged Higgs bosons and of technipions with exotic quantum numbers.

Effects of Lepton Compositeness

It is still possible that the mysteries of physics on the 1 TeV mass scale will be resolved by the discovery that the quarks and leptons are composite objects on this scale. If the electron is indeed composite, the new interactions which bind its constituents will eventually influence the cross-sections for e^+e^- annihilation. Let us characterize the physical size of an electron as the inverse of a mass scale Λ which gives the strength of these interactions. Then, by the time $\sqrt{s}$ becomes as large as Λ, the interactions of the electron's constituents, rather than the electron itself, should determine the physics of e^+e^- annihilation. Since this is so, we should expect that, even at much lower energies, these constituent-binding interactions significantly alter the predictions of the standard model. Let me now discuss what sort of effects might be visible in e^+e^- annihilation up to TeV energies.

The simplest way to discuss the new interactions of electrons and other fermions which might arise from constituent-binding forces is to represent these as 4-fermion contact interactions; this should be a good approximation when $\sqrt{s} \ll \Lambda$. In general, such interactions will lead to correction terms in the amplitude for a given process of order

$$\frac{1}{\alpha} \cdot \frac{s}{\Lambda^2} \tag{14}$$

relative to the standard model contribution. The factor Λ^{-2} is the coefficient of a 4-fermion interaction of correct dimension. The factor α^{-1} reflects the fact that the new contribution is competing only with an electroweak process.[22] This enhancement factor makes the contact interactions visible for quite large values of Λ. Current experiments at PEP and PETRA, at values of $\sqrt{s}$ near 30 GeV, yield lower bounds greater than 1 TeV on the Λ parameters which characterize the corrections to Bhabha scattering.[23] This extreme sensitivity to the effects of contact interactions should also be a property of e^+e^- annihilation at very high energies. One aspect of this is shown in Fig. 20. Let us imagine that left-handed electrons and muons have a common constituent, one, say, which carries the weak $SU(2)$ charge. Then one would expect to find a contact interaction which could mediate the reaction $e_L^+e_L^- \rightarrow \mu_L^+\mu_L^-$.[24] This interaction will modify not only the total cross-section for $e^+e^- \rightarrow \mu^+\mu^-$; it will also modify the angular distribution (and, of course, the various polarization asymmetries). As an example, Fig. 20 displays the effect on the forward-backward asymmetry. Notice that this effect can be very large, even for values of Λ ten times the center-of-mass energy.

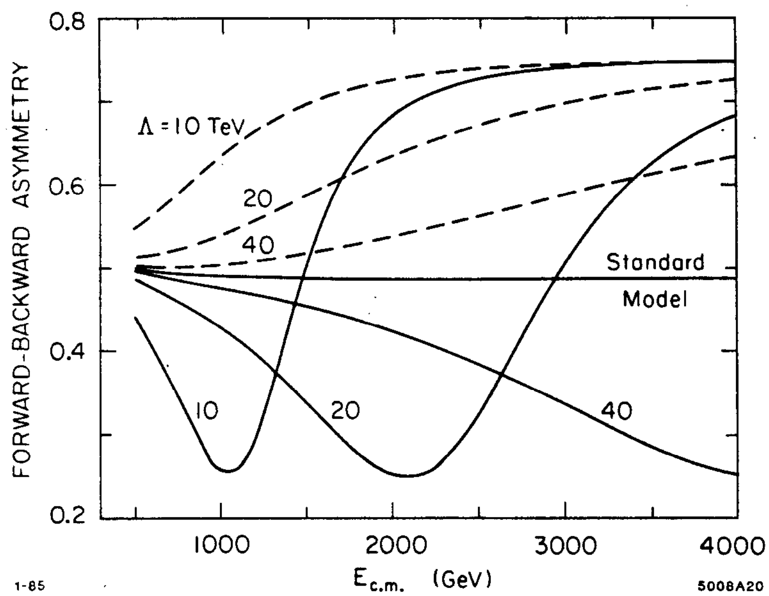

Fig. 20. Effect of a contact interaction linking left-handed electrons and muons on the forward-backward asymmetry in $e^+e^- \rightarrow \mu^+\mu^-$, for various values of Λ. The two sets of curves correspond to the two different signs for the interference term.

What if the electron is in fact quite large, corresponding to Λ values of, say, 3 TeV? My first comment should be that the presence of the contact interactions becomes obvious quickly as one moves up in energy: Fig. 21 shows the behavior of the $e^+e^- \rightarrow \mu^+\mu^-$ total cross-section in the model described in the previous paragraph. For such a low value of Λ, the effects are dramatic even for relatively low values of $\sqrt{s}$. What is more interesting, though, is what happens in the TeV region of energies. This is shown (schematically) in Fig. 22. As the amplitude becomes dominated by the contact interaction, the cross-section rises linearly. At values of $\sqrt{s}$ of order Λ, one expects to find resonances corresponding to higher-mass excited states of the leptons' constituents. At higher values of $\sqrt{s}$, the μ pair production cross-section should decrease as some power of s and the Bhabha scattering cross-section should tend to a constant, as a consequence of Regge asymptotic behavior. The order of magnitude of these strong interaction cross-sections may be estimated geometrically as:

$$\sigma \sim \Lambda^{-2} \sim 0.1 \text{ nb}, \tag{15}$$

for $\Lambda \sim 3$ TeV. This is a huge cross-section by the standards of eq. (3), a striking signature of the emergence of a new scale of physics.

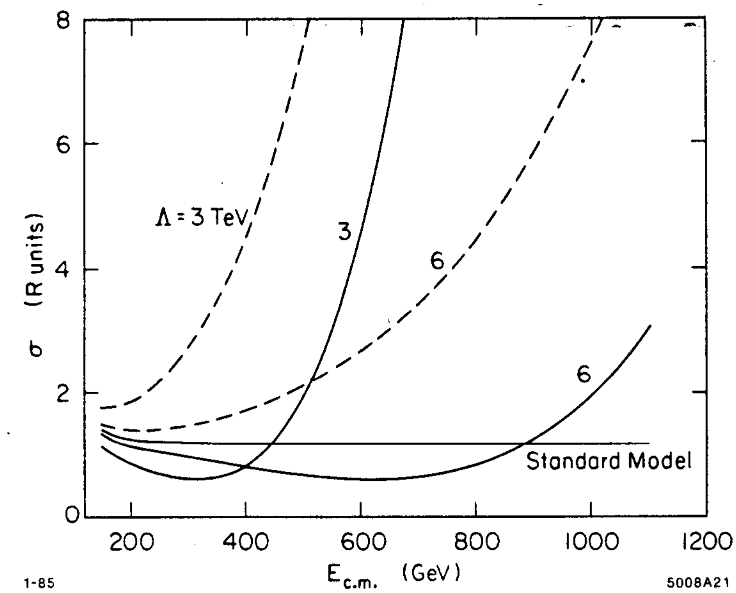

Fig. 21. Effect of a contact interaction linking left-handed electrons and muons on the total cross-section for $e^+e^- \to \mu^+\mu^-$. The notation is as in Fig. 20.

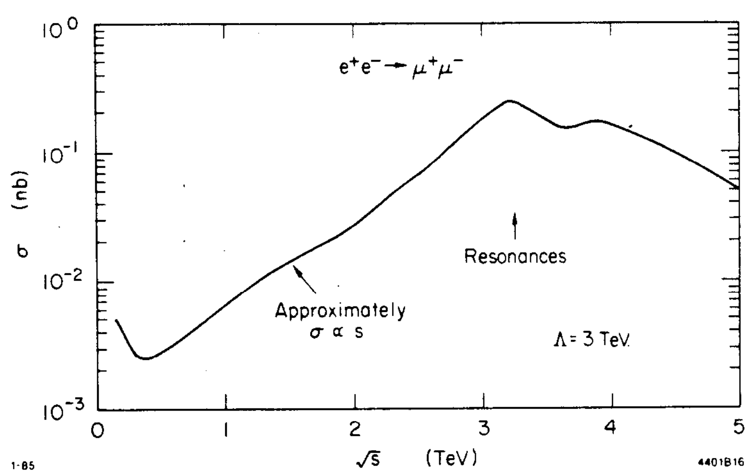

Fig. 22. Behavior of the cross-section for $e^+e^- \to \mu^+\mu^-$ if the electron is composite at a relatively small mass scale ($\Lambda \approx 3$ TeV). Note that the cross-section is plotted in absolute units, rather than units of R.

Dreams or Reality?

I have now argued that there are many remarkable effects to be observed at a TeV-energy, R-luminosity e^+-e^- collider. The experiments which might be done at such a facility would continue the exploration for new scales of physics up to TeV energies and would give definitive information on the mechanism of the weak-interaction symmetry breaking. Unfortunately, no reasonable design for a TeV-energy collider now exists, and great technical problems remain to be overcome before such an accelerator can be built. Why, then, should we even begin to think about a future which lies in this direction?

I might justifiably be criticized as a dreamer for taking this route of argument, but I find in the technical problems of high-energy e^+-e^- colliders a very attractive picture of the future of our field. The barrier set up by synchrotron-radiation losses pushes us away from synchrotrons to clashing linear accelerators. The barrier set up by limitations on the expenditure of power pushes us to compact accelerating devices with high frequency accelerating fields, high gradients, and high energy density over a very small volume. Laser, plasma beat-wave, and wake-field accelerators, to name only a few of the methods now under investigation, offer the promise of providing such small-scale devices. None of these is yet sufficiently well developed to be the basis of a serious design. But one can plausibly imagine that some such method will make the next step in accelerator design a step down from gargantuan facilities to machines that fit on small patches of ground and use novel technologies to eliminate the expense of huge construction projects. At several crucial junctures in the history of physics, we have found ways to bridge a gap in technology, in order to continue our exploration of the fundamentals of Nature. It is time that we set ourselves to this task again, to turn this fond dream into a reality.

Appendix: How to Compare the Energies of e^+-e^- and p-p Colliders

Let me append to this report some remarks on the comparison of center-of-mass energies of electron and proton colliders. It is clear that the maximum energy at which a proton collider can access new physics is much less than its nominal center-of-mass energy, because the proton is a composite object from the viewpoint of exploring the physics of quarks and leptons. One should then pose the question: at what fraction of the center-of-mass energy of a p-p collider does an e^+-e^- collider have an equivalent ability to discover new particles? The reader should note that my discussion of this question will not properly account the many advantages of doing physics in the environment of e^+e^- annihilation—the cleanliness of events, the well-defined parton center-of-mass energy, and, most importantly, the ability to follow up on a discovery by observing a new state in many decays modes beyond the one which gives the best signature. This is simply a comparison of the brute strength of these two methods in the specific mode of new particle production.

A somewhat naive answer to the question goes as follows: Since new particles are produced in proton-proton collisions by the interaction of constituents of each proton, we should determine the average value of the momentum fraction of the relevant constituents, $\langle x \rangle$, and write the equivalence:

$$\sqrt{s}(e^+e^-) \sim \langle x \rangle \cdot \sqrt{s}(pp). \tag{16}$$

A reasonable value for $\langle x \rangle$ would be in the range $\frac{1}{6}$–$\frac{1}{10}$; this value would change slowly with $\sqrt{s}$ due to logarithmic scaling violations.

This argument, however, ignores the ferocious nature of the *p*-*p* environment. Studies for the SSC have shown that there is a practical limit to the luminosity at which general-purpose 4π detectors will function at a *p*-*p* collider, a limit set not by the usual background processes such as beam-gas interactions but rather by the huge number of true *p*-*p* collisions resulting in uninteresting two-jet and multi-jet final states. This limit comes at roughly 10^{33} or 10^{32} $\mathrm{cm}^{-2}\mathrm{sec}^{-1}$; the lower figure probably applies if a vertex chamber is essential to the experimental program. On the other hand, the parton-parton cross-sections for the production of new particles of mass $\mathcal{M}$ characteristically fall as $\sigma(\mathcal{M}) \sim \mathcal{M}^{-2}$. At fixed luminosity, the production of particles whose mass is a given fraction of $\sqrt{s}$ becomes rarer and rarer with increasing $\sqrt{s}$, so that the signatures for the production of such particles become increasingly more difficult to recognize above their backgrounds.

To quantify this picture, let me present the results of calculations of new particle production at high-energy proton colliders in a form appropriate to a comparison with e^+e^- annihilation. The presentation is similar in form to one done some time ago by Ellis.[2] Recently, however, Eichten, Hinchliffe, Lane, and Quigg (EHLQ) have presented a definitive compilation of predictions for high-energy *p*-*p* collisions;[1] I will base my analysis on their results.

Table I. Reactions for a Comparison of e^+e^- and pp Collider Energies

	Object Probed	Criterion of EHLQ	Equivalent $\sqrt{s}(e^+e^-)$
W	new W boson	1000 W's with $\|y\| < 1.5$	m_W
Q	new heavy quark	500 pairs with $\|y_a\| < 1.5$*	$2m_Q$
$\tilde{g}$	gluino	10^4 events with $\|y_{\tilde{g}}\| < 1.5$	$3m_{\tilde{g}}$
L	new heavy lepton	50 excess of $L^{\pm}N^0$; 10% efficiency for $L^{\pm}$ †	$2m_L$
Λ	qq contact interaction‡	factor of 2 effect + 50 events/bin of $\Delta p_{\perp} = 100$ GeV	$\Lambda/30$

* EHLQ quote $50/\epsilon$ pairs.

† EHLQ do not assume an efficiency.

‡ Result depends on the sign of interference; I take the geometric mean.

To compare effective center-of-mass energies for e^+-e^- and *p*-*p* colliders, I have chosen five reactions which span the range of the types of new physics which have been investigated by theorists: discovery of a new W boson, a new heavy quark, and a new heavy lepton, discovery of the gluino, the supersymmetric partner most accessible in *p*-*p* collisions, and observation of a contact interaction in quark-quark scattering. EHLQ have assessed the likelihood of each of these discoveries in high-energy *p*-*p* collisions and have given reasonable criteria for the visibility of each signal above background. In Table I, I have listed, for each of these reactions, the discovery criterion of EHLQ and also my estimate of the center-of-mass energy needed in e^+e^- annihilation to make the same discovery. A few of these equivalences require comment: A new W boson can appear in e^+e^- annihilation only above the pair-production threshold, $\sqrt{s} = 2m_W$; however, the existence of a new W generally implies the existence of a new Z boson, very close in mass, which would be seen as a dramatic resonance in e^+e^- annihilation. The gluino is difficult to produce in e^+e^- annihilation; however, if the gluino has a mass as large as 1 TeV, there is no reason why it should be much lighter than the supersymmetric partners of quarks and leptons.

These particles, though, are bosons, so I have allowed some distance above pair-production threshold. The equivalent value of $\sqrt{s}$ for the contact interaction reflects current experience at PEP and PETRA.[23]

This assignment of equivalences allows us to plot the results obtained by EHLQ for the reach of p-p colliders as a function of their center-of-mass energies as a comparison of center-of-masses energies of proton and electron colliders. The results depend on the assumption of a limiting luminosity for p-p collisions, and on the value of that limiting luminosity. Figures 23 and 24 show this comparison for the cases in which this luminosity is set at 10^{32} and 10^{33} $\text{cm}^{-2}\text{sec}^{-1}$, respectively. This comparison shows that the mass reach available from the SSC will be roughly that of a 2-4 TeV e^+-e^- collider, for the class of processes which are most easily accessed in p-p collisions. These curves also show that, in extrapolating to higher energies, this comparison roughly follows the law:

$$\sqrt{s}(e^+e^-) \propto (\sqrt{s}(pp))^{\frac{1}{2}}. \tag{17}$$

This is the same relation as that between the available $\sqrt{s}$ from colliding-beam as opposed to fixed target experiments. As we reach into the TeV regime of energies, we must begin to take e^+-e^- colliders seriously as our path to the future.

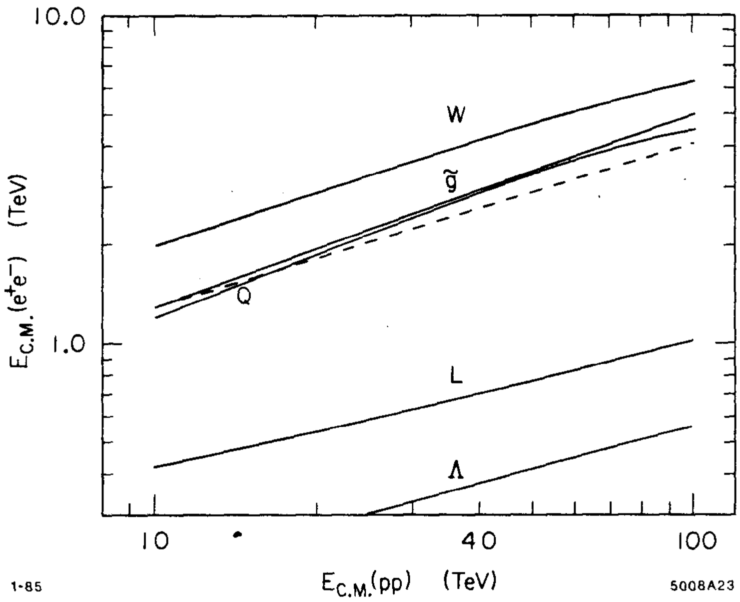

Fig. 23. Comparison of effective center-of-mass energies for e^+-e^- and p-p colliders, for the five processes listed in Table I, assuming a luminosity for p-p collisions of 10^{32} $\text{cm}^{-2}\text{sec}^{-1}$ integrated over a year of 10^7 seconds. The dotted line is a suggested fit to the data: $\text{E}_{\text{CM}}(e^+e^-) = \sqrt{\text{E}_{CM}(pp)/6}$, with all energies in TeV.

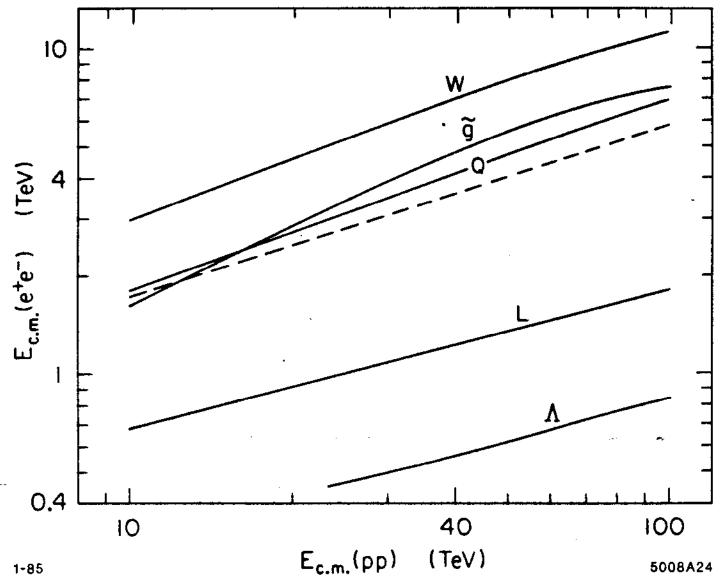

Fig. 24. Comparison of effective center-of-mass energies for e^+-e^- and p-p colliders, as in Fig. 23, but assuming a luminosity of 10^{33} $cm^{-2}sec^{-1}$. The dotted line is a suggested fit to the data: $E_{CM}(e^+e^-) = \sqrt{E_{CM}(pp)/3}$, with all energies in TeV.

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