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# ACCELERATION OF ELECTRONS BY THE INTERACTION OF A BUNCHED ELECTRON BEAM WITH A PLASMA*

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## ABSTRACT

A new scheme for accelerating electrons, employing a bunched relativistic electron beam in a cold plasma, is analyzed. We show that energy gradients can exceed 1 GeV/m and that the driven electrons can be accelerated from $\gamma_0 mc^2$ to $3\gamma_0 mc^2$ before the driving beam slows down enough to degrade the plasma wave. If the driving electrons are removed before they cause the collapse of the plasma wave, energies up to $4\gamma_0^2 mc^2$ are possible. A non-collinear injection scheme is suggested in order that the driving electrons can be removed.

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In the past several years, the laser-plasma interaction as a mechanism for charged particle acceleration has attracted interest[1] because of the large electric field which a plasma can support ($\sim$ 100 GeV/m). However, these "beat-wave" accelerators[2,3] rely heavily on the state of the art of laser technologies. For example, the scheme requires a fine tuning[4] between the plasma frequency $\omega_p$ and the beat-wave frequency of the laser in order that the wake plasma wave excited by the laser beat-wave grows linearly. This in turn either puts a severe constraint on the uniformity of the plasma density, or relies on very high power lasers to shorten the time of growth. In addition, it may be necessary to deliver the laser energy in a pulse shorter than 10 pecoseconds in order to avoid competing instabilities.[5]

This letter presents another scheme for a plasma accelerator. Lasers are not required and large energy gradients are attained. The idea is to inject a sequence of bunched high energy electrons into a cold plasma. As in the two stream instability, the streaming electrons lose energy to the background plasma by exciting a wake plasma wave. If a late coming electron bunch rides on the wave at a proper phase, it will be boosted to a higher energy due to the longitudinal electric field in the wave.

Consider a system in which a chain of relativistic electron bunches with initial $\beta_0 = v_b/c \lesssim 1$ stream through a cold, uniform plasma along the $z$-axis with a constant separation $d$. Assuming that the longitudinal spread $l_b$ of each electron bunch is much smaller than the plasma wavelength, $\lambda_p$, the whole bunch of $q$ electrons behaves as a single particle with charge $Q = qe$.

The linearized equation of motion and equation of continuity for the cold, non-relativistic background plasma are $\partial_t \vec{v}_{p1} = -(e/m)\vec{E}_1$ and $\partial_t n_{p1} +$

$n_{p0}\nabla \cdot \vec{v}_{p1} = 0$, respectively, where $\vec{E}_1$ is the electric field of plasma and beam: $\vec{E}_1 = \vec{E}_{p1} + \vec{E}_{b1}$, where the plasma velocity is $\vec{v}_p = \vec{v}_{p0} + \vec{v}_{p1}$ $(\vec{v}_{p0} = 0)$, and the plasma density is $n_p = n_{p0} + n_{p1}$ $(n_{p0} \gg n_{p1})$. For $N$ driving electron bunches, the charge and current densities are

$$\rho_1(\vec{x}) = -en_{p1}(\vec{x}) - Q\sum \delta(\vec{x} - \vec{x}_i) \ , \tag{1}$$

$$\vec{J}_1(\vec{x}) = -en_{p0}\vec{v}_{p1}(\vec{x}) - Q\vec{v}_b \sum \delta(\vec{x} - \vec{x}_i) \ , \tag{2}$$

respectively, where $\vec{x} \equiv \rho e_1 + z e_3$ in cylindrical coordinates, and $\vec{x}_i$'s are the instantaneous positions of the $N$ bunches: $\vec{x}_i \equiv [v_b t + (N - i)d]e_3$, and the summations are over $i = 1, \cdots, N$.

The longitudinal electric field in the wake of these $N$ bunches is $\vec{E}_1 = -(1/c)\partial_t \vec{A}_1 - \nabla\phi_1$. For an ultra-relativistic electron beam, where $\beta_0 \simeq 1$, $v_b$ is approximately constant over several plasma wavelengths, even though substantial energy is transferred to the plasma wave. It is thus convenient to work with the variable $\varsigma \equiv z - v_b t \leq 0$ which measures the distance behind the last bunch, and we may put $\partial_t = -v_b\partial_\varsigma$ and $\partial_z = \partial_\varsigma$. In the Coulomb gauge, the equation for the scalar potential is $\nabla^2\phi_1 = -4\pi\rho_1$, and that for the vector potential is $\nabla_\perp^2 \vec{A}_1 = -(4\pi/c)\vec{J}_1 - \beta_0\nabla\partial_\varsigma\phi_1$, where $\nabla_\perp^2$ is the transverse Laplacian and $(1 - \beta_0^2)$ is neglected.

To solve for $\phi_1$, we take the $\varsigma$-derivative twice and combine the result with the equations of motion and continuity, and Eq. (1) to obtain

$$\nabla^2\left(\partial_\varsigma^2 + k_p^2\right)\phi_1 = 4\pi Q \sum \partial_\varsigma^2 \delta(\vec{x} - \vec{x}_i) \ , \tag{3}$$

where $k_p \equiv \omega_p/v_b = (4\pi n_{p0}e^2/mv_b^2)^{1/2}$, and $\vec{x} - \vec{x}_i = \rho e_1 - [(N - i)d - \varsigma]e_3$.

The solution of this equation requires that we solve

$$\left(\partial_\varsigma^2 + k_p^2\right)\phi_1 = -Q\sum \partial_\varsigma^2 (1/|\vec{x} - \vec{x}_i|) \ , \tag{4}$$

which has the solution[6]

$$\phi_1(\rho,\varsigma) = Q\sum \left\{ -(1/|\vec{x}-\vec{x}_i|) + k_p \int_\varsigma^\infty d\varsigma' \ \sin k_p(\varsigma'-\varsigma)/|\vec{x}\,'-\vec{x}\,'_i| \right\} \ . \tag{5}$$

where $\varsigma' \equiv z' - v_b t$ and $|\vec{x}\,' - \vec{x}\,'_i| \ \equiv \{\rho^2 + [(N-i)d - \varsigma']^2\}^{1/2}$. A favorable aspect of our acceleration scheme is that the phase velocity, $\omega_p/k_p$, of this plasma wave is the same as the beam velocity, $v_b$. This contrasts with the two stream instability observed in a continuous beam. In that case, even though the phase velocity is shifted slightly below the beam velocity, the corresponding $\gamma$ may be significantly below the $\gamma$ of the beam.[7] This would be a serious disadvantage for the purpose of collective particle acceleration.

Next, turn to the vector potential $\vec{A}_1$. Taking the $\varsigma$-derivative on both sides of the equation for the vector potential, and invoking the equation of motion for the current term, we obtain

$$\partial_\varsigma\left(\nabla_\perp^2 - \beta_0^2 k_p^2\right)\vec{A}_1 = -\beta_0 \nabla\left(\partial_\varsigma^2 + k_p^2\right)\phi_1 + 4\pi Q\vec{\beta}_0 \sum \partial_\varsigma \delta(\vec{x}-\vec{x}_i) \ . \tag{6}$$

Combining with Eq. (4), the above equation decouples entirely from the scalar potential. Removing the $\varsigma$-derivative common to each term, this equation further reduces to a inhomogeneous modified Helmholtz equation in two dimensions for each component of $\vec{A}_1$.

We are interested in the wake field trailing behind the $N$ bunches on the $z$ axis. However, the integral in Eq. (5) includes $\varsigma = 0$ and therefore is logarithmically divergent when $\rho = 0$. This is certainly unphysical. The origin of this symptom is due to the lack of any thermal effects in our cold fluid model. Thus Eq. (5) is not applicable to the region within a Debye length from the axis. Bearing this in mind, Eq. (5) reads

$$\phi_1(\rho,\varsigma) = \frac{2\pi Q}{\lambda_p}\sum\left\{-\frac{1}{k_p\sqrt{\rho^2+[(N-i)d-\varsigma]^2}}+\int_{\varsigma}^{\infty} d\varsigma' \frac{\sin k_p(\varsigma'-\varsigma)}{\sqrt{\rho^2+[(N-i)d-\varsigma']^2}}\right\}, \tag{7}$$

where $\lambda_p \equiv 2\pi/k_p$. By the same token the vector potential in Eq. (6) is not applicable to the region within a Debye length behind the plane $\varsigma = 0$ and

$$\begin{aligned} A_{1z}(\rho,\varsigma) = & -\frac{2\pi\beta_0 Q}{\lambda_p}\sum\int_0^{2\pi}\frac{d\theta}{2\pi}\int_0^{\infty}\rho' d\rho'\ K_0\left(\beta_0 k_p\sqrt{\rho^2+\rho'^2-2\rho\rho'\cos\theta}\right) \\ & \times \frac{\partial^2}{\partial\rho'^2}\frac{1}{k_p\sqrt{\rho'^2+[(N-i)d-\varsigma]^2}} \quad , \end{aligned} \tag{8}$$

where $K_0$ is the modified Bessel function of order zero. Plots of $\phi_1$ and $A_{1z}$ as functions of $|\varsigma|$ evaluated at $\rho = c/\omega_p$ are shown in Fig. 1, where $N = 5$ and $d = \lambda_p$.

The longitudinal electric field is computed by taking $\varsigma$-derivatives since $E_{1z} \approx \partial_\varsigma(A_{1z} - \phi_1)$. The maxima of $E_{1z}$ are at $|\varsigma| \simeq (n+1/2)\lambda_p$, where $n$ is any non-negative integer, and the contribution to the maximum $E_{1z}$ comes predominantly from the scalar potential. If the separation between the driven bunch and the last driving bunch is such that $|\varsigma|$ is around $\lambda_p/2$, the energy gradient attainable for each electron in the driven bunch is

$$\epsilon = -eE_{1z} \simeq 16\pi^2 eQ/\lambda_p^2 \quad . \tag{9}$$

As an example, consider a plasma of density $n_{p0} = 10^{15}\text{cm}^{-3}$ (which sets the limit on the longitudinal bunch spread: $l_b \ll \lambda_p \simeq 1.0\text{mm}$). If each bunch consists of $q \simeq 2.5 \times 10^{10}$ particles, Eq. (9) shows that $\mathcal{E} \simeq 2.4$ GeV/m. This treatment ignores non-linear plasma effects and self-consistency effects that act to slow the driving bunches. It is only valid if the electric field does not approach the cold wave-breaking amplitude, and if the electric energy is small compared to the free energy of the driving bunches. The first condition provides an upper limit on the maximum allowed energy gradient: $\mathcal{E}_{\max} \simeq \sqrt{n_{p0}}$ eV/cm $\simeq 3.2$ GeV/m$>2.4$ GeV/m, so our linear theory is still reasonable. The second condition requires that $(E_{1z}^2/8\pi) \cdot L < q\gamma_0 mc^2/\text{Area}$, where $L$ is the length of the beam-plasma interaction region. For the above example, and $100\mu$ radii bunches, this limits the effective acceleration length to $L < 0.37\gamma_0$ cm.

To complement the above analytic treatment, self-consistent numerical simulations have been done, using a one-and-two-halves dimensional $(x, v_x, v_y, v_z)$, relativistic, fully electromagnetic, particle code. Two bunched electron beams with mean densities of $10^{-3}$ and $10^{-5}$ relative to the background electron plasma served as the driving and driven beams, respectively. Bunching was represented by density profiles of the form $1 + \sin kx$, $180°$ out of phase for the two beams. Each beam was initially mono-energetic with momentum $(\beta\gamma)_0 = 5.9$, and had 16 bunches within the 2048-gridpoint periodic system. The speed of light was chosen to satisfy $k\beta c = \omega_p$ in order to best excite the plasma wave. Since the background plasma had an initial temperature $T/mc^2 = 10^{-3}$, this wave had essentially zero group velocity.

Figure 2 shows the momentum distribution of the two beams at the time ($\omega_p t = 88$) when the maximum momentum $(\beta\gamma)_{\max} = 16$, was attained. The wave grew so that the bulk of the driving and driven electrons were in the regions of greatest negative and positive force, respectively. The driving electrons lost momentum to the wave until they were significantly slower than the wave, i.e., until $\beta\gamma \approx 1$. They quickly fell behind the wave until they reached the accelerating region, where they regained momentum and energy from the wave until it essentially vanished. Except for the relatively short interval when the driving electrons fell behind the wave, the force on the bulk of the driven electrons was equal in magnitude to that on the driving electrons, but always positive. Thus we expect $(\beta\gamma)_{\max} \approx 3(\beta\gamma)_0 - 2$, which was well satisfied in our simulation.

The cycle of wave growth and decline repeated as the simulation continued. However, further acceleration of the driven electrons did not occur. In falling behind the wave by a half wavelength, most of the driving electrons had come into phase with the maximally accelerated driven electrons, and both groups were decelerated together as the wave was re-created during the second cycle.

One way to continue the acceleration of the driven electrons is to remove the driving beam from the plasma before it begins to destroy the wave. Therefore, the previous simulation was repeated (but with $T/mc^2 = 10^{-6}$), and when the maximum wave amplitude was attained ($\omega_p t = 48$), the charge of the driving electrons was set to zero.

Figure 3 shows the momentum distribution at $\omega_p t = 512$, when the driven momenta were as large as $\beta\gamma = 69.5$ and were still increasing. A more recent run gave maximum attainable value of $(\beta\gamma)_{\max} = 130$ at $\omega_p t = 1568$. This agrees well with the modified[9] wave-breaking limit of Tajima and Dawson, $(\beta\gamma)_{\max} \approx 4(\beta\gamma^2)_0 = 142$. The behavior of the lowest energy driven electrons suggests that the velocity of the wave had increased to $\beta\gamma \approx 9$ for the later stages of the simulation, probably because these lower energy driven electrons were serving as a weak driver for the wave. We speculate that this effect may enable one to achieve still higher energies.

In the first simulation, the maximum energy was gained by those electrons in the region of maximum positive force, one half wavelength behind the bulk of the driving electrons. In the second, the maximum energy was gained by those electrons initially at the potential-energy peak of the wave, three quarter wavelengths behind the driving bunches. Using beams 270° out of phase would double the relative number of high-energy electrons. However, the large number of low-energy driven electrons can be reduced only by using significantly shorter driven bunches.

Experimentally, it may be possible to remove the driving electrons from the acceleration region by employing a non-collinear injection scheme.[10] The basic idea is to inject two sets of electron bunches at a slight angle, $2\theta$, towards each other and then accelerate particles down the axis of symmetry in the superposed wake fields of the two intersecting beams. In this way, the electrons in the driving bunches may pass out of the interaction region before they can reabsorb the accelerating waves. Such a scheme also allows one to stage the driving beams by bringing in fresh driver beams at different positions.

An advantage of this non-collinear injection scheme is that the effective phase velocity $\tilde{v}_{ph}$ of the superposed wakefield can be controlled by selecting the angle of the two intersecting waves, i.e., $\tilde{v}_{ph} \equiv (v_{ph}/\cos\theta) > v_{ph}(= v_b)$. In principle, one can choose an angle $\theta$ such that $\tilde{v}_{ph}$ is larger than $c$. However, since the energy gain is limited by the finite spatial extent of the interaction region, which is proportional to $\sin^{-1}\theta$, there should be an optimum angle that gives maximum energy gain per stage. A detailed discussion will be reported elsewhere.

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9. Allowing for particles to ride from the top to the bottom of a potential well rather than from the top to the zero level provides the factor 4 in this expression instead of the factor 2 given by Tajima and Dawson (cf Ref. 2).
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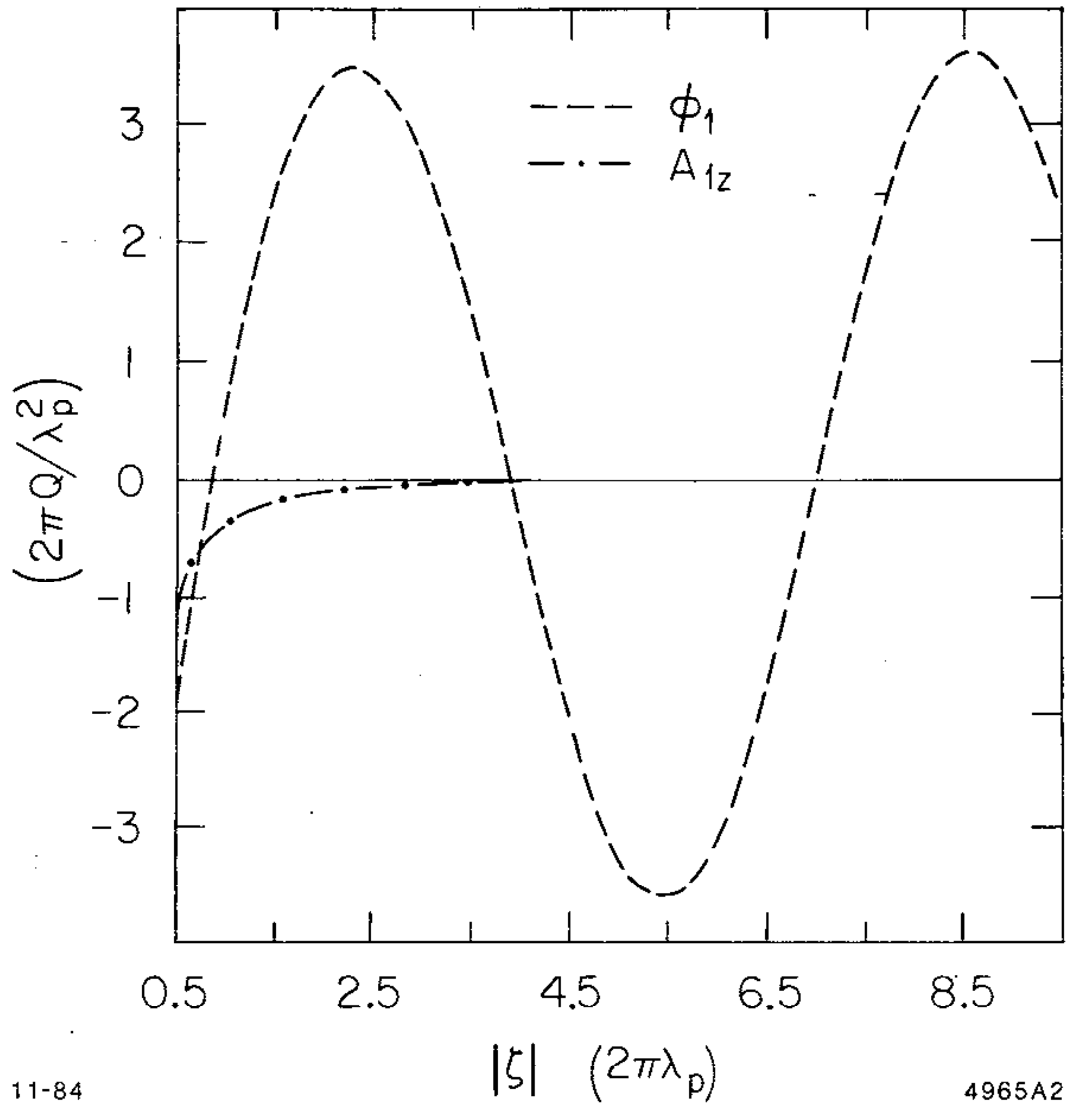
## FIGURE CAPTION

Fig. 1. Potentials as functions of distance behind the last bunch.

Fig. 2. Momentum distribution of the driving and driven electron beams when the latter has attained its maximum upper limit.

Fig. 3. Momentum distribution of the driven electrons before the maximum upper limit is attained. This case differs from that of the preceding figure in that the driving beam is removed from the system when the plasma wave reaches its maximum amplitude, at which time the driving beam has the momentum distribution shown. Note the different scales in the two figures.



$\phi_1$
$A_{1z}$
3
2
1
0
-1
-2
-3
$\left(2\pi Q/\lambda_p^2\right)$
0.5
2.5
4.5
6.5
8.5
$|\zeta| \quad (2\pi\lambda_p)$
11-84
4965A2


Fig. 1

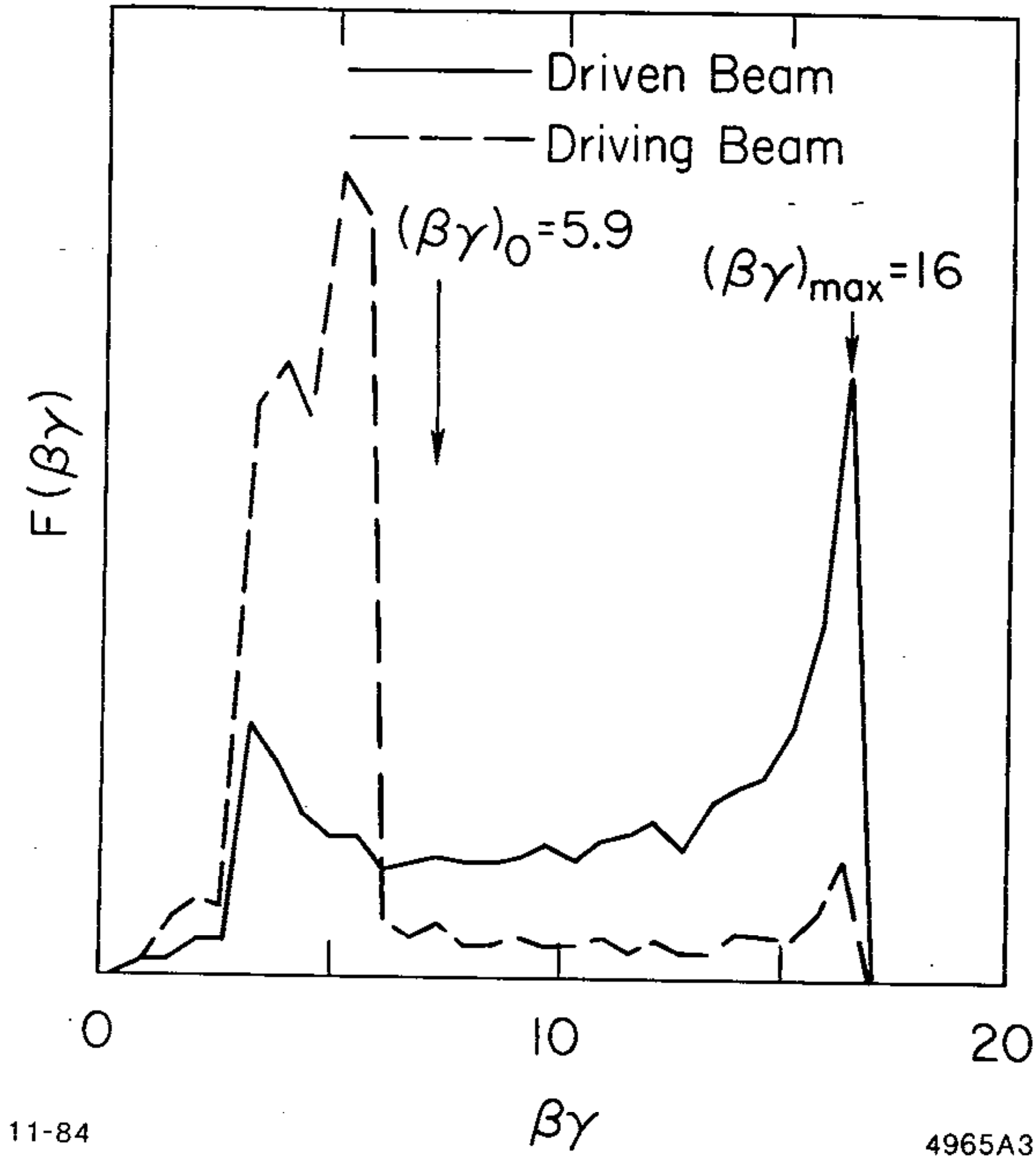
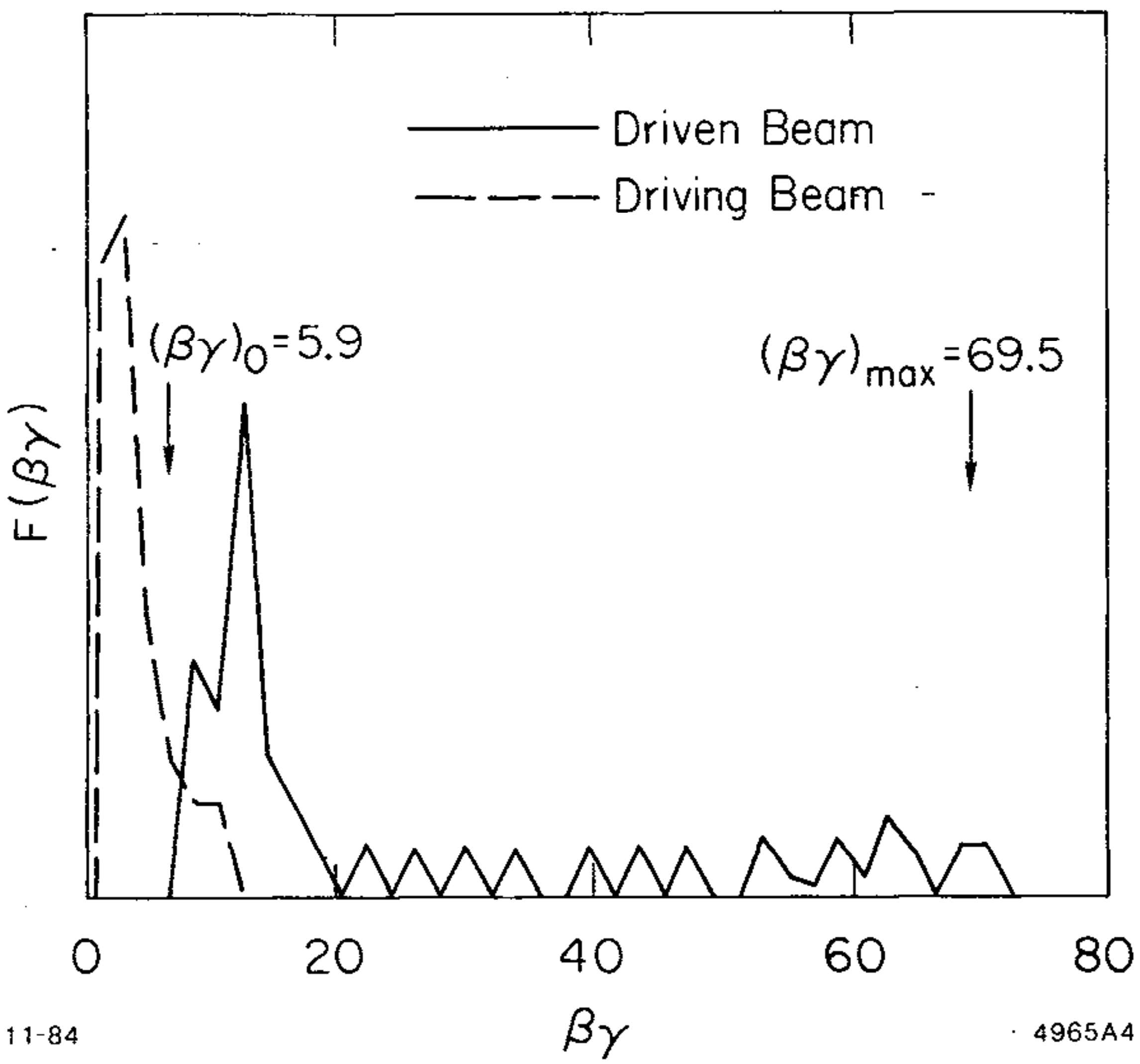

Driven Beam
Driving Beam
$(\beta\gamma)_0 = 5.9$
$(\beta\gamma)_{max} = 16$
$F(\beta\gamma)$
0
10
20
$\beta\gamma$
11-84
4965A3

Fig. 2



Driven Beam
Driving Beam
$(\beta\gamma)_0$=5.9
$(\beta\gamma)_{max}$=69.5
F($\beta\gamma$)
0
20
40
60
80
$\beta\gamma$
11-84
4965A4


Fig. 3

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# ERRATUM

## ACCELERATION OF ELECTRONS BY THE INTERACTION OF A BUNCHED ELECTRON BEAM WITH A PLASMA*

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[ *Phys. Rev. Lett.* **54**, 693 (1985) ]

Equation (5) should read

$$\phi_1(\rho,\varsigma) = Q \sum \left\{ -(1/|\vec{x}-\vec{x}_i|) + k_p \int_{\varsigma}^{\infty} d\varsigma' \, \sin k_p(\varsigma'-\varsigma)/|\vec{x}\,'-\vec{x}\,'_i| \right\} . \qquad (5)$$

* Work supported in part by the National Science Foundation, Grant PHY 83-04641, and the Department of Energy, Contract DE-AC03-76SF00515.

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The paragraph starting line 14 on p. 694 is incorrect and should be replaced by the following:

" We are interested in the wake field trailing behind the $N$ bunches on the $z$ axis. However, the integral in Eq. (5) includes $\varsigma = 0$ and therefore is logarithmically divergent when $\rho = 0$. This is certainly unphysical. The origin of this symptom is due to the lack of any thermal effects in our cold fluid model. Thus Eq. (5) is not applicable to the region within a Debye length from the axis. Bearing this in mind, Eq. (5) reads

$$\phi_1(\rho,\varsigma) = \frac{2\pi Q}{\lambda_p} \sum \left\{ -\frac{1}{k_p\sqrt{\rho^2 + [(N-i)d - \varsigma]^2}} + \int_{\varsigma}^{\infty} d\varsigma' \frac{\sin k_p(\varsigma' - \varsigma)}{\sqrt{\rho^2 + [(N-i)d - \varsigma']^2}} \right\}, \tag{7}$$

where $\lambda_p \equiv 2\pi/k_p$. By the same token the vector potential in Eq. (6) is not applicable to the region within a Debye length behind the plane $\varsigma = 0$ and

$$\begin{aligned} A_{1z}(\rho,\varsigma) = & -\frac{2\pi\beta_0 Q}{\lambda_p} \sum \int_0^{2\pi} \frac{d\theta}{2\pi} \int_0^{\infty} \rho' d\rho' \; K_0\left(\beta_0 k_p \sqrt{\rho^2 + \rho'^2 - 2\rho\rho'\cos\theta}\right) \\ & \times \frac{\partial^2}{\partial \rho'^2} \frac{1}{k_p\sqrt{\rho'^2 + [(N-i)d - \varsigma]^2}} \quad , \end{aligned} \tag{8}$$

where $K_0$ is the modified Bessel function of order zero. Plots of $\phi_1$ and $A_{1z}$ as functions of $|\varsigma|$ evaluated at $\rho = c/\omega_p$ are shown in Fig. 1, where $N = 5$ and $d = \lambda_p$. "

Equation (9) should read

$$\epsilon = -eE_{1z} \simeq 16\pi^2 eQ/\lambda_p^2 \quad . \tag{9}$$

In 4 lines below Eq. (9), the number of particles should be $q \simeq 2.5 \times 10^{10}$. In the last line of the same paragraph, $L$ should be $< 0.37\gamma_0$ cm.