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CABIBBO-ANGLE FAVORED TWO-BODY DECAYS OF D-MESON*

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ABSTRACT

The branching ratios for $D \to PP$ Cabibbo-angle favored decays are studied in a model independent manner. A hybrid model, a mixture of spectator model (with color suppression) and exchange model, is proposed to provide acceptable parameters. Conclusions are derived for $D \to VP$ decays. Finally the effect of final state interactions on these decays is studied.

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1. Introduction

Data on two-body pseudoscalar (PP) decays of the D-meson have existed in literature for quite some time.[1] New data on the PP and the vector-pseudoscalar (VP) decay modes, with much higher statistics, are expected soon from MARK III.[2,3] It is appropriate at this time to take a fresh look at the two-body decays of the D-mesons.

In this paper we study the Cabibbo-angle favored two-body decays. First we analyze the $D \to PP$ branching ratios in a model independent manner *without final state interactions (FSI)*. In the light of this analysis we show why the commonly used models such as the spectator model (with color suppression) and the exchange model, individually, do not work. A hybrid model is then developed, which in the absence of FSI, provides a parametrization of $D \to PP$ data. We then go on to derive conclusions on $D \to PP$ and $D \to VP$ branching ratios and compute $\Gamma(D^0 \to \overline{K}^0\eta)$, $\Gamma(D^0 \to \overline{K}^0\omega)$ and $\Gamma(D^0 \to \overline{K}^0\phi)$. Finally we show what role FSI plays in the two-body Cabibbo-angle favored decays of the D-meson.

2. Analysis without Final State Interactions

The hard gluon corrected Hamiltonian for the Cabibbo-angle favored charm decays is,

$$H_W = \frac{G}{\sqrt{2}}\cos^2\theta_c\left[\frac{1}{2}(f_+ + f_-)(\overline{u}d)(\overline{s}c) + \frac{1}{2}(f_+ - f_-)(\overline{s}d)(\overline{u}c)\right] \qquad (1)$$

where θ_c is the Cabibbo angle in the four-quark model and the coefficients f_+

and f_- are computed to be[4–6]

$$f_+ = 0.69 \qquad f_- = 2.09 \ . \tag{2}$$

In the numerical work in this paper results are also computed for $f_+ = f_- = 1$ for the sake of comparison. As H_W changes isospin by one unit the following sum rule among the decay amplitudes is satisfied,

$$A(D^0 \to K^-\pi^+) + \sqrt{2}\, A(D^0 \to \overline{K}{}^0\pi^0) = A(D^+ \to \overline{K}{}^0\pi^+) \ . \tag{3}$$

In terms of the amplitudes with final states in $I = 1/2$ and $3/2$, these decay amplitudes are,

$$\begin{aligned} A(D^0 \to \overline{K}{}^0\pi^0) &= \frac{1}{\sqrt{3}}(\sqrt{2}\, A_3 + A_1) \\ A(D^0 \to K^-\pi^+) &= \frac{1}{\sqrt{3}}(A_3 - \sqrt{2}\, A_1) \\ A(D^+ \to \overline{K}{}^0\pi^+) &= \sqrt{3}\, A_3 \ . \end{aligned} \tag{4}$$

A_1 and A_3 are real in absence of FSI as they would be in the spectator or the exchange models also.

Using Eq. (4) we define the following ratios,

$$R_{00}(K\pi) \equiv \frac{\Gamma(D^0 \to \overline{K}{}^0\pi^0)}{\Gamma(D^0 \to K^-\pi^+)} = \frac{(1+\sqrt{2}\, r)^2}{(\sqrt{2}-r)^2} \tag{5}$$

$$R_{0+}(K\pi) \equiv \frac{\Gamma(D^0 \to K^-\pi^+)}{\Gamma(D^+ \to \overline{K}{}^0\pi^+)} = \frac{1}{9}\left(1 - \frac{\sqrt{2}}{r}\right)^2 \tag{6}$$

where $r \equiv A_3/A_1$ and the subindices refer to the D-meson charges in the ratios. Experimentally

$$R_{00}(K\pi) = 0.35 \pm 0.07 \pm 0.07 \,^{1,2} \tag{7}$$

$$R_{0+}(K\pi) = 3.7 \pm 1.0 \pm 0.8 \ .^{3} \tag{8}$$

In Eq. (8), $\tau(D^+)/\tau(D^0) = 2.5 \pm 0.6$, all errors treated as statistical, is used.

Using Eq. (6) and (8) one can solve for r from $R_{0+}(K\pi)$. We obtain two solutions

$$r = -0.3 \begin{array}{l} +0.07 \\ -0.15 \end{array} \tag{9}$$

or

$$r = 0.21 \begin{array}{l} +0.07 \\ -0.03 \end{array} \ . \tag{10}$$

The curve marked 0° in Fig. 1 shows $R_{0+}(K\pi)$ as a function of r in absence of FSI. The two solutions shown in Eq. (9) and (10) are readily identified from this curve.

We can also solve for r from Eq. (5) and (7) using $R_{00}(K\pi)$. We obtain two solutions again,

$$r = -0.08 \begin{array}{l} +0.08 \\ -0.11 \end{array} \tag{11}$$

or

$$r = -2.23 \begin{array}{l} +0.50 \\ -0.55 \end{array} \ . \tag{12}$$

The curve marked 0° in Fig. 2 shows $R_{00}(K\pi)$ as a function of r in absence of FSI. The two solutions shown in Eq. (11) and (12) are readily identified from the curve.

From the four solutions, Eq. (9)-(12), it is evident that there is *no common solution* that satisfies both $R_{00}(K\pi)$ and $R_{0+}(K\pi)$ of Eq. (7) and (8) for *real* decay amplitudes. One may take the view that a common solution with $r \approx -0.2$

could result from a slightly more liberal treatment of errors. Such a solution would imply that most of the decay occurs through the [6] of SU(3) which is contained in [20] of SU(4) and only a small fraction goes through [15*] of SU(3) contained in [84] of SU(4). This is indeed to be expected on grounds of $\Delta I = \frac{1}{2}$ rule extended to SU(4) where one would expect [20] representation in H_W to dominate over the [84]. There is, of course, the possibility that no common solution will be found with *real amplitudes* once data with better statistics are available. We show later that data require that complex amplitudes be used. We now turn to the commonly used models.

Spectator Model (with color suppression):

The model is shown in Fig. 3. In this model the amplitudes are *real* and one obtains, up to an overall factor,

$$
\begin{aligned}
A(D^0 \to \overline{K}^0\pi^0) &= \frac{1}{3\sqrt{2}}\,(2f_+ - f_-) \\
A(D^0 \to K^-\pi^+) &= \frac{1}{3}\,(2f_+ + f_-) \\
A(D^+ \to \overline{K}^0\pi^+) &= \frac{4}{3}\,f_+
\end{aligned}
\tag{13}
$$

Using Eqs. (4) and (13) one gets,

$$
A_3 = \frac{4}{3\sqrt{3}}\,f_+ \qquad A_1 = -\frac{1}{3\sqrt{6}}\,(2f_+ + 3f_-)\ . \tag{14}
$$

Using $f_+ = 0.69$ and $f_- = 2.09$, one obtains, $r = -0.51$. If one uses $f_+ = f_- = 1$, one obtains $r = -1.13$. Clearly both these solutions are ruled out by Eq. (9)-(12). For $r = -0.51$, $D^0 \to \overline{K}^0\pi^0$ is very strongly suppressed resulting in $R_{00}(K\pi) \approx 1/50$. For $r = -1.13$, one gets $R_{00}(K\pi) \approx 1/18$. Incidentally, in this model, $r = -0.2$ would require $f_-/f_+ = 8.75$.

Exchange model:

Normally the amplitude resulting from this process, shown in Fig. 4, would be helicity suppressed; however such suppression could be obviated by soft gluon emission from the initial state. In this model one gets real amplitudes again, and up to an overall factor,

$$\begin{aligned} A(D^0 \to \overline{K}^0\pi^0) &= -\frac{g}{3\sqrt{2}} \\ A(D^0 \to K^-\pi^+) &= \frac{g}{3} \\ A(D^+ \to \overline{K}^0\pi^+) &= 0 \end{aligned} \tag{15}$$

where g is a dimensionless parameter. Clearly the exchange model gives,

$$r = 0 \ . \tag{16}$$

With $r = 0$, $R_{00}(K\pi)$ is at the margin of acceptability (see Eq. (11)) but not $R_{0+}(K\pi)$ which blows up at $r = 0$. Thus exchange model by itself will not satisfy the two ratios.

A Hybrid model:

One can construct a hybrid model which is a mixture of the spectator model, with color suppression, and the exchange model. In this model, up to an overall factor, one obtains the following real amplitudes,

$$\begin{aligned} A(D^0 \to \overline{K}^0\pi^0) &= \frac{1}{3\sqrt{2}}\,(2f_+ - f_-) - \frac{g}{3\sqrt{2}} \\ A(D^0 \to K^-\pi^+) &= \frac{1}{3}\,(2f_+ + f_-) + \frac{g}{3} \\ A(D^+ \to \overline{K}^0\pi^+) &= \frac{4}{3}\,f_+ \ . \end{aligned} \tag{17}$$

Defining,

$$a \equiv \frac{f_- + g}{2f_+} \tag{18}$$

and using Eqs. (4) and (17) we get,

$$r \equiv \frac{A_3}{A_1} = \frac{-2\sqrt{2}}{1+3a} \ . \tag{19}$$

For $r = -0.2$ this leads to,

$$g = 3.95, \quad \text{for } f_+ = 0.69 \ , f_- = 2.09 \tag{20}$$

and

$$g = 7.75, \quad \text{for } f_+ = f_- = 1 \ . \tag{21}$$

The knowledge of g together with the assumed knowledge of f_+ and f_- now allows us to study other $D \to PP$ and $D \to VP$ decays. Consider first $(D^0 \to \overline{K}^0 \eta)$ and $(D^0 \to \overline{K}^0 \eta')$. If we ignore $\eta - \eta'$ mixing and treat η as a pure SU(3) octet and η' as a singlet, then the hybrid model leads to the following real amplitudes, up to an overall factor,

$$A(D^0 \to \overline{K}^0 \eta) = \frac{1}{3\sqrt{6}} (2f_+ - f_- + g) \tag{22}$$

$$A(D^0 \to \overline{K}^0 \eta') = \frac{1}{3\sqrt{3}} (2f_+ - f_- + g) \ . \tag{23}$$

The difference is only in the Clebsch-Gordan coefficient in the $u\overline{u}$ content of η and η'. We caution the reader that Eq. (22) and (23) are model dependent relations. The two decays can be related at the SU(4) level with a different result.[7] With the S-wave phase space ratio of 1.37 one obtains

$$\frac{\Gamma(D^0 \to \overline{K}^0 \eta)}{\Gamma(D^0 \to \overline{K}^0 \eta')} \simeq 0.68 \ . \tag{24}$$

As the final states in both these decays involve only a single amplitude this ratio will not be effected by FSI which we discuss later.

If we assume that the overall factors arising from hadronization in the $K\pi$ and $K\eta$ final states are the same then, *without FSI*, one finds,

$$\frac{\Gamma(D^0 \to \overline{K}^0 \eta)}{\Gamma(D^0 \to \overline{K}^0 \pi^0)} = 0.28 \ \frac{(2f_+ - f_- + g)^2}{(2f_+ - f_- - g)^2} \tag{25}$$

$$= 0.13, \quad \text{for } f_+ = 0.68, \ f_- = 2.09, \ g = 3.95 \tag{26}$$

$$= 0.46, \quad \text{for } f_+ = f_- = 1, \ g = 7.75 \ . \tag{27}$$

The effect of FSI on this ratio is discussed later in this paper.

VP two-body decays:

Since the quark content of the final states in $D \to K\rho$ and $D \to K^*\pi$ decays is the same as that in $D \to K\pi$ decay we expect the decay amplitudes for the VP modes to bear a flavor-similarity to the PP modes through the hybrid model. The spin dependence will arise through hadronization and result in a different overall Lorentz structure. We then anticipate that up to an overall Lorentz structure factor,

$$\begin{aligned} A(D^0 \to \overline{K}^0 \rho^0) &= \frac{1}{3\sqrt{2}}(2f_+ - f_-) - \frac{g}{3\sqrt{2}} \\ A(D^0 \to K^- \rho^+) &= \frac{1}{3}(2f_+ + f_-) + \frac{g}{3} \\ A(D^+ \to \overline{K}^0 \rho^+) &= \frac{4}{3} f_+ \end{aligned} \tag{28}$$

Similar relations can be written for the $\overline{K}^{*0}\pi^0$, $K^{*-}\pi^+$ and $\overline{K}^{*0}\pi^+$ modes. If hadronization were to result only in an overall factor then we anticipate,

$$R_{00}(K\pi) \approx R_{00}(K\rho) \approx R_{00}(K^*\pi) \tag{29}$$

and

$$R_{0+}(K\pi) \approx R_{0+}(K\rho) \approx R_{0+}(K^*\pi) \tag{30}$$

where the VP ratios are defined in an analogous manner to Eq. (5) and (6). The data at present[1] are not very precise,

$$\begin{aligned}
BR(D^0 \to \overline{K}^0\rho^0) &= 0.1 \pm {}^{0.6}_{0.1}\ \% \\
BR(D^0 \to K^-\rho^+) &= 7.2 \pm {}^{3.0}_{3.1}\ \% \\
BR(D^0 \to \overline{K}^{*0}\pi^0) &= 1.4 \pm {}^{2.3}_{1.4}\ \% \\
BR(D^0 \to K^{*-}\pi^+) &= 3.4 \pm 1.4\ \% \ .
\end{aligned} \tag{31}$$

Data on all the VP charge states are expected[2,3] soon from Mark III. In particular data on $D \to K\rho$ and $D \to K^*\pi$ will allow us to analyze the VP modes in the same manner as the PP modes. The data at present are quite consistent with Eq. (29).

Next we estimate $\Gamma(D^0 \to \overline{K}^0\phi)$. The final state being a pure $I = 1/2$ state, this decay proceeds via the exchange process only, if one were to ignore the OZI-violating contribution. The amplitude, assuming for the moment an SU(3) symmetric vacuum so that $s\bar{s}$-pairs are excited with the same probability as the

$u\overline{u}$- or $d\overline{d}$-pair, is (up to an overall factor containing the Lorentz structure)

$$A(D^0 \to \overline{K}^0\phi) = \frac{g}{3} \,. \tag{32}$$

If we assume that hadronization results only in an overall factor, containing the Lorentz structure which is the same for all VP modes, then one obtains,

$$\frac{\Gamma(D^0 \to \overline{K}^0\phi)}{\Gamma(D^0 \to \overline{K}^0\rho^0)} = \frac{0.9\, g^2\, F_{s\overline{s}}}{(2f_+ - f - g)^2} \tag{33}$$

$$\frac{\Gamma(D^0 \to \overline{K}^0\phi)}{\Gamma(D^0 \to K^-\rho^+)} = \frac{0.45\, g^2\, F_{s\overline{s}}}{(2f_+ + f_- + g)^2} \tag{34}$$

where we have used a P-wave phase space ratio of 0.45 and $F_{s\overline{s}} \approx 1/3$ is the inhibition factor[8] for exciting an $s\overline{s}$-pair from the vacuum. We get (without FSI)

$$\frac{\Gamma(D^0 \to \overline{K}^0\phi)}{\Gamma(D^0 \to \overline{K}^0\rho^0)} = 0.22, \quad \text{for } f_+ = 0.69,\ f_- = 2.09 \text{ and } g = 3.95 \tag{35}$$

$$= 0.4\ , \quad \text{for } f_+ = f_- = 1 \text{ and } g = 7.75 \tag{36}$$

$$\frac{\Gamma(D^0 \to \overline{K}^0\phi)}{\Gamma(D^0 \to K^-\rho^+)} = 0.04, \quad \text{for } f_+ = 0.69,\ f_- = 2.09 \text{ and } g = 3.95 \tag{37}$$

$$= 0.08, \quad \text{for } f_+ = f_- = 1 \text{ and } g = 7.75 \tag{38}$$

These ratios are comparable to those derived earlier.[9]

The decay $D^0 \to \overline{K}^0\omega$ can be treated in a similar way. Up to an overall Lorentz factor the hybrid model, without FSI, generates the following real amplitude,

$$A(D^0 \to \overline{K}^0\omega) = \frac{1}{3\sqrt{2}}\,(2f_+ - f_- + g) \tag{39}$$

with the same assumptions as those made for $D^0 \to \overline{K}^0\phi$, one obtains

$$\frac{\Gamma(D^0 \to \overline{K}^0\omega)}{\Gamma(D^0 \to \overline{K}^0\rho^0)} = \frac{(2f_+ - f_- + g)^2}{(2f_+ - f_- - g)^2} \tag{40}$$

For $r = -0.2$, without FSI, one gets

$$\frac{\Gamma(D^0 \to \overline{K}^0\omega)}{\Gamma(D^0 \to \overline{K}^0\rho^0)} = 0.48, \quad \text{for } f_+ = 0.69,\ f_- = 2.09,\ g = 3.95 \tag{41}$$

$$= 1.68, \quad \text{for } f_+ = f_- = 1,\ g = 7.75\ . \tag{42}$$

In the next section the effect of FSI on these ratios is studied.

3. The Role of Final State Interactions (FSI)

FSI is expected to play an important role in D-decays since strangeness 1 resonances are known[10–12] to exist at 1.4–1.5 GeV. Theoretical analysis is, however, far from clear since most of the final states are highly inelastic. For the $K\pi$ decay mode, however, a fairly reliable analysis can be made since the s-wave $K\pi$ scattering in 0^+ state appears[10,11] to be elastic up to an energy of about 1.4 GeV. The only other competing channel is $K\eta$. The inelasticity at the D-meson mass is small. If one were to assume that $K\pi$ scattering in 0^+ state is elastic at 1.86 GeV then the phases of A_1 and A_3 in Eq. (4) are the elastic scattering phases δ_1 and δ_3. One can then write[13,14]

$$A(D^0 \to \overline{K}^0\pi^0) = \frac{1}{\sqrt{3}}\left(\sqrt{2}\,A_3e^{i\delta_3} + A_1e^{i\delta_1}\right)$$

$$A(D^0 \to K^-\pi^+) = \frac{1}{\sqrt{3}}\left(A_3e^{i\delta_3} - \sqrt{2}\,A_1e^{i\delta_1}\right) \tag{43}$$

$$A(D^+ \to \overline{K}^0\pi^+) = \sqrt{3}\,A_3e^{i\delta_3}$$

The ratios $R_{00}(K\pi)$ and $R_{0+}(K\pi)$ of Eq. (5) and (6) now read,

$$R_{00}(K\pi) = \frac{\left\{1+\sqrt{2}\,r\cos(\delta_1-\delta_3)\right\}^2 + 2r^2\sin^2(\delta_1-\delta_3)}{\left\{\sqrt{2}-r\cos(\delta_1-\delta_3)\right\}^2 + r^2\sin^2(\delta_1-\delta_3)} \tag{44}$$

$$R_{0+}(K\pi) = \frac{1}{9}\left[\left\{1-\frac{\sqrt{2}}{r}\cos(\delta_1-\delta_3)\right\}^2 + \frac{2}{r^2}\sin^2(\delta_1-\delta_3)\right] . \tag{45}$$

The $I = 1/2, 0^+$ channel is known to resonate (kappa meson) at 1.4 GeV[11]. The phase δ_1 crosses 90° at this energy. δ_3 is known to be −25° to −30° at 1.4 GeV[11]. It is probably very reasonable to assume $\delta_1 - \delta_3$ in the range of 120°-180° at the D-meson mass.

If we assume a particular value for $\delta_1 - \delta_3$ then one can solve for r from $R_{00}(K\pi)$ and $R_{0+}(K\pi)$ of Eq. (44) and (45) by using the experimental values for these ratios given in Eq. (7) and (8).

We choose $\delta_1 - \delta_3 = 150°$ for numerical estimates. $R_{0+}(K\pi)$ then yields the following two solutions,

$$r = 0.29\ {}^{+0.15}_{-0.06} \tag{46}$$

or

$$r = -0.21 \begin{array}{l} +0.03 \\ -0.07 \end{array} \tag{47}$$

In Fig. 1 the curve marked 150° shows $R_{0+}(K\pi)$ as a function of r for $\delta_1 - \delta_3 =$ 150°. The solutions shown in Eq. (46) and (47) are easily recognized from the diagram. It is also seen from Fig. 1 that FSI boosts $R_{0+}(K\pi)$ for $r > 0$ and suppresses it for $r < 0$.

For $\delta_1 - \delta_3 = 150°$, $R_{00}(K\pi)$ yields the following two solutions,

$$r = 0.1 \pm \begin{array}{l} 0.12 \\ 0.10 \end{array} \tag{48}$$

or

$$r = 1.91 \begin{array}{l} +0.5 \\ -0.5 \end{array} \tag{49}$$

The curve marked 150° in Fig. 2 shows $R_{00}(K\pi)$, for $\delta_1 - \delta_3 = 150°$, as a function of r. The two solutions, Eqs. (48) and (49), are evident from this curve. In Fig. 2 we have also plotted $R_{00}(K\pi)$ for $\delta_1 - \delta_3 = 120°$ and 180°.

The solutions for r, with FSI, Eq. (46)-(49), are almost sign-reversed to those without FSI, Eq. (9)-(12). This reversal of sign would be exact for $\delta_1 - \delta_3 = 180°$, since for this choice of the phase shifts the decay amplitudes are again real, however the sign of A_3 is reversed with respect to A_1. For $(\delta_1 - \delta_3) = 150°$ the situation is not much different. Comparing Eq. (46) and (48) we see that a common solution can *almost* be found for $r \approx 0.23$. In fact, for $\delta_1 - \delta_3$ *smaller* than 150° there will indeed be a common solution at $r \approx 0.23$. This, one can understand by looking at the trend of $R_{00}(K\pi)$ as a function of $\delta_1 - \delta_3$ from Fig. 2. For the numerical work that follows we have used $r = 0.23$.

In Fig. 5 we show what FSI can do to the ratio $R_{00}(K\pi)$. Without FSI the acceptable values of r are negative; however, FSI boosts this ratio *above* the experimental limits for *all* negative values of r. For positive values of r, $R_{00}(K\pi)$ is too large without FSI but FSI pulls it down to within experimental limits by simultaneously suppressing $D^0 \to \overline{K}^0\pi^0$ and enhancing $D^0 \to K^-\pi^+$.

The strategy next is to use the hybrid model, which does not have FSI, to generate decay amplitudes with $r = 0.23$ and let FSI pull the ratio $R_{00}(K\pi)$ within acceptable limits. By setting $r = 0.23$ in Eq. (19) we obtain $a = -4.43$. This, together with Eq. (18), yields (incidentally, to generate $r = 0.23$ from spectator model alone would require $f_-/f_+ = -8.86$)

$$g = -8.21, \quad \text{for } f_+ = 0.69,\ f_- = 2.09 \tag{50}$$

and

$$g = -9.86, \quad \text{for } f_+ = f_- = 1\ . \tag{51}$$

With these parameters one can now compute $\Gamma(D^0 \to \overline{K}^0\phi)$ with FSI, which plays no role in $D^0 \to \overline{K}^0\phi$ since there is only one amplitude involved, one obtains

$$\frac{\Gamma(D^0 \to \overline{K}^0\phi)}{\Gamma(D^0 \to \overline{K}^0\rho^0)} = \frac{0.9\, g^2\, F_{s\bar{s}}}{(2f_+ - f_- - g)^2}\, F_{00} \tag{52}$$

$$\frac{\Gamma(D^0 \to \overline{K}^0\phi)}{\Gamma(D^0 \to K^-\rho^+)} = \frac{0.45\, g^2\, F_{s\bar{s}}}{(2f_+ + f_- + g)^2}\, F_{+-} \tag{53}$$

Eqs. (52) and (53) are the same as Eq. (33) and (34) except for the FSI "enhancement factors" F_{00} and F_{+-} for $D^0 \to \overline{K}^0\rho^0$ and $D^0 \to K^-\rho^+$ respectively.

These factors are defined as,

$$
\begin{aligned}
F_{00} &= \frac{\Gamma(D^0 \to \overline{K}{}^0\rho^0) \text{ without } FSI}{\Gamma(D^0 \to \overline{K}{}^0\rho^0) \text{ with } FSI} \\
&= \frac{(1+\sqrt{2}\,r)^2}{\left\{1+\sqrt{2}\,r\cos(\phi_1-\phi_3)\right\}^2 + 2r^2\sin^2(\phi_1-\phi_3)}
\end{aligned}
\tag{54}
$$

and

$$
F_{+-} = \frac{(\sqrt{2}-r)^2}{\left\{\sqrt{2}-r\cos(\phi_1-\phi_3)\right\}^2 + r^2\sin(\phi_1-\phi_3)} \, . \tag{55}
$$

ϕ_1 and ϕ_3 are the phases of A_1 and A_3 respectively for $D^0 \to K\rho$ decays defined analogously to Eq. (43) for $D^0 \to K\pi$ decays. If the $K\rho$ scattering in 0^- state were elastic then ϕ_1 and ϕ_3 would be the scattering phase shifts. In contrast to the $K\pi$ system where phase shift analysis in the 0^+ state exists,[11] no comparable phase shift analysis exists for the $K\rho$ (or $K^*\pi$) 0^- state. Nevertheless, a radial excitation of the K-meson is known to exist at 1.46 GeV[12] which couples to $K\rho$, $K^*\pi$ and $K\epsilon$ channels. Thus each of these channels is inelastic being coupled to other channels via $K(1.46)$. The phase of the decay amplitude, in such cases, cannot be identified with the scattering phase shifts. Strictly, a coupled channel analysis[14] is required with its own uncertainties and ambiguities.

Despite the remarks of the previous paragraph it is reasonable to expect that for small inelasticities ϕ_1 and ϕ_3 would not be much different from the scattering phase shifts. Indeed in a model where *all* of the decay goes through the resonant state, and *all* of the scattering goes through the same resonant state, the phase of the decay amplitude will be equal to the phase of the scattering amplitude even in the inelastic case. Because of the presence of $K(1.46)$ the scattering phase shift in $I = 1/2$, 0^- state is expected to cross 90° at 1.46 GeV. $I = 3/2$, 0^-

state, on the other hand, being an "exotic" channel is expected to have a small phase shift, possibly negative as in the $K\pi$ case. Bearing all these remarks in mind, it is reasonable to expect $\phi_1 - \phi_3 \approx 120° - 180°$ at the D-meson mass. The "enhancement factors" are then (with $r = 0.23$),

$$\begin{aligned} \phi_1 - \phi_3 = 120° &: \quad F_{00} = 2.25\ , \quad F_{+-} = 0.59 \\ \phi_1 - \phi_3 = 150° &: \quad F_{00} = 3.24\ , \quad F_{+-} = 0.54 \\ \phi_1 - \phi_3 = 180° &: \quad F_{00} = 3.86\ , \quad F_{+-} = 0.52 \end{aligned} \tag{56}$$

with these "enhancement factors" we obtain for $\phi_1 - \phi_3 = 120°$, $f_+ = 0.69$, $f_- = 2.09$, $g = -8.21$ (Eq. (50)):

$$\frac{\Gamma(D^0 \to \overline{K}^0 \phi)}{\Gamma(D^0 \to \overline{K}^0 \rho^0)} = 0.8 \tag{57}$$

$$\frac{\Gamma(D^0 \to \overline{K}^0 \phi)}{\Gamma(D^0 \to K^- \rho^+)} = 0.26 \tag{58}$$

For $\phi_1 - \phi_3 = 120°$, $f_+ = f_- = 1.0$, $g = -9.86$ (Eq. (51)):

$$\frac{\Gamma(D^0 \to \overline{K}^0 \phi)}{\Gamma(D^0 \to \overline{K}^0 \rho^0)} = 0.56 \tag{59}$$

$$\frac{\Gamma(D^0 \to \overline{K}^0 \phi)}{\Gamma(D^0 \to K^- \rho^+)} = 0.18 \tag{60}$$

For $\phi_1 - \phi_3 = 180°$ the ratios of Eq. (57) and (59) will rise by 70% while those of (58) and (60) will decrease by 12%.

If the arguments following Eq. (55) regarding the phases of the VP-amplitudes hold then the equality of the ratios expressed in Eq. (29) and (30) is expected to remain intact.

The effect of FSI on $D^0 \to \overline{K}{}^0\omega$ can be studied in an analogous manner to that of $D^0 \to \overline{K}{}^0\phi$. In $D^0 \to \overline{K}{}^0\omega$ decay only a single amplitude is involved also and therefore, FSI has no effect on $\Gamma(D^0 \to \overline{K}{}^0\omega)$. One obtains, with FSI ($r = 0.23$ is used),

$$\frac{\Gamma(D^0 \to \overline{K}{}^0\omega)}{\Gamma(D^0 \to \overline{K}{}^0\rho^0)} = \frac{(2f_+ - f_- + g)^2}{(2f_+ - f_- - g)^2} F_{00} \tag{61}$$

$$= 3.2, \quad \text{for } f_+ = 0.69,\ f_- = 2.09,$$

$$g = -8.21,\ \phi_1 - \phi_3 = 120^\circ \tag{62}$$

$$= 1.5, \quad \text{for } f_+ = f_- = 1,\ g = -0.86,\ \phi_1 - \phi_3 = 120^\circ\ . \tag{63}$$

The $\overline{K}{}^0\omega$ mode should therefore be observable.

FSI effects $\Gamma(D^0 \to \overline{K}{}^0\eta)/\Gamma(D^0 \to \overline{K}{}^0\pi^0)$ in the following manner ($r = 0.23$ is used),

$$\frac{\Gamma(D^0 \to \overline{K}{}^0\eta)}{\Gamma(D^0 \to \overline{K}{}^0\pi^0)} = 0.28\, \frac{(2f_+ - f_- + g)^2}{(2f_+ - f_- - g)^2} F_{00} \tag{64}$$

where F_{00} is given in Eq. (54); however, one should read $\delta_1 - \delta_3$ for $\phi_1 - \phi_3$. We obtain

$$\frac{\Gamma(D^0 \to \overline{K}{}^0\eta)}{\Gamma(D^0 \to \overline{K}{}^0\pi^0)} = 0.88, \quad \text{for } f_+ = 0.69,\ f_- = 2.09,$$

$$g = -8.21, \delta_1 - \delta_3 = 120^\circ \tag{65}$$

$$= 0.40, \quad \text{for } f_+ = f_- = 1\ ,\ g = -0.86,\ \delta_1 - \delta_3 = 120^\circ \tag{66}$$

For $\delta_1 - \delta_3 = 150^\circ$ and 180°, F_{00} given in Eq. (56) should be used in Eq. (64). Note that since both $D^0 \to \overline{K}{}^0\eta$ and $D^0 \to \overline{K}{}^0\eta'$ involve only a single amplitude, FSI has no effect on the ratio $\Gamma(D^0 \to \overline{K}{}^0\eta)/\Gamma(D^0 \to \overline{K}{}^0\eta')$.

In a recent paper Scadron[15] has evaluated the $D \to PP$ amplitudes in the vacuum saturation approximation. In this approximation the amplitudes for neutral decay modes, such as $\overline{K}{}^0\pi^0$, are not calculable. However, the amplitudes for $D^+ \to \overline{K}{}^0\pi^+$ and $D^0 \to K^-\pi^+$ calculated in Ref. 15 lead to $r = -0.42$. From Fig. 1 with the curve marked 0° it is clear that this leads to $R_{0+}(K\pi) \approx 2.0$ which is acceptable. FSI will pull this ratio down to $\lesssim 1$ which is below the lower bound for $R_{0+}(K\pi)$. From Fig. 2 with the curve marked 0° it is clear that $r = -0.42$ leads to $R_{00}(K\pi) \approx 0.05$, well below the acceptable limits. The effect of FSI (see Fig. 5) is to boost $R_{00}(K\pi)$ *above* the acceptable limits.

4. Summary

We started this paper with a study of the three decay modes $D^0 \to \overline{K}{}^0\pi^0$, $D^0 \to K^-\pi^+$ and $D^+ \to \overline{K}{}^0\pi^+$ *without FSI.* We found that *real* decay amplitudes almost fail to yield a value of $r \equiv A_3/A_1$ which would simultaneously fit $R_{00}(K\pi)$ and $R_{0+}(K\pi)$. With only a slightly more generous error in $R_{00}(K\pi)$ and/or $R_{0+}(K\pi)$ a solution with $r = -0.2$ could be found. This value of r was then used in the hybrid model in computing branching ratios without FSI. It could very well be that with better statistics data no solution to a simultaneous fit to $R_{00}(K\pi)$ and $R_{0+}(K\pi)$ will be found with *real* amplitudes and one would have to use complex amplitudes.

Without FSI and with $r = -0.20$ we obtain

$$\frac{\Gamma(D^0 \to \overline{K}{}^0\eta)}{\Gamma(D^0 \to \overline{K}{}^0\pi^0)} = 0.13, \quad \text{for } f_+ = 0.69,\ f_- = 2.09,\ g = 3.95 \tag{67}$$

$$= 0.46, \quad \text{for } f_+ = f_- = 1,\ g = 7.75 \tag{68}$$

$$\frac{\Gamma(D^0 \to \overline{K}^0\phi)}{\Gamma(D^0 \to \overline{K}^0\rho^0)} = 0.22, \quad \text{for } f_+ = 0.69,\ f_- = 2.09,\ g = 3.95 \tag{69}$$

$$= 0.40, \quad \text{for } f_+ = f_- = 1,\ g = 7.75 \tag{70}$$

$$\frac{\Gamma(D^0 \to \overline{K}^0\omega)}{\Gamma(D^0 \to \overline{K}^0\rho^0)} = 0.48, \quad \text{for } f_+ = 0.69,\ f_- = 2.09,\ g = 3.95 \tag{71}$$

$$= 1.68, \quad \text{for } f_+ = f_- = 1,\ g = 7.75\ . \tag{72}$$

Once FSI is included we found that a fit to $R_{00}(K\pi)$ and $R_{0+}(K\pi)$ could be found with $r \approx 0.23$ provided that the phase difference $\delta_1 - \delta_3 \lesssim 150°$. With the inclusion of FSI and with $r = 0.23$, we predict

$$\frac{\Gamma(D^0 \to \overline{K}^0\eta)}{\Gamma(D^0 \to \overline{K}^0\pi^0)} = 0.88, \quad \text{for } f_+ = 0.69,\ f_- = 2.09,$$

$$g = -8.21,\ \delta_1 - \delta_3 = 120° \tag{73}$$

$$= 0.40, \quad \text{for } f_+ = f_- = 1,\ g = -9.86,\ \delta_1 - \delta_3 = 120° \tag{74}$$

$$\frac{\Gamma(D^0 \to \overline{K}^0\phi)}{\Gamma(D^0 \to \overline{K}^0\rho^0)} = 0.8, \quad \text{for } f_+ = 0.69,\ f_- = 2.09,$$

$$g = -8.21,\ \phi_1 - \phi_3 = 120° \tag{75}$$

$$= 0.56, \quad \text{for } f_+ = f_- = 1,\ g = -9.86,\ \phi_1 - \phi_3 = 120° \tag{76}$$

$$\frac{\Gamma(D^0 \to \overline{K}^0\omega)}{\Gamma(D^0 \to \overline{K}^0\rho^0)} = 3.2, \quad \text{for } f_+ = 0.69,\ f_- = 2.09,$$

$$g = -8.21,\ \phi_1 - \phi_3 = 120° \tag{77}$$

$$= 1.5, \quad \text{for } f_+ = f_- = 1,\ g = -9.86,\ \phi_1 - \phi_3 = 120°\ . \tag{78}$$

The ratio $\Gamma(D^0 \to \overline{K}^0\eta)/\Gamma(D^0 \to \overline{K}^0\eta')$ in our hybrid model, unaffected by FSI, is 0.68 which is determined only by the Clebsch-Gordan coefficients and the phase space.

The three rates $\Gamma(D^0 \to \overline{K}^0\pi^0)$, $\Gamma(D^0 \to K^-\pi^+)$ and $\Gamma(D^+ \to \overline{K}^0\pi^+)$ with better statistics will prove very useful in the amplitude analysis as *real* amplitudes are hard put to satisfy data already.

Finally we don't expect color suppression to occur for $\overline{K}^0\rho^0$ and $\overline{K}^{*0}\pi^0$ modes as it indeed does not occur for $\overline{K}^0\pi^0$.

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FIGURE CAPTIONS

1. $R_{0+}(K\pi)$ versus r, plotted for $\delta_1 - \delta_3 = 0°$ and $150°$. Shaded area shows the allowed region defined by Eq. 8. Curves for $\delta_1 - \delta_3 = 120°$ and $180°$ are not significantly different from that for $150°$.

2. $R_{00}(K\pi)$ versus r, plotted for $\delta_1 - \delta_3 = 0°$, $120°$, $150°$, and $180°$. Shaded area shows the allowed region defined by Eq. 7. For $\delta_1 - \delta_3 = 0°$, R_{00} stays above 2 for all positive values of r not plotted.

3. The spectator (a) and the color suppressed (b) graphs.

4. The exchange graph.

5. $R_{00}(K\pi)$ versus r, plotted for $\delta_1 - \delta_3 = 0°$, $120°$, $150°$ and $180°$ over a wider range than in Fig. 2.

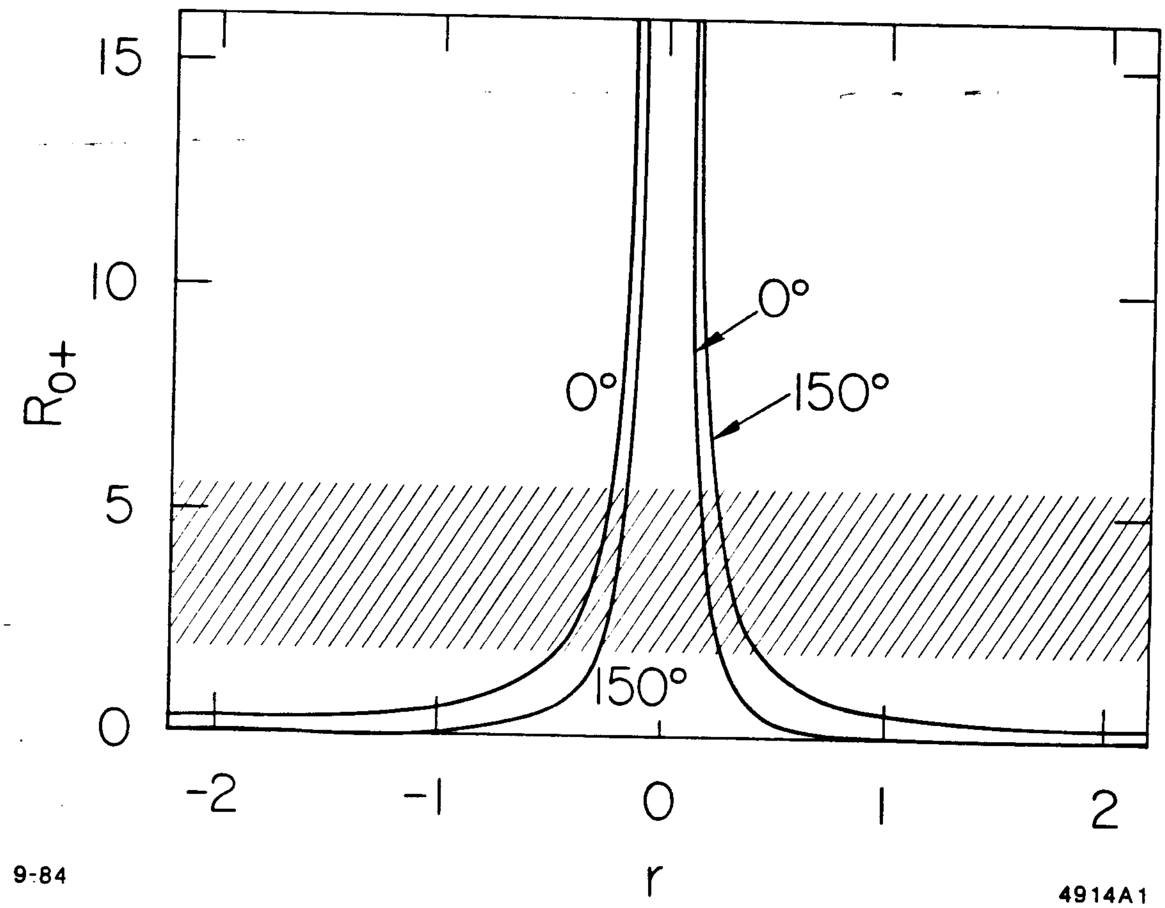


Fig. 1

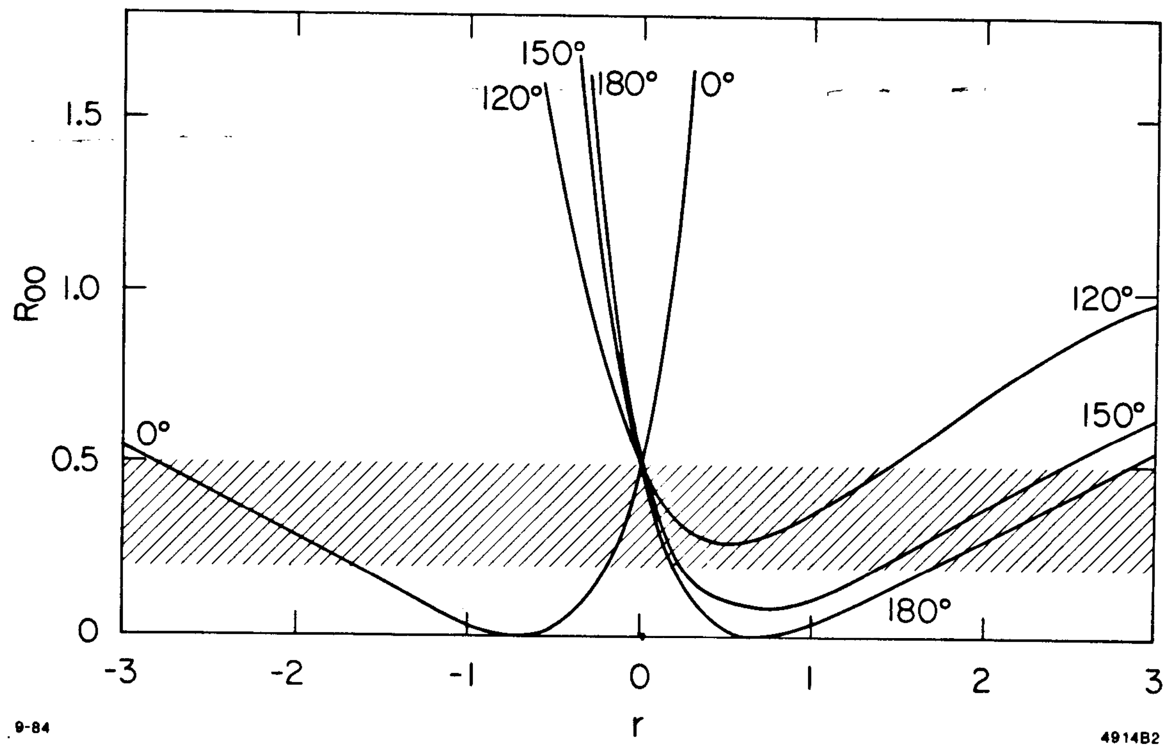
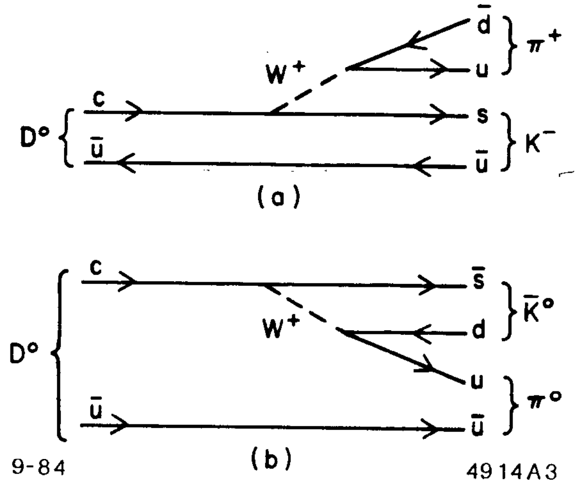


Fig. 2



d̄
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(a)
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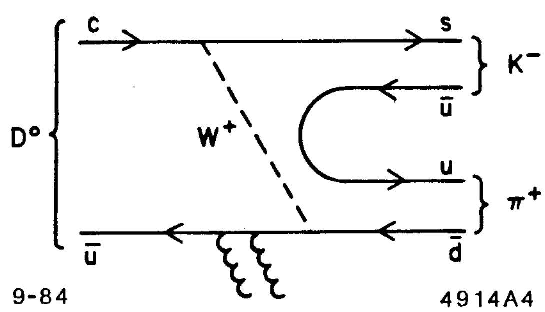
9-84
(b)
4914A3

Fig. 3



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4914A4

Fig. 4

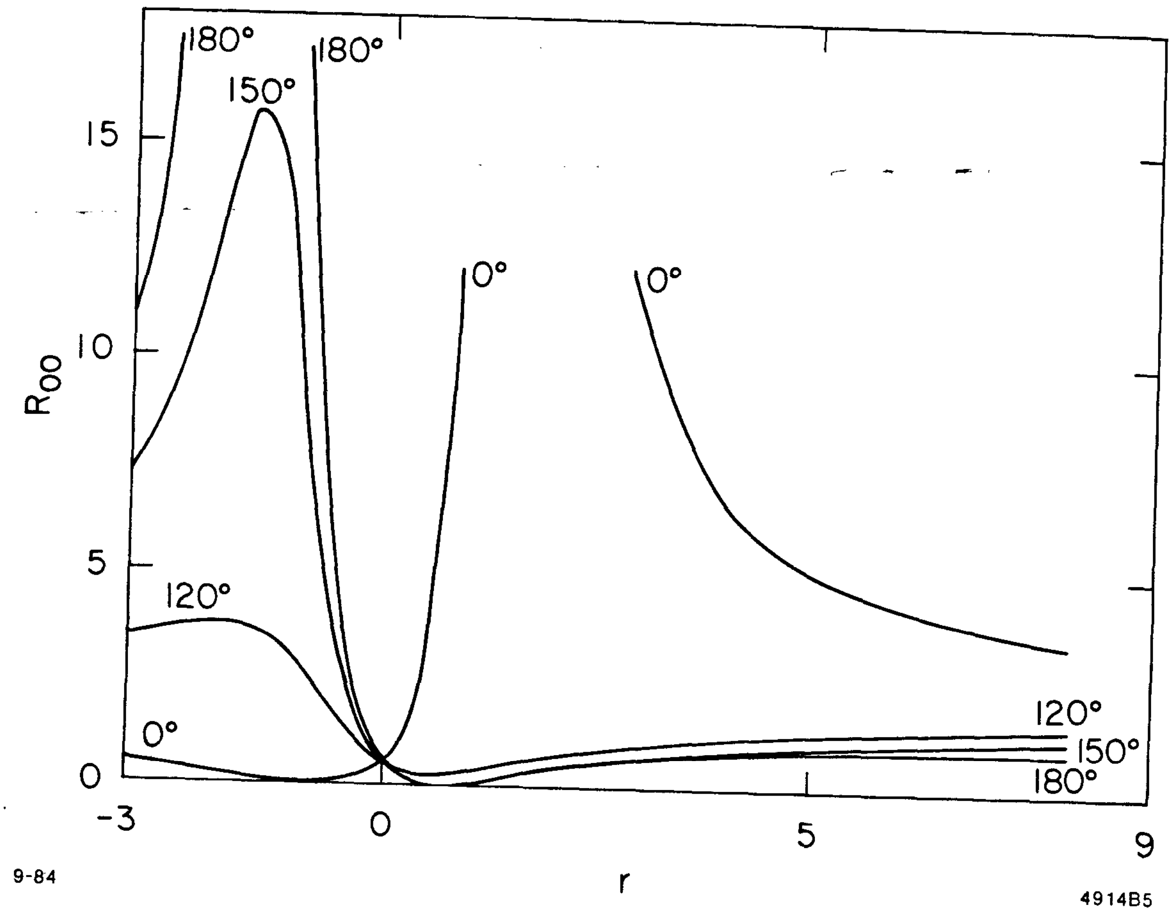

Fig. 5