SLAC – PUB – 3457
September 1984
T

NONPERTURBATIVE PROPAGATORS IN QUANTUM CHROMODYNAMICS*

TOMAS I. LARSSON

Stanford Linear Accelerator Center
Stanford University, Stanford, California, 94305

ABSTRACT

Leading nonperturbative corrections to the quark and gluon propagators are derived following the assumption that the nonperturbative QCD vacuum can be described in terms of non-vanishing vacuum expectation values for the composite operators $[\bar{\psi}\psi]$ and $[G^2_{\mu\nu}]$. The nonperturbative quark propagator can be described in terms of a running quark mass and a running normalization function. These quantities are shown to be gauge independent. The nonperturbative gluon propagator can be described in terms of a running gauge parameter and a running normalization function.

Submitted to *Physical Review D*

* Work supported by the Department of Energy, contract DE – AC03 – 76SF00515.

1. Introduction

In this article we study quark and gluon propagation in the presence of quark and gluon condensates. The basic assumption underlying our analysis is that the nonperturbative QCD vacuum can be described in terms of non-vanishing vacuum expectation values (vev's) of gauge invariant composite operators such as the quark condensate $[\bar{\psi}\psi]$ and the gluon condensate $[G^2_{\mu\nu}]$. This hypothesis has been used extensively in successful attempts to account for nonperturbative phenomena. It is the key assumption in the so called QCD-sum rules[1] which have been used to study (among other subjects) charmonium decays, SU(3)-symmetry breaking effects and hadron wavefunctions[2]. Another application of the same assumption is a recent analysis[3] of the MIT bag model where it is found that the existence of quark and gluon condensates allows for the possibility of having a small strong coupling constant inside the bag.

Mueller[4] has recently shown that it is possible to define the nonperturbative condensates in a consistent way. Fukuda and Kazama[5] have demonstrated condensation of $[G^2_{\mu\nu}]$ by constructing the effective potential through the trace anomaly equation. However, we feel that this latter derivation remains somewhat ambiguous since it is not based on a rigorous definition of the gluon condensate.

In the following analysis, the composite operators will appear in the operator product expansion[6] (OPE) of the quark and gluon propagators respectively. The operators in this expansion must carry vacuum quantum numbers, and from general grounds one may conclude that they must be gauge singlets. The operators in the OPE are furthermore characterized by their dimension. The contribution from an operator with a higher dimension falls off more rapidly with momentum than a lower-dimension operator.

Apart from the unit operator, the operators $[\bar{\psi}\psi]$ (dimension 3) and $[G^2_{\mu\nu}]$ (dimension 4) are the gauge independent operators of lowest dimension in QCD. In the following we neglect higher-dimension operators. It should be mentioned that if higher-dimension operators acquire non-zero vev's, their influence can be neglected only for large momenta. We also note that the OPE can be taken seriously only for hard momenta in the propagator to be expanded.

In Refs. 1 and 2 the following phenomenological values for the vev's of $[\bar{\psi}\psi]$ and $[G^2_{\mu\nu}]$ are given:

$$\begin{cases} \langle\Omega|\,[\bar{u}u]\,|\Omega\rangle = \langle\Omega|\,[\bar{d}d]\,|\Omega\rangle = 1.3\,\langle\Omega|\,[\bar{s}s]\,|\Omega\rangle \;\simeq\; -(0.25GeV)^3\;; \\ \langle\Omega|\,[\bar{c}c]\,|\Omega\rangle = \langle\Omega|\,[\bar{b}b]\,|\Omega\rangle = \langle\Omega|\,[\bar{t}t]\,|\Omega\rangle \;\simeq 0\;; \\ \langle\Omega|\,[\frac{\alpha_s}{\pi}G^2_{\mu\nu}]\,|\Omega\rangle \;\simeq +0.012\,GeV^4\;. \end{cases} \tag{1}$$

The authors of Ref. 1 use a value close to 1 for α_s, corresponding to $Q^2 = 0.2\,GeV$ and $\Lambda_{QCD} = 0.1\,GeV$.

The effect of the quark condensate on the quark propagator was analyzed in 1976 by Politzer[7] and the analysis was later revised by Pascual and de Rafael[8] who pointed out some numerical errors in Politzers paper. Politzers main interest was in the derivation of the so called nonperturbative quark mass, obtained through the nonperturbative quark propagator. Here we revise the analysis in Refs. 7 and 8 and we extend it to include the gluon condensate contribution to the quark propagator. The nonperturbative quark propagator can be described in terms of a running (momentum dependent) normalization function and a nonperturbative quark mass. We derive these quantites and compare the result to that of Refs. 7 and 8.

We evaluate the nonperturbative corrections to the gluon propagator in a general covariant gauge. The corrected propagator can be described in terms of running normalization function and a running gauge parameter.

The ghost propagator is not influenced by $[\bar{\psi}\psi]$ or $[G^2_{\mu\nu}]$. In the following our analysis will be done at the tree level and all the calculations are done in Euclidean space.

2. The Nonperturbative Quark Propagator

The operator product expansion of the inverse quark propagator in the presence of non-vanishing vev's for the operators $[\bar{\psi}\psi]$ and $[G^2_{\mu\nu}]$ is:

$$
\begin{aligned}
{}^{ab}_{\alpha\beta}S^{-1}_{np} \approx\ & {}^{ab}_{\alpha\beta}C^{\mathbb{1}}(p)\ \mathbb{1} + {}^{ab}_{\alpha\beta}\, C^{[\bar{\psi}\psi]}(p)\ \langle\Omega|\,[\bar{\psi}\psi]\,|\Omega\rangle \\
& + \ {}^{ab}_{\alpha\beta}C^{[G^2_{\mu\nu}]}(p)\ \ \langle\Omega|\,[G^2_{\mu\nu}]\,|\Omega\rangle\,.
\end{aligned}
\tag{2}
$$

The coefficients carry the spinor and color quantum numbers as indicated by the indices α, β (spinor) and a,b (quark color). The coefficients are calculated perturbatively. Here they will be calculated to first order in α_s. ${}^{ab}_{\alpha\beta}C^{\mathbb{1}}(p)$, the coefficient associated with the unit operator, is equal to the perturbative inverse quark propagator. In Euclidean space we hence have:

$$
{}^{ab}_{\alpha\beta}C^{\mathbb{1}}(p) = i\ \delta^{ab}\ (\not{p} + m).
\tag{3}
$$

In order to calculate ${}^{ab}_{\alpha\beta}C^{[\bar{\psi}\psi]}(p)$, we will follow the standard recipe in the literature.[9] The coefficient is obtained by equating a (2+n)-point, one-particle irreducible Greens function—where two of the external legs have hard momentum and the remaining n external legs are assigned zero momentum—with the coefficient times an n-point Greens function with an insertion of the operator under study at zero momentum. The number n corresponds to the number of elementary fields contained in the composite operator.

In our present case, we chose to study the quark-antiquark, one gluon exchange, four-point function. The operator product expansion of the four-point function is illustrated in Fig. 1. It reads:

$$P^{cd}_{\gamma\delta}\ {}^{abcd}_{\alpha\beta\gamma\delta}\Gamma^{(2+2)}(p,p,0,0) \approx\ {}^{ab}_{\alpha\beta}C^{[\bar{\psi}\psi]}(p)\ \ P^{cd}_{\gamma\delta}\ {}^{cd}_{\gamma\delta}\Gamma^{(2)}_{[\bar{\psi}\psi(0)]}(0). \tag{4}$$

In this equation, the one-particle irreducible Greens function ${}^{abcd}_{\alpha\beta\gamma\delta}\Gamma^{(2+2)}(p,p,0,0)$ is the diagram on the left hand side in Fig. 1. The role of the contraction operator $P^{cd}_{\gamma\delta}$ is to ensure that the collective quantum numbers of the soft legs are those of vacuum. It also effectuates the necessary summation over color and spinor indices. In this case $P^{cd}_{\gamma\delta}$ is simply equal to $\delta^{cd}\delta_{\gamma\delta}$. Note that $[\bar{\psi}\psi]$ is defined as $[\bar{\psi}^a_\alpha\psi^a_\alpha]$.

The t-channel does not contribute to the left-hand side in Fig. 1 since it is identical to zero when the two lower legs are assigned zero momentum. The fact that only the s-channel enters the calculation implies that only condensates with the same flavor as the quark with hard momentum (the propagating quark) contributes. To second order in the coupling constant g ($g^2 = 4\pi\alpha_s$) the s-channel, one-gluon exchange diagram is gauge independent (see the separate chapter on this issue) , and the correct gauge independent expression is automatically obtained in Feynman gauge. When contracted in spinor and color space, the diagram on the left-hand side in Fig. 1 yields:

$$P^{cd}_{\gamma\delta}\ {}^{abcd}_{\alpha\beta\gamma\delta}\Gamma^{(2+2)}(p,p,0,0) = \frac{16}{3}\, g^2\, \delta^{ab}\, \delta_{\alpha\beta}\, \frac{1}{p^2}. \tag{5}$$

An insertion of $[\bar{\psi}\psi]$, at zero momentum in the inverse quark propagator, $\Gamma^{(2)}(0)$, is obtained by differentiation with respect to minus the quark mass:

$$\Gamma^{(2)}_{[\bar{\psi}\psi(0)]}(0) = \frac{\partial}{\partial(-m)}\, \Gamma^{(2)}(0). \tag{6}$$

After contraction with $P^{cd}_{\gamma\delta}$ we have:

$$P^{cd}_{\gamma\delta}\ {}^{cd}_{\gamma\delta}\Gamma^{(2)}_{[\bar{\psi}\psi(0)]}(0) \ = \ -12\ i. \tag{7}$$

Combining (4), (5) and (7) we now obtain:

$${}^{ab}_{\alpha\beta}C^{[\bar{\psi}\psi]}(p) \ = \ \frac{4}{9}\ i\ g^2\ \delta^{ab}\ \delta_{\alpha\beta}\ \frac{1}{p^2}. \tag{8}$$

In order to obtain ${}^{ab}_{\alpha\beta}C^{[G^2_{\mu\nu}]}$ in (2), we study the expansion of a six-point function with two hard quark legs and four soft gluon legs. The OPE expansion of this diagram is illustrated in Fig. 2, and it reads:

$$P^{ABCD}_{\mu\nu\rho\sigma} \; {}^{ab,ABCD}_{\alpha\beta,\mu\nu\rho\sigma}\Gamma^{(2+4)}(p,p,0,0,0,0) \approx$$

$$ {}^{ab}_{\alpha\beta}C^{[G^2_{\mu\nu}]}(p) \; P^{ABCD}_{\mu\nu\rho\sigma} \; {}^{ABCD}_{\mu\nu\rho\sigma}\Gamma^{(4)}_{[G^2_{\mu\nu}(0)]} \,. \tag{9}$$

The contraction operator, $P^{ABCD}_{\mu\nu\rho\sigma}$, plays the same role here as in the previous case. It ensures that the collective quantum numbers of the soft gluon legs are those of vacuum and it effectuates the summation over gluon color- and space/time- indices compatible with that requirement. $P^{ABCD}_{\mu\nu\rho\sigma}$ is identical to the contraction operator in the four-gluon vertex:

$$\begin{aligned} P^{ABCD}_{\mu\nu\rho\sigma} &= f^{ABE} f^{CDE}(\delta_{\mu\rho}\delta_{\nu\sigma} - \delta_{\nu\rho}\delta_{\mu\sigma}) + \\ &\quad + f^{CBE} f^{ADE}(\delta_{\mu\rho}\delta_{\nu\sigma} - \delta_{\nu\mu}\delta_{\rho\sigma}) + f^{DBE} f^{CAE}(\delta_{\mu\nu}\delta_{\rho\sigma} - \delta_{\nu\rho}\delta_{\mu\sigma}). \end{aligned} \tag{10}$$

$\Gamma^{(2+4)}$ is a one-particle irreducible six-point function and $\Gamma^{(4)}_{[G^2_{\mu\nu}(0)]}$ is the gluon four-point vertex with an insertion of $[G^2_{\mu\nu}]$ at zero momentum. Such an insertion in a one-particle irreducible Greens function is obtained through the following formula[10]:

$$\Gamma_{[G^2_{\mu\nu}(0)]} \;=\; -4\left(-\frac{g}{2}\frac{\partial}{\partial g} + \alpha_G\frac{\partial}{\partial \alpha_G} + \frac{n_g}{2}\right)\Gamma \tag{11}$$

Here, n_g is the number of external gluon legs and α_G is the gauge parameter. In our case $n_g = 4$. With

$$ {}^{ABCD}_{\mu\nu\rho\sigma}\Gamma^{(4)} \;=\; -g^2 \, P^{ABCD}_{\mu\nu\rho\sigma}\,, \tag{12}$$

we hence get:

$$ {}^{ABCD}_{\mu\nu\rho\sigma}\Gamma^{(4)}_{[G^2_{\mu\nu}(0)]} \;=\; 4g^2 \, P^{ABCD}_{\mu\nu\rho\sigma}\,, \tag{13}$$

and

$$P^{ABCD}_{\mu\nu\rho\sigma} \;\; {}^{ABCD}_{\mu\nu\rho\sigma}\Gamma^{(4)}_{[G^2_{\mu\nu}(0)]}(0) \;\; = \;\; 31104 \; g^2. \tag{14}$$

The left hand side of (9) is evaluated to be:

$$\begin{aligned} P^{ABCD}_{\mu\nu\rho\sigma} \;\; {}^{ab,ABCD}_{\alpha\beta,\mu\nu\rho\sigma}\Gamma^{(2+4)}(p,p,0,0,0,0) = \;& 36 \; i \; g^4 \; \delta^{ab} \; \frac{1}{(p^2+m^2)^3} \\ & \times (2p^2\not{p} + 3mp^2 + 3m^2\not{p} + 4m^3)_{\alpha\beta}. \end{aligned} \tag{15}$$

From (9), (14) and (15) we now get:

$$\begin{aligned} {}^{ab}_{\alpha\beta}C^{[G^2_{\mu\nu}]}(p) \approx \frac{i}{864} \; g^2 \; \delta^{ab} \; \frac{1}{(p^2+m^2)^3} \; & (2p^2\not{p} + 3mp^2 + \\ & + 3m^2\not{p} + 4m^3)_{\alpha\beta} \end{aligned} \tag{16}$$

We are now in a position to obtain the nonperturbative inverse quark propagator. From (2), (3), (8) and (16) we get:

$$\begin{aligned} {}^{ab}_{\alpha\beta}S^{-1}_{np} \; & \approx i \; \delta^{ab}(\not{p} + m)_{\alpha\beta} \; + \\ & + \frac{4}{9} \; i \; g^2 \; \delta^{ab}\delta_{\alpha\beta}\frac{1}{p^2} \; \langle\Omega| \, [\bar{\psi}\psi] \, |\Omega\rangle \; + \\ & + \frac{1}{864} \; i \; g^2 \; \delta^{ab} \; \frac{1}{(p^2+m^2)^3} \\ & \times (2p^2\not{p} + 3mp^2 + 3m^2\not{p} + 4m^3)_{\alpha\beta} \; \langle\Omega| \, [G^2_{\mu\nu}] \, |\Omega\rangle \, . \end{aligned} \tag{17}$$

In Fig. 3 we give a diagrammatic interpretation of this equation. The nonperturbative inverse propagator can be expressed in terms of a running normalization function, $N_q(p^2)$, and a running mass, $M(p^2)$, in the following way:

$$ {}^{ab}_{\alpha\beta}S^{-1}_{np} \;=\; i\,\delta^{ab}\frac{1}{N_q(p^2)}(\not p + M(p^2))_{\alpha\beta}. \tag{18} $$

This corresponds to Politzer's definition of the nonperturbative mass[7] . We have:

$$ N_q(p^2) \;=\; \Big(1 \;+\; \frac{1}{864}g^2\frac{1}{(p^2+m^2)^3}(2p^2+3m^2)\,\langle\Omega|\,[G^2_{\mu\nu}]\,|\Omega\rangle\Big)^{-1} \tag{19} $$

and

$$ \begin{aligned} M(p^2) \;=\; &\Big(m \;+\; \frac{4}{9}g^2\frac{1}{p^2}\,\langle\Omega|\,[\bar\psi\psi]\,|\Omega\rangle + \\ &+\; \frac{1}{864}g^2\frac{1}{(p^2+m^2)^3}(3mp^2+4m^3)\,\langle\Omega|\,[G^2_{\mu\nu}]\,|\Omega\rangle\Big)\;N_q(p^2). \end{aligned} \tag{20} $$

Inverting (18) we obtain the nonperturbative quark propagator:

$$ {}^{ab}_{\alpha\beta}S_{np} \;=\; N_q(p^2)\Big(\frac{-i\delta^{ab}}{\not p + M(p^2)}\Big)_{\alpha\beta}. \tag{21} $$

For timelike momenta (p^2 negative) the running mass is a positive quantity ($\langle\Omega|\,[\bar\psi\psi]\,|\Omega\rangle$ is negative). $M(p^2)$ approaches m in both limits $-p^2 \mapsto m^2$ and $-p^2 \mapsto \infty$. For massless quarks (m=0) $M(p^2)$ has a maximum at $-p^2 = \sqrt{\frac{1}{12}\pi\alpha_s\,\langle\Omega|\,[G^2_{\mu\nu}]\,|\Omega\rangle}$. With the phenomenological value of $\langle\Omega|\,[G^2_{\mu\nu}]\,|\Omega\rangle$ in (1), the maximum occurs at $\sqrt{-p^2} \simeq 0.3GeV$. If we let both m and $\langle\Omega|\,[G^2_{\mu\nu}]\,|\Omega\rangle$ be zero, we see that this expression agrees with that of Ref. 8, specialized to Feynman gauge. We note that for large p^2, the mass is identical to Politzers mass (up to a constant factor, see Ref. 8). For low momenta, where the gluon condensate is important, our mass goes like p^2, differing drastically from Politzer's mass which goes like $\frac{1}{p^2}$ for all momenta. However we want to stress that for low momenta the nonperturbative quark mass might get large contributions from higher-dimension operators. The only region where we can feel confident about our results is in the asymptotic region where our result coincides with that of Politzer.

3. The Nonperturbative Gluon Propagator

The OPE for the inverse gluon propagator is:

$$\begin{aligned}(D_{np}^{\mu\nu,AB})^{-1} &\approx {}^{AB}_{\mu\nu}C^{\mathbb{1}}(p)\,\mathbb{1} \\ &+ {}^{AB}_{\mu\nu}C^{[\bar\psi\psi]}(p)\ \langle\Omega|\,[\bar\psi\psi]\,|\Omega\rangle + {}^{AB}_{\mu\nu}C^{[G^2_{\mu\nu}]}(p)\ \langle\Omega|\,[G^2_{\mu\nu}]\,|\Omega\rangle \qquad (22)\end{aligned}$$

We evaluate this expression in a general covariant gauge where we have the perturbative inverse propagator:

$$ {}^{AB}_{\mu\nu}C^{\mathbb{1}}(p) = \delta^{AB}p^2\left(\delta_{\mu\nu} - (1-\frac{1}{\alpha_G})\frac{p_\mu p_\nu}{p^2}\right). \qquad (23)$$

Here α_G is the gauge parameter and the notation is such that $\alpha_G = 1$ corresponds to Feynman gauge. The coefficient ${}^{AB}_{\mu\nu}C^{[\bar\psi\psi]}(p)$ is obtained from the following equation, graphically represented in Fig. 4:

$$P^{ab}_{\alpha\beta}\ {}^{AB,ab}_{\mu\nu,\alpha\beta}\Gamma^{(2+2)}(p,p,0,0) \approx {}^{AB}_{\mu\nu}C^{[\bar\psi\psi]}(p)\,P^{ab}_{\alpha\beta}\ {}^{ab}_{\alpha\beta}\Gamma^{(2)}_{[\bar\psi\psi(0)]}(0)\ . \qquad (24)$$

Here $\Gamma^{(2+2)}$ is the sum of the two diagrams on the left-hand side in Fig. 4. The contraction operator $P^{ab}_{\alpha\beta}$ is the same as that we used in the derivation of the quark condensate contributions to the quark propagator. After contraction with $P^{ab}_{\alpha\beta}$ we have:

$$P^{ab}_{\alpha\beta}\ {}^{AB,ab}_{\mu\nu,\alpha\beta}\Gamma^{(2+2)}(p,p,0,0) = -12ig^2\delta^{AB}\delta_{\mu\nu}\ \frac{m}{(p^2+m^2)}\ . \tag{25}$$

Combining with (7) and (23) we have:

$$ {}^{AB}_{\mu\nu}C^{[\bar{\psi}\psi]}(p) \approx g^2\delta^{AB}\delta_{\mu\nu}\frac{m}{(p^2+m^2)}\ . \tag{26}$$

In order to obtain the contribution from the gluon condensate, we study the following equation, graphically represented in Fig. 5:

$$\begin{aligned} &P^{CDEF}_{\rho\sigma\tau\lambda}\ {}^{AB,CDEF}_{\mu\nu,\rho\sigma\tau\lambda}\Gamma^{(2+4)}(p,p,0,0,0,0) \approx \\ &\qquad {}^{AB}_{\mu\nu}C^{[G^2_{\mu\nu}]}(p)\ P^{CDEF}_{\rho\sigma\tau\lambda}\ {}^{CDEF}_{\rho\sigma\tau\lambda}\Gamma^{(4)}_{[G^2_{\mu\nu}(0)]}(0)\ . \end{aligned} \tag{27}$$

Here $\Gamma^{(2+4)}$ is the sum of the five diagrams on the left hand side in Fig. 5. The left hand side in (27) is evaluated[11] to be (in a general covariant gauge, Euclidean space):

$$\begin{aligned} P^{CDEF}_{\rho\sigma\tau\lambda}\ {}^{AB,CDEF}_{\mu\nu,\rho\sigma\tau\lambda}\Gamma^{(2+4)}(p,p,0,0,0,0) &= \frac{27}{2}\ \delta^{AB}\ g^4\ \frac{1}{p^2} \\ &\times\left(\frac{p_\mu p_\nu}{p^2}(1-\alpha_G)^2 - 31\frac{p_\mu p_\nu}{p^2}(1-\alpha_G)\right. \\ &\left. -\ \delta_{\mu\nu}(1-\alpha_G)^2 + 28\delta_{\mu\nu}(1-\alpha_G) - 13\frac{p_\mu p_\nu}{p^2} + 13\delta_{\mu\nu}\right)\ . \end{aligned} \tag{28}$$

From (14) and (23) we get:

$$
\begin{aligned}
{}^{AB}_{\mu\nu}C^{[G^2_{\mu\nu}]}(p) \approx & \frac{1}{2304}\,\delta^{AB}\,g^2\,\frac{1}{p^6} \\
& \times\left(\frac{p_\mu p_\nu}{p^2}(1-\alpha_G)^2 - 31\frac{p_\mu p_\nu}{p^2}(1-\alpha_G)\right. \\
& \left. -\,\delta_{\mu\nu}(1-\alpha_G)^2 + 28\delta_{\mu\nu}(1-\alpha_G) - 13\,\frac{p_\mu p_\nu}{p^2} + 13\delta_{\mu\nu}\right).
\end{aligned} \tag{29}
$$

We now get the nonperturbative inverse gluon propagator from (22), (23), (26) and (29):

$$
\begin{aligned}
(D^{\mu\nu,AB}_{np})^{-1} \approx & \;\delta^{AB}p^2\left(\delta_{\mu\nu} - (1-\frac{1}{\alpha_G})\frac{p_\mu p_\nu}{p^2}\right) + \\
& +\;\delta^{AB}g^2\delta_{\mu\nu}\sum_{q=d,u,s,c,b,t}\frac{m_q}{(p^2+m_q^2)}\quad\langle\Omega|\,[\bar{q}q]\,|\Omega\rangle + \\
& +\frac{1}{2304}\,\delta^{AB}\,g^2\,\frac{1}{p^2}\,\left(\frac{p_\mu p_\nu}{p^2}(1-\alpha_G)^2 - 31\,\frac{p_\mu p_\nu}{p^2}(1-\alpha_G)\right. \\
& \left. -\,\delta_{\mu\nu}(1-\alpha_G)^2 + 28\delta_{\mu\nu}(1-\alpha_G) - 13\,\frac{p_\mu p_\nu}{p^2} + 13\delta_{\mu\nu}\right)\;\langle\Omega|\,[G^2_{\mu\nu}]\,|\Omega\rangle\,.
\end{aligned} \tag{30}
$$

A diagrammatic representation of this equation is given in Fig. 6. In analogy with the inverse quark propagator, we write the inverse gluon propagator in the following form:

$$
(D^{\mu\nu,AB}_{np})^{-1} \;=\; \frac{1}{N_g(p^2)}\,\delta^{AB}\,p^2\,\left(\delta_{\mu\nu} - (1-\frac{1}{A_G(p^2)})\frac{p_\mu p_\nu}{p^2}\right), \tag{31}
$$

where we have introduced a running normalization function:

$$
\begin{aligned}
N_g(p^2) &= \Big(1 + \sum_{q=d,u,s,c,b,t} \frac{g^2 m_q \langle\Omega|\,[\bar{q}q]\,|\Omega\rangle}{p^2(p^2+m_q^2)} + \\
&+ \frac{g^2 \langle\Omega|\,[G_{\mu\nu}^2]\,|\Omega\rangle}{2304p^4}(13-(1-\alpha_G)^2+28(1-\alpha_G))\Big)^{-1},
\end{aligned}
\tag{32}
$$

and a running gauge parameter, $A(p^2)$. The expression for the running gauge parameter is rather complicated in a general covariant gauge. The gauge in which the physics is most transparent is Landau gauge where $\alpha_G = A_G(p^2) \equiv 0$. In this gauge all the nonperturbative effects are contained in the running normalization function and the expression for the nonperturbative gluon propagator reads:

$$
\begin{aligned}
D_{np}^{\mu\nu,AB} &= \Big(1 + \sum_{q=d,u,s,c,b,t} \frac{g^2 m_q \langle\Omega|\,[\bar{q}q]\,|\Omega\rangle}{p^2(p^2+m_q^2)} + \frac{5g^2 \langle\Omega|\,[G_{\mu\nu}^2]\,|\Omega\rangle}{288p^4}\Big)^{-1} \\
&\times \frac{\delta^{AB}}{p^2}\left(\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right).
\end{aligned}
\tag{33}
$$

4. The Nonperturbative Quark Mass and the question of gauge dependence

The so called nonperturbative quark mass was introduced by Politzer[7] who obtained the mass by first calculating the nonperturbative quark propagator through the operator product expansion. Politzer neglected the contribution from the gluon condensate and the analysis was done for massless quarks only. Unfortunately, the analysis was somewhat unclear and not specific on details. In an attempt to review Politzers analysis, Pascual and de Rafael pointed out some spinor and color factors missing in Ref. 7. They also claimed that the nonperturbative quark mass is gauge parameter dependent when evaluated in a general covariant gauge. Politzers analysis was done in Landau gauge and he does not comment on the question of whether or not the result is gauge dependent. In a recent article[12], Elias and Scadron argues that the nonperturbative quark mass is gauge independent and that the correct result is automatically obtained—as in Politzers analysis—in Landau gauge.

Because of the confusion surrounding the nonperturbative quark mass, we want to make some comments on the issue of gauge dependence of the nonperturbative quark propagator in Eq. 17.

Suppose the derivation of the nonperturbative quark propagator is done in a general covariant gauge. If the gauge parameter α_G survives throughout the analysis the quark propagator will be gauge dependent, otherwise not. The truncated six-point function in Fig. 2 does not contain any gluon propagators and hence not α_G. Since the four-gluon vertex does not contain α_G either, this means that the contribution from the gluon condensate does not contain α_G.

In the case of the contribution from the quark condensate, it would seem that α_G is introduced through the diagram on the left hand side in Fig. 1, since it contains the gluon propagator. However, in that diagram, the gluon propagator is sandwiched between quark propagators. It is well known[13] that when the quarks are on the mass-shell, the second term of the gluon propagator, sandwiched between the quark propagators, disappear by virtue of the Dirac equation for free quarks. We therefore know that at least in the limit of on-shell quarks, the diagram is gauge independent and that the correct result is automatically obtained in Feynman gauge.

Furthermore, the Dirac equation can be used also in the case of off-shell quarks. Compared to the case of free quarks, the equation is now modified by a term proportional to the coupling constant g. When the gluon propagator is sandwiched between off-shell quarks, the same trick as in the on-shell case can be used. In this case the contribution originating from the second term in the gluon propagator does not vanish, but it is proportional to g^2, so this contribution will be of higher order in the coupling constant than the contribution from the first term. This means that to first order in α_s the diagram in Fig. 1 is gauge independent also for off-shell quarks and the correct result is that automatically obtained in Feynman gauge without using the Dirac equation. It follows that Elias and Scadron must be wrong when they claim that the nonperturbative quark mass should be calculated in Landau gauge without using the Dirac equation to eliminate the contribution from the second term in the gluon propagator.

5. Discussion

Our main result are the expressions for the nonperturbative quark and gluon propagators, Eqs. (21) and (33). For low values of momenta, these expressions will have to be corrected by contributions from any operators of higher dimension than four that acquire non-zero vev's, and from multiple insertions of $[\bar{\psi}\psi]$ and $[G^2_{\mu\nu}]$. We suggest (quite arbitrarily and probably very conservatively) that as long as the nonperturbative corrections that we have calculated here are one order of magnitude smaller than the regular propagator, further nonperturbative corrections can probably safely be neglected. With the phenomenological values of the vev's of $[\bar{\psi}\psi]$ and $[G^2_{\mu\nu}]$ in (1), we thus estimate that the nonperturbative corrections that we have calculated here can be taken seriously for $\sqrt{-p^2} \geq 0.60$

GeV in the case of the gluon propagator and $\sqrt{-p^2} \geq 0.80$ GeV in the case of the quark propagator. One way to lower the limit of validity of the nonperturbative analysis is of cause to calculate, or at least estimate, the contribution from higher dimension operators and from multiple operator insertions. These questions are subject to current research.

ACKNOWLEDGEMENTS

I gratefully acknowledge many inspiring discussions with Professor S. J. Brodsky who have provided encouraging support throughout this project. I am also grateful for many fruitful discussions with Professor H. R. Quinn, Professor A. Mueller, Dr. H. Evertz, Dr. C. Peterson and Dr. M. Scadron.

Financial support for this project has been provided by the Swedish Natural Science Research Council, The American-Scandinavian Foundation and The Fulbright Commission.

REFERENCES

1. M. A. Shifman, A. I. Vainshtein and V. I. Zakharov, Nucl. Phys. B147 (1979) 385, 448, 519.
2. For a recent review on the applications of the QCD-sum rules see V. L. Chernyak and A. R. Zhitnitsky, Novosibirsk reports IYF-83-103–108 (1983).
3. T. H. Hansson, CERN report TH.3921-CERN.
4. A Mueller, Report No. CU-TP-289.
5. R. Fukuda and Y. Kazama, Phys. Rev. Lett. 45 (1980) 1142.
6. K. Wilson, Phys. Rev. 179, 1499 (1969); W. Zimmermann, in Lectures in Elementary Particles and Quantum Field Theory, edited by S. Deser, M. Grisau, and H. Pendleton (MIT, Cambridge, Mass. 1971).
7. H. D. Politzer, Nucl. Phys. B177, 397 (1976).
8. P. Pascual and E. de Rafael, Z. Phys. C12, 127 (1982).
9. For a comprehensive description of the method we use here, see C. Taylor and B. McClain, Phys. Rev. D28, 1364 (1983), where the case of ϕ^4 theory is studied.
10. H. Kluberg-Stern and J.B. Zuber, Phys. Rev. D12, 467 (1975).
11. In the evaluation we used REDUCE, a system for carrying out algebraic operations accurately on a computer, developed by A. C. Hearn.
12. V. Elias and M. Scadron, Phys. Rev. D30, 647 (1984).
13. See for instance C. Itzykson and J. B. Zuber, Quantum Field Theory, p. 277, McGraw Hill (1980).

FIGURE CAPTIONS

1. Graphical illustration of Eq.(4). The bars on the lower legs indicates that the corresponding propagator is assigned zero momentum. The upper diagram in the middle part of the equation is defined to be the graphical representation of the coefficient ${}^{ab}_{\alpha\beta}C^{[\bar{\psi}\psi]}(p)$.
2. Graphical representation of Eq.(9). Notations are the same as in Fig. 1.
3. Graphical interpretation of Eq. (17).
4. Graphical representation of Eq.(24). Notations are the same as in Fig. 1.
5. Graphical representation of Eq.(27). Notations are the same as in Figs. 1 and 2. The small quadrats in the diagram represents four-gluon couplings and the triangles three gluon couplings.
6. Graphical interpretation of Eq. (30).

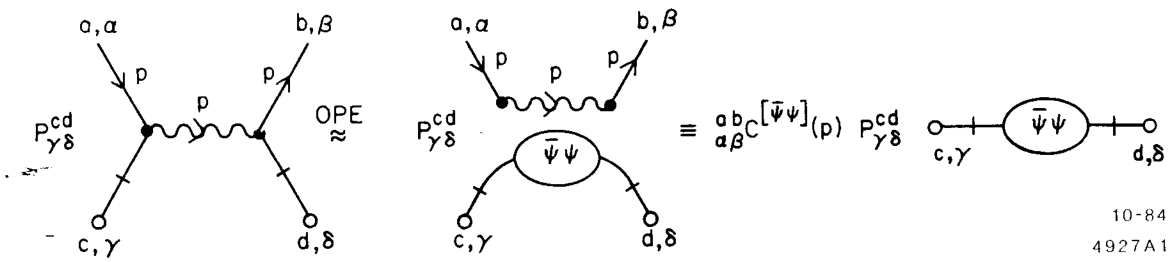

a, α
b, β
p
p
p
$P^{cd}_{\gamma\delta}$
OPE
≈
c, γ
d, δ
a, α
b, β
p
p
p
$P^{cd}_{\gamma\delta}$
$\bar{\psi}\psi$
c, γ
d, δ
$\equiv {}^{ab}_{\alpha\beta}C^{[\bar{\psi}\psi]}(p)\ P^{cd}_{\gamma\delta}$
$\bar{\psi}\psi$
c, γ
d, δ
10-84
4927A1

Fig. 1

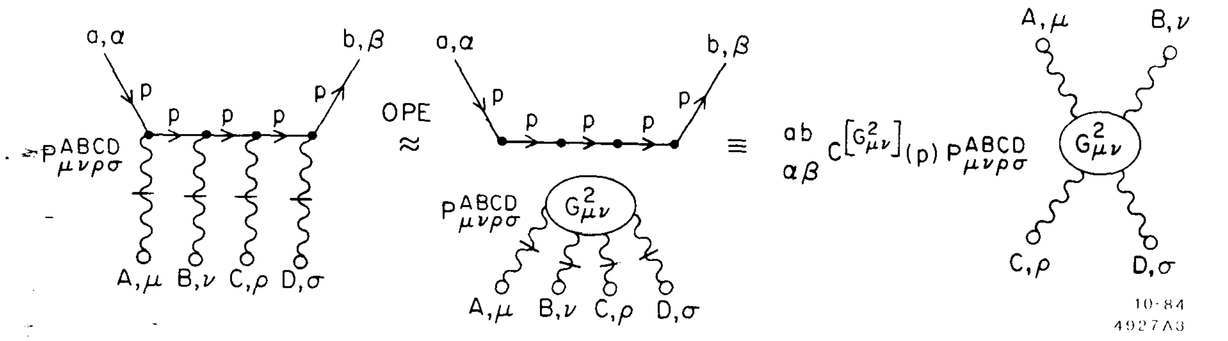

a, α
b, β
p
$P^{ABCD}_{\mu\nu\rho\sigma}$
A, μ
B, ν
C, ρ
D, σ
OPE
≈
$G^2_{\mu\nu}$
≡
$C^{ab}_{\alpha\beta}[G^2_{\mu\nu}](p)\, P^{ABCD}_{\mu\nu\rho\sigma}$
10-84
4927A3

Fig. 2

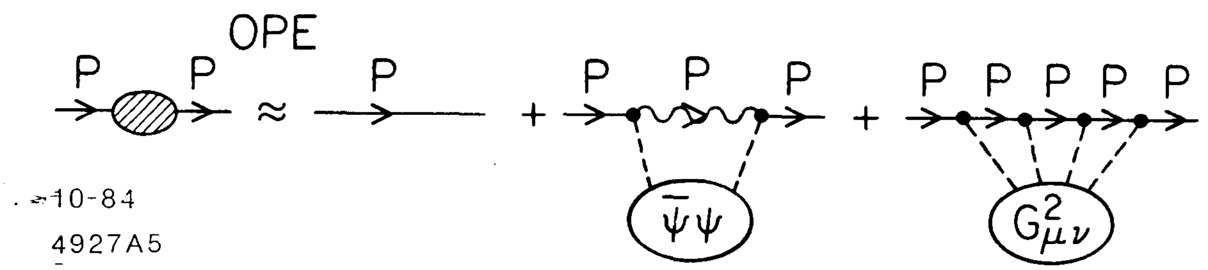


Fig. 3

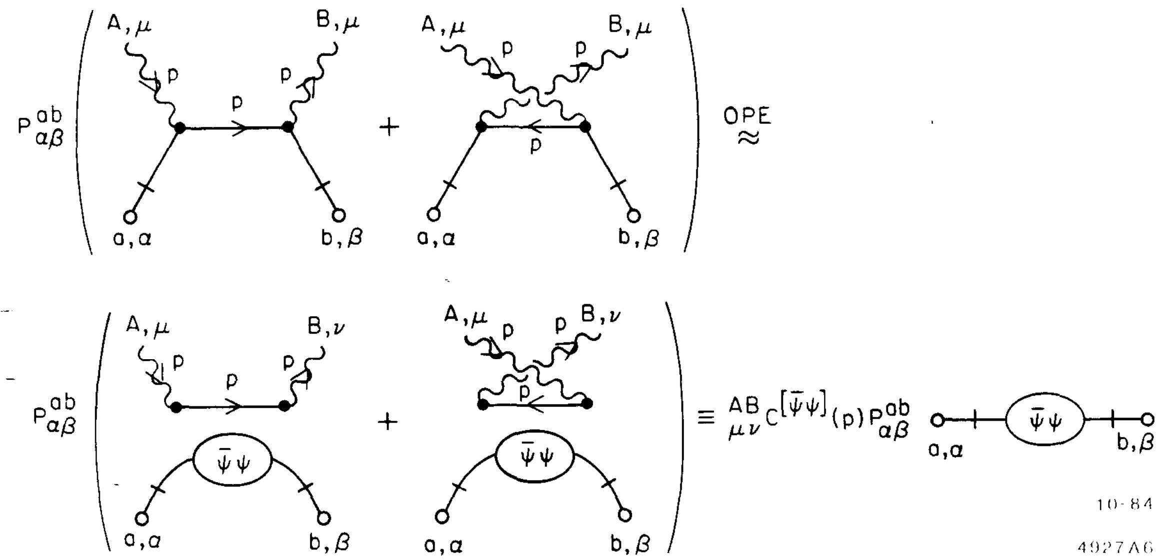


Fig. 4

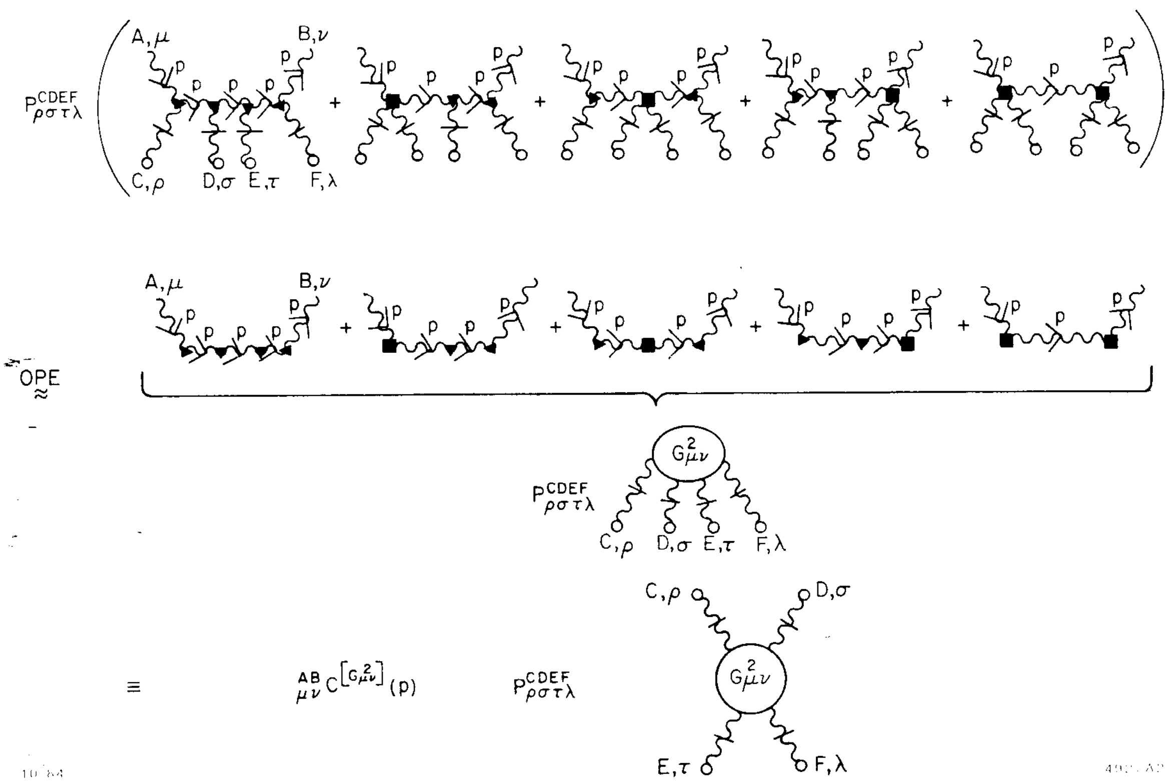
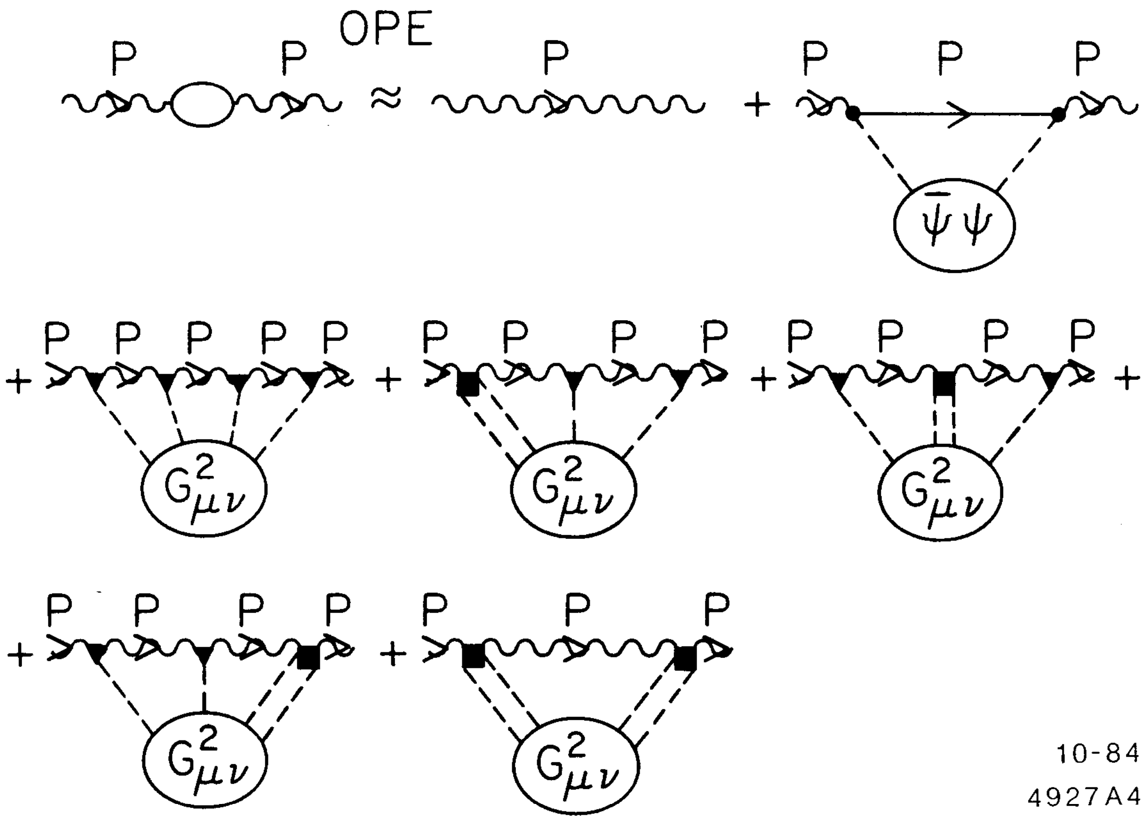

$P^{CDEF}_{\rho\sigma\tau\lambda}$
A, μ
B, ν
p
C, ρ
D, σ
E, τ
F, λ
OPE
≈
≡
$^{AB}_{\mu\nu}C^{[G^2_{\mu\nu}]}(p)$
$G^2_{\mu\nu}$

Fig. 5



OPE
P
P
≈
P
+
P
P
P
$\bar{\psi}\psi$
P P P P P
+
P P P P
+
P P P P
+
+
$G^2_{\mu\nu}$
$G^2_{\mu\nu}$
$G^2_{\mu\nu}$
P P P P
+
P P P
+
$G^2_{\mu\nu}$
$G^2_{\mu\nu}$
10-84
4927A4

Fig. 6