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A REALISTIC THEORY OF FAMILY UNIFICATION*

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ABSTRACT

We develop an O(18) theory of family unification consistent with all established phenomenology and cosmology. The theory makes firm predictions that will be tested soon. It implies that five new families live below 265 GeV. One of the new families is left-handed, and the other four are right-handed. The new left-handed family is lighter than its right-handed counterparts. The lightest right-handed quark should have a mass of less than 130 GeV. All the charged leptons should be lighter than 55 GeV; the lightest should be less than 40 GeV. The five new neutrinos have Dirac masses of less than 40 GeV, so a total of eight doublet neutrinos contribute to the width of the Z^0. We study the decays of new families, and show that proton decay into $e^+ \pi^0$ proceeds at an observable but acceptable rate, with a lifetime on the order of $10^{32\pm1}$ years.

1. Introduction

Despite its stunning success, the standard SU(3) $\times$ SU(2) $\times$ U(1) model is not a fundamental theory of the strong, weak and electromagnetic interactions. It leaves open many important questions. The standard model does not explain why there are three forces, nor why the weak interactions are purely $V - A$. It does not tell us why quarks and leptons come in families, nor why the families repeat.

Conventional grand unified theories begin to answer these questions.[1] They unify the known forces by collecting the SU(3) $\times$ SU(2) $\times$ U(1) gauge bosons into a single irreducible representation of a simple group G. The quarks and leptons, however, are not treated so well. Several irreducible representations are required to account for the known families of quarks and leptons. Conventional grand unified theories unify forces – but not families. They do not tell us who ordered the muon.

Family unified theories provide a natural answer to these questions.[2–4] In family unified theories, the forces *and* the families are both incorporated into irreducible representations of the underlying gauge group G. The most appealing theories of family unification are based on the group O(18). There are many reasons for this. In an O(18) theory, all the known families are incorporated into just one representation, the 256-dimensional spinor. This spinor is complex, so superheavy masses for ordinary fermions are forbidden. Furthermore, the group O(18) is anomaly-free, so the anomalies of each representation cancel among themselves.

Previous attempts to construct theories based on O(18) were plagued by serious difficulties.[2] These stem from the fact that the 256-dimensional spinor contains too many families. This may be easily seen by decomposing the 256 under O(10) $\times$ O(8), $256 \rightarrow (16, 8') + (\overline{16}, 8'')$. Here O(10) is the usual grand unification group, and O(8) is a horizontal family symmetry. Under O(10) $\times$ O(8), the 256 contains eight left- and eight right-handed families. With 16 light families, the color coupling blows up at a few hundred TeV. These theories are not perturbatively unifiable.

To avoid this problem, it is necessary to split the O(18) spinor and give some families mass at the grand unification scale M_{GUT}. In Reference [3] we have shown how to do this, and we review the mechanism in Section 2 of this paper. We find that four left- and four right-handed families survive down to the weak scale M_W. The extra families give dramatic experimental signatures, so O(18) will be tested soon.

In this paper we systematically study the low-energy phenomenology of our O(18) theory. Our most important results include upper bounds on the masses and mixings of the right-handed families. We shall see that there must be one right-handed quark below 130 GeV, and one right-handed charged lepton below 40 GeV. Furthermore, we shall show that eight neutrinos must contribute to the width of the Z^0. These new particles are all within the range of accelerators such as the Sp$\bar{\text{p}}$S collider, the Tevatron, the SLC, LEP and the SSC.

The plan of this paper is as follows. In Section 2 we review the model of Reference [3]. We show how to split the heavy families from the light families, and how to split the left-handed families from their right-handed counterparts. In Section 3 we analyze the constraints that arise from cosmology and low-energy phenomenology. We find that proton decay experiments limit us to one or two Higgs doublets. We explain the neutrino counting, investigate the decays of right-handed matter, and discuss left-handed Kobayashi-Maskawa mixing. In Section 4 we treat the one Higgs case. We find bounds on the masses and mixings of the heavy families, and show that the one Higgs case gives eight ultralight neutrinos. In Section 5 we analyze the two Higgs case. We present a model with three ultralight neutrinos, realistic Kobayashi-Maskawa mixing, and no stable right-handed matter. We place bounds on the right-handed masses, and show why the two-Higgs model is the preferred version of the theory. (Some technical details are collected in the appendices.) In Section 6 we discuss some experimental signatures of the two-Higgs model. We examine proton decay channels, flavor violating processes and the decays of the new families. We conclude in Section 7 with a summary of our most important results.

2. Splitting the Spinor

In the previous section we have seen that the 256-dimensional spinor of O(18) contains eight left- and eight right-handed families, $256 \rightarrow (16,8') + (\overline{16},8'')$. Because of perturbative unification, not all these families can survive down to low energies. As least half of the families must join together and gain mass at the grand unification scale M_{GUT}.

In Reference [3] we have shown how to split the O(18) spinor. We classified all continuous symmetries $H \subseteq O(8)$ under which the 256 contains a complex representation of $SU(3) \times SU(2) \times U(1) \times H$. Fermions in real representations of $SU(3) \times SU(2) \times U(1) \times H$ gain masses of order M_{GUT}, while fermions in complex representations remain massless down to the weak scale M_W.[5]

As shown in Reference [3], only a few symmetries H protect some families from acquiring mass at the grand unification scale. The requirement of perturbative unification restricts the choice still further. It limits H to one of two possible symmetries. Both of these symmetries are abelian, and both lead to four left- and four right-handed families in the low-energy world. The $U(1)_H$ charges of various O(8) representations are shown in Table 1.

Since the family symmetry H is a gauge symmetry, it cannot be preserved all the way down to the weak scale M_W. Otherwise, there would be flavor-changing neutral currents in conflict with experiment. The symmetry H must be broken at a scale greater than 10^5 GeV. To take this into account, the family $U(1)_H$ is broken to a Z_N subgroup at the grand unification scale M_{GUT}. Of course, the choice of N must be such that the Z_N symmetry still protects the light left- and right-handed families from joining together and acquiring mass.

The situation is summarized in Figure 1. The group O(18) breaks to O(10) $\times$ O(8) at a scale $\langle\omega\rangle$, perhaps of order the Planck mass M_P. The family group O(8) then breaks to Z_N at a scale $\langle\chi\rangle$ of order M_{GUT}. The flavor symmetry O(10) breaks to SU(3) $\times$ SU(2) $\times$ U(1) at a similar scale $\langle\psi\rangle$. Four left- and four right-handed families survive down to the weak scale M_W.

O(18) predicts both left- and right-handed families in the low-energy world, so we must explain why the right-handed families are heavier than their left-handed counterparts. This is easy to do via O(8) group theory. The crucial point is that the left- and right-handed families transform under *different* representations of O(8). The O(8) multiplication laws ensure that if all Weinberg-Salam Higgs doublets are contained in the $35''$ of O(8), they couple *only* to the right-handed families. The right-handed families receive direct masses at the weak scale M_W, while the masses for the left-handed families are generated by one-loop radiative corrections (see Figures 2 and 3).

3. Phenomenological and Cosmological Constraints

The O(18) theory discussed above has four left- and four right-handed families at the weak scale M_W. The families are protected from acquiring superheavy masses by a discrete Z_N family symmetry. The possible choices for N – and the family charges themselves – are strongly constrained by cosmology and low-energy phenomenology. In this section we discuss the constraints that arise from proton decay, the neutrino spectrum, the decays of right-handed matter, and the left-handed Kobayashi-Maskawa matrix.

3.1 Proton Lifetime

Limits on the proton lifetime exclude the minimal SU(5) grand unified theory. They exclude many other theories as well. O(18) escapes this fate because it has eight families that survive down to low energies. For eight families, and only for eight families, the color beta function is dominated by its two-loop contribution.

The two-loop contribution increases the unification mass from its minimal SU(5) value. Of course, with eight families α_{GUT} also increases from its standard SU(5) value. The increase in M_{GUT} offsets the increase in α_{GUT} and prolongs the proton lifetime.

The O(18) proton lifetime was calculated in Reference [6] for the case of one Higgs doublet. Additional Higgs doublets decrease the proton lifetime because they increase the speed with which the SU(2) coupling approaches the SU(3) coupling. We have repeated the analysis of Reference [6] for an arbitrary number of Higgs doublets. The pertinent results are collected in Table 2.

The most important thing to note about Table 2 is the last column, where we have tabulated the O(18) partial lifetime* $\tau(p \rightarrow e^+ \pi^0)$, relative to the SU(5) case.[7] To be in accord with experiment, τ/τ_5 must be greater than 200. We see that the one- and two-Higgs models give acceptable proton lifetimes for $\Lambda_{\overline{MS}}$ greater than 100 and 150 MeV, respectively. These models lead to lifetimes on the order of $10^{32\pm1}$ years. Since experiments now measure[8] $\tau(p \rightarrow e^+ \pi^0) \gtrsim 2 \times 10^{32}$ years, O(18) predicts that proton decay could soon be seen.

The results of Table 2 exclude models with three or more Higgs doublets that have decays predominantly through the $\pi^0 e^+$ channel – they give too small values of τ/τ_5 and $\sin^2\theta_W$. Since Higgs triplets count as four doublets, models with triplets are excluded as well. Proton decay experiments suggest that the low-energy Higgs content of our O(18) theory include just one or two weak doublets, and an arbitrary number of gauge singlets.

3.2 Neutrino Counting

In Sections 4 and 5 we will see that the low-energy neutrino spectrum provides a crucial test of our O(18) theory. In preparation for these sections, we review here the general features of O(18) neutrino counting.

* Proton decay channels are discussed in Section 6.

The O(18) spinor contains 32 neutrinos. Sixteen are in superheavy generations. As before, they get $O(10) \times Z_N$ -invariant masses and disappear from the low-energy spectrum. Eight of the remaining neutrinos are $SU(2) \times U(1)$ singlets, and eight are doublets. Ultralight neutrino masses arise by the Gell-Mann–Ramond–Slansky–Yanagida mechanism.[9] Every ultraheavy singlet-singlet mass gives an ultralight neutrino doublet with mass of order $M_W^2/M_{GUT} \gtrsim 0.1$ eV.

The singlet-singlet mass matrix determines the number of ultralight neutrinos. This matrix depends crucially on the Z_N family charges introduced in Reference [3]. In view of the importance of the low-energy neutrino spectrum, we extend the Z_N family symmetry to a new Z_N' , where

$$Z_N' \subset U(1)_{H'} = U(1)_H + f\,U(1)_{B-L} \,. \tag{3.1}$$

The discrete symmetry Z_N' is contained in a U(1) symmetry H', generated by a linear combination of H and the $B-L$ generator of O(10).† The new Z_N' symmetry does not change the superheavy left-right mass spectrum because $B-L$ acts oppositely on left- and right-handed families.

The singlet-singlet neutrino masses depend on the value of f. When $f=0$, the Z_N' charges reduce to those in Table 1, modulo N. These charges come in opposite pairs, so all eight singlet neutrinos acquire superheavy masses. In this case, the low-energy spectrum contains eight ultralight isodoublet neutrinos.

Models with eight ultralight neutrinos are in conflict with the simplest version of big bang nucleosynthesis. The abundance of primordial helium and deuterium increases with the number of light neutrinos. The situation is summarized in Figure 4, where we have graphed the primordial helium and deuterium abundances, $X(^4He)$ and $X(^2H)$, for $N_\nu = 2,\ 4$ or 8 ultralight neutrinos.[10]

From Figure 4 we see that observational evidence suggests $N_\nu \lesssim 4$. This constraint, however, can be evaded. For example, the primordial helium abundance

† $B-L$ denotes the generator of O(10) that commutes with SU(5).

might be as large as 0.30. Since the number of neutrinos will soon be measured in Z^0 decays, we do not wish to exclude the possibility that $N_\nu = 8$.

If we take the standard nucleosynthesis constraints seriously, however, we are permitted at most four ultralight neutrinos. The other four, if stable, must have masses greater than 2 GeV.[11] This implies that $f \neq 0$ in equation (3.1). In Sections 4 and 5 we will see that low-energy phenomenology strongly constrains the choices for f. We will be led to consider models with two Higgs doublets and three ultralight neutrinos.*

3.3 The Stability of Right-Handed Matter

There are strong observational limits on the relative abundance of right-handed matter in the universe today.[12] In this section we use these limits to place additional constraints on our O(18) theory.

The heaviest right-handed families are not affected by the cosmological bounds because they decay rapidly into their lighter right-handed counterparts. These decays occur via the ordinary weak interactions with lifetimes between 10^{-16} and 10^{-21} seconds. The lighter right-handed particles F decay into ordinary matter by induced operators of various dimensions d. The effective Lagrangians are of the form

$$\mathcal{L}_{\text{eff}} \simeq \frac{1}{M_{GUT}^{d-4}} \, F f_1 f_2 \, ... \, , \tag{3.2}$$

where f_1 and f_2 are ordinary light particles. The lifetimes for such decays are estimated to be

$$\tau_F \simeq \frac{(M_{GUT}/10^{15}\text{ GeV})^{2(d-4)}}{(M_F/10^2\text{ GeV})^{2(d-4)+1}} \, 10^{26(d-5)} \text{ seconds} \, . \tag{3.3}$$

Dimension-four and dimension-five operators lead to relatively short lifetimes. Operators of higher dimensions give lifetimes longer than the age of the universe.

* Another way to raise the mass of some of the neutrinos is to introduce an isotriplet of Higgs scalars. This possibility is ruled out by proton decay experiments.

If right-handed families cannot decay via operators of dimension four or five, one must face the prospect of essentially stable right-handed fermions.

If such particles existed in the early universe, would they be seen today? If the excess of right-handed matter over antimatter is comparable to that of ordinary matter – as in models with right-handed CP-violation – the answer is yes. Right-handed matter would be as common as ordinary matter and would certainly have been observed.

Even if there is no CP-violation in the right-handed sector, some right-handed matter would still survive by having failed to annihilate after decoupling. The relative right versus left quark number density can be estimated to be of order[12]

$$\frac{N_Q}{N_q} \simeq \frac{N_{\bar{Q}}}{N_q} \simeq 10^{-10} , \tag{3.4}$$

where Q is the lightest right-handed quark with mass of approximately 100 GeV. This number is small, but it is not small enough to have escaped detection.

Stable right-handed fermions give rise to heavy nuclei composed of left- and right-handed quarks. For example, the lightest right-handed quark Q can combine with ordinary up and down quarks to form mesons M and baryons B. The lightest hadrons with one right-handed quark include the isodoublet mesons and isosinglet baryons listed in Table 3.‡ These stable heavy hadrons combine with ordinary hadrons to form hydrogenic nuclei, such as heavy hydrogen and heavy deuterium, as shown in Table 4. Experimental searches in water limit the relative abundance of heavy hydrogen[13] to be less than 10^{-30}.

If more than one in 10^{20} heavy quarks gives rise to a hydrogenic nucleus, this bound is violated. Of course, not all quarks form hydrogenic nuclei – some are processed into higher-Z nuclei. The burning of heavy mesons M into higher

‡ The isosinglet baryon should be lighter than its isotriplet partners because of the color magnetic forces that account for the lower mass of the isosinglet Λ compared to the isotriplet Σ.

nuclei is analogous to the burning of neutrons. About one in 10^5 neutrons fails to participate in nucleosynthesis,[14] so we expect one out of every 10^5 mesons M^+ to form a heavy hydrogenic nucleus. The case of the heavy baryons B^+ is even more clear. The baryons B carry no isospin, so their interactions with nucleons are suppressed relative to those of the mesons M. Their efficiency for burning into high-Z nuclei is smaller than that of the mesons M. Therefore at least one out of every 10^5 M^+ and B^+ escapes being processed into a higher-Z nucleus. It remains as a hydrogenic nucleus, and violates the observational bounds by 15 orders of magnitude. The abundance of composite hydrogenic nuclei – such as B^0p and M^0p – is expected to exceed the observational bound[13] by several orders of magnitude.

Since late annihilation of heavy hadrons cannot significantly reduce their number density, we conclude that the abundance of heavy hydrogenic nuclei cannot be suppressed to one part in 10^{20}. The bounds on heavy hydrogen imply that right-handed quarks must decay by operators of dimension five or less. In Sections 4 and 5 we will see that the heavy hydrogen limits constrain the Z_N' charges of our O(18) theory. We must make sure that no stable right-handed matter remains today.

3.4 Left-Handed Mixing

The Z_N' family symmetry discussed above also leads to serious constraints on the interactions of the left-handed families. The Z_N' symmetry must not prevent Kobayashi-Maskawa mixings of the left-handed quarks.[15] One must ensure that the left-handed Kobayashi-Maskawa matrix allows $s - u$ and $b - c$ transitions in the charged weak current. Models with two Higgs doublets must also satisfy strong constraints from flavor-changing neutral currents.

In the next two sections we will study the phenomenology of the one- and two-Higgs doublet models. We will see that the combined restrictions of proton decay, no heavy hydrogen and sufficient left-handed mixing place strong constraints on our O(18) theory.

4. One Higgs Doublet

4.1 General Considerations

In the previous section we found that proton decay experiments restrict O(18) theories to contain just one or two Higgs doublets. In this section we study the case of one Higgs doublet. To simplify the discussion that follows, we redefine the left-handed H charges of Table 1 to be $\pm c$ and $\pm d$. This changes the right-handed H charges to be $\pm\frac{1}{5}(3c-4d)$ and $\pm\frac{1}{5}(4c+3d)$, for either U(1) symmetry.

The phenomenological issues discussed in Section 3 give strong restrictions in the one-Higgs case. The most important constraint comes from the Kobayashi-Maskawa matrix. If there is to be sufficient left-handed mixing, the Z'_N charges of the left-handed quark doublets must all be the same. In the previous section we defined the Z'_N charges to be given by a discrete subgroup of a U(1) symmetry H' generated by a linear combination of H and the $B-L$ generator of O(10),

$$\mathrm{U}(1)_{H'} \;=\; \mathrm{U}(1)_H \;+\; f\,\mathrm{U}(1)_{B-L}\,. \tag{4.1}$$

If the left-handed Z'_N charges are to be the same, the left-handed H charges must also be the same. This implies that $2c = 2d = 0$, modulo N.

The fact that $2c = 2d = 0$ (mod N) severely restricts the pattern of right-handed Kobayashi-Maskawa mixing. It is not hard to show that this condition implies that all right-handed families must have *different* H charges if light families are to be protected from acquiring mass at the grand unification scale M_{GUT}. Since the right-handed families have different H charges, there is no Cabibbo mixing and no CP-violation in the right-handed sector.

Further constraints on the one-Higgs case come from the neutrino spectrum. The fact that ν_e, ν_μ and ν_τ are so light implies that the left-handed singlet-singlet neutrino mass matrix must have at least three nonzero eigenvalues. This leads to additional restrictions on the Z'_N charges listed in Table 5. Since $2c = 2d = 0$

(mod N), the left-handed singlet-singlet mass matrix has nonzero eigenvalues if and only if $10f = 0$ (mod N). When $10f = 0$ (mod N), the left-handed singlet-singlet matrix has four nonzero eigenvalues. In this case it is easy to see that the corresponding right-handed matrix also has four nonzero eigenvalues. With a total of eight nonzero eigenvalues, the Gell-Mann–Ramond–Slansky–Yanagida mechanism yields eight ultralight neutrinos. In the case of one Higgs doublet, we are left with eight neutrinos of mass $\gtrsim 0.1$ eV, in apparent conflict with the simplest version of big bang nucleosynthesis.

As discussed in the previous section, one must also consider the stability of right-handed matter. With only one Higgs doublet, the only possible dimension-five operators are of the form

$$\bar{Q}\, q\, (\phi^\dagger \phi)\ , \qquad \bar{L}\, \ell\, (\phi^\dagger \phi)\ , \tag{4.2}$$

where ϕ is the Weinberg-Salam Higgs doublet. These operators are excluded by the Z_N' family symmetry. Therefore the only way right-handed matter can decay is through Higgs singlets. These decays can occur either through dimension-four or dimension-five operators.

The combined constraints of realistic Kobayashi-Maskawa mixing, no heavy hydrogen and at least three light neutrinos severely restrict the one-doublet model. We find that the left-handed Z_N' charges must all be equal, and that $2c = 2d = 0$ (mod N). We discover that there is no Cabibbo mixing and no CP-violation in the right-handed sector. We also find that there must be eight ultralight neutrinos and extra Higgs singlets.

4.2 Right-Handed Masses

As explained in Section 2, the Weinberg-Salam Higgs doublet ϕ couples directly to the right-handed quarks and leptons through Yukawa matrices Y. O(18) radiative corrections induce couplings to the left-handed families, as shown in Figure 3. If the left-handed masses are to agree with experiment, the direct Yukawa couplings in the matrix Y must be of order one.

The right-handed quark and lepton Yukawa matrices are specified at the unification scale M_{GUT}. The physical right-handed masses are obtained by evolving the Yukawa couplings to low energies via the SU(3) $\times$ SU(2) $\times$ U(1) renormalization group equations.

The SU(3) $\times$ SU(2) $\times$ U(1) renormalization group equations are given in Reference [16]. They split naturally into two pieces. The first accounts for the renormalization of the Yukawa couplings by the gauge bosons. It tends to increase the Yukawas at low energies. The second describes the renormalization of the Yukawas by the Higgs scalars. It tends to decrease the Yukawas at low energies. For Yukawas of order one, the two contributions compete and lead to a fixed-point behavior. The essential features of the right-handed mass spectrum are given by the infrared fixed points of the SU(3) $\times$ SU(2) $\times$ U(1) renormalization group equations.[17,18] They do not depend on the details of the O(18) Yukawa couplings at the unification scale.

The renormalization group analysis for an arbitrary number of families and one Higgs doublet was discussed in Reference [18]. Assuming perturbative unification and a SU(3) $\times$ SU(2) $\times$ U(1) desert, it was shown that the quark and lepton masses obey strict bounds,

$$
\begin{aligned}
\sum_Q {M_Q}^2 &\lesssim (350 \text{ GeV})^2 \\
\sum_E {M_E}^2 &\lesssim (330 \text{ GeV})^2 ,
\end{aligned}
\qquad (4.3)
$$

as a consequence of the infrared fixed-point behavior. In our O(18) theory, we will see that the sum over all heavy quarks saturates (4.3), while the lepton bound is far from being reached.

To improve our knowledge of the right-handed mass spectrum, let us examine the right-handed couplings in more detail. The first thing to note is that Fermi statistics require the O(18) Higgs representation to be symmetric. Since the $35''$ of O(8) is symmetric, the O(10) Higgs representation must be symmetric as well. This limits ϕ to lie in either the 10 or the 126 of O(10).

The phenomenological constraints discussed above imply that the Weinberg-Salam Higgs doublet has H charge zero. The H charges of the right-handed families are $\pm\frac{1}{5}(3c-4d)$ and $\pm\frac{1}{5}(4c+3d)$, where $2c = 2d = 0 \pmod N$ and the values of the charges are all different. They yield right-handed up, down, electron and neutrino matrices of the following form:

$$Y \simeq \begin{pmatrix} 0 & x & 0 & 0 \\ x & 0 & 0 & 0 \\ 0 & 0 & 0 & y \\ 0 & 0 & y & 0 \end{pmatrix} . \tag{4.4}$$

The Yukawa matrices are symmetric,* so the right-handed masses come in degenerate pairs. Note that the Yukawa matrices are very sparse. There is no Cabibbo mixing and no CP-violation in the right-handed sector.

The fact that the Weinberg-Salam Higgs lies in either a 10 or a 126 of O(10) gives further restrictions on the right-handed Yukawa matrices. If the Higgs lies in the 10, then the up, down, electron and neutrino matrices U, D, E and N are related as follows:

$$U = N , \quad D = E . \tag{4.5}$$

* In principle, x and y are related by O(18) Clebsch-Gordan coefficients. Since the low-energy right-handed masses are determined by fixed point conditions, we shall ignore the O(18) relations between x and y.

If the Higgs lies in the 126, the relations are

$$U = -\frac{1}{3}N, \quad D = -\frac{1}{3}E. \tag{4.6}$$

The right-handed Yukawa matrices are completely determined by four input parameters.

Because of the fixed-point structure of the renormalization group equations, the low-energy quark and lepton masses are essentially independent of the Yukawa couplings at the unification scale. To see this more clearly, we perform a Monte Carlo with different sets of initial conditions at the unification scale M_{GUT}. We throw uniform distributions for the up and down Yukawas, with values chosen randomly from the interval 0.5 – 10, and compute the electron and neutrino Yukawas from (4.5) or (4.6). We then evolve all Yukawas down to the weak scale M_W using the SU(3) $\times$ SU(2) $\times$ U(1) renormalization group equations. In Figures 5 - 7 we collect the low-energy masses for 1000 sets of initial conditions. The resulting histograms exhibit the regularities of the right-handed spectra.

The low-energy up-type quark masses are collected in Figure 5*a* for the Weinberg-Salam Higgs in the 126 of O(10). The down distributions are essentially identical, as are the quark distributions arising from the 10. The peaking of the distribution shows the fixed point behavior. Note that the right-handed quarks all lie below 190 GeV. Although the quark distribution is relatively peaked, it is not sharp enough to predict individual quark masses. However, in Reference [18] we have shown that the *sum* of the *squares* of the quark masses should be sharply peaked. These sums are collected in Figure 5*b*. We see that the quark bound (4.3) is saturated in the right-handed sector,

$$\sum_Q {M_Q}^2 \simeq (350\ \text{GeV})^2. \tag{4.7}$$

This implies that the lightest right-handed quark must lie below 125 GeV.

The charged-lepton distributions are shown in Figures 6 and 7 for ϕ in the 10 and the 126, respectively. Experiments at PETRA exclude $\phi \in 10$, as shown in Figure 6, but they do not exclude $\phi \in 126$, as illustrated in Figure 7*a*. For $\phi \in 126$, the charged leptons have masses less than 120 GeV. In Figure 7*b* we collect the sums of the squares of the charged lepton masses. We see that the charged lepton bound (4.3) is far from being saturated. A better limit is given by

$$\sum_E {M_E}^2 \lesssim (105 \text{ GeV})^2 . \tag{4.8}$$

The bound (4.8) implies that at least one right-handed charged lepton must live below 55 GeV.

Thus we have seen that the masses of the right-handed families are strongly constrained by the fixed points of the SU(3) $\times$ SU(2) $\times$ U(1) renormalization group equations. We find that the Weinberg-Salam Higgs doublet must lie in the (126, 35$''$) of O(10) $\times$ O(8). We also learn that the sums of the squares of the right-handed quark and charged lepton masses obey

$$\begin{aligned} \sum_Q {M_Q}^2 &\simeq (350 \text{ GeV})^2 \\ \sum_E {M_E}^2 &\lesssim (105 \text{ GeV})^2 . \end{aligned} \tag{4.9}$$

These relations imply that one right-handed quark must live below 125 GeV, and one right-handed charged lepton must lie below 55 GeV. The fourth left-handed family is even lighter.

5. Two Higgs Doublets

5.1 General Considerations

In the previous section we showed that full left-handed Kobayashi-Maskawa mixing leads to eight ultralight neutrinos in the one-Higgs case. In this section we introduce a second Higgs doublet. This allows us to reconcile the Kobayashi-Maskawa matrix with standard big bang nucleosynthesis. However, it also opens the possibility that physical scalars might mediate flavor-changing neutral currents. This is very dangerous in light of the stringent experimental limits on $K^0 - \bar{K}^0$ mixing.

One way to avoid problems with flavor-changing neutral currents is to ensure that one Higgs couples primarily to up-type quarks, and that the other Higgs couples primarily to down-type quarks. This is called the Glashow-Weinberg-Paschos mechanism;[19] it leads to an approximate Peccei-Quinn symmetry[20] in the Yukawa sector,

$$\mathcal{L}_Y \;\simeq\; (\phi_u \;+\; \epsilon\phi_d^*)\,\bar{u}\,q \;+\; (\phi_d \;+\; \epsilon'\phi_u^*)\,\bar{d}\,q \;+\; \ldots\;, \tag{5.1}$$

where $\epsilon, \epsilon' \ll 1$. The approximate Peccei-Quinn symmetry guarantees that flavor-changing neutral currents are suppressed by powers of ϵ or ϵ'.

The preceeding scenario arises naturally in our O(18) theory, provided both Higgs doublets lie in the (126, 35″) of O(10) $\times$ O(8). The 126-dimensional representation of O(10) contains two Higgs doublets that can be identified by their SU(5) quantum numbers, $\overline{5}_H$ and 45_H. If left-handed masses are generated by effective operators of the form $16\times16\times\langle126^*\rangle$, the Glashow-Weinberg-Paschos mechanism automatically occurs. This is easy to see by decomposing the effective operators under SU(5),

$$16\times16\times\langle126^*\rangle \;\supset\; 10\times10\times\langle\overline{5}_H^*\rangle \;+\; \overline{5}\times10\times\langle45_H^*\rangle\;, \tag{5.2}$$

where $\overline{5}$ and 10 are the SU(5) fermion representations. As we see in (5.2), the Higgs doublet contained in the $\overline{5}_H$ couples only to up-type quarks. It should be identified with ϕ_u^*. The doublet in the 45_H couples only to down-type quarks; it should be identified with ϕ_d^*. The Glashow-Weinberg-Paschos mechanism is automatic provided all fermion masses arise from the operators (5.2). The same holds true if the masses are generated by higher-dimensional operators of the form

$$\frac{1}{M^{n+m+k}} \; 16 \times 16 \times \langle 126^* \rangle \times \langle 1 \rangle^n \times \langle 45 \rangle^m \times \langle 210 \rangle^k \; \ldots \tag{5.3}$$

For all such operators, the $\overline{5}_H$ contains ϕ_u^*, and the 45_H contains ϕ_d^*.

The O(18) graph of Figure 3*a* induces an effective operator of precisely the form (5.3). If Figure 3*a* gives the dominant contribution to the left-handed masses, the Glashow-Weinberg-Paschos mechanism naturally suppresses flavor-changing neutral currents.

Violations of the Glashow-Weinberg-Paschos mechanism arise from operators of the form

$$\frac{1}{M^{m+k}} \; 16 \times 16 \times \langle 126 \rangle \times \langle 45 \rangle^m \times \langle 210 \rangle^k \; \ldots \tag{5.4}$$

These operators couple the 45_H to up-type quarks and the $\overline{5}_H$ to down-type quarks. Such an operator is induced by the O(18) graph of Figure 3*b*. It gives rise to the terms proportional to ϵ and ϵ' in equation (5.1). If the magnitude of the graph in Figure 3*b* is suppressed relative to that of Figure 3*a*, then ϵ and ϵ' are small numbers and the violation of the Glashow-Weinberg-Paschos mechanism is acceptably small. Below, we will show that ϵ and ϵ' must be of order 0.1. Note that if the diagram of Figure 3*b* dominates the graph of Figure 3*a*, the Glashow-Weinberg-Paschos mechanism is still in force. All that is necessary is for *one* of the graphs to dominate the other.

The implementation of the Glashow-Weinberg-Paschos mechanism in O(18) relies crucially on the fact that the 126-dimensional representation of O(10) is *complex*. If the Higgs were in a real representation, there would be no way to distinguish the two diagrams of Figure 3, and the Glashow-Weinberg-Paschos mechanism could not be implemented easily.*

In the case of one Higgs doublet, full left-handed Kobayashi-Maskawa mixing implied the existence of eight ultralight neutrinos and extra Higgs singlets. The various Higgs singlets are not very appealing, and more than four ultralight neutrinos are in conflict with the simplest version of standard big bang nucleosynthesis. The addition of a second Higgs doublet allows us to do away with the extra Higgs singlets. It also permits us to raise the masses of five ultralight neutrinos. The resulting Z_N' quantum numbers of the two-Higgs theory are very tightly constrained.

In Appendix A we examine the Z_N' charges in the two-Higgs case. We prove that in order to have dimension-five operators and realistic Kobayashi-Maskawa mixing one has at most *three* ultralight neutrinos. Furthermore, we show that there are only two sets of Z_N' charges that realize this possibility. The Z_N' charges for these models are listed in Tables 6 and 7. A significant difference between the two cases is that the Z_5 charges allow CP-violation in the right-handed sector, while the Z_{10} charges do not.

As we see from Tables 6 and 7, both sets of Z_N' charge assignments lead to the following textures† for the left-handed quark and lepton Yukawa matrices:*

$$u \sim m \begin{pmatrix} 0 & 0 & 0 & \epsilon \\ 0 & 0 & \epsilon & 1 \\ 0 & \epsilon & 1 & 0 \\ \epsilon & 1 & 0 & 0 \end{pmatrix} , \qquad d \sim e \sim m \begin{pmatrix} 0 & 0 & \epsilon & 1 \\ 0 & \epsilon & 1 & 0 \\ \epsilon & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix} . \tag{5.5}$$

These textures allow CP-violation in the left-handed sector. They give Kobayashi-Maskawa mixings between successive generations of order ϵ, and mixings of order

* Note that this excludes the Higgs doublets from lying in the (10,35″) of O(10) × O(8).
† By "texture" we mean the general pattern of zero and nonzero entries of a matrix.
* Here we assume that ϕ_d has H charge zero. We could have chosen ϕ_u to have charge zero.

ϵ^2 between generations twice removed. Given the strong bound on $b-u$ mixing, this pattern is qualitatively correct. On the basis of the Cabibbo angle and $b-c$ mixing, we expect that $\epsilon \sim 0.1$. Note that the exact left-handed mass matrices are difficult to determine because they are generated by radiative corrections.

The textures (5.5) do not depend on whether we are considering the Z_5 or Z_{10} version of the model. The right-handed textures, however, depend on the value of N. For $N=5$, we have

$$U \sim N \sim M \begin{pmatrix} \eta & \eta & \eta & \eta \\ \eta & \eta & \eta & \eta \\ \eta & \eta & \eta & \eta \\ \eta & \eta & \eta & \eta \end{pmatrix}, \qquad D \sim E \sim M \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{pmatrix}, \quad (5.6a)$$

while for $N=10$, we find

$$U \sim N \sim M \begin{pmatrix} 0 & \eta & 0 & \eta \\ \eta & 0 & \eta & 0 \\ 0 & \eta & 0 & \eta \\ \eta & 0 & \eta & 0 \end{pmatrix}, \qquad D \sim E \sim M \begin{pmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{pmatrix}. \quad (5.6b)$$

In either case, only ϕ_d couples to the right-handed textures. Since ϕ_d lies in a 45^*_H of SU(5), the down and electron matrices receive direct contributions of order M, as shown in Figure 8*a*. The up and neutrino matrices receive radiative contributions of order ηM. This is illustrated in Figure 8*b*. Both U and N are η-suppressed relative to D and E. In general we expect $\eta \lesssim 1$, but we should not exclude the possibility that $\eta \simeq 1$. If $\eta \ll 1$, then ϕ_u and ϕ_d cannot be interchanged because the new charged leptons would be seen at PETRA. However, for $\eta \simeq 1$, the roles of ϕ_u and ϕ_d can be reversed.

5.2 RIGHT-HANDED MASSES

As shown in Appendix A, the combined constraints of dimension-five operators, realistic Kobayashi-Maskawa mixing and three ultralight neutrinos severely restrict the family charges. They require one Higgs doublet to have H charge zero, and the other to have charge ± 1 or ± 3. For definiteness, we choose ϕ_d to have charge zero. Since the left-handed masses are generated radiatively, ϕ_d must couple to the right-handed families with Yukawas of order one. As in Section 4, these Yukawas are large enough for the low-energy masses to be governed by the infrared fixed points of the SU(3) $\times$ SU(2) $\times$ U(1) renormalization group equations. Because only one Higgs couples to the right-handed families, the general analysis is similar to that discussed in Section 4. The only difference is that the fixed point relations hold for the Yukawas, rather than the masses. This is because $\langle\phi_u\rangle/\langle\phi_d\rangle$ is undetermined in a two-Higgs theory. A general analysis of the one-Higgs renormalization group equations shows that there are two independent bounds,[18]*

$$\begin{aligned} \sum_Q {g_Q}^2 &\lesssim 4.3 \\ \sum_L {g_L}^2 &\lesssim 3.6 \,. \end{aligned} \tag{5.7}$$

Here g_Q and g_L are the the eigenvalues of the right-handed quark and lepton Yukawa matrices. Since $\langle\phi_d\rangle \leq 175$ GeV, equations (5.7) can be converted into bounds on the masses,

$$\begin{aligned} \sum_Q {M_Q}^2 &\lesssim (365 \text{ GeV})^2 \\ \sum_L {M_L}^2 &\lesssim (335 \text{ GeV})^2 \,. \end{aligned} \tag{5.8}$$

These limits are independent of $\langle\phi_u\rangle/\langle\phi_d\rangle$.

⋆ We have modified the results of Ref. [18] to account for two Higgs doublets.

To get a more detailed picture of the low-energy spectrum, we again resort to the Monte Carlo techniques introduced in the previous section. As before, we renormalize 1000 Yukawa matrices from the unification scale M_{GUT} down to the weak scale M_W. At the unification scale we impose the O(10) relation,

$$D = -\frac{1}{3}E , \qquad (5.9)$$

where E is a symmetric matrix. Because the up and neutrino masses are generated radiatively, U and N are not related. The entries in the E matrix are chosen randomly from the interval 1.0 – 10.0. The entries in the D matrix are computed from (5.9). The η-analysis discussed above is incorporated by choosing the entries in the N and U matrices from the interval η – 10η. Since we are primarily interested in finding upper bounds on the masses, we set $\eta = 1.0$.†

The matrices U, D, E and N are renormalized from M_{GUT} to M_W using the $SU(3) \times SU(2) \times U(1)$ renormalization group equations. The results of the Monte Carlo are collected in Figures 9 through 11 (for $\eta = 1.0$). In Figure 9, separate histograms display the up, down, electron and neutrino Yukawas at the weak scale M_W. These histograms are directly comparable to the mass histograms in the one-doublet case. Note that although the up Yukawas are much smaller than the down Yukawas at the unification scale M_{GUT}, their distributions are similar at M_W.

Since $\langle \phi_d \rangle \leq 175$ GeV, the distributions in Figure 9 imply upper bounds on the quark and lepton masses.* From Figures 9a and 9b we see

$$\begin{aligned} M_U &\lesssim 265 \text{ GeV} \\ M_D &\lesssim 230 \text{ GeV} , \end{aligned} \qquad (5.10)$$

† For simplicity, we choose E to have the form (4.4). We also choose U and N to have the texture (4.4), without symmetric entries.

* The up and neutrino bounds are tighter if η is smaller. For example, $\eta = 0.1$ implies $M_U \lesssim 160$ GeV and $M_N \lesssim 30$ GeV.

while Figures 9c and 9d give

$$\begin{aligned} M_N &\lesssim 40 \text{ GeV} \\ M_E &\lesssim 55 \text{ GeV} . \end{aligned} \tag{5.11}$$

The fixed-point behavior is evident in the peaking of the up and down distributions in Figure 9. To see the fixed-point behavior more clearly, we turn to Figure 10a, where we have collected $\sum_Q {g_Q}^2$ for each of the 1000 sets of initial conditions. We see that the quark sum rule is saturated at the weak scale M_W:

$$\sum_Q {g_Q}^2 \simeq 4.3 . \tag{5.12}$$

In Figure 10b we plot $\sum_L {g_L}^2$ for the same initial conditions. We find that the bound (5.7) can be tightened considerably,

$$\sum_L {g_L}^2 \lesssim 0.36 . \tag{5.13}$$

The fact that the quark sum rule is saturated while the lepton sum rule is not can be understood by the fixed point behavior of the SU(3) × SU(2) × U(1) renormalization group equations.[18]

Because $\langle \phi_d \rangle$ is undetermined, the sum rules (5.12) and (5.13) give only bounds on the quark and lepton masses,

$$\begin{aligned} \sum_Q {M_Q}^2 &\simeq (365 \text{ GeV})^2 \\ \sum_L {M_L}^2 &\lesssim (105 \text{ GeV})^2 . \end{aligned} \tag{5.14}$$

These limits tell us that there should be at least one right-handed quark below 130 GeV and one right-handed lepton below 40 GeV.

The separate Yukawa sums are illustrated in Figure 11. Figures 11*a* and 11*b* allow us to bound the up and down Yukawas,

$$\sum_U {g_U}^2 \lesssim 4.0$$
$$\sum_D {g_D}^2 \lesssim 2.6 \ . \tag{5.15}$$

These equations imply upper limits on the lightest right-handed quarks:

$$M_U \lesssim 175 \text{ GeV}$$
$$M_D \lesssim 140 \text{ GeV} \ . \tag{5.16}$$

Figures 11*c* and 11*d* give bounds on the neutrino and charged lepton Yukawas,

$$\sum_N {g_N}^2 \lesssim 0.16$$
$$\sum_E {g_E}^2 \lesssim 0.16 \ . \tag{5.17}$$

These limits imply that the lightest right-handed leptons have masses

$$M_N \lesssim 40 \text{ GeV}$$
$$M_E \lesssim 40 \text{ GeV} \ . \tag{5.18}$$

The bounds (5.10) – (5.18) are very stringent. Equation (5.11) indicates that at least four right-handed neutrinos and one charged lepton should contribute to the width of the Z^0. Together with (5.18), this implies that at least one right-handed lepton doublet should be seen in the decays of the W.

Since these bounds are determined for $\eta = 1$, our conclusions do not change if we switch ϕ_u and ϕ_d. If η is much smaller than 1, the upper bound on M_E violates the PETRA limits when ϕ_u has H charge zero.

6. Experimental Signatures

In the preceeding sections we outlined the structure of the one- and two-Higgs versions of the O(18) theory. We found that nucleosynthesis constraints led us to prefer the two-Higgs version of the model. In this section we discuss the most important experimental signatures that follow from O(18) family unification.

6.1 Proton Decay Channels

In ordinary grand unified theories, the family assignments of quarks and leptons are not unique. While it is certainly possible to assign u, d and e to the same family, it is by no means necessary. For example, the τ can be placed in the same family as the u and the d. With this assignment, the dominant proton decay mode is not $e^+\pi^0$. This ambiguity is absent in our O(18) theory because of the Z_N' family symmetry. The family symmetry constrains the quark and lepton masses. It tells us how quarks and leptons combine into families, and allows us to predict the precise decay modes of the proton.

In this section we show how the Z_N' charges determine the decay channels of the proton. To find the dominant decays, we work in the limit $\epsilon = 0$. Since $B - L$ is conserved by dimension six operators, we only consider the H part of the Z_N' symmetry. We shall see that the H charges are such that proton decay can proceed through the $e^+\pi^0$ channel.

In Appendix A we showed that two-Higgs models require the ordinary families to have H charges $\pm\frac{1}{2}$ and $\pm\frac{7}{2}$. For definiteness, we take the H charge of the SU(2) singlet $\bar{u}$ to be $\frac{1}{2}$. The textures (5.5) then imply that the doublet u has charge $\frac{1}{2}$. Since H commutes with SU(2), the doublet d must also carry charge $\frac{1}{2}$. Because of (5.5), this fixes the charge of the singlet $\bar{d}$ to be $-\frac{1}{2}$. In terms of SU(5) representations, u, $\bar{u}$ and d all belong to a decouplet with H charge $\frac{1}{2}$, which we denote by $10_{\frac{1}{2}}$. In a similar fashion, $\bar{d}$ is contained in a $\overline{5}_{-\frac{1}{2}}$.

The H charges of the singlet and doublet electrons can also be determined from (5.5). This is because the charged-lepton and down-quark mass matrices have the same texture, and their matrix elements are related by SU(5) Clebsch-Gordan coefficients at the scale M_{GUT}. Since the electron is so light, it is natural to assign the doublet e and ν to the $\overline{5}_{-\frac{1}{2}}$, and the singlet $\bar{e}$ to the $10_{\frac{1}{2}}$.

Proton decay is mediated by SU(5) gauge bosons and Higgs scalars through effective operators of dimension six. Here we consider the gauge boson contributions to the decays. In terms of SU(5) representations, the induced dimension-six operators are

$$\begin{aligned} q\ l\ \bar{u}^{\dagger}\ \bar{d}^{\dagger} &\subset 10_{\frac{1}{2}} \times \overline{5}_{-\frac{1}{2}} \times 10^{\dagger}_{\frac{1}{2}} \times \overline{5}^{\dagger}_{-\frac{1}{2}} \\ q\ q\ \bar{u}^{\dagger}\ \bar{e}^{\dagger} &\subset 10_{\frac{1}{2}} \times 10_{\frac{1}{2}} \times 10^{\dagger}_{\frac{1}{2}} \times 10^{\dagger}_{\frac{1}{2}}\ , \end{aligned} \tag{6.1}$$

where $q = (u,\ d)$ and $l = (\nu,\ e)$. These operators conserve H, so the $e^+\pi^0$, $e^+\rho^0$, and $e^+\omega^0$ final states are not suppressed.

The preceeding arguments apply when ϕ_d has H charge zero (mod N). When ϕ_u has charge zero, the matrix textures for the up and down quarks are exchanged. Then d, $\bar{d}$, u, e and $\bar{e}$ have identical H charges, opposite to that of $\bar{u}$. This implies that all gauge mediated four-fermion operators are forbidden, so other families are needed for the effective operators to conserve the H symmetry. This implies that the $e^+\pi^0$ decay mode is Cabbibo suppressed when ϕ_d has non-zero H charge.

6.2 Flavor Violation

An important consequence of O(18) is that lepton-number violations are not necessarily suppressed by M_W/M_{GUT}. In our two-Higgs model, for example, lepton number is violated by Cabibbo mixings between the three ultralight left-handed neutrinos. These Cabibbo mixings are induced by the Dirac mass of the fourth left-handed neutrino.

Additional flavor violations arise because the left-handed singlet neutrino has the same family charge as at least two of its right-handed counterparts (see

Tables 6 and 7). The left- and right-handed singlets mix, and induce further mixings between the left- and right-handed neutrino doublets. Since left- and right-handed families have opposite weak couplings, these mixings induce direct flavor violations in the couplings of the Z^0. Such off-diagonal couplings can give rise to monojet events.[21]

Violations of lepton number are known to be very small. For example, the branching ratio for $\mu \to e\gamma$ has been measured to be less than 2×10^{-10}. In O(18), the dominant contribution to this process is given by a W loop, where the internal fermion is one of the Dirac neutrinos N. This contribution to the branching ratio is given by[22]

$$\mathrm{BR}\ (\mu \to e\gamma) \ \simeq \ \frac{3}{32}\,\frac{\alpha}{\pi}\,\frac{x^2}{\sin^2\xi\,\cos^2\xi}\,|\theta_\mu^*\,\theta_e|^2\ , \tag{6.2}$$

where $\tan\xi = \langle\phi_u\rangle/\langle\phi_d\rangle$ and θ_l denotes the Cabibbo mixing between the lepton l and the neutrino N. The generalized GIM cancellation mechanism leads to the factor $x = (M_N/M_W)^2$ in the $\mu \to e\gamma$ amplitude.

The suppression of $\mu \to e\gamma$ restricts the range of neutrino mixing angles and consequently the Z'_N family charges. For example, when $M_N \simeq 40$ GeV and $\tan\xi \simeq 1$, Equation (6.2) implies that $|\theta_\mu^*\theta_e|$ must be less than 10^{-3}. A scenario with sequential mixing to the fourth neutrino has $\theta_\mu \simeq \epsilon^2$ and $\theta_e \simeq \epsilon^3$ and yields a contribution to (6.2) well inside the experimental limit. A complete analysis of the relationship between the Z'_N charge assignments and the experimental limits on flavor violation is presented in Reference [23].

The O(18) theory presented here also has flavor-changing neutral currents in the quark sector. There are very strict limits on such currents. An analysis of the $K^0 - \bar{K}^0$ transition amplitude is presented in Appendix B. We show that the matrix textures (5.5) give flavor-changing neutral currents suppressed by ϵ^2 times the difference of the Higgs masses squared. The $K^0 - \bar{K}^0$ transition amplitude can be made sufficiently small for reasonable values of ϵ.

6.3 DECAYS OF NEW FAMILIES

Perhaps the most striking consequence of O(18) family unification is the existence of five new families. In this section we discuss their dominant decays.

As shown in Section 2, one of the five new families is left-handed. Its quarks and charged lepton cascade down to ordinary fermions by Cabibbo-suppressed weak interactions. The lifetimes of these particles are on the order of $2 \times 10^{-6}\ (m_\mu/M_F)^5 \simeq\ 10^{-16} - 10^{-21}$ seconds. The fourth left-handed charged lepton is lighter than half the Z^0 mass. It should therefore contribute to the width of the Z^0.

The other four new families are right-handed. They are heavier than their left-handed counterparts. The heaviest right-handed fermions cascade weakly to their lighter right-handed partners with lifetimes on the order of 10^{-16} to 10^{-21} seconds.

The decays of the lighter right-handed fermions are more complicated. Since only ϕ_d contributes to the right-handed masses, and since ϕ_d has H charge zero, the right-handed sector has an unbroken (discrete) H symmetry at low energies. This symmetry forbids mixings between right-handed fermions with different H charges. Therefore the lightest fermion in each H sector is potentially long-lived.

As discussed in Section 3, cosmological constraints imply that extra right-handed quarks and leptons cannot be seen today. This prompted us to arrange the Z_N' family charges so that the lightest right-handed quarks and leptons can decay by dimension-five operators,

$$\mathcal{L}_{\rm eff} \ \simeq\ \frac{1}{M_{GUT}} \bar{Q}\, q\, \phi_u\, \phi_d \ +\ \frac{1}{M_{GUT}} \bar{L}\, l\, \phi_u\, \phi_d \ . \tag{6.3}$$

These operators induce decays into Higgs scalars with lifetimes on the order of a few seconds – provided, of course, that the decays are energetically allowed. The operators (6.3) also induce Cabibbo mixings between the left- and right-handed fermions of order $\langle\phi_W\rangle^2/(M_F M_{GUT})\ \simeq\ 10^{-12} - 10^{-13}$. These mixings permit the right-handed fermions to decay into real or virtual W bosons (see Table 8). Real W emission implies a lifetime of approximately 10 seconds. Virtual W's decay into fermion pairs with a lifetime of order 10^4 to 10^7 seconds.

The decays of the lightest neutrinos depend on the Z_N' charges. In the Z_5 models, all singlet neutrinos have the same family charge. The singlet neutrinos induce mixings between the left- and right-handed doublet neutrinos, so all right-handed neutrinos decay into ordinary matter with typical weak lifetimes, $\tau_N \simeq 10^{-16}\,(10\ \mathrm{GeV}/M_N)^5$ seconds. In the Z_{10} model, the singlet neutrinos have Z_N' charges 1 and -4. The neutrinos with charge 1 mix with the left-handed sector, so they decay as above. The neutrinos with charge -4 cascade to the lightest lepton in the same charge sector. If energetically allowed, this lepton decays by a dimension-five operator with a lifetime of about $10 \times (10\ \mathrm{GeV}/M_L)$ seconds. Otherwise, it decays through by virtual W emission, with a lifetime of order $10^7 \times (10\ \mathrm{GeV}/M_L)^3$ seconds.

The most important point about the neutrino spectrum is that all eight neutrinos are lighter than half the Z^0 mass. They all contribute to its width. In the two-Higgs case, the electron, mu and tau neutrinos are ultralight, with masses $\gtrsim 0.1$ eV. The other five neutrinos are heavier, with masses less than 40 GeV. Subject to the Z_N' symmetry, the extra neutrinos decay into ordinary leptons. Their lifetimes depend on their mass. If the unstable neutrinos decay rapidly enough, they do not contribute to the invisible width of the Z^0.

In contrast, O(18) models with only one Higgs doublet have eight ultralight neutrinos. One-Higgs models enhance the invisible width of the Z^0 by a factor of 8/3. In Section 4 we showed that one-Higgs models also contain Higgs singlets s. The lightest right-handed quarks decay via $Q \rightarrow q + s$ with lifetimes of approximately 10^{-26} seconds.

7. The Bottom Line

In this paper we have developed an O(18) theory of family unification. The model is consistent with all established phenomenology and cosmology. It explains why quarks and leptons come in families, and why the families repeat.

The most striking feature of O(18) family unification is that it predicts eight families below the weak scale. Four of the families are left-handed, with $V - A$ weak interactions. The other four are right-handed, with $V + A$ couplings. Furthermore, the left-handed families are lighter than their right-handed counterparts. O(18) family unification has a rich phenomenology that will be tested very soon. In this section we summarize our most significant results.

The first thing to note is that O(18) predicts proton decay at an observable but acceptable rate. O(18) models with one or two Higgs doublets give lifetimes on the order of

$$\tau\,(p \;\rightarrow\; e^+\,\pi^0\,) \;\simeq\; 10^{32\pm1}\ \text{years}\ , \tag{7.1}$$

and values of $\sin^2\theta_W$ of approximately

$$\sin^2\,\theta_W \;\simeq\; 0.216\ . \tag{7.2}$$

Models with three or more Higgs doublets (and $e^+\pi^0$ as a dominant decay channel) are excluded, for they give too small values of τ and $\sin^2\theta_W$. Since current experimental limits indicate that $\tau \gtrsim 2 \times 10^{32}$ years, we predict that proton decay could soon be seen.

The preferred version of our O(18) theory contains two Higgs doublets. In addition to the three standard families, the fermion spectrum contains:

- **One new left-handed family**: O(18) family unification predicts the existence of one new left-handed family. The particles in this fourth family are somewhat lighter than their right-handed counterparts. The neutrino has mass less than 40 GeV and contributes to the width of the Z^0. The fourth neutrino decays into standard fermions through ordinary charged-current processes. Depending on its lifetime, it might not contribute to the invisible decays of the Z^0.

- **Four right-handed quark doublets:** The masses of the right-handed quarks are determined by the fixed-point structure of the renormalization group equations. In Section 5 we found

$$\begin{aligned} M_U &\lesssim 265 \text{ GeV} \\ M_D &\lesssim 230 \text{ GeV} . \end{aligned} \tag{7.3}$$

At least one right-handed quark should have mass less than 130 GeV. The heavier right-handed quarks cascade to their lightest partners by standard charged-current processes, with lifetimes on the order of 10^{-21} seconds. If kinematically allowed, the lightest right-handed quarks decay via dimension-five operators,

$$Q \rightarrow q + \text{scalars} , \tag{7.4}$$

with lifetimes of order a second. Otherwise, the Cabbibo mixing of right- and left-handed quarks allows decays into real and virtual W bosons,

$$\begin{aligned} Q &\rightarrow q + W \\ Q &\rightarrow q + f + \bar{f}' , \end{aligned} \tag{7.5}$$

with lifetimes on the order of 10 seconds and 10^4 seconds, respectively. The processes (7.4) and (7.5) ensure that no stable right-handed matter is seen today.

- **Four right-handed charged leptons:** The renormalization group analysis discussed above gives bounds on the masses of the right-handed charged leptons:

$$M_E \lesssim 55 \text{ GeV} . \tag{7.6}$$

At least one of the right-handed charged leptons should contribute to the widths of the W and Z^0. The right-handed charged leptons decay to their neutrino partners by ordinary weak interactions. The lifetimes for these decays are of order 10^{-19} seconds. If the lightest right-handed lepton is charged, it decays through a dimension-five operator,

$$E \rightarrow e + \text{scalars} . \tag{7.7}$$

The lifetimes for the decays (7.7) are on the order of a few seconds. If the lepton is lighter than the scalars, it decays through a virtual W,

$$E \rightarrow e + f + \bar{f}' \tag{7.8}$$

with a lifetime of order 10^5 seconds.

- **Four right-handed neutrinos**: The four right-handed neutrinos all have masses less than 40 GeV. Their masses are all less than half that of the Z^0.

$$M_N \lesssim \frac{1}{2} M_Z . \tag{7.9}$$

The heaviest right-handed neutrinos decay into lighter charged leptons by charged-current processes, with lifetimes of order 10^{-16} $(10 \text{ GeV}/M_N)^5$ seconds. The light right-handed neutrinos mix with their left-handed partners.* Because of this mixing, the right-handed neutrinos also decay into ordinary matter. The lifetimes for these decays are of order 10^{-16} $(10 \text{ GeV}/M_N)^5$ seconds. If M_N is large enough, the right-handed neutrinos do not contribute to the invisible decays of the Z^0.

⋆ All right-handed neutrinos mix in the Z_5 models. In the Z_{10} case, the story is more complicated. See Section 6.

The O(18) models with one Higgs doublet give eight ultralight neutrinos, all with masses $\gtrsim .1$ eV. Such a large number of ultralight neutrinos is in apparent conflict with the simplest version of big-bang nucleosynthesis. Models with one Higgs doublet also require the existence of light Higgs singlets. The charged fermion spectrum, however, is similar to that of the two-Higgs model. In the one-Higgs case, all eight neutrinos contribute to the invisible decays of the Z^0. This provides an important distinction between the one- and two-Higgs models.

Both the one- and two-Higgs versions of O(18) family unification predict that eight neutrinos should contribute to the width of the Z^0. Furthermore, it is quite likely that at least one right-handed charged lepton also contributes to its width. Given the striking signatures discussed here, O(18) cannot help but be tested soon.

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APPENDIX A

In this appendix we prove the incompatibility of dimension-five operators with four ultralight left-handed neutrinos and realistic Kobayashi-Maskawa mixing. We find a model with three ultralight neutrinos that satisfies our constraints. We show that the H charge assignments of this model are unique.

The proof that dimension-five operators are incompatible with four ultralight neutrinos and realistic left-handed mixing relies on the H charge assignments of the fermions. Given the left-handed H charges $\pm c$, $\pm d$, and the right-handed charges $\pm\frac{1}{5}(3c-4d)$, $\pm\frac{1}{5}(4c+3d)$, we will show that there are simply too many constraints to be simultaneously satisfied.

Four ultralight left-handed neutrinos require four singlet-singlet masses in the left-handed sector. This implies that one of the following equations must hold:

1. $2c = -2c = 2d = -2d$,
2. $2c = 2d = -(c+d)$, or
3. $(c+d) = -(c+d)$.

These charge conditions give strong restrictions on the allowed dimension-five operators. Since operators of the form $\bar{Q}\,q\,(\phi_u\phi_d)$ automatically conserve hypercharge and $B-L$, we only discuss the possible H charges. Let ΔH be the H charge of $\phi_u\phi_d$. Subject to the above restrictions, it is easy to find all possible values of ΔH. Dimension-five operators require at least one of the combinations $\pm c+\Delta H$ or $\pm d+\Delta H$ to be different from $\pm c$ or $\pm d$. It is straightforward to show that cases 1) and 2) return only the left-handed charges $\pm c$ and $\pm d$. Case 3, however, is a little more complicated. At least one more condition must be imposed to guarantee realistic left-handed mixing. It is not hard to show that any extra condition takes left-handed charges to left-handed charges. This forces us to conclude that dimension-five operators are incompatible with four ultralight neutrinos and left-handed mixing. Models with four ultralight neutrinos need additional Higgs singlets to ensure that right-handed matter decays quickly.

If we relax the charge conditions to allow just *three* ultralight left-handed neutrinos, we can have dimension-five operators and realistic left-handed mixing. We will show that this leads to one of two Z'_N symmetries with identical H charge assignments.

Three ultralight left-handed neutrinos require three singlet-singlet masses in the left-handed sector. This leads to a unique relation between the left-handed charges (up to family permutations),

$$2c \;=\; -\,(c\;+\;d). \tag{A.1}$$

By examining the left-handed Yukawa matrices, one can show that sufficient Kobayashi-Maskawa mixing requires one of the two Higgs doublets to have H charge zero (mod N). It then follows that the other doublet must have charge $\pm 2c$ (mod N).

The lightest mirror fermions must decay via dimension-five operators. As before, we consider the combinations $\pm c + \Delta H$ and $\pm d + \Delta H$, where $\Delta H = \pm 2c$. Because of $(A.1)$, dimension-five operators exist only if the right-handed fermions have charge $\pm(c+2d)$. This restricts the H charges of mirror families to be equal (modulo N, up to a sign). In terms of the parameters c and d, we have

$$\frac{1}{5}(4c+3d) \;=\; \pm\;\frac{1}{5}(3c-4d)\;. \tag{A.2}$$

Together with $(A.1)$, equation $(A.2)$ determines the family charges in terms of c or d. The values of c and d are restricted to be integer or a half-integer. For $d = 1/2$, we obtain

$$N \;=\; 5,\;10$$

$$\text{Left}-\text{Handed Family Charges } H: \quad \pm\frac{7}{2},\ \pm\frac{1}{2} \tag{A.3}$$

$$\text{Right}-\text{Handed Family Charges } H: \quad \pm\frac{5}{2},\ \pm\frac{5}{2}\;.$$

If d is any other integer or half-integer, equation $(A.3)$ should be rescaled appropriately. For $N = 5$, equation $(A.1)$ is satisfied for $c = \pm\frac{1}{2}, \pm\frac{7}{2}$. For $N = 10$, equation $(A.1)$ implies $c = \pm\frac{7}{2}$. We find that one Higgs doublet has charge 0, and the other has charge ± 3 for Z_{10}, or ± 1, ± 3 for Z_5. In either case, only the doublet with H charge zero contributes to the right-handed masses.

For any of the allowed Higgs charges and for either discrete symmetry, the left-handed mass matrices can be written in the form of equation (5.2). As described in Section 5, these matrices have realistic Kobayashi-Maskawa mixing. They also adequately suppress Higgs-induced flavor-changing neutral currents.

APPENDIX B

In Appendix A we demonstrated that models with three ultralight neutrinos are consistent with dimension-five operators and realistic Kobayashi-Maskawa mixing. We now examine the extent to which the Yukawa matrices (5.5) suppress flavor-changing neutral currents. We compute the contributions from direct flavor-changing neutral couplings and compare the results to the standard contributions from $W^{\pm}$ exchanges.[24]

Let us parameterize the Yukawa couplings for the left-handed families as follows,

$$\mathcal{L}_Y \;=\; \overline{U}_L[\phi_u A_1 + \epsilon\phi_d{}^* A_2]U_R \;+\; \overline{D}_L[\phi_d B_1 + \epsilon\phi_u{}^* B_2]D_R \;+\; h.c.\ , \qquad (B.1)$$

where U and D contain the four up- and down-type quarks. We factor out ϵ explicitly, so the matrices A_n and B_n are of comparable magnitude. We assume that $H(\phi_d) = 0$ and $H(\phi_u) \neq 0$. Our final conclusions are not sensitive to this choice.

The vacuum expectation values of the Higgs fields ϕ_u and ϕ_d are defined as follows,

$$\langle\phi_u\rangle \;=\; v_u\ , \qquad \langle\phi_d\rangle \;=\; v_d\ . \qquad (B.2)$$

It is convenient to set $\cos\xi = v_d/v$, where $v^2 = v_u{}^2 + v_d{}^2$. In this notation, the quark mass matrices are

$$\begin{aligned} M_U &= (\sin\xi A_1 + \epsilon\cos\xi A_2)\, v \;\equiv\; E_U\, v \\ M_D &= (\cos\xi B_1 + \epsilon\sin\xi B_2)\, v \;\equiv\; E_D\, v\ . \end{aligned} \qquad (B.3)$$

The two neutral scalars ϕ_1 and ϕ_2 couple to linear combinations of A_n and B_n ,

$$\begin{aligned} F_U &= \cos\xi\, A_1 - \epsilon\sin\xi\, A_2 \;=\; \cot\xi\, E_U - \epsilon A_2/\sin\xi \\ F_D &= \sin\xi\, B_1 - \epsilon\cos\xi\, B_2 \;=\; \tan\xi\, E_D - \epsilon B_2/\cos\xi\ , \end{aligned} \qquad (B.4)$$

so diagonalizing E_U and E_D also diagonalizes F_U and F_D to order one. Therefore, flavor violations are suppressed by powers of ϵ – and by the fact that the matrices A_2 and B_2 are very sparse.

For ease of presentation, we parameterize the Yukawa matrices in the following way:

$$\sin\xi\, A_1 = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \widetilde{B} \\ 0 & 0 & \widetilde{C} & 0 \\ 0 & \widetilde{D} & 0 & 0 \end{pmatrix} \qquad \cos\xi\, A_2 = \begin{pmatrix} 0 & 0 & 0 & \widetilde{\kappa} \\ 0 & 0 & \widetilde{\beta} & 0 \\ 0 & \widetilde{\gamma} & 0 & 0 \\ \widetilde{\delta} & 0 & 0 & 0 \end{pmatrix}$$

$$\cos\xi\, B_1 = \begin{pmatrix} 0 & 0 & 0 & A \\ 0 & 0 & B & 0 \\ 0 & C & 0 & 0 \\ D & 0 & 0 & 0 \end{pmatrix} \qquad \sin\xi\, B_2 = \begin{pmatrix} 0 & 0 & \kappa & 0 \\ 0 & \beta & 0 & 0 \\ \gamma & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}. \qquad (B.5)$$

A, B, C and D are essentially the masses for the down-type quarks d, s, b and b', respectively. (We make a similar assignment for the up-type quarks.) Simple rotations of $\sin\xi\, A_1$ and $\cos\xi\, B_1$ to diagonal form allow one to read off the flavor-violating elements of F.

The $K_L - K_S$ mass difference gives the most stringent limit on neutral flavor-changing couplings. We will show that the contribution from the conventional box diagram (Figure 12a) dominates the direct contribution from our Yukawa couplings (Figure 12b).

The neutral Higgs couplings give tree-level contributions to the $K^0 - \bar{K}^0$ transition amplitude,

$$|\langle K^0 \mid H_{FC} \mid \bar{K}^0\rangle| = \frac{(\epsilon\kappa)^2}{2\sin^2\xi\cos^2\xi} \left| \frac{1}{M_1^2} - \frac{1}{M_2^2} \right| |\langle K^0 \mid \bar{s}_R d_L \bar{s}_R d_L \mid \bar{K}^0\rangle| , \qquad (B.6)$$

where M_1 and M_2 are the scalar and pseudoscalar Higgs masses. In (B.6) we have used the $s - d$ element of F, $\epsilon\kappa/(\sin\xi\cos\xi)$. This is related to the sine of the Cabbibo angle, $\sin\theta_C \simeq \epsilon \mid \widetilde{\kappa}/\widetilde{B} - \kappa/B \mid$, in the approximation $A^2 \ll B^2$. To the extent that $\kappa \simeq \widetilde{\kappa}$, we may substitute $\epsilon\kappa \simeq \sin\theta_C \, m_s/v$.

The standard result for the $K^0 - \bar{K}^0$ transition amplitude is

$$\mid \langle K^0 \mid H_{WW} \mid \bar{K}^0 \rangle \mid = \frac{g^4}{2^7\pi^2} \frac{m_c^2}{M_W^4} \sin^2\theta_C \mid \langle K^0 \mid \bar{s}_L \gamma_\mu d_L \, \bar{s}_L \gamma^\mu d_L \mid \bar{K}^0 \rangle \mid \ . \quad (B.7)$$

We evaluate the matrix elements of the four-quark operators using the vacuum insertion method.[25] Requiring $\mid \langle K^0 \mid H_{FC} \mid \bar{K}^0 \rangle \mid \lesssim \mid \langle K^0 \mid H_{WW} \mid \bar{K}^0 \rangle \mid$ implies

$$\frac{50}{\sin^2\xi \cos^2\xi} M_W^2 \mid \frac{1}{M_1^2} - \frac{1}{M_2^2} \mid \lesssim 1 \ , \quad (B.8)$$

where we have taken $m_s = 150$ MeV and $m_c = 1.5$ GeV. This bound is easily satisfied, so we conclude that the flavor-changing neutral currents can be sufficiently suppressed in the effective low-energy theory.

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Table Captions

1. The charges of various O(8) representations under the two U(1) symmetries that leave four light left-right families. The values of a and b are determined by the vacuum expectation values of certain Higgs fields. Any Higgs potential will restrict a and b to be integers. Note that it is also possible for the left- and right-handed charges to be interchanged.

2. Proton lifetime in O(18) models with one or two Higgs doublets. In the last column, τ and τ_5 refer to the partial lifetimes of the $e^+ \, \pi^0$ decay mode. Our preferred predictions are underlined.

3. The lightest hadrons composed of one right-handed quark and either one or two ordinary quarks. Here, I denotes the strong isospin of the multiplet.

4. Hydrogenic nuclei formed from the heavy hadrons of Table 3.

5. The Z'_N charges of the left- and right-handed singlet neutrinos that can participate in the Gell-Mann–Ramond–Slansky–Yanagida mechanism. These charges are derived for the one-Higgs case, where $2c = 2d = 0$, modulo N.

6. The family charges of the light left- and right-handed fermions, for $f = \frac{3}{10}$ and $N = 5$ or 10. With these assignments, ϕ_u has H charge 3 or ± 1. The H charges that correpond to the four columns of the table are $\frac{7}{2}$, $-\frac{7}{2}$, $\frac{1}{2}$, and $-\frac{1}{2}$ for the left-handed fermions, and $\frac{5}{2}$, $-\frac{5}{2}$, $\frac{5}{2}$, and $-\frac{5}{2}$ for the right-handed fermions. Our SU(5) conventions are such that $16 \rightarrow \overline{5} + 10 + 1$.

7. The family charges of the light left- and right-handed fermions, for $f = \frac{1}{10}$ and $N = 5$. With these assignments, ϕ_u has H charge 1 or ± 3. The H charges of the left- and right-handed families are those of Table 6.

8. Decay modes for the lightest right-handed fermions F into ordinary fermions f. Here, $\theta = \langle \phi_W \rangle^2 \, / \, M_F \, M_{GUT}$, N_C denotes the number of decay channels, and $\Gamma_\mu = \Gamma(\mu \rightarrow e\bar{\nu}\nu)$. In estimating the lifetimes, we have assumed $M_Q = 100$ GeV, $M_E = 30$ GeV, $M_N = 10$ GeV, $M_{GUT} = 10^{15}$ GeV and $N_C = 10$.

Table 1

O(8) Representation	U(1) Charges	
$8'$	$\pm\frac{1}{2}(3a+b),\ \pm\frac{1}{2}(a-3b),$ $\pm\frac{1}{2}(a+b),\ \pm\frac{1}{2}(a-b)$	$\pm\frac{1}{2}(5a+3b),\ \pm\frac{1}{2}(5a-b),$ $\pm\frac{1}{2}(3a+b),\ \pm\frac{1}{2}(a-b)$
$8''$	$\pm\frac{1}{2}(3a-b),\ \pm\frac{1}{2}(a+3b),$ $\pm\frac{1}{2}(a+b),\ \pm\frac{1}{2}(a-b)$	$\pm\frac{1}{2}(a+3b),\ \pm\frac{1}{2}(7a+b),$ $\pm\frac{1}{2}(3a+b),\ \pm\frac{1}{2}(a-b)$
8_V	$\pm a,\ \pm b,\ \pm(a+b),\ \pm(a-b)$	$\pm 2a,\ \pm(a+b),\ \pm(a-b),\ \pm(3a+b)$
$35'$	$0,\ \pm a,\ \pm b,\ \pm 2a,\ \pm 2b,$ $\pm(a+b),\ \pm(a-b),\ \pm(2a+b),$ $\pm(2a-b),\ \pm(a+2b),\ \pm(a-2b),$ $\pm(3a+b),\ \pm(a-3b)$	$0,\ \pm 2a,\ \pm 2b,\ \pm 4a,$ $\pm(a+b),\ \pm(a-b),\ \pm(3a+b),$ $\pm(3a-b),\ \pm(5a+b),\ \pm(5a-b),$ $\pm(5a+3b),\ \pm 2(a+b),\ \pm 2(2a+b)$
$35''$	$0,\ \pm a,\ \pm b,\ \pm 2a,\ \pm 2b,$ $\pm(a+b),\ \pm(a-b),\ \pm(2a+b),$ $\pm(2a-b),\ \pm(a+2b),\ \pm(a-2b),$ $\pm(3a-b),\ \pm(a+3b)$	$0,\ \pm 2a,\ \pm 2b,\ \pm 4a,$ $\pm(a+b),\ \pm(a-b),\ \pm(3a+b),$ $\pm(3a-b),\ \pm(5a+b),\ \pm(7a+b),$ $\pm(a+3b),\ \pm 2(a+b),\ \pm 2(2a+b)$
56_V	$0,\ \pm a,\ \pm b,\ \pm 2a,\ \pm 2b,$ $\pm 3a,\ \pm 3b,\ \pm(a+b),\ \pm(a-b),$ $\pm(a+2b),\ \pm(a-2b),\ \pm(2a+b),$ $\pm(2a-b),\ \pm 2(a+b),\ \pm 2(a-b)$	$0,\ \pm 2a,\ \pm 2b,\ \pm 4a,\ \pm 6a,$ $\pm(a+b),\ \pm(a-b),\ \pm(3a+b),$ $\pm(3a-b),\ \pm(5a+b),\ \pm 2(3a+b),$ $\pm 2(a+b),\ \pm 2(a-b),\ \pm 3(a+b),$ $\pm 2(2a+b)$

Table 2

Model	$\Lambda_{\overline{MS}}$ (MeV)	M_X (10^{15} GeV)	$\alpha_{GUT}(M_X)$	$\sin^2\theta$	τ/τ_5
Standard SU(5)	100	0.13	0.024	0.216	1
One Doublet	50	0.52	0.11	0.218	1.2×10^1
One Doublet	100	1.46	0.14	0.214	4.9×10^2
One Doublet	150	3.18	0.17	0.210	7.4×10^3
One Doublet	200	5.69	0.21	0.208	5.2×10^4
Two Doublets	50	0.32	0.11	0.222	1.7×10^0
Two Doublets	100	0.88	0.14	0.218	6.4×10^1
Two Doublets	150	1.86	0.17	0.215	9.0×10^2
Two Doublets	200	3.23	0.20	0.213	5.7×10^3

Table 3

Hadrons	U Lightest	D Lightest
Mesons $(I = \frac{1}{2})$	$\overline{U}u = M^0$ $U\bar{u} = \overline{M}^0$	$\overline{D}u = M^+$ $D\bar{u} = M^-$
Baryons $(I = 0)$	$U(ud - du) = B^+$ $\overline{U}(\bar{u}\bar{d} - \bar{d}\bar{u}) = B^-$	$D(ud - du) = B^0$ $\overline{D}(\bar{u}\bar{d} - \bar{d}\bar{u}) = \overline{B}^0$

Table 4

Hadrons	U Lightest	D Lightest
Mesons	M^0p, M^0pn, M^0pnn	M^+, M^+n, M^+nn
Baryons	B^+, B^+n, B^+nn	B^0p, B^0pn, B^0pnn

Table 5

Left – Handed Charges	Right – Handed Charges
$c - 5f$	$\frac{1}{5}(4c + 3d) + 5f$
$c - 5f$	$-\frac{1}{5}(4c + 3d) + 5f$
$c - 5f$	$\frac{1}{5}(3c - 4d) + 5f$
$c - 5f$	$-\frac{1}{5}(3c - 4d) + 5f$

Table 6

SU(5) Representation	Z'_N Charges			
$\overline{5}$	$\frac{22}{5}$	$-\frac{13}{5}$	$\frac{7}{5}$	$\frac{2}{5}$
10	$\frac{16}{5}$	$-\frac{19}{5}$	$\frac{1}{5}$	$-\frac{4}{5}$
1	2	-5	-1	-2
5	$\frac{8}{5}$	$-\frac{17}{5}$	$\frac{8}{5}$	$-\frac{17}{5}$
$\overline{10}$	$\frac{14}{5}$	$-\frac{11}{5}$	$\frac{14}{5}$	$-\frac{11}{5}$
1	4	-1	4	-1

Table 7

SU(5) Representation	Z'_N	Charges		
$\bar{5}$	$\frac{19}{5}$	$-\frac{16}{5}$	$\frac{4}{5}$	$-\frac{1}{5}$
10	$\frac{17}{5}$	$-\frac{18}{5}$	$\frac{2}{5}$	$-\frac{3}{5}$
1	3	-4	0	-1
5	$\frac{11}{5}$	$-\frac{14}{5}$	$\frac{11}{5}$	$-\frac{14}{5}$
$\overline{10}$	$\frac{13}{5}$	$-\frac{12}{5}$	$\frac{13}{5}$	$-\frac{12}{5}$
1	3	-2	3	-2

Table 8

Process	Estimate of Width	Lifetime (seconds)
$F \to f + \phi_u + \phi_d$	$\Gamma_F \simeq M_F^3/M_{GUT}^2$	$\tau_Q \sim 1$ $\tau_E \sim 10$ $\tau_N \sim 10^3$
$F \to f + \phi$	$\Gamma_F \simeq \langle\phi_W\rangle^2 M_F/M_{GUT}^2$	$\tau_Q \sim 1$ $\tau_E \sim 1$ $\tau_N \sim 10$
$Q \to q + W$	$\Gamma_Q \simeq \alpha_2\,\theta^2 M_Q$	$\tau_Q \sim 10$
$Q \to q + W$ $\to q + f + \bar{f}'$	$\Gamma_Q \simeq (m_\mu/M_Q)^{-5}\,\theta^2\,\Gamma_\mu N_C$	$\tau_Q \sim 10^4$
$E \to e + W$ $\to e + f + \bar{f}'$	$\Gamma_E \simeq (m_\mu/M_E)^{-5}\,\theta^2\,\Gamma_\mu N_C$	$\tau_E \sim 10^5$
$N \to \nu + W$ $\to \nu + f + \bar{f}'$	$\Gamma_N \simeq (m_\mu/M_N)^{-5}\,\theta^2\,\Gamma_\mu N_C$	$\tau_N \sim 10^7$

Figure Captions

1. A schematic picture illustrating the hierarchy of symmetry breakings. The vacuum expectation value $\langle \chi \rangle$ breaks $U(1)_H$ to Z_N at a scale of order M_{GUT}.

2. Four right-handed families acquire masses at the weak scale $\langle \phi \rangle \simeq M_W$.

3. The left-handed families obtain masses through mixings with the right-handed families. Dominant contributions include the loop graphs illustrated here.

4. The predicted mass fractions X(^{2}H) and X(^{4}He) for 2,4 and 8 light neutrinos. The curves vary with the baryon-to-photon ratio η. The popular limits for primordial helium and deuterium productions are indicated. These graphs are compiled from the figures in Reference [10].

5. (a) A histogram of 4000 right-handed up-type Yukawas, evaluated at the weak scale $M_W = 80$ GeV. The single Higgs doublet is assumed to lie in a 126 of O(10). The upper bound on g_U implies $M_U \lesssim 180$ GeV. The down-type distribution is essentially identical. (b) The sum of the squares of all right-handed quark Yukawas.

6. A histogram of 4000 right-handed charged-lepton Yukawas, evaluated at the scale M_W. The Higgs doublet is assumed to lie in a 10 of O(10). This case is excluded by the PETRA bound.

7. (a) Charged-lepton Yukawas for one Higgs doublet in the 126 of O(10). (b) The sums of squares of the charged-lepton Yukawas.

8. Masses of the light right-handed families are generated by the coupling of the 45_H to (a) down quarks and (b) up quarks. The parameter η, introduced in Section 5, denotes the ratio of the contributions (b) to (a).

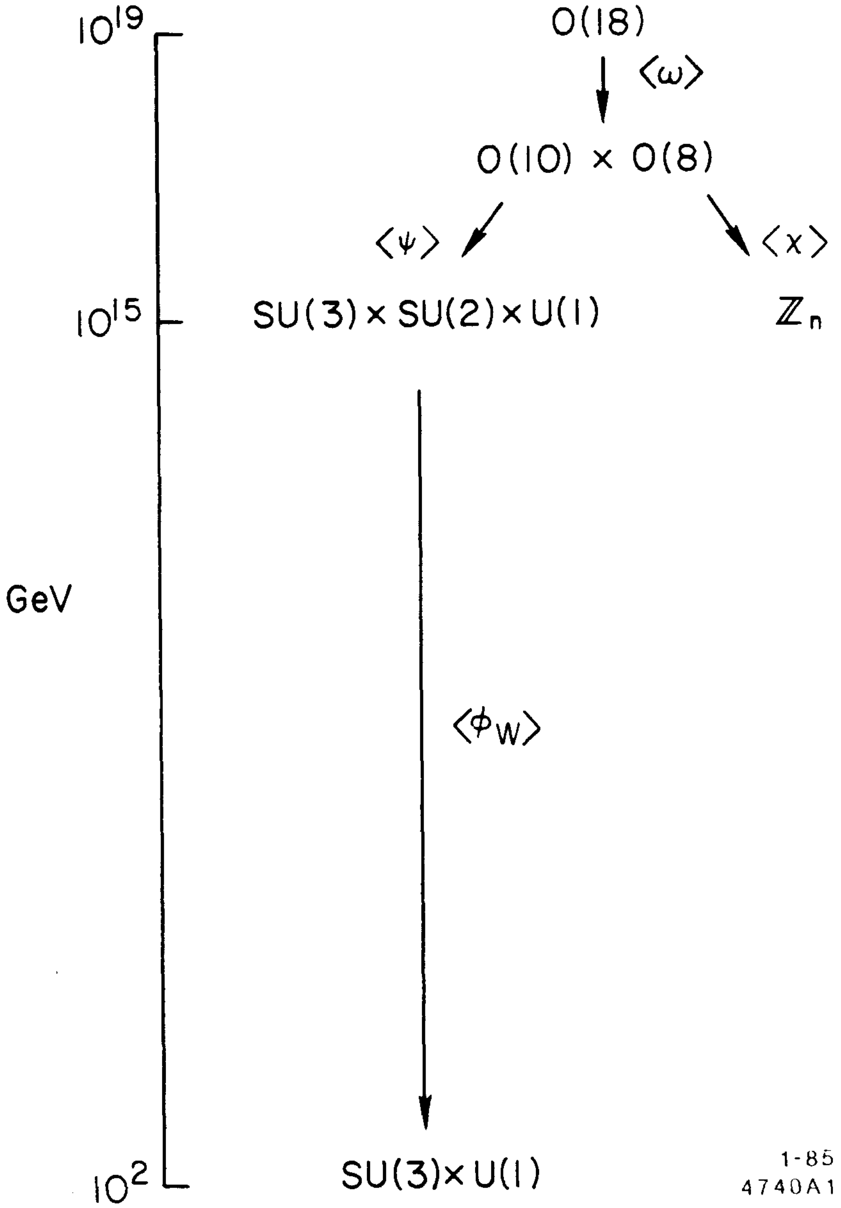
9. Histograms for right-handed (a) up, (b) down, (c) electron and (d) neutrino Yukawas at the weak scale in the model with two Higgs doublets.

10. The sums of the squares of the right-handed (a) quark and (b) lepton

Yukawas for the two-Higgs model of Section 5. Each histogram contains 1000 runs.

11. Histograms of the sums of the squares of the right-handed (a) up, (b) down, (c) electron and (d) neutrino Yukawas for the two-Higgs model.

12. (a) One of the standard $W^{\pm}$ exchange graphs contributing to the $K^0 - \bar{K}^0$ amplitude. (b) The tree-level flavor-changing Higgs contribution to the $K^0 - \bar{K}^0$ amplitude.



10^{19}
O(18)
⟨ω⟩
O(10) × O(8)
⟨ψ⟩
⟨χ⟩
10^{15}
SU(3) × SU(2) × U(1)
$\mathbb{Z}_n$
GeV
$\langle\phi_W\rangle$
10^2
SU(3) × U(1)
1-85
4740A1

Fig. 1

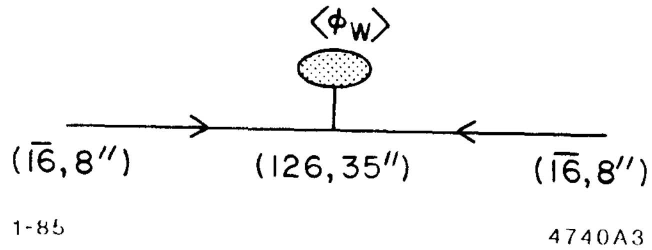


Fig. 2

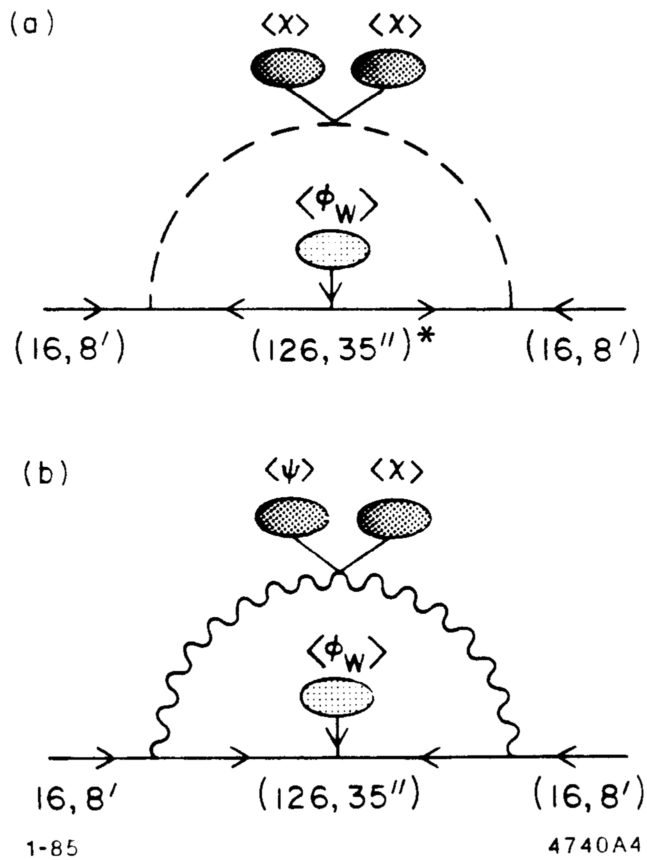
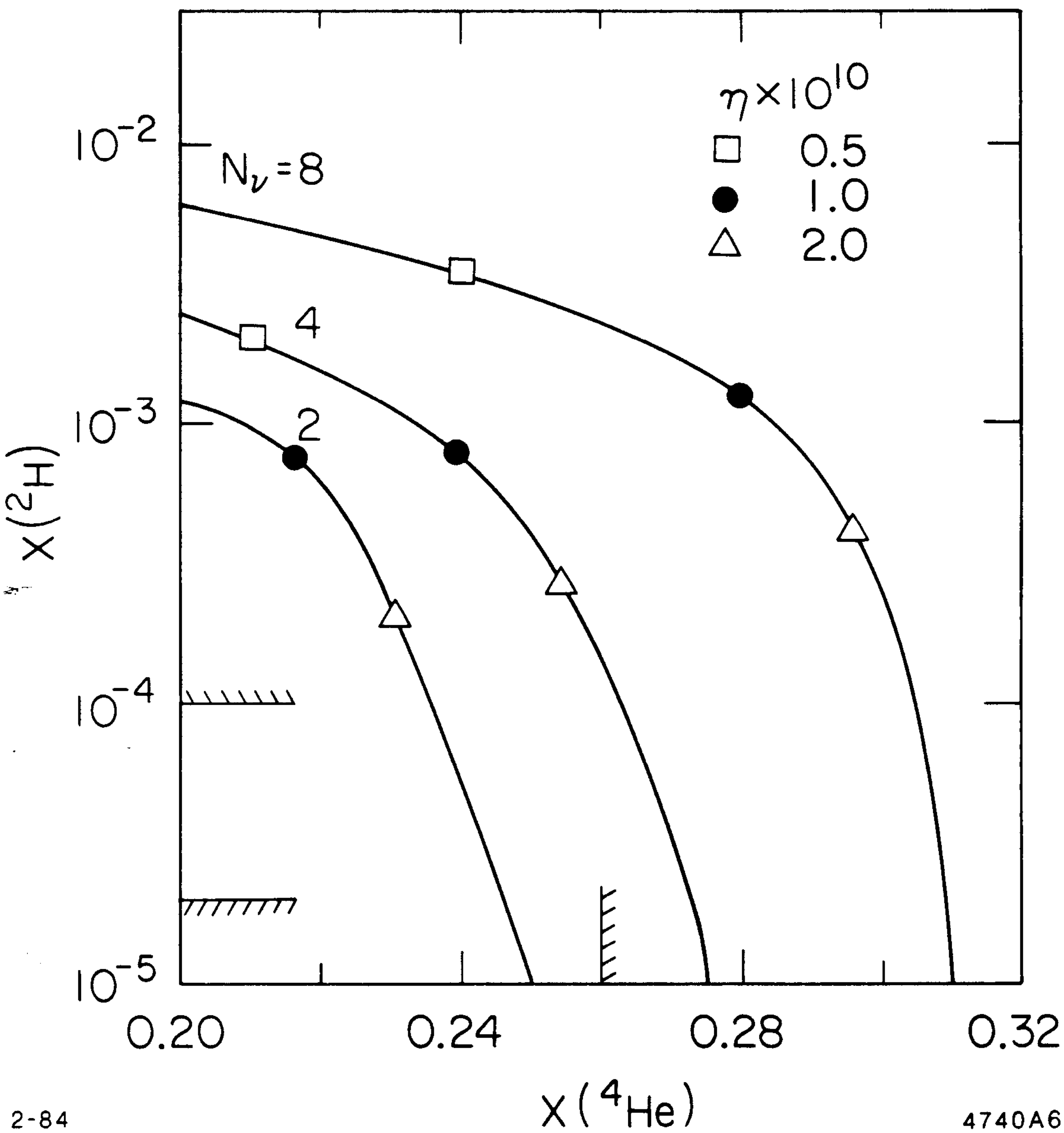


Fig. 3



$\eta \times 10^{10}$
0.5
1.0
2.0
$N_\nu = 8$
4
2
10^{-2}
10^{-3}
10^{-4}
10^{-5}
$X(^2H)$
0.20
0.24
0.28
0.32
$X(^4He)$
2-84
4740A6

Fig. 4

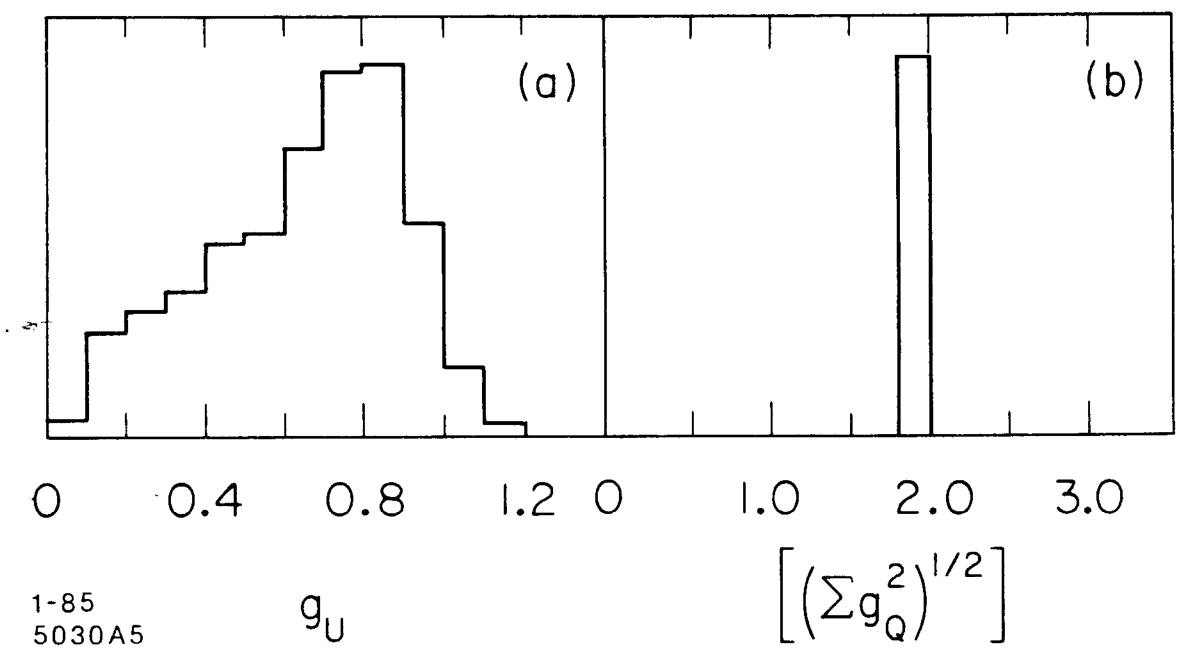
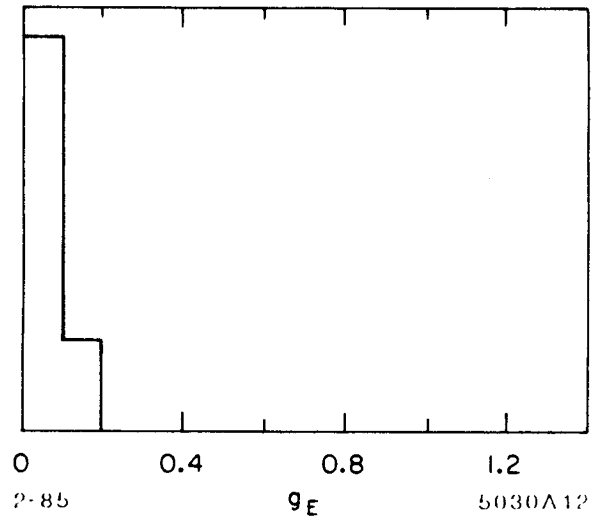
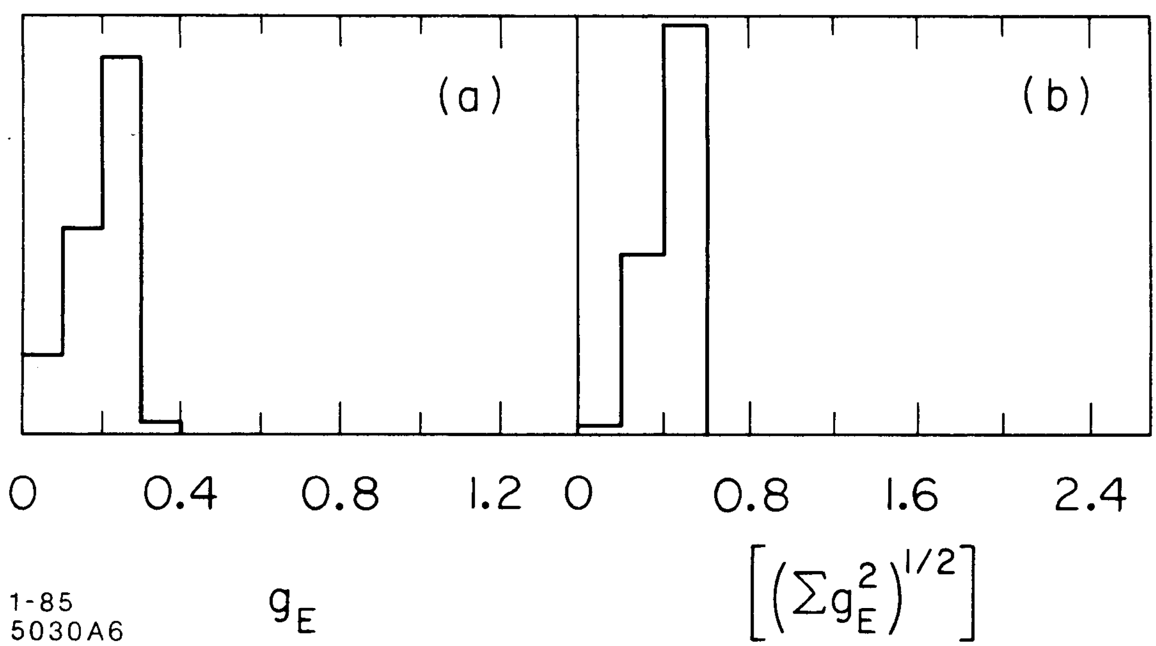

(a)
(b)
0
0.4
0.8
1.2
0
1.0
2.0
3.0
1-85
5030A5
g_U
$\left[\left(\sum g_Q^2\right)^{1/2}\right]$

Fig. 5



0
0.4
0.8
1.2
2-85
g_E
5030A12

Fig. 6



(a)
(b)
0
0.4
0.8
1.2
0
0.8
1.6
2.4
1-85
5030A6
g_E
$\left[\left(\sum g_E^2\right)^{1/2}\right]$

Fig. 7

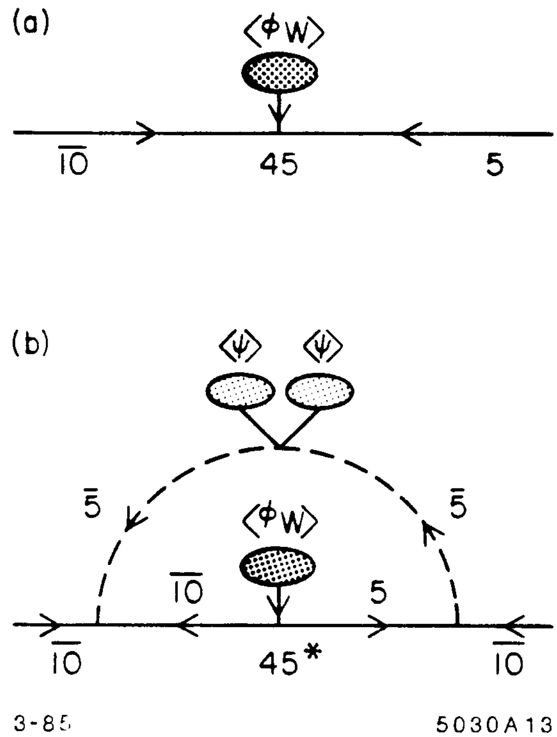


Fig. 8

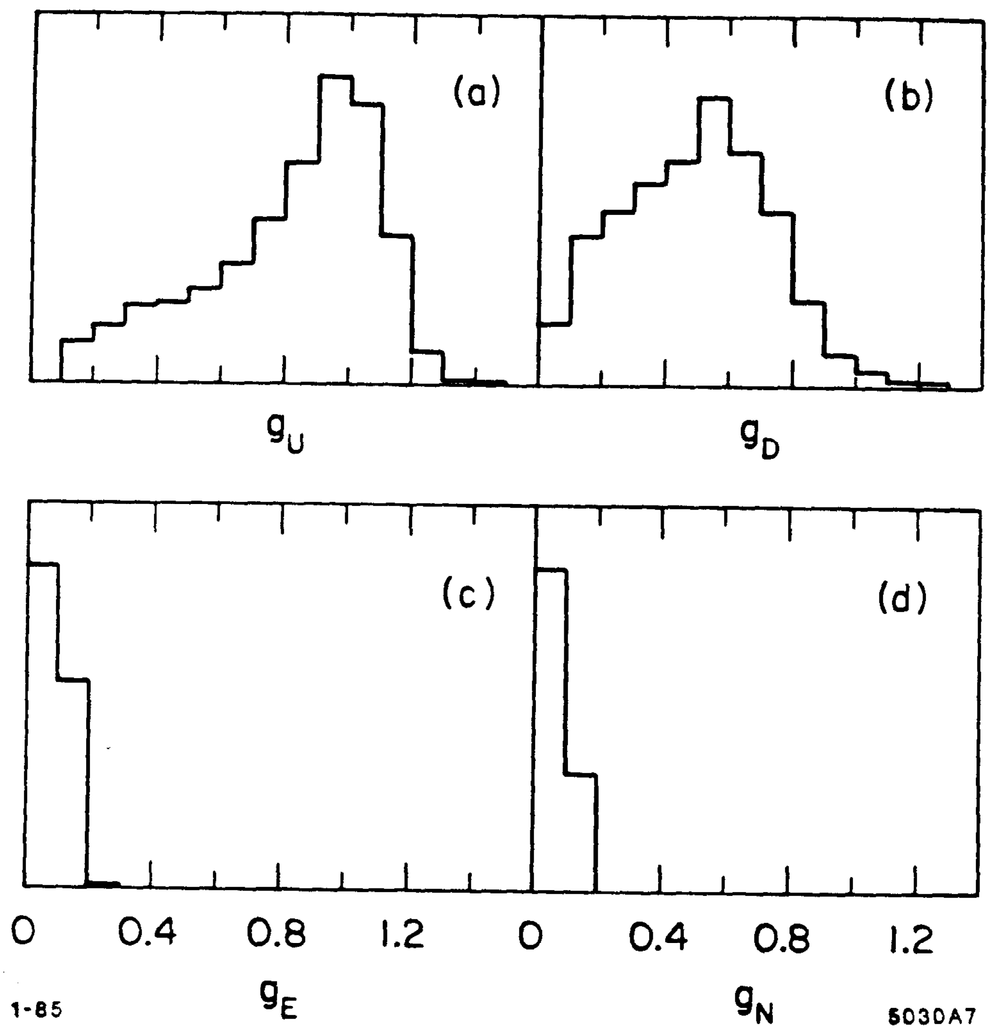


Fig. 9

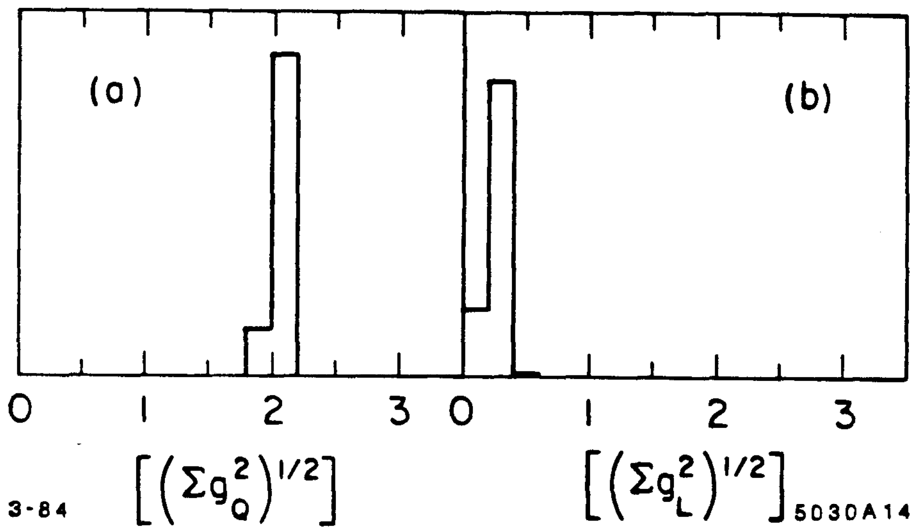

(a)
(b)
0
1
2
3
0
1
2
3
$\left[\left(\Sigma g_Q^2\right)^{1/2}\right]$
$\left[\left(\Sigma g_L^2\right)^{1/2}\right]$
3-84
5030A14

Fig. 10

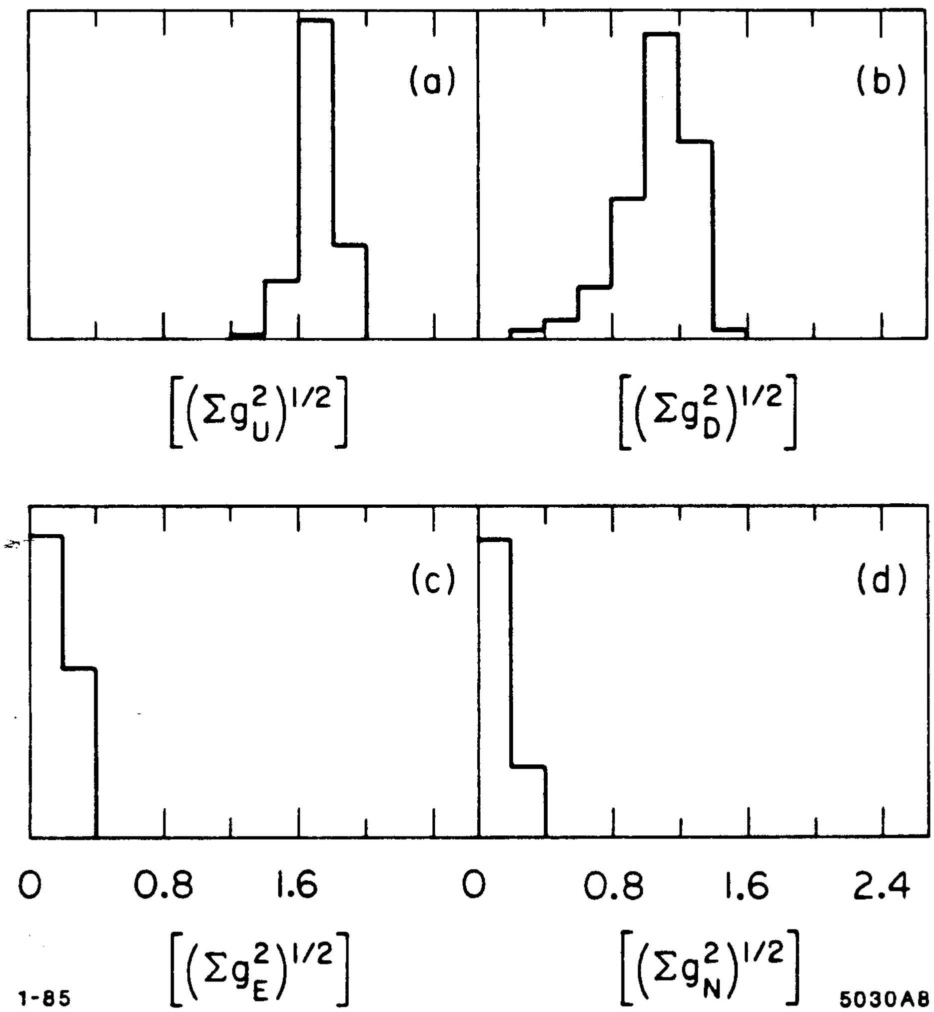

(a)
(b)
(c)
(d)
$[(\Sigma g_U^2)^{1/2}]$
$[(\Sigma g_D^2)^{1/2}]$
0
0.8
1.6
2.4
$[(\Sigma g_E^2)^{1/2}]$
$[(\Sigma g_N^2)^{1/2}]$
1-85
5030A8

Fig. 11

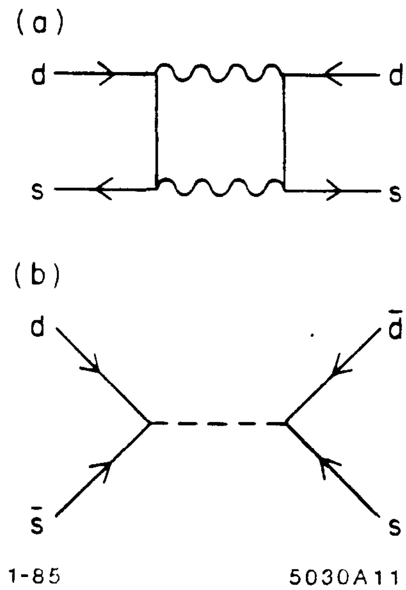


Fig. 12