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CONSTRAINTS ON THE MIXING OF A FOURTH FAMILY OF QUARKS*

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ABSTRACT

This paper studies the constraints on the mixing of a possible fourth family of quarks. A mixing angle convention is introduced in which $s_{ij} \equiv \sin\theta_{ij}$ represents to a good approximation the ij element of the quark mixing matrix ($i < j$). A range of parameters is found, $s_{14} < 0.06$, $s_{24} < 0.1$; $s_{14}s_{24} \sim 0(10^{-4} - 10^{-3})$ and/or $s_{14}s_{34} \gtrsim O(10^{-2})$, for which the contribution of the fourth family to the $K_L - K_S$ mass difference is negligible yet may lead to dominant effects in the $K^0 - \overline{K}^0$ CP impurity parameter ϵ. This may be realized in a scheme in which the four quark generations are mixed mostly in pairs. A possible signature of such a scheme may be an abundant rate of same-sign dileptons at $\Upsilon(4S)$ if the fourth up-type quark is sufficiently heavy.

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1. Introduction

The standard $SU(3) \times SU(2) \times U(1)$ six quark model of the strong, weak and electromagnetic interactions describes all charged current weak interaction phenomena in terms of a 3×3 unitary matrix which represents the mixing between the quark mass eigenstates and the weak interaction eigenstates. The mixing matrix is parametrized in terms of three angles and a phase, conventionally known as the Kobayashi-Maskawa[1] (K-M) angles $\theta_i (i = 1,2,3)$ and phase δ. While accurate measurements of one of the mixing angles θ_1 (the Cabibbo angle) have existed for quite a few years, the other two angles were first measured about a year ago.[2,3] These angles θ_2 and θ_3 while not very accurately known at present, are considerably smaller than θ_1, thereby giving rise to a relatively large B meson lifetime and to quite a small semileptonic decay branching ratio of B into uncharmed hadrons. The phase δ is supposed to be responsible for the CP violation observed in the $K^0 - \overline{K}^0$ system. In the standard calculation of the CP-impurity parameter ϵ,[4] based on the short distance dominated box diagram, it is found to be proportional to $\sin\theta_2 \sin\theta_3 \sin\delta$ and to a growing function of the t quark mass. It was pointed out some time ago[5] that to account for the measured value of ϵ sufficiently small values of θ_2 and θ_3 would require large values of m_t. Very recently a few events were reported by the UA1 group at CERN,[6] which may be the first indication for the existence of a t quark in the mass range $30 \leq m_t \leq 50$ GeV. The present uncertainty in the experimental determination of θ_2, θ_3 and m_t and the theoretical ambiguity in the absolute magnitude estimate of the box diagram matrix element (B_K) leave sufficient freedom to account for the measured value of ϵ. In general this requires the parameters θ_2 and θ_3 to lie in the upper parts of their allowed ranges and some preference is given to the upper range of possible values of m_t. Future improvements in the measurements of these three parameters, and in the theoretical estimate of B_K, may however indicate a different situation and may potentially lead to a problem with the standard six quark model.

Since with merely two generations CP is conserved in the single Higgs doublet $SU(2) \times U(1)$ model, it is traditionally believed that the observed CP violation phenomena are intimately related to the existence of the third generation of quarks.[1] It is however obvious that these phenomena exist whenever the number of generations is larger than two. The number of generations N, just as the quark masses, mixing angles and phases, is among the questions for which the $SU(2) \times U(1)$ model has no answer. Some cosmological arguments indicate that N may not be larger than four.[7] If there exist altogether four generations it is not inconceivable that the dominant source of CP violation in the $K^0 - \overline{K}^0$ system is the fourth generation. Off hand this seems an unlikely possibility if one assumes that the mixing between the first two lowest generations and the hypothetical fourth family is considerably smaller than their measured mixing with the third generation. However with our modest understanding of quark masses and mixing one should keep an open mind to other possibilities.

The purpose of this paper is to study the implications of the existence of a fourth generation on the quark mixing matrix and on the related phenomena.[8–10] At first sight it seems that with the proliferation of the number of quark mixing angles and phases such a study would contain too many arbitrary parameters to be useful. We will in fact show that there exists a physically intuitive way to define an extension of the quark mixing matrix to four generations, such that the information gathered within the three generation model about the corresponding mixing angles would not be lost. Our arguments will be based on the unitarity of the mixing matrix and on the calculation of the $K_L - K_S$ mass difference. Then, since one of the motivations of this work is to anticipate a potential problem with explaining the value of ϵ in the three generation model, we will seek a range of mixing parameters in the four generation model which may resolve this problem if it does occur. Whereas this search will be purely phenomenological we will also study a few typical forms of the $N = 4$ mixing matrix obtained by extrapolation from the measured elements of the $N = 3$ matrix.

The paper is organized as follows. In Section 2 we introduce a convention for the quark mixing matrix for any N and derive expressions for the matrix elements in the $N = 4$ case. The values of the mixing angles within the first three generations and certain bounds on the mixing of the fourth generation with the first two are derived in Section 3. For the latter we use the unitarity of the mixing matrix and the $K_L - K_S$ mass difference. In Section 4 we obtain further constraints on the latter mixing angles based on the measured value of ϵ. We point out the range of values of these parameters which are required to make the fourth generation the dominant source of CP violation in the $K^0 - \overline{K}^0$ system. Some remarks are added about the effect of the fourth generation on the value of ϵ', which is a measure of the direct CP violation in $K^0 \to 2\pi$. The effect of a fourth generation on $B^0 - \overline{B}^0$ mixing is studied in Section 5. Section 6 describes a few hierarchy schemes of the four generation mixing parameters which we regard as plausible extrapolations of the $N = 3$ mixing matrix. Finally Section 7 contains a brief summary of our results.

2. The quark mixing matrix

The hadronic charged current weak Lagrangian for N generations is

$$\mathcal{L}_{cc} = \frac{ig}{\sqrt{2}} W_\mu^+ \sum_{i,j=1}^{N} U_{ij} \overline{u}_{Li} \gamma_\mu d_{Lj} + \text{h.c.} \tag{2.1}$$

where $u_{Li}(d_{Li})$ is the ith generation left-handed doublet quark field with charge 2/3 (-1/3). For parametrizing the unitary matrix U it is convenient to introduce "complex rotations" connecting each possible pair of generations.[11] The rotation ω_{12} between the first and second generation, for example, is given by

$$\omega_{12} = \begin{bmatrix} \cos\theta_{12} & e^{i\phi_{12}}\sin\theta_{12} & 0 & \cdots \\ -e^{-i\phi_{12}}\sin\theta_{12} & \cos\theta_{12} & 0 & \cdots \\ 0 & 0 & 1 & \cdots \\ \vdots & \vdots & \vdots & \end{bmatrix} \tag{2.2}$$

θ_{12}, the rotation angle, expresses the amount of mixing between the first two generations, while ϕ_{12} is a (potentially) CP violating phase. The rotations ω_{ij} for general i and j are defined analogously. Then U may be simply expressed as

$$U = \prod_{i<j} \omega_{ij} \,. \tag{2.3}$$

Any particular order of factors may be used on the right-hand side. To the extent that θ_{ij} are small the order is actually irrelevant to first approximation. There are of course $N(N-1)/2$ angles θ_{ij} and the net phase arbitrariness in the definition of the quark fields may be characterized as follows. Introduce N real arbitrary parameters α_i (for convenience only constrained to satisfy

$$\sum_{i=1}^{N} \alpha_i = 0) \ . \tag{2.4}$$

Then we are free to replace each potentially CP violating phase ϕ_{ij} by

$$\phi'_{ij} = \phi_{ij} + \alpha_i - \alpha_j \ . \tag{2.5}$$

In particular we may set $(N-1)$ judiciously chosen phases $\phi'_{ij} = 0$ leading to $\frac{1}{2}(N-1)(N-2)$ independent CP violating phases.

For the first three generations we choose the mixing matrix to be

$$\omega_{23}\omega_{12}\omega_{13} = \begin{bmatrix} c_{12}c_{13} & s_{12} & c_{12}s_{13}e^{i\phi_{13}} & 0 & \cdots \\ [-c_{23}c_{13}s_{12} & c_{12}c_{23} & [-c_{23}s_{12}s_{13}e^{i\phi_{13}} & 0 & \cdots \\ -s_{23}s_{13}e^{-i\phi_{13}}] & & +c_{13}s_{23}] & & \\ [c_{13}s_{12}s_{23} & -c_{12}s_{23} & [c_{13}c_{23} & 0 & \cdots \\ -c_{23}s_{13}e^{-i\phi_{13}}] & & +s_{12}s_{23}s_{13}e^{i\phi_{13}}] & 0 & \cdots \\ 0 & 0 & 0 & 1 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \end{bmatrix} \tag{2.6}$$

Here the abbreviations $c_{ij} = \cos\theta_{ij}$, $s_{ij} = \sin\theta_{ij}$ were used and the two phases ϕ_{12} and ϕ_{23} were made to vanish by Eqs. (2.4) and (2.5). A parametrization of the three generation model similar to Eq. (2.6) was first suggested by Maiani[12] with a somewhat different phase convention. Very recently Chau and Keung[13] proposed modifying Maiani's phase convention into that of Eq. (2.6).

For $N = 4$ the mixing matrix is defined as

$$U = (\omega_{34}\omega_{24}\omega_{14})(\omega_{23}\omega_{12}\omega_{13}) \ . \tag{2.7}$$

In addition to ϕ_{12} and ϕ_{23} also ϕ_{34} will be chosen to vanish, thus leaving us with three independent CP violating phases. At this point we wish to note that another convention $\phi_{12} = \phi_{23} = \phi_{13} = 0$, which would eliminate the CP phases from the first three generations, may be easily shown not to be consistent with Eq. (2.5). As will be shown in the next section all the angles θ_{ij} except θ_{34} must be small, and the approximation $\cos\theta_{ij} \approx 1$ will be applied to them. One finds:

$$
\begin{aligned}
U_{11} &\approx 1 \\
U_{12} &\simeq s_{12} \\
U_{13} &\simeq s_{13}e^{i\phi_{13}} \\
U_{14} &\simeq s_{14}e^{i\phi_{14}} \\
U_{21} &\simeq -s_{12} - s_{23}s_{13}e^{-i\phi_{13}} - s_{14}s_{24}e^{i(\phi_{24}-\phi_{14})} \\
U_{22} &\simeq 1 - s_{12}s_{14}s_{24}e^{i(\phi_{24}-\phi_{14})} \\
U_{23} &\simeq s_{23} - s_{13}e^{i\phi_{13}}\left(s_{12} + s_{14}s_{24}e^{i(\phi_{24}-\phi_{14})}\right) \\
U_{24} &\simeq s_{24}e^{i\phi_{24}} \\
U_{31} &\simeq c_{34}\left(s_{12}s_{23} - s_{13}e^{-i\phi_{13}}\right) - s_{14}s_{34}e^{-i\phi_{14}} \\
&\qquad + s_{24}s_{34}e^{-i\phi_{24}}\left(s_{12} + s_{23}s_{13}e^{-i\phi_{13}}\right) \\
U_{32} &\simeq -c_{34}s_{23} - s_{34}\left(s_{24}e^{-i\phi_{24}} + s_{12}s_{14}e^{-i\phi_{14}}\right) \\
U_{33} &\simeq c_{34}\left(1 + s_{12}s_{23}s_{13}e^{i\phi_{13}}\right) + \ldots \\
U_{34} &\simeq s_{34} \\
U_{41} &\simeq -c_{34}s_{14}e^{-i\phi_{14}} + c_{34}s_{24}e^{-i\phi_{24}}\left(s_{12} + s_{23}s_{13}e^{-i\phi_{13}}\right) \\
&\qquad + s_{34}\left(-s_{12}s_{23} + s_{13}e^{-i\phi_{13}}\right) \\
U_{42} &\simeq -c_{34}s_{24}e^{-i\phi_{24}} + s_{23}s_{34} - c_{34}s_{12}s_{14}e^{-i\phi_{14}} \\
U_{43} &\simeq -s_{34} - c_{34}\left(s_{23}s_{24}e^{-i\phi_{24}} + s_{13}s_{14}e^{i(\phi_{13}-\phi_{14})}\right)
\end{aligned}
$$

$$- s_{12}s_{13}s_{24}e^{i(\phi_{13}-\phi_{24})}\Big)$$

$$U_{44} \simeq c_{34} \ . \tag{2.8}$$

The advantage of our convention is made obvious by the observation that the sines of the six mixing angles θ_{ij} are good approximations to the corresponding measured matrix elements U_{ij} $(i < j)$.

$$\begin{array}{lll} U_{us} \simeq s_{12} & |U_{ub}| \simeq s_{13} & U_{cb} \simeq s_{23} \\ |U_{ub'}| \simeq s_{14} & |U_{cb'}| \simeq s_{24} & U_{tb'} \simeq s_{34} \end{array} \tag{2.9}$$

Here (t', b') denotes the fourth doublet of quarks. The result $U_{cb} \simeq s_{23}$ follows from the small measured ratio of $|U_{ub}/U_{cb}| \ll 1$.[3]

3. Constraints on s_{i4} from unitarity and from the $K_L - K_S$ mass difference

The most precise values of U_{ud} and U_{us} were obtained recently[14]

$$\begin{array}{l} |U_{ud}| = 0.9735 \pm 0.0015 \\ |U_{us}| = 0.231 \pm 0.003 \ . \end{array} \tag{3.1}$$

Using as the lowest possible value for these parameters a 1σ value, one may derive from the unitarity of the mixing matrix a limit on U_{ui}, $i > 2$.

$$|U_{ui}| < 0.06 \qquad i = b, b', \ldots \tag{3.2}$$

An almost order of magnitude stronger upper bound for one of the mixing parameters

$$|U_{ub}| < 0.007 \tag{3.3}$$

was obtained from the B-lifetime determination of U_{cb} [2,15]

$$|U_{cb}| = 0.05 \pm 0.01 \tag{3.4}$$

and from the measured upper limit[3,16] $|U_{ub}/U_{cb}| < 0.11$.

Due to the large uncertainty in the determination of U_{cs} ($|U_{cs}| > 0.8$ at best[17]) a similar unitarity argument when applied to $U_{cb'}$ does not lead to a useful upper limit. Equations (3.1)-(3.4) may be expressed in terms of the mixing angles by using Eqs. (2.8):

$$\begin{aligned} s_{12} &= 0.23 \ , \qquad s_{23} = 0.05 \pm 0.01 \\ s_{13} &< 0.007 \ , \quad s_{14} < 0.06 \ . \end{aligned} \tag{3.5}$$

The existence of a fourth family may affect the calculation of $\Delta M_K = M_{K_L} - M_{K_S}$.[4] It has been stressed for some time[18] that in order to preserve the original success in relating the c quark mass to ΔM_K in the four quark model,[19] the contribution to the mass difference from quarks beyond the second family should not exceed the c quark contribution. It was furthermore conjectured[20] that the existence of very heavy quarks ($m_Q \gg 1$ GeV) should not affect the kaon system except for CP violation which is supposed to owe its existence to $N > 2$ quarks. It then follows that the masses of such quarks and their mixing to the light quarks must be correlated in a manner which gives rise to negligible contributions to ΔM_K. In fact with Eqs. (3.3) and (3.4) it turns out that as long as say $m_t < M_W$ the t quark contribution lies two orders of magnitude below the measured value of ΔM_K. We will assume that the above conjecture applies also to the t' contribution.

The various quark contributions to ΔM_K may be read from the expression derived in Ref. 4 for the short distance dominated $2W$ exchange diagrams (the so-called "box diagram"). Generalized to any number of generations it reads:

$$\Delta M_K^{\mathrm{box}} = 2\mathrm{Re}\, M_{12}^{\mathrm{box}} = \frac{B_K G_F^2 f_K^2 M_K M_W^2}{6\pi^2} \sum_{i,j=c,t,t'\ldots} \eta_{ij} E(x_i, x_j) \mathrm{Re}\, \lambda_i \lambda_j \ . \quad (3.6)$$

M_{12} is the off diagonal $K^0 - \overline{K}^0$ mass matrix element and B_K is the conventional parameter which represents the matrix element of the $\Delta S = 2$ short distance operator between the K^0 and $\overline{K}^0$ states. The value of $B_K = 0.33$ with a possible theoretical uncertainty of 50% may be regarded as a reasonable value derived by current algebra techniques.[21] In the following we will use this value although somewhat larger estimates are not entirely excluded.[22] The parameters η_{ij} are QCD correction factors of order one[23] $\eta_{cc} = 0.7$, $\eta_{tt} = 0.6$, $\eta_{ct} = 0.4$. The λ_i are products of quark mixing elements $\lambda_i = U_{is}^* U_{id}$, and the dimensionless box diagram functions E are given in terms of the quark masses $x_i = m_{q_i}^2 / M_W^2$:[24]

$$E(x_i, x_i) = x_i \left[\frac{1}{4} + \frac{9}{4(1-x_i)} - \frac{3}{2(1-x_i)^2}\right] - \frac{3}{2}\left(\frac{x_i}{1-x_i}\right)^3 \ell n\, x_i$$

$$E(x_i, x_j) = x_i x_j \left\{ \left[\frac{1}{4} + \frac{3}{2(1-x_i)} - \frac{3}{4(1-x_i)^2}\right] \frac{\ell n\, x_i}{x_i - x_j} \right. \quad (3.7)$$

$$\left. + (x_i \leftrightarrow x_j) - \frac{3}{4(1-x_i)(1-x_j)} \right\} .$$

One often uses the approximation

$$E(x_i, x_i) \simeq x_i \qquad \text{for } x_i \ll 1$$

$$E(x_i, x_j) \simeq x_i \ell n(x_j/x_i) \quad \text{for } x_i \ll x_j \ll 1 \ . \quad (3.8)$$

We note in passing that with $B_K = 0.33$, $\eta_{cc} = 0.7$, $m_c = 1.5\ GeV$, $Re\lambda_c^2 \simeq s_{12}^2$, the c quark term in Eq. (3.6) provides merely a fraction ($\sim \frac{1}{4}$) of the measured value of ΔM_K. Certain long-distance contributions are also needed to account for the mass difference.[25]

As suggested above we will require that each of the t' contributions to the right-hand side of Eq. (3.6) is much smaller than that of the c quark. First consider

$$\eta_{t't'} E(x_{t'}, x_{t'}) \, \mathrm{Re} \, \lambda_{t'}^2 \ll \eta_{cc} E(x_c, x_c) \mathrm{Re} \, \lambda_c^2 \ . \tag{3.9}$$

For a range of interest $m_{t'} = 40 - 150 \ GeV$ $E(x_{t'}, x_{t'})$ varies by an order of magnitude between 0.21 and 2.1. The parameters η_{ij} depend only weakly on the number of generations and on the quark masses;[23] so we take $\eta_{t't'} \sim 0.5$. $Re \, \lambda_{t'}^2$ may be expressed in terms of the mixing angles by use of the last two of Eq. (2.8). Disregarding accidental cancellations between the various terms we single out the term $s_{12}^2 c_{34}^4 s_{24}^4$ for which we find from Eq. (3.9)

$$c_{34}^4 s_{24}^4 \ll 10^{-4} - 10^{-3} \tag{3.10}$$

depending somewhat on $m_{t'}$. We therefore conclude that if θ_{34} is not too large

$$s_{24} < 0.1 \ . \tag{3.11}$$

We checked that requiring the smallness of all other s_{i4} contributions to ΔM_K, including those of the t quark, the tt' and the ct' diagrams, does not lead to any further constraint on s_{i4} beyond Eqs. (3.5) and (3.11). No constraint may be obtained for s_{34} which in principle may be even larger than s_{12}.

4. Constraints on s_{i4} from CP violation in the $K^0 - \overline{K}^0$ system

The two parameters ϵ and ϵ' which describe CP violation in the $K^0 - \overline{K}^0$ system are given by[26]

$$\frac{2\eta_{+-} + \eta_{00}}{3} \equiv \epsilon \simeq \frac{1}{\sqrt{2}} e^{i(\pi/4)} \left(\frac{\mathrm{Im}\, M_{12}}{\Delta M_K} + \xi \right) \tag{4.1}$$

$$\frac{\eta_{+-} - \eta_{00}}{3} \equiv \epsilon' \simeq \frac{1}{\sqrt{2}} e^{i\left(\frac{\pi}{2} + \delta_2 - \delta_0\right)} \left(\frac{\mathrm{Im}\, A_2}{\mathrm{Re}\, A_0} - \omega \xi \right) \tag{4.2}$$

$$\omega = \frac{\mathrm{Re}\, A_2}{\mathrm{Re} A_0} \qquad \xi = \frac{\mathrm{Im}\, A_0}{\mathrm{Re}\, A_0}$$

A_I and δ_I ($I = 0, 2$) are respectively the complex $K \to 2\pi$ amplitudes and the (additional) $\pi\pi$ phase shift in the isospin I state. In the quark basis adopted by us in Section 2 (just as in the K-M basis), in which U_{ud} and U_{us} are real, A_0 obtains an imaginary part from the so-called "penguin" diagrams[27] while A_2 remains real. Thus $|\epsilon'| \simeq |\omega\xi|/\sqrt{2}$ and the second term in Eq. (4.1) is bounded by the recently measured upper limit $|\epsilon'/\epsilon| < 0.01$:[28]

$$|\xi/\sqrt{2}| \simeq |\epsilon'/\omega| \simeq 20\, |\epsilon'| < 0.2\, |\epsilon| \tag{4.3}$$

where the value $\omega = 1/20$ was used.[29]

Recently Donoghue and Holstein[30] have estimated the long-distance contributions to $\mathrm{Im}\, M_{12}$ and have shown that with the present upper limit on $|\epsilon'/\epsilon|$ they may not lead to more than a 20% correction to ϵ. Since this contribution and the ξ term are correlated they partially overlap and their overall contribution is expected to be less than 30%. Disregarding this uncertainty, which may be further reduced by future improvements of the limits on $|\epsilon'/\epsilon|$, one may approximate ϵ by the short-distance dominated box diagrams calculation of $\mathrm{Im}\, M_{12}$.

The expression for ϵ^{box}, similar to the one for ΔM_K^{box} in Eq. (3.6), is given by[4]

$$\epsilon^{\text{box}} = e^{i(\pi/4)} \frac{B_K G_F^2 f_K^2 M_K M_W^2}{12\sqrt{2}\,\pi^2 \Delta M_K} \sum_{i,j=c,t,t'} \eta_{ij} E(x_i, x_j)\, \mathrm{Im}\,\lambda_i \lambda_j \ . \tag{4.4}$$

Using the values[29] $G_F = 1.178 \times 10^{-5}\, GeV^{-2}$, $\Delta M_K / M_K = 0.71 \times 10^{-14}$, $f_K = 160\, MeV$, $|\epsilon| = 2.27 \times 10^{-3}$ and[31] $M_W = 82\, GeV$ one obtains the sum rule (assuming $\epsilon \simeq \epsilon^{\text{box}}$):

$$B_K \sum_{i,j=c,t,t'} \eta_{ij} E(x_i, x_j)\, \mathrm{Im}\,\lambda_i \lambda_j \simeq 10^{-7} \ . \tag{4.5}$$

It was pointed out in Ref. 5 that within the three generation model the present limits on the mixing parameters, given in our convention by Eqs. (3.5), would provide stringent upper limits on the quantities $\mathrm{Im}\,\lambda_i \lambda_j$ $(i, j = c, t)$ which by Eq. (4.5) impose lower bounds on the function $E(x_t, x_t)$. Since this function is increasing with m_t one may use this to derive lower bounds on this mass.[32]

To illustrate this point let us note that in the angle-and-phase convention introduced in Section 2 one obtains in the three generation model, using Eqs. (3.5)

$$\begin{aligned} -\mathrm{Im}\,\lambda_c^2 &= 2\mathrm{Im}\,\lambda_c \lambda_t \simeq 2 s_{12} s_{23} s_{13} \sin\phi_{13} < 2 \times 10^{-4} \\ \mathrm{Im}\,\lambda_t^2 &= -\mathrm{Im}\,\lambda_c^2 \Big(s_{23}^2 - \frac{s_{23} s_{13}}{s_{12}} \cos\phi_{13} \Big) < 8 \times 10^{-7} \ . \end{aligned} \tag{4.6}$$

Furthermore one has $E(x_c, x_c) = 3.3 \times 10^{-4}$ $(m_c = 1.5$ GeV$)$ and $E(x_c, x_t) = (2.0 - 2.6)\, 10^{-3}$ where the slight variation corresponds to the mass range $m_t = 30 - 100\ GeV$. The function which exhibits the strongest growth with m_t is $E(x_t, x_t)$ which obtains the values 0.12 - 1.0 in the above mass range. Using the rather conservative value $B_K = 0.5$ it is then found that the sum rule of Eq.

(4.5) may be saturated only for $m_t > 40$ GeV. Furthermore, a mass of 40 GeV requires that s_{13} and s_{23} take their upper limit values of Eq. (3.5) and that the CP violating phase is "maximal", i.e. $\phi_{13} \simeq \pi/2$.

What do these considerations imply for the mixing of a possible fourth family of quarks if it exists? Disregarding the unlikely possibility that the t' terms in Eq. (4.5) cancel those of the first three generations, their contribution is expected to be smaller that the right-hand side of Eq. (4.5). The η_{ij} parameters for t' do not differ substantially from those of t. The same quark mass functions describe the t and t' contributions. Since Eq. (4.5) is saturated by the first three generations with $m_t = 40$ GeV if the values on the right-hand side of Eqs. (4.6) are actually achieved, similar upper bounds then apply to $\mathrm{Im}\,\lambda_c\lambda_{t'}$ and $\mathrm{Im}\,\lambda_{t'}^2$ $(2\,\mathrm{Im}\,\lambda_t\lambda_{t'})$, respectively, if $m_{t'} = 40$ GeV.

$$\begin{aligned} \mathrm{Im}\,\lambda_c\lambda_{t'} &< 10^{-4} \\ \mathrm{Im}\,\lambda_{t'}^2, 2\,\mathrm{Im}\,\lambda_t\lambda_{t'} &< 4\times 10^{-7}\ . \end{aligned} \tag{4.7}$$

These limits become stronger for higher values of $m_{t'}$. For instance for $m_{t'} =$ 150 GeV the bound on $\mathrm{Im}\,\lambda_{t'}^2$ is stronger by an order of magnitude.

To find out what Eqs. (4.7) imply for the mixing parameters s_{i4} one may write down the explicit expressions for the quantities in Eq. (4.7) in terms of these angles and the phases ϕ_{ij}. Being imaginary parts, they will contain terms multiplying $\sin\phi_{ij}$. Therefore Eqs. (4.7) are useless unless some kind of "maximal" phase assumption is made. In the following we will adopt such an assumption, considering unlikely the alternative possibility that the t' couples to the first two generations with large mixing angles and small phases. We will also disregard the unlikely possibility that different terms in $\mathrm{Im}\,\lambda_c\lambda_{t'}$, etc. may cancel each other. It remains then a matter of simple examination and use of Eqs. (3.5) to determine the strongest constraints implied by Eqs. (4.7) on s_{i4}. Writing down only the relevant terms

$$\begin{aligned}
\operatorname{Im}\lambda_c\lambda_{t'} &= -s_{12}c_{34}^2 s_{14}s_{24}\sin(\phi_{24}-\phi_{14}) - s_{12}s_{23}c_{34}s_{34}s_{14}\sin\phi_{14}+\dots \\
\operatorname{Im}\lambda_t\lambda_{t'} &= -s_{12}s_{23}^2c_{34}^4 s_{14}s_{24}\sin(\phi_{24}-\phi_{14}) - s_{12}s_{23}^3c_{34}^3 s_{34}s_{14}\sin\phi_{14}+\dots \qquad (4.8)\\
\operatorname{Im}\lambda_{t'}^2 &= c_{34}^4 s_{14}^2 s_{24}^2\sin 2(\phi_{24}-\phi_{14}) - s_{23}^2c_{34}^2 s_{34}^2 s_{14}^2\sin 2\phi_{14}+\dots
\end{aligned}$$

one obtains for $m_{t'} = 40$ GeV

$$c_{34}^2 s_{14}s_{24} < 5\times 10^{-4}\ , \qquad c_{34}s_{34}s_{14} < 10^{-2}\ . \qquad (4.9)$$

The limits on a heavier t', e.g. $m_{t'} = 150$ GeV, are stronger by about a factor of 2. The first limit of Eq. (4.9) provides an order of magnitude extension of the limits derived in Eqs. (3.5) and (3.11) if one assumes $c_{34} \simeq 1$.

At this point we wish to note that the existence of a fourth family of quarks with very strong mixing to the third generation, $s_{34} \sim 0(1)$, would have reduced the t quark contribution to ϵ in Eq. (4.4). It is straightforward to show that with four generations the expressions for $\operatorname{Im}\lambda_c\lambda_t$ and $\operatorname{Im}\lambda_t^2$ in Eqs. (4.6) get multiplied by c_{34}^2 and c_{34}^4 respectively. As the discussion which follows Eqs. (4.6) illustrates, with c_{34} significantly smaller than one, the sum rule (4.5) might not be satisfied even if m_t is somewhat heavier than 40 GeV.

The inequalities

$$s_{14}s_{24} < 5\times 10^{-4}\ , \qquad s_{34}s_{14} < 10^{-2} \qquad (4.10)$$

turn into strong inequalities ($\ll$) if one assumes the t' contribution to ϵ to be negligible compared to that of the t quark (as is the case for the t term in ΔM_K relative to that of c).

One may turn around the argument which led to Eqs. (4.10) and note that the t' contribution to ϵ may be as large as the measured value of ϵ. Namely, if $s_{14}s_{24} \sim 0(10^{-4} - 10^{-3})$ and/or $s_{14}s_{34} \gtrsim 0(10^{-2})$ and the CP violating phases of the fourth generation were "maximal", the t' could be the dominant source of CP violation. This may be required within the $SU(2) \times U(1)$ single Higgs model if future experiments support a t quark mass in the vicinity of 40 GeV plus at least one of the following developments:

1. The present upper limit on U_{ub} is considerably reduced (or a more precise value is measured for τ_B well above 10^{-12} sec).
2. A theoretical value for B_K is reliably calculated and turns out to be considerably smaller than 0.5.

Let us briefly discuss ϵ' which measures CP violation in $K \to 2\pi$. In the three generation model it is proportional to $s_{23}s_{13}\sin\phi_{13}$ which, as argued above, is anticipated to lie in the vicinity of the upper limits of Eqs. (3.5) with $\phi_{13} \simeq \pi/2$ if $m_t \sim 40$ GeV. This would then lead to a nonzero value of ϵ', for which the theoretical prediction suffers from uncertainties in the hadronic matrix element estimate.[33] If this theoretical ambiguity were to be resolved and if the present experimental upper limit[28] on ϵ' were decreased there might be a potential problem for the three generation model. The extension to four families would not resolve this difficulty. If the t' quark is to be the dominant source of CP violation in $K \to 2\pi$, ϵ' will be proportional to $s_{14}s_{24}\sin(\phi_{24} - \phi_{14})$ rather than to $s_{23}s_{13}\sin\phi_{13}$. To obtain the value of ϵ would again lead to a prediction for ϵ' which is too large. This prediction may be made somewhat smaller by choosing a large value of $m_{t'}$ since the increase of the penguin amplitude with $m_{t'}$ is slower than that of $\epsilon^{\rm box}$. There are some indirect arguments within the three generation model for a positive real value of ϵ'/ϵ.[34] The recent measurement of this ratio,[28] still consistent with zero, seems to favor a negative value. It is straightforward to show that within an extension to four families, in which ϵ is dominated by the t' contribution, the sign of ϵ'/ϵ will be the same as in the three generation model.

5. Constraints from $B^0 - \overline{B}^0$ Mixing

In analogy with Eq. (3.6) one obtains for the mass difference of two opposite-CP neutral B mesons:[18,35]

$$\Delta M_B \simeq \Delta M_B^{\rm box} = 2|M_{12}^{\rm box}| = \frac{B_B G_F^2 f_B^2 M_B m_W^2}{6\pi^2} \sum_{i,j=c,t,t'} \eta_{ij}^B E^B(x_i, x_j)|\lambda_i^B \lambda_j^B| \, . \tag{5.1}$$

Here the various factors are defined in the B system in analogy to the corresponding factors of Eq. (3.6) defined in the K system. Considering in particular the state $B^0 = B_d = b\overline{d}$ we take $M_B = 5.2$ GeV and $\lambda_i^B = U_{ib}^* U_{id}$. To be conservative we use for the B_B parameter and for the B decay constant the rather low values[36] $B_B = 0.33$, $f_B = f_K = 160$ MeV. The box diagram functions $E^B(x_i, x_j)$ are somewhat complicated by their extra dependence on the external heavy b quark mass.[37] For the two terms of interest to us $i = j = t'$ and $i = t$, $j = t'$ this complication may be disregarded since for them $E^B = E$ holds within 10% for m_t, $m_{t'} > 25$ GeV.[38] The QCD factor $\eta_{tt}^B \simeq 0.85$ was calculated in Ref. 35 and a similar value will be taken for $\eta_{tt'}^B$, $\eta_{t't}^B$.

$B^0 - \overline{B}^0$ mixing is described by the dimensionless parameters $x \equiv \Delta M_B/\Gamma$, where Γ is the (average) B^0 decay rate. With the above values of the various parameters one finds

$$x \equiv \frac{\Delta M_B}{\Gamma} \simeq 10^3 \tau_B \sum_{i,j=t,t'} E(x_i, x_j)\, |\lambda_i^B \lambda_j^B| \tag{5.2}$$

where τ_B stands for the B lifetime in units of 10^{-12} sec. The cc and ct terms were neglected since their box diagram functions are much smaller than that of tt, whereas all three quark mixing factors $\lambda_i^B \lambda_j^B$ $(i, j = c, t)$ are comparable in magnitude:

$$\begin{aligned} \lambda_c^B &\simeq -s_{12}s_{23} \\ \lambda_t^B &\simeq c_{34}^2 \left(s_{12}s_{23} - s_{13}e^{-i\phi_{13}}\right) + c_{34}s_{34}\left(s_{12}s_{24}e^{-i\phi_{24}} - s_{14}e^{-i\phi_{14}}\right) \ . \end{aligned} \tag{5.3}$$

We have also dropped the ct' term since it too comes with a small $E(x_c, x_{t'})$ and, as we checked, it would not lead to any new constraint beyond the ones obtained from Eq. (5.2). The t' mixing factor is

$$\lambda_{t'}^B \simeq c_{34}s_{34}\left(s_{14}e^{-i\phi_{14}} - s_{12}s_{24}e^{-i\phi_{24}}\right) + \cdots \tag{5.4}$$

where we omit terms of order 10^{-4} or smaller by using Eqs. (3.5) and (3.11).

As a result of $B^0-\overline{B}^0$ mixing one expects same sign dileptons to be emitted in semileptonic decays of $B^0\overline{B}^0$ pairs produced in e^+e^- annihilation.[39] The number of same sign dileptons divided by opposite sign dileptons from $B^0\overline{B}^0$ decays at the $\Upsilon(4S)$, i.e. just above $B\overline{B}$ threshold, is given to a good approximation (neglecting CP violation) by:[38,39]

$$y \equiv \frac{N(\ell^+\ell^+) + N(\ell^-\ell^-)}{N(\ell^-\ell^+)} \simeq \frac{x^2}{2 + x^2} \ . \tag{5.5}$$

The present experimental upper limit on y is about 30%, if one assumes that the $\Upsilon(4S)$ state decays to neutral and charged B meson pairs with a ratio 2:3

and that the neutral and charged B mesons have equal semileptonic branching ratios.[40] Note that within the standard three generation model one expects y to be a few percent at most if $m_t \sim 40$ GeV.[41] This estimate may be easily obtained from Eqs. (3.5), (3.7), (5.2), (5.3) and (5.5) and depends on the values assumed for B_B and f_B.

The experimental limit $y < 0.3$ (and its possible future improvement) imposes certain constraints on s_{i4}, which may be obtained from Eqs. (5.2), (5.4) and (5.5). For instance, the modest requirement ($y < 0.3$)

$$x_{t'} \equiv 10^3 \tau_B \, E(x_{t'}, x_{t'}) |\lambda_{t'}^B|^2 < 1 \tag{5.6}$$

leads to $s_{34}s_{14} < 0.1,\ 0.03$ for $m_{t'} = 40,\ 150$ GeV respectively. It does not give rise to any constraint beyond Eq. (4.10) obtained from ϵ.

At this point we wish to readdress the possibility that t' is the dominant source of CP violation in the K system. As pointed out in Section 4, this would be the case if for instance $s_{14}s_{34} \gtrsim 0\left(10^{-2}\right)$. It is interesting to note that in such a case (which requires s_{34} to be at least as large as s_{12}) one may have rather large $B^0 - \overline{B}^0$ mixing effects. For instance if $s_{14}s_{34} = (1-2)\,10^{-2}$ and if $m_{t'} =$ 150 GeV, x may take values close to one and correspondingly one would expect same sign dileptons to be abundant at $\Upsilon(4S)$.

6. Hierarchy schemes of mixing among the four generations.

The three measured mixing matrix elements

$$\begin{aligned} s_{12} &= |U_{us}| = 0.23 \\ s_{23} &\simeq |U_{cb}| \simeq 0.05 \\ s_{13} &= |U_{ub}| < 0.01 \end{aligned} \qquad (6.1)$$

may suggest a few kinds of hierarchy structure in the mixing matrix. Here we extrapolate the structure to the hypothetical fourth generation and study the consequences, with particular attention given to the t' contribution to ϵ. We start by listing the suggested types of hierarchy. Since our convention has the advantage that $|U_{ij}| \simeq s_{ij}$ $(i < j)$ we may use s_{ij} directly to characterize the structure of the mixing matrix.

a) The first and most obvious scheme suggested by Eqs. (6.1) was proposed by Wolfenstein:[42]

$$s_{12} \sim \lambda \qquad s_{23} \sim \lambda^2 \qquad s_{13} \sim \lambda^3 \qquad (\lambda = 0.2)\ . \qquad (6.2)$$

Within this scheme s_{13} lies near its present experimental upper limit as required to account for ϵ in the framework of three generations. The pattern which emerges

$$1 \leftarrow \lambda \rightarrow 2 \leftarrow \lambda^2 \rightarrow 3 \leftarrow \lambda^3 \rightarrow 4 \qquad (6.3)$$

leads to the expectations[43]

$$s_{34} \sim \lambda^3 \qquad s_{24} \sim \lambda^5 \qquad s_{14} \sim \lambda^6\ . \qquad (6.4)$$

b) A simpler scheme, which is at least a rough approximation, is

$$s_{12} \sim s_{23} \sim \lambda\ , \qquad s_{13} \sim \lambda^2 \qquad (\lambda = 0.1)\ . \tag{6.5}$$

Drawn schematically as

$$1 \leftarrow \lambda \rightarrow 2 \leftarrow \lambda \rightarrow 3 \leftarrow \lambda \rightarrow 4 \tag{6.6}$$

this leads to

$$s_{34} \sim \lambda\ , \qquad s_{24} \sim \lambda^2\ , \qquad s_{14} \sim \lambda^3\ . \tag{6.7}$$

To fit Eqs. (6.1) more accurately, Eqs. (6.5) may be replaced by a two parameter description[44]

$$s_{12} \sim \alpha_1\ , \qquad s_{23} \sim \alpha_2\ , \qquad s_{13} \sim \alpha_1\alpha_2 \tag{6.8}$$

and when a third parameter α_3 is introduced to represent s_{34} the corresponding expectations are

$$s_{34} \sim \alpha_3\ , \qquad s_{24} \sim \alpha_2\alpha_3\ , \qquad s_{14} \sim \alpha_1\alpha_2\alpha_3\ . \tag{6.9}$$

c) If a fourth generation exists it is possible that Eqs. (6.1) are telling us merely that the mixing occurs mostly within each of the two pairs $1 \leftrightarrow 2$, $3 \leftrightarrow 4$. In terms of a single parameter ($\lambda \sim 0.2$) this may be represented by

$$s_{12} \sim s_{34} \sim \lambda\ , \qquad s_{13}, s_{23}, s_{14}, s_{24} \leq \lambda^2\ . \tag{6.10}$$

In Section 4 we have shown that in order that the t' quark ($m_{t'} \leq 150$ GeV) make a significant contribution to ϵ one must have either $s_{14}s_{24} > 10^{-4}$

or $s_{14}s_{34} > 10^{-2}$. This contribution is seen to be extremely tiny in scheme a) (Eqs. (6.4)) and still unnoticeable in the single parameter version of scheme b) (Eq. (6.7)). If one assumes a large s_{34} mixing in the three parameter version of scheme b) (Eqs. (6.9)) the t' contribution to ϵ may be appreciable. Finally in the pair-associated scheme c) (Eqs. (6.10)) the t' may even be the dominant source of CP violation if one assumes s_{14} to be larger than s_{13}, say $s_{14} \sim s_{23}$. As pointed out at the end of Section 5, in such a scheme with a heavy t' ($m_{t'} =$ 150 GeV chosen to illustrate the point) same sign dileptons at $\Upsilon(4S)$ may be abundant.

7. Summary

In this paper we studied the restrictions on the mixing of a fourth generation of quarks based mostly on the neutral kaon system. To do so we first introduced a parametrization of the mixing matrix, which may be easily generalized to any number of quark families and which is very convenient for direct mixing angle determination from experiments. A range of mixing angles is found, $s_{14} < 0.06$ $s_{24} < 0.1$ $s_{14}s_{24} \sim 0(10^{-4} - 10^{-3})$ and/or $s_{14}s_{34} \gtrsim 0(10^{-2})$, for which the contribution of the hypothetical t' quark to the $K_L - K_S$ mass difference is negligible, yet the t' may have a large effect on the CP impurity parameter ϵ. Within the few schemes of the mixing matrix that we studied, the one in which the four families are mostly mixed in pairs can lead to a t' dominated ϵ. This may resolve a potential problem foreseen in the three generation model, which could materialize from future improvements of the mixing angle and m_t measurements.

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