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A REVIEW OF POLARIZED PROTON BEAM TECHNIQUES* AND A REPORT ON THE WORKSHOP ON POLARIZED BEAMS

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Résumé – Après une introduction aux techniques de base concernant les faisceaux de protons polarizés (changement de spin, saut de résonances, "Serpent de Sibérie"), un rapport sur le récent atelier tenu sur l'Accélération et Stockage des Faisceaux de Protons et Électrons dans les Très Grandes Machines est présenté.

Abstract – This paper first introduces the basic techniques for polarized proton beams: spin flip, resonance jumps, Siberian Snakes, etc., and then concludes with a report on the Workshop on Acceleration and Storage of Polarized Protons and Electrons in Very Large Machines.

I. INTRODUCTION

The purpose of this paper is two-fold. First it is to introduce the reader to the problems associated with the acceleration and storage of polarized protons and to discuss the solutions to these problems. Second, it is to review briefly the Workshop on Acceleration and Storage of Polarized Protons and Electrons in Very Large Machines. Notice that the discussion in the first part of this paper is limited to polarized protons. The basic physics of electron polarized beams is quite different from that for protons. In Table 1 you see the contrast between the two. For electrons both the spin and the orbit experience a competition between polarizing (damping) effects and depolarizing (quantum excitation) effects. For polarized protons the problem is quite different in that the polarizing effects (and quantum spin diffusion) are absent. For the orbit the aim is to transport phase space without dilution during acceleration or storage. For the spin degree of freedom the situation is completely analogous. We begin with a polarized beam and attempt to avoid depolarization. The problem, for both the orbit and the polarization and for both electrons and protons, is resonance. For the orbit we simply set the basic frequency so that resonances are avoided; however, the situation is different for the spin because the natural precession frequency is proportional to the energy.

Table 1. The Comparison of Electrons and Protons

	Protons	Electrons
$\frac{g}{2} - 1$	$G = 1.793$	$a = .00116$
Precession frequency	$\kappa = \gamma G$	$\nu_{sp} = \gamma a$
Orbit	transport emittance without dilution	damping + quantum excitation $\Rightarrow$ emittance
Spin	transport polarization without depolarization	radiative polarization + quantum spin diffusion $\Rightarrow$ polarization
Problem: Orbit	resonances	resonances, quantum lifetime, etc.
Problem: Spin	resonances $\kappa = k_1 + k_2\nu_x + k_3\nu_z + k_4\nu_s$ resonance crossing	resonances $\nu_{sp} = k_1 + k_2\nu_x + k_3\nu_z + k_4\nu_s$ enhanced diffusion

In the first section of the first part of this paper the basic physics is discussed and the resonances enumerated. The second section treats the spin resonance in detail while the third discusses the standard 'cures' for depolarization in low energy machines (~ 25 GeV). In the next section the cure for high energy proton storage rings, the 'Siberian Snake,' is discussed in some detail and in the final section of the first part we mention some outstanding problems and examine the possibilities at very high energy. The second part of the paper presents a brief report on the workshop on polarized beams. The topics mentioned there will be covered in much more detail in the individual contributions to the conference proceedings.

II. A REVIEW OF POLARIZED PROTON BEAM TECHNIQUES[1]

A. The Basic Physics[2]

To understand the basic physics consider a circular accelerator. The particles are kept on an essentially circular orbit by a guide field which is mostly a vertical magnetic field. In this field the spin of a particle precesses about the vertical with a frequency

$$\kappa = \gamma G = \gamma(\frac{g}{2} - 1) \quad . \tag{1}$$

i.e., for a bend angle $\Delta\theta$, the spin precesses by

$$\Delta\theta_{spin} = \gamma G \Delta\theta \tag{2}$$

when compared to the bent orbit. In a perfectly vertical magnetic field the motion is a pure precession, and thus, the vertical projection of the spin vector $\vec{S}$ is preserved

$$S_z = \text{const.} \tag{3}$$

Therefore, if a vertically polarized beam were injected, the polarization would be maintained.

However, the story is not yet complete. In order to keep particles in the neighborhood of the ideal orbit, focusing fields are also necessary (in particular, vertical focusing). So from time to time a particle must pass through horizontal magnetic fields which bend it back to the neighborhood of the design orbit. This causes the spin to precess out of the vertical. On the other hand, these horizontal fields must average to zero since the average position is nearly a planar circle.

Thus we have the situation that the field experienced by a particle on its orbit is a vertical magnetic field plus fluctuating horizontal fields. This leads to a precession of the spin around the vertical plus small fluctuations out of the vertical. These fluctuating terms average to zero unless the precession frequency is the same as the frequency of oscillation of the horizontal fields (as seen by the particle). If these frequencies are the same, the resonance condition, then on each turn, the small precession around the horizontal axis adds in phase with the previous turns, a situation which can lead to depolarization.

What are the frequencies at which resonance occurs? The vertical orbit of a particle in an accelerator is composed of two parts. Let z be the vertical deviation from an ideal design orbit.

$$z = z_{co} + z_\beta \tag{4}$$

An accelerator is never built quite perfectly. These errors (magnet misalignments, etc.) drive the oscillations in the vertical direction at the frequency with which they occur. In terms of the turning angle of the accelerator, the errors are periodic, and thus a Fourier decomposition yields all integer frequencies. Thus z_{co} (co – closed orbit), the "inhomogeneous response" of this oscillator is a function of the accelerator only, and

$$z_{co} \rightarrow \text{ integer frequencies, } k \quad \text{"imperfection resonances"} \tag{5}$$

On the other hand, not all particles are exactly on this orbit, and therefore, they oscillate about it with the natural frequency of the focusing system, "the homogeneous solution." This betatron oscillation, z_β, consists of an oscillation at a frequency ν_z, the vertical tune, modulated periodically in phase and amplitude. Thus, if the accelerator has P identical periods, then

$$z_\beta \rightarrow \text{ frequencies } kP \pm \nu_z, \; k \text{ an integer} \quad \text{"intrinsic resonances"} \tag{6}$$

If the imperfections in quadrupole gradients are also included, then the exact periodicity is unity, and we find

$$\text{Gradient errors, } z_\beta \rightarrow k \pm \nu \ . \tag{7}$$

This discussion concerned the orbits, but since the field on the orbit is just linearly related to the orbit, then these are also the frequencies of the magnetic field on the orbit. Of course, the actual situation is even more complex since there are higher order effects also. The general resonance condition is

$$\kappa = \gamma G = k_1 + k_2\nu_z + k_3\nu_x + k_4\nu_s \ . \tag{8}$$

where ν_x and ν_s are the horizontal and synchrotron oscillation tunes respectively.

To illuminate the problem consider just the imperfection resonances. Then during acceleration a resonance is encountered every 0.52 GeV, clearly a frequent occurrence for a high energy accelerator. In the next sections we give a quantitative treatment of these resonances to understand the depolarization mechanism and to understand the methods that have been developed for avoiding depolarization.

B. The Spin Resonance

1. The Equations of Motion. The spin of a particle, taken as a classical normalized vector, in a static magnetic field obeys the equation

$$\begin{aligned} \frac{d\vec{S}}{dt} &= \frac{e}{\gamma mc}\vec{S} \times \left[(1+\gamma G)\vec{B}_\perp + (1+G)\vec{B}_\parallel\right] \\ &= \vec{S} \times \vec{\Omega} \end{aligned} \tag{9}$$

where $\vec{S}$ is in the rest frame, t and $\vec{B}$ are in the lab, and $\vec{B}_\perp(\vec{B}_\parallel)$ are the components of the magnetic field which are perpendicular (parallel) to the instantaneous velocity. The orbit equations are given by

$$\frac{d\vec{v}}{dt} = \frac{e}{\gamma mc}\vec{v} \times \vec{B} \ . \tag{10}$$

The difference between the bending of the orbit and precession of the spin in a transverse magnetic field is clear from the difference in the coefficients in Eqs. (9) and (10).

It has become customary and also convenient to express the above equations in spinor notation. To do this we let

$$\vec{S} = \psi^\dagger \vec{\sigma} \psi \tag{11}$$

where $\vec{\sigma}$ stands for the Pauli matrices and ψ is a two-component spinor.Then the equation of motion for ψ is given by

$$\frac{d\psi}{dt} = \frac{i}{2}(\vec{\sigma}\cdot\vec{\Omega})\psi \ . \tag{12}$$

Equations (11) and (12) are exactly equivalent to Eq. (9).

If in addition we go to a coordinate system which rotates with the velocity of the particle and change the independent variable to the bending angle, then Eq. (12) becomes[2]

$$\frac{d\psi}{d\theta} = \frac{i}{2}\begin{pmatrix} -\kappa & \varsigma(\theta) \\ \varsigma^*(\theta) & \kappa \end{pmatrix}\psi \tag{13}$$

where

$$\varsigma = -(1+\gamma G)(\rho z'' + iz') + i\rho(1+G)(z/\rho)' + \text{higher order terms } = \sum_k \epsilon_k\, e^{-i\kappa_k\theta} \ . \tag{14}$$

Thus, the horizontal fields on the orbit of the particle have been expressed in terms of the vertical deviation from the ideal orbit, z, and the instantaneous bending radius, ρ. The primes indicate d/ds where s is the distance along the ideal orbit. This expression for the perturbing fields is convenient for calculations since commonly the orbit in an accelerator is well known.

Notice that ς has been expressed as a series with coefficients, ϵ_k. In general for the first order effects, we need to include all frequencies mentioned in the introduction. However, it is useful, provided the frequencies are well separated, to consider each resonance separately. To begin, consider a perfect machine.

2. The Perfect Machine. For the perfect machine there is only a vertical field; thus, setting ς equal to zero in Eq. (13) yields

$$\frac{d\psi}{d\theta} = -\frac{i}{2}\kappa\sigma_z\psi \tag{15}$$

which has the solution

$$\psi(\theta) = \exp\left[\frac{-i}{2}\int_0^\theta \kappa d\theta'\sigma_z\right]\psi_0 \tag{16}$$

or if we define

$$\chi(\theta) = \int_0^\theta \kappa d\theta' \ , \tag{17}$$

then Eq. (16) becomes, explicitly,

$$\psi(\theta) = \begin{pmatrix} e^{-i\chi/2} & 0 \\ 0 & e^{i\chi/2} \end{pmatrix}\psi_0 \ . \tag{18}$$

This is a precession around the z axis at an instantaneous frequency, κ. Thus, the projection on the z axis is preserved, *i.e.*

$$S_z = \psi^\dagger\sigma_z\psi = \psi_0^\dagger\sigma_z\psi_0 = S_{z0} \ . \tag{19}$$

3. An Isolated Resonance. The problem for an isolated resonance is only slightly more difficult than that for a pure vertical field. For this problem let κ be constant ($\gamma =$ const.) and consider one resonance with strength ϵ at frequency κ_0 (κ_0 can be any one of the resonances discussed previously). Thus we must solve the equation

$$\frac{d\psi}{d\theta} = \frac{i}{2}\begin{pmatrix} -\kappa & \epsilon e^{-i\kappa_0\theta} \\ \epsilon e^{i\kappa_0\theta} & \kappa \end{pmatrix}\psi \ . \tag{20}$$

Now change to a coordinate system which rotates at the frequency of the perturbation, κ_0. In this frame the perturbing field appears stationary rather than oscillating. Explicitly we let

$$\psi = \exp(-i\kappa_0\theta\sigma_z/2)\,\phi \tag{21}$$

and obtain

$$\frac{d\phi}{d\theta} = \frac{i}{2}\begin{pmatrix} -\delta & \epsilon \\ \epsilon & \delta \end{pmatrix}\phi = -\frac{i}{2}\lambda(\vec{n}\cdot\vec{\sigma})\phi \tag{22}$$

where

$$\begin{aligned} &\delta = \kappa - \kappa_0, \qquad \lambda = \sqrt{\epsilon^2+\delta^2} \\ &\vec{n} = \epsilon/\lambda\,\hat{x} + \delta/\lambda\,\hat{z} \ . \end{aligned} \tag{23}$$

The solution is simply

$$\phi(\theta) = \exp(-i\lambda(\vec{n}\cdot\hat{\sigma})\theta/2)\phi_0 \ . \tag{24}$$

This is a precession about $\vec{n}$ and the projection along $\vec{n}$ is invariant; however, now $\vec{n}$ is no longer along the vertical (see Fig. 1). (In addition if we transform back to the original coordinates, $\vec{n}$ is precessing about the vertical.) Thus the perturbation creates an effective field which is precessing at frequency κ_0 and tipped out of the vertical by an amount ϵ/λ.

Consider the following thought experiment: let δ be very large and negative so that $\vec{n} = -z$. Now vary δ adiabatically through zero (resonance) to a large positive value ($\gg \epsilon$). Then $\vec{n}$ rotates and finally $\vec{n}$ and $-\vec{n}$ change places. However, since it is the projection along $\vec{n}$ which is preserved, this means that the spin flips. This spin flip is completely analogous to that which takes place in Nuclear Magnetic Resonance.

Thus, we have an interesting situation: If $\epsilon = 0$ the resonance does nothing, and if $\epsilon \neq 0$, then the spin flips (provided we change γ and thus κ slowly enough to be adiabatic). This is quite a contrast to orbit resonances which can drive the beam out of the beam pipe. In fact this process of adiabatic spin slip has been used successfully at the Argonne ZGS, Saturne II, and the AGS at BNL.

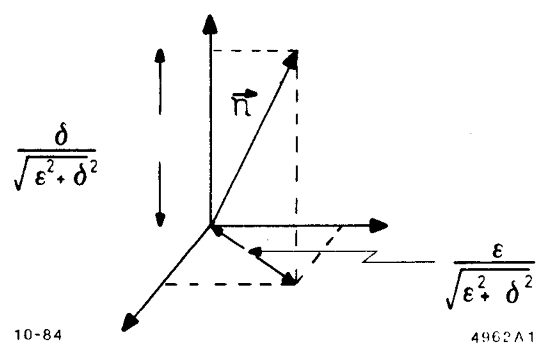

Fig. 1. The rotation of $\vec{n}$ in the neighborhood of a resonance.

The question is now to understand what "adiabatic" is in the context of acceleration through a spin resonance. In any system an adiabatic variation is one which occurs during a time which is large compared to the inverse of the frequency. Near a spin resonance the frequency λ is $\sim \epsilon$; in addition ϵ is the "width" of the resonance since it determines the variation in δ necessary to flip $\vec{n}$. Thus if we change δ at rate $\alpha = d(\kappa - \kappa_0)/d\theta$, the "time", $\Delta\theta$, for passage through the resonance is

$$\Delta\theta = 2\epsilon/\alpha \ . \tag{25}$$

For this passage to be adiabatic there should by many oscillations within this "time," *i.e.*

$$\epsilon\Delta\theta \gg 2\pi \Rightarrow \epsilon^2/\alpha \gg \pi \ . \tag{26}$$

Therefore, the resonance must be sufficiently strong or the rate of passage sufficiently slow for spin flip to take place. On the other hand it is clear that if ϵ is small enough, the resonance has no effect at all. The question is then what happens in the intermediate cases; the answer is depolarization.

4. Passage Through Resonance. The depolarization effect of a single isolated resonance can be calculated in terms of known special functions in the case of linear variation of the energy and the precession frequency κ. The result is particularly simple if we integrate the spin motion from $-\infty$ to ∞; *i.e.* if we start far from the resonance and end far from the resonance on the opposite side. The depolarization in this case is given by[3,2]

$$S_z^{final} = \left(2e^{-\pi\epsilon^2/2\alpha} - 1\right) S_z^{initial} \ . \tag{27}$$

Thus, we see again that the parameter ϵ^2/α plays the key role. Notice also that both spin flip and no spin flip are contained in this formula; if ϵ^2/α is small enough then the polarization is unchanged, while, if ϵ^2/α is sufficiently large, we find the spin flip described previously. In the first case the spin precession frequency is changing so rapidly that the spin vector does not have time to follow the change in direction of the effective field; while for the second case the change in frequency is sufficiently slow to allow the precessing spin to follow the reversal of the effective field.

C. Cures for Small Machines

There are basically four approaches based on the Froissart and Stora formula, Eq. (27), for eliminating the depolarizing effects of resonances.

1. Make ϵ Small (spin transparency or harmonic matching). In the case of imperfection resonances this is accomplished by tuning out the harmonics of the closed orbit which are causing the trouble by using correction dipoles. In practice, the change in the orbit is so small that the polarization of the beam must be used as an indicator. This method was used quite successfully at the Argonne ZGS, is now used at Saturne II for some of the depolarizing resonances,[4] and has been recently tested once again at the AGS at BNL.[5] In addition this method has now also been used successfully for electrons at PETRA where the polarization was improved from 20% to 80%.[6]

In principle this idea can also be used to tune out the effects of intrinsic resonances.[7] In this case, correction quadrupoles are necessary and the correction requires a calculation with the existing lattice. Correction quadrupoles are also necessary to eliminate the resonances due to errors in quadrupole gradients. Thus far, correction quadrupoles have been used successfully at Saturne II to correct errors in gradients; however, this method has not yet been attempted with intrinsic resonances.

2. Increase α. From Eq. (27) we see that decreasing the strength of the resonance, ϵ, is equivalent to increasing the rate of passage through the resonance, α. Since there are limitations on the acceleration rate in any accelerator, this method is not very useful for imperfection resonances. However, this method can be and has been used quite successfully for intrinsic resonances. The important difference here is that for intrinsic resonances, the frequency depends upon the vertical tune ν_z.

Therefore, it is possible to cross resonances by changing the tune of the machine abruptly as the precession frequency comes close to a resonance. This is illustrated in Fig. 2. Note that it is necessary to maintain the tune shift for some time until the normal acceleration rate has separated the spin precession frequency from the resonance.

This method has been used quite successfully at the Argonne ZGS and is now being used at the AGS at BNL.[5] There are, however, limitations to this method in that the tune shifts must be small enough not to damage the orbit of the beam yet large enough to clear the "tails" of the resonance. Of course, the rate of the tune shift must be quite high also. The design at the AGS is probably close to the feasible limits. In this case the tune shift is 0.25 in a time of about 2 μsec. With these finite changes in frequency, it is also necessary to use a modified formula for calculating the depolarizing effects.[8,9]

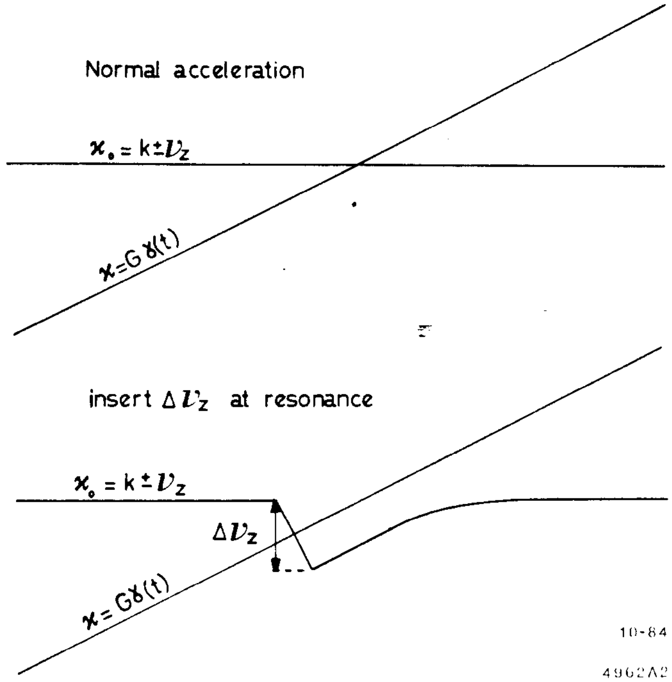

Fig. 2. The resonance jump.

3. Make ϵ larger. If an individual resonance is quite strong, then, provided that other resonances are well separated, it is possible to use the resonance to flip the polarization of the beam. This can be accomplished in the case of imperfection resonances by the same method of closed orbit correction; however, in this case the "correction" is shifted in phase to enhance the resonance. This method is used successfully at Saturne II (100% spin flip) and has been successfully tested also at the AGS at BNL.

4. Decrease α. By similar arguments one can change the rate of passage through a resonance to enhance its effect. One can, of course, vary the acceleration rate, but in addition it is possible to again use a tune shift to change the rate of passage through the resonance. This method was used with only moderate success at the Argonne ZGS. It has been tested also at the AGS at BNL and preliminary results yielded 60% depolarization.[5]

Finally we should mention that there have been new interference effects observed at Saturne II which have now been understood theoretically.[10,4] These are caused by synchrotron oscillations. These effects have not limited performance at Saturne II and have so far not been observed at the AGS.

D. The Cure for Large Machines – The "Siberian Snake"

For high energy accelerators or storage rings such as FNAL and the SPS at CERN, there are hundreds of resonances which are stronger than those for lower energy machines. Thus, the standard approaches of resonance jumping and spin flip are not so attractive. In particular, resonance jumping becomes technically unfeasible. One can imagine a spin flip at each resonance; however, a global solution which would eliminate all the resonances is desirable. This solution exists in the form of the "Siberian Snake."[11]

~~1.~~ **The Single Siberian Snake.** To understand the basic principle consider Fig. 3 (taken from Ref. 12). Assume that at the point S_y on the circumference of the accelerator there is a device which: a) precesses the spin by 180° **around the longitudinal** direction, and b) yields no net orbit deflection or displacement.

Now imagine the spin of a particle pointing in some arbitrary direction traversing the ring once, from the point A and back. During this circuit the spin gets precessed first by an angle around the z axis

$$\Delta\theta_{1/2} = \gamma G\pi \quad , \tag{28}$$

then it gets precessed around the longitudinal axis by

$$\Delta\theta = \pi \quad , \tag{29}$$

and finally it once again precesses by $\Delta\theta_{1/2}$ in returning to its starting point. Figure 3 shows how each of the three projections of the spin are treated by such a succession of transformations. The key point is that the snake effectively reverses the vertical direction so that the two $\Delta\theta_{1/2}$ precessions exactly cancel, with the net effect that the spin is rotated about the longitudinal direction by 180°. Thus, as you see in Fig. 3 the longitudinal projection of the spin is preserved at the point A. In addition, the spin tune, has been changed to $\nu_p = 1/2$ independent of energy, because the other projections each repeat after two revolutions.

Thus the normal precession frequency has been changed to a non-resonant point independent of energy and the mode of operation has been changed; a polarized beam is injected so that it is **longitudinal** at the point A. Knowing the polarization at A, it is then easy to calculate it (as a function of energy) at other points in the ring.

2. The Double Siberian Snake There is another possibility which has some advantages over the single Siberian Snake. In this case there are two snakes directly opposite (180° bending between the snakes). One snake, S_y rotates 180° about the longitudinal direction while the other, S_x rotates 180° about the horizontal direction.

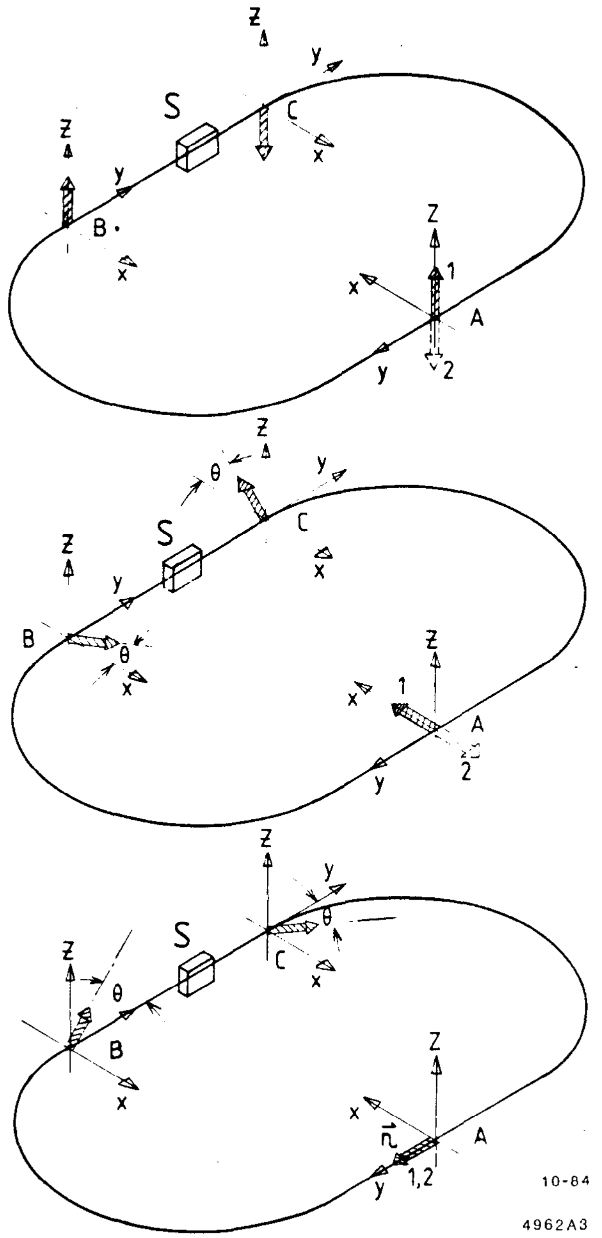

Fig. 3. Spin motion with a single Siberian Snake.[12]

Figure 4 demonstrates how each projection of the spin moves in a ring with a double Siberian Snake.

For this scheme the spin tune is again changed to 1/2 independent of energy; however, now it is the vertical projection of the polarization which is preserved, up in one-half of the ring and down in the other half. This has the distinct advantage that the invariant spin direction is independent of energy at **all** points along the circumference.

The key point about both snake schemes is that they shift the spin tune to a non-resonant point **independent** of the energy. This places the spin tune on an equal footing with betatron and synchrotron tunes; however, there are still problems not solved by the snake.

3. Non-linear Spin Resonance with a Snake In the presence of a snake the resonance condition in Eq. (8) becomes

$$\frac{1}{2} = k_1 + k_2\nu_z + k_3\nu_x \tag{30}$$

where $k_4\nu_s$ has been suppressed. Equation (30) is valid for a coasting beam and approximately valid if ν_s is sufficiently small. (For ν_s larger the following analysis is easily extended.) Written as in Eq. (30) the spin resonance condition becomes a restriction in two dimensional betatron tune space. In Fig. 5 you see a set of possible resonance lines which must be avoided. This includes all resonances which satisfy

$$|k_2| + |k_3| < 3 \; . \qquad (31)$$

All of these lines correspond to higher order betatron resonance lines if one includes both sum and difference non-linear betatron resonances. For example the resonance with $|k_2| = |k_3| = 1$ corresponds to betatron resonances given by

$$\pm 2\nu_x + \pm 2\nu_z = \text{odd integer} \; . \qquad (32)$$

This is a subset of the stable and unstable cubic betatron resonances.

In addition each of these resonance lines has a width which depends strongly on the k's. Those resonances in which k_1 is a multiple of the periodicity are strong and should be avoided by a judicious choice of a working point. The previous discussion is especially relevant to storage rings. If the working point is chosen on a (low order) resonance then even if the resonance is very weak, the beam will eventually depolarize. In fact slow non-linear resonance has been experimentally observed at the ZGS where a small shift in the extraction energy eliminated the problem.[13]

The calculation of the widths of these non-linear resonances is difficult; however, recent progress with tracking both orbit and spin at DESY to discover non-linear effects will help considerably for testing individual storage rings.[14] These non-linear resonances are also excited by the beam-beam effect.

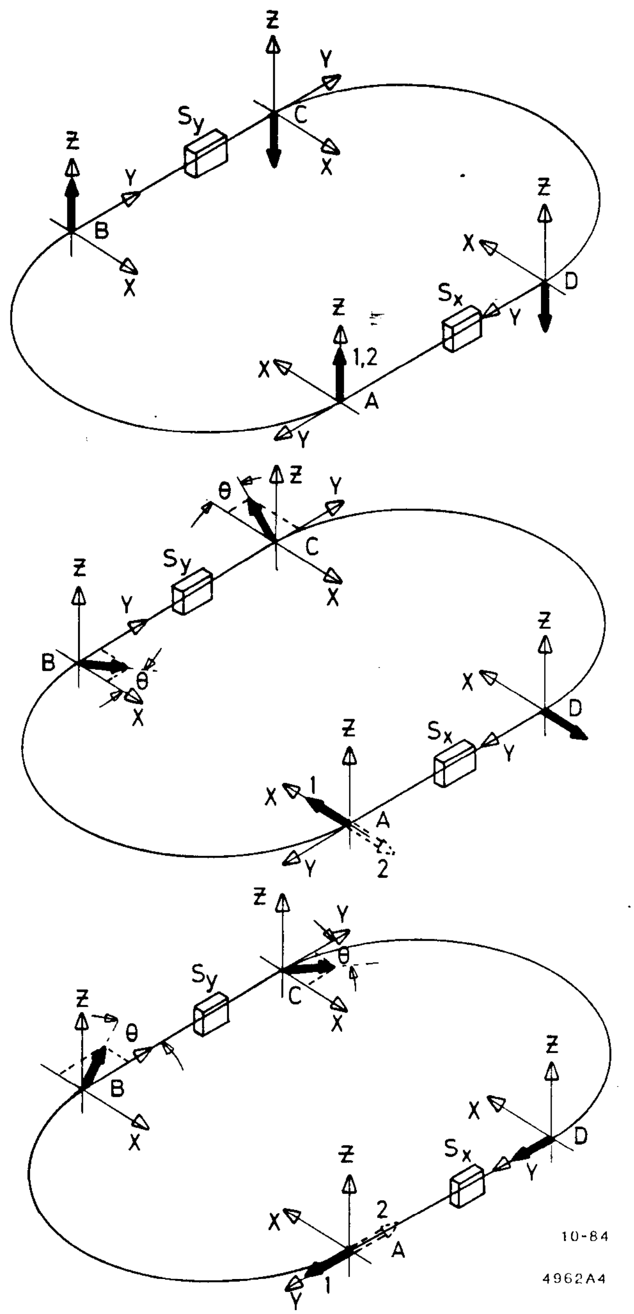

Fig. 4. Spin motion in a double Siberian Snake.

4. Spin Tune Shifts in Siberian Snakes If the arcs of a storage ring were just bends, then all of the previous discussion would be exact. However, we know that the horizontal fields can have large cumulative effects near the spin resonances discussed in the first section. This means that near the 'old' spin resonances given by Eq. (8), the horizontal fields must be taken into account.

Following Ref. 15 consider an isolated resonance of complex width ϵ. Then if the precession frequency in the arcs is exactly the frequency of the perturbation, the spin tune is

Single snake:

$$\nu_p = \frac{1}{2} + |\,\epsilon\,| \cos\phi \qquad (33)$$

Double snake:

$$\cos \pi\nu_p = \cos(2\phi)\sin^2(\pi\,|\,\epsilon\,|\,/2) \qquad (34)$$

where ϕ is a phase factor. In addition to the change in tune there is a change in the $\vec{n}$ direction also. Thus, the old resonances which were eliminated by the Siberian Snake scheme cannot simply be forgotten. They lead to two important effects:

1) integer resonance can reappear if $|\,\epsilon\,| > 1/2$ for the single snake or if $|\,\epsilon\,| > 1$ for the double snake and
2) they shift the non-linear resonance condition in Eq. (30) and thus shift also the intercepts of the resonance lines in Fig. 5.

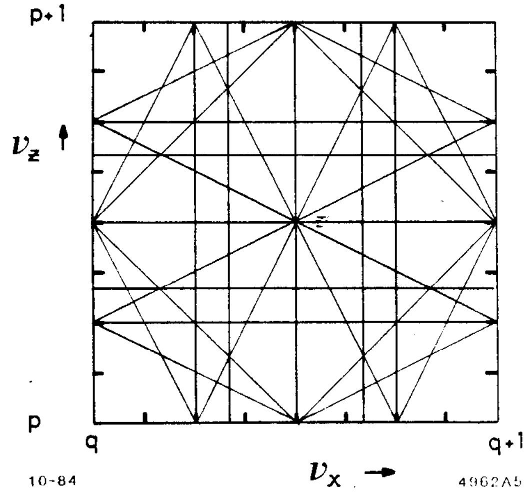

Fig. 5. Nonlinear spin resonances with a Snake.

Therefore the choice of a working point must also include the choice of an energy which is far enough from large resonances to ensure that the snake works properly. If, however, the spin resonances are quite strong, integer resonances occur in spite of the snake.

5. Scaling for ϵ The resonance strengths ϵ_k of the linear resonances mentioned can be calculated for any given machine.[2] The details of the sizes depend upon the layout of the linear lattice and upon the size of the errors. This strong dependence can be exploited to correct the lattice and diminish the effects.[7] As was mentioned earlier this has been demonstrated in both proton and electron machines and goes by several names, spin transparency, harmonic matching, etc. In addition, though, there are some scaling factors which are valid for any machine. Inspecting Eq. (14) one finds for intrinsic resonances

$$\epsilon_{int} \simeq \epsilon_0(\mathcal{E}/\mathcal{E}_0)^{1/2}(\beta/\beta_0)^{1/2}(E/E_0)^{1/2} \qquad (35)$$

where $\mathcal{E}$, β, and E are the invariant emittance, average beta function and energy respectively. The magnetic fields scale linear with particle energy; however, adiabatic damping reduces the effect on the spin by a factor $E^{-1/2}$. For imperfection resonances

$$\epsilon_{imp} \simeq \epsilon_0 \left(\frac{z_{co}}{z^0_{co}}\right)_{rms} (E/E_0) \ . \qquad (36)$$

In this case the scaling is linear with energy for given closed orbit. To estimate the ϵ's for a high energy storage ring consider a calculation for the AGS at BNL.[2] Then assuming that the invariant emittance, beta functions and closed orbits vary little one finds

$$\epsilon_{int} \simeq (1-15)\times 10^{-2}\left(\frac{E[GeV]}{25}\right)^{1/2} \qquad (37)$$

$$\epsilon_{imp} \simeq (1-25)\times 10^{-3}\left(\frac{E[GeV]}{25}\right) \ . \qquad (38)$$

These scaling laws compare well with actual calculations done in Ref. 15 for a 400 GeV storage ring.

It is clear that at energies higher than 400-500 GeV it will be necessary to use the snakes and harmonic matching to keep the ϵ's well below unity.

6. A Typical Snake Design Many combinations of transverse bends have been invented to create the rotation of the spin direction by π while returning the orbit. One of the best designs for a snake of type S_y is by Steffen.[16] The problem is to use a combination of bends which keeps the excursion of the orbit small at injection energy. At higher energy, since the snake magnets do their job at fixed field, the orbit excursion in the magnets drop inversely with the energy.

The detailed design here is taken from Ref. 17 which also appears in the proceedings of the Workshop on SP fixed target physics.[18] It consists of a series of ten dipoles and has a total length of 20.5 m. Table 2 presents the basic parameters of such a snake.[17,19] Notice the large aperture necessary. This is due to the relatively low injection momentum of 14 GeV/c. Otherwise the snake presents no technical problems and fits easily into the SPS.

Table 2. Snake Dipole Magnets for the SPS[27,28]

Type	Short Dipole	Long Dipole
Number	6	4
Spin rotation angle	$\pi/4$	$\pi/2$
$\int$ Bdl	1.375 Tm	2.75 Tm
Gap diameter	20.0 cm	20.0 cm
Overall		
length	125 cm	200 cm
width	160 × 85 cm	160 × 85 cm
weight	8.4 t	15.5 t
Total snake power		1.5 MW
Power for SPS physics		80 MW

E. Some Outstanding Problems

There are two very important outstanding problems which are essential for the operation of a high energy polarized proton storage ring: Beam intensity and measurement of the polarization.

There are standard methods for the measurement of the polarization for protons which can be applied up to about 25 GeV.[20] However, at higher energies there have been a few proposals but no clear cut winner as yet.[21] There is much work to be done here both for high energy fixed target experiments or for polarized storage rings.

The maximum intensity of a proton storage ring is limited in normal operation by either coherent instabilities or the beam-beam effect. For polarized operation this is also true; however, so far, sources have not yet supplied enough current to reach these limits. The solution chosen at the AGS at BNL is $H^- \uparrow$ injection. With the present source current, the intensity is in excess of 10^{10} $p \uparrow$. Saturne II has a similar intensity. However there are plans to increase the intensity of both these machines by an order of magnitude. One interesting new development at CERN uses polarized stable atomic hydrogen to obtain intense atomic beams.[22,23] This holds great promise for application to polarized sources. For application to storage rings intensities should be increased to levels approaching those obtained with unpolarized beams (AGS $\sim 10^{13}p$). However, if stacking techniques are combined with low emittance polarized beams, somewhat lower intensities should be acceptable.

Finally, if we assume we have high intensity and can measure polarization, will the beams remain polarized while in collision in a storage ring? The field due to the other beam not only changes the orbit of the particle, it also directly precesses the spin by a small amount. For electrons there is experimental evidence that depolarization takes place near the beam-beam limit;[6] however, there is no experimental data on polarized colliding proton beams! There have been theoretical studies that also suggest that beam-beam depolarization should occur near the beam-beam limit,[24] and there is a new proposal for a method to make the spin immune to the effects of the beam-beam force.[25]

The Siberian Snake "cures" these beam-beam effects also; however, the non-linear resonances discussed earlier are further enhanced by the beam-beam effect.

Can these techniques discussed here be extended to very high energy, say 20 TeV? If we examine the scaling laws in Eqs. (37) and (38), we see that the resonance strengths will be considerably greater than unity. However, there has been work by Kondratenko which suggests that these large resonances can be cured by multiplying the number of snakes. The number of snakes in this case scales roughly linear with the energy of the machine. Thus, a 20 TeV SSC might need about 50 Siberian Snakes. However, there is much work to be done before this issue can be settled.

Summary

The summary to this part of the paper can be very brief. The standard techniques for curing depolarization are well understood both theoretically and experimentally. We look forward to new source developments to bring polarized intensities up to those for normal operation or least sufficient for injection into large storage rings. The cure for large machines, the "Siberian Snake," is well understood theoretically but now needs an accelerator or storage ring as a home. The question is: who will take the next step, polarized protons at about 300 GeV?

III. WORKSHOP ON ACCELERATION AND STORAGE OF POLARIZED PROTONS AND ELECTRONS IN VERY LARGE MACHINES

The program of the workshop at the symposium consisted of four parts:

1. Introductory Talks
2. Exotic Spin Manipulation
3. Problems Specific to Electrons
4. Problems Specific to Protons and Scaling to Very High Energy

The emphasis was to have been on the last section; however, the contributions covered all topics roughly equally. The format left time for informal discussion as well as contributed talks. There were many talks, and thus it was necessary to meet the next Monday, Sept. 17, to continue the discussion on beam-beam depolarization. In the next section I will highlight some of the contributions made at the workshop. For the details of the contributions below, please refer to the conference proceedings.

Discussions and Talks at the Mini-Workshop

A. L. NAKACH

A. Nakach discussed some very interesting experimental and theoretical results from Saturne II concerning the depolarization from intrinsic resonances. They have shown that the correlation between betatron amplitude and spin flip can be exploited to obtain a partial recovery of the apparent depolarization. This is done by crossing two successive intrinsic resonances. In this case the large betatron amplitude particles experience a double spin flip while those at very small amplitude do not flip at all. It is only those at intermediate amplitudes that suffer depolarization, and thus the 'depolarization' is partially recovered. This has also been shown to happen at Saturne if they accelerate through an intrinsic resonance and decelerate through the same resonance.

L. RATNER

Larry Ratner discussed the recent success of the AGS at BNL in accelerating polarized protons up to 16.5 GeV with 40% polarization and intensities of about 10^{10}. They have found that spin flip does work at imperfection resonances; however, spin flip for intrinsic resonances yields some depolarization. Their choice for controlling depolarization is, therefore, to correct imperfection resonances and to jump intrinsic resonances with fast tune jump quadrupoles. They so far have only 8 tune jump quads powered in the AGS, but they plan to add more power supplies soon to bring the number up to 12.

D. BARBER

Barber discussed the possibility of a solenoidal spin rotator for HERA. He has shown that it is possible to match such a rotator both for the spin and for the orbit. In doing this he has developed some very general techniques making use of algebraic programs such as REDUCE. The rotator suggested consists of about 40 meters of 7 Tesla solenoids. This option has not yet been adopted for HERA. In spite of the high fields necessary, this scheme may be attractive because the geometry of the device is independent of energy and helicity orientation.

J. BUON

Jean Buon discussed several problems which he has been studying recently. He first talked about the general optimization of fully anti-symmetric mini-rotator schemes for LEP and HERA. Although Siberian Snakes have been suggested primarily for solving depolarization for large proton storage rings, they might also be used to control some aspects of depolarization in electron storage rings. Jean studied some aspects of this problem and found that they do not solve the problem of depolarization for electrons. To treat the problem of beam-beam depolarization, Jean suggested at the 12^{th} International Conference on High Energy Accelerators a scheme for curing the problem which cancels the effect in one damping time. In the talk here he showed that this technique is incompatible with normal spin matching techniques. Finally he discussed some attempts at using spin matching to reduce or enhance intrinsic resonances for the AGS. This work was done with Philip Bambade. They had only moderate success because of the lack of freedom in the AGS lattice.

BEAM-BEAM DISCUSSION (J. KEWISH)

The talk by Jean Buon stimulated a discussion of the effect of the beam-beam interaction on the beam polarization. The discussion was led by J. Kewish but many people contributed. Alex Chao made the point that the linear part of the beam-beam effect could be cancelled using the normal spin matching conditions but with the linear part of the beam-beam force present. Jean Buon further discussed his technique which differs in that it attempts to cancel the beam-beam effect over many revolutions. Kewish however pointed out that the nonlinear beam-beam kick yields a beam blow up before a damping time. This implies strong nonlinear effects on the spin motion. It was generally agreed that the problem needs further study. This is of course hampered by the general lack of understanding of the beam-beam effect on the orbit since it is the fields on this orbit which cause depolarization.

R. D. RUTH

I gave a general introduction to techniques for maintaining polarization for protons up to about 500 GeV. At very much higher energies, say 20 TeV, the resonances become very strong. So far these strengths have not been calculated, but there are some scaling laws which suggest that the techniques which work at low energy (tune jumps, orbit corrections and two Siberian Snakes) will not work at such high energies. It has been suggested that many Siberian Snakes could be used to solve the problem. The number of snakes needed rises linearly with energy, thus one would need on the order of 50 snakes for an SSC.

E. D. COURANT

Ernest Courant led a discussion on the scaling of techniques to very high energy. There is now a long list of possible accelerators for high energy polarized protons: the Sp$\bar{p}$S, the Tevatron, UNK, the Large Hadron Collider (LHC) and the SSC. It is clear that terrain following for the large machines is essentially incompatible with polarization. It is also clear that the simple multiplication of snakes may not work at energies much higher than 1 TeV. There is also the unsolved problem of beam-beam depolarization. In spite of this there are good reasons to believe that with multiple Siberian Snakes, polarized beams at very high energy will be possible; however, the problem needs much more study.

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