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Bound-State Effects in $\Upsilon \rightarrow \gamma +$ Resonance

JAMES PANTALEONE*

Laboratory of Nuclear Studies, Cornell University
Ithaca, N. Y. 14853

and

MICHAEL E. PESKIN†

Stanford Linear Accelerator Center
Stanford University, Stanford, California, 94305

and

S.-H. HENRY TYE*

Laboratory of Nuclear Studies, Cornell University
Ithaca, N. Y. 14853

ABSTRACT

We study the effect of b-$\bar{\text{b}}$ bound state dynamics on the amplitude for the reaction $\Upsilon \rightarrow \gamma +$Resonance. We argue from our results that the recently discovered $\varsigma(8320)$ must have a scalar, rather than a pseudoscalar, coupling to the b quark.

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The Crystal Ball collaboration has recently announced the discovery of a new resonance, the $\varsigma(8320)$, which appears in Υ decays recoiling against a single photon with branching ratio of roughly 0.5%[1] . The discovery is clearly one of great interest, since there is no particle called for in the standard gauge theory of strong, weak, and electromagnetic interactions which would be produced in this mode at the large rate required by the Crystal Ball data. It is therefore important to ask whether theoretical analysis can add any constraints on the nature of the ς to those which follow straightforwardly from the observations.

In this paper, we will concentrate on the one piece of experimental data which strikes us as unusual, not only from the viewpoint of the standard model, but also from the viewpoint of any likely modification of this model. The Crystal Ball group has reported that the ς is not observed at the Υ', and has placed a stringent upper bound:

$$\frac{BR(\Upsilon' \to \gamma + \varsigma)}{BR(\Upsilon \to \gamma + \varsigma)} < 0.22 \quad (90\% \text{ confidence}). \tag{1}$$

We do not find it plausible to interpret the ς as a b-$\bar{\text{b}}$ state: this would require a hyperfine splitting at least a factor of 30 larger than that predicted by QCD, even though the corresponding prediction for the ψ-η_c splitting works quite well[2] . Any other interpretation, however, requires that the b and $\bar{\text{b}}$ annihilate as the ς is being produced. Conventional wisdom would then dictate that the amplitude for formation of the ς from any Υ resonance would be given by a universal factor times $|\psi(0)|$, where $\psi(r)$ is the bound-state wavefunction of that particular resonance. This picture gives a definite prediction for the ratio which appears in (1). Let $BR(\text{cascade})$ be the fraction of Υ' decays which involve photon or 2-pion cascades to lower resonances rather than b-$\bar{\text{b}}$ annihilation. Including this correction[3] , and phase space, we find:

$$\begin{aligned}\frac{BR(\Upsilon' \to \gamma + \varsigma)}{BR(\Upsilon \to \gamma + \varsigma)} &= (1 - BR(\text{cascade})) \cdot \frac{(1 - m_\varsigma^2/m_{\Upsilon'}^2)}{(1 - m_\varsigma^2/m_\Upsilon^2)} \\ &= 0.76 \pm 0.05,\end{aligned} \tag{2}$$

which is clearly in disagreement with (1).

To understand why (1) is so small, we must somehow understand why the amplitude for ς production depends on the detailed structure of each b-$\bar{\text{b}}$ bound state rather than only on the wavefunction at the origin. This more involved dependence on the bound state wavefunctions might, in principle, arise from the b-$\bar{\text{b}}$ dynamics before the annihilation or from the annihilation amplitude itself. To investigate how large such an effect might be, we have studied the amplitude for formation of the ς in three different models—assuming, in turn, that the ς is (1) a Higgs boson with scalar coupling to b-$\bar{\text{b}}$, (2) a Higgs boson with pseudoscalar coupling, and (3) a pseudoscalar bound state of a pair of new colored fermions. We find that the last two models are clearly incompatible with the result (1). In the case of a scalar Higgs, we find a substantial cancellation in the amplitude for $\Upsilon' \to \gamma + \varsigma$, making this model at least plausibly compatible with the experimental constraint. (An alternative, much more devious, explanation of the suppression of the Υ' decay, can be constructed if the ς is a bound state of colored scalars; this model has been described by one of us elsewhere[4] .)

Before analyzing these models in detail, we should comment briefly on their plausibility. It has been known since the work of Wilczek[5] that Higgs bosons might be produced in the radiative decays of the Υ. The expected branching ratio for a standard Higgs of the mass of the ς is 5×10^{-5}, roughly two orders of magnitude smaller than that observed for the ς. However, Wilczek also observed[6] that models with more than one Higgs boson contain an additional parameter, the ratio of the Higgs vacuum expectation values, which allows the rate of this process to be adjusted arbitrarily. This is thus a natural interpretation of the ς, as several authors have already noted[7,8] . In most such models, the light Higgs is a pseudoscalar (the axion is a notable example), but this is certainly not necessary. Our third alternative constrains the rate for $\Upsilon \to \gamma + \varsigma$ considerably more tightly: The ς is formed by annihilation of the b and $\bar{\text{b}}$ into a photon and two gluons and the subsequent recombination of the two gluons into a new fermion-antifermion pair. The large energy of the annihilation process should justify estimating the rate by perturbative QCD. Using the general computation of this rate by Guberina and Kühn[9], three groups[10–12] have estimated the branching ratio for the radiative decay of the Υ into a gluino-gluino bound state to be roughly 5×10^{-4}. For Dirac color octet fermions, this branching ratio would be 0.1%, a number probably compatible with observation, considering the errors of both theory and experiment. This hypothesis will of course be ruled out if the ς is confirmed to decay to $\tau^+\tau^-$.

Let us now begin our analysis of the corrections to the picture in which the amplitude for b-$\bar{\text{b}}$ annihilation processes depend only on the value of the wavefunction at the origin. Building on eq. (2), we define a "zeta function", $Z(m_\varsigma)$, by writing:

$$\frac{BR(\Upsilon' \to \gamma + \varsigma)}{BR(\Upsilon \to \gamma + \varsigma)} = (1 - BR(\text{cascade})) \cdot \frac{(1 - m_\varsigma^2/m_{\Upsilon'}^2)}{(1 - m_\varsigma^2/m_\Upsilon^2)} \cdot Z(m_\varsigma). \tag{3}$$

As with the more familiar zeta-function of Riemann, our main interest in $Z(m)$ will be to locate its zeros. We would be pleased if $Z(m)$ had a zero in the vicinity of m_ς; in any event, we must find $Z(m_\varsigma) < 0.3$ to explain the experimental result (1).

We consider first the corrections to (2) arising from the b-$\bar{\text{b}}$ bound state dynamics before annihilation*. Wilczek, in his original computation of the rate for $\Upsilon \to \gamma + \text{Higgs}$[5], considered the simple process shown in Fig. 1(a), in the approximation that one could ignore bound-state effects in the annihilation process. Later, Haber, Kane, and Sterling[14] and Ellis, Gaillard, Nanopoulos, and Sachrajda[15] pointed out that this approximation is not adequate when the energy of the photon is small, since in that case, the mass of the b-$\bar{\text{b}}$ state after emission of the photon may be sufficiently close to the Υ region that one

* After completing this analysis, we learned that a very similar computation had been done by Polchinski, Sharpe, and Barnes[13]. This computation has also been done for the pseudoscalar case by Lane, Meshkov, and Wilczek[7].

cannot ignore the potential binding the b and $\bar{b}$. These authors discussed the mixing of the Higgs with specific quarkonium states, but did not attempt a calculation valid for general Higgs masses. We will make that extension of their work here. We must compute the amplitude indicated schematically in Fig. 1(b): the initial Υ radiates a photon by a multipole transition, propagates in the resulting quarkonium state, and eventually annihilates to the Higgs via a pointlike coupling.

Wilczek's computation gave the width for Higgs production as†

$$\Gamma(\Upsilon \to \gamma + \mathrm{H}) = 2\sqrt{2}\alpha\, G_F\, Q_{EM}^2 \cdot |\psi(0)|^2 \cdot \left(1 - \frac{m_{\mathrm{H}}^2}{m_{\Upsilon}^2}\right). \tag{4}$$

In the limit of small photon energy, the process shown in Fig. 1(b) gives a formula which differs from (4) only in that the term $|\psi(0)|^2$ is replaced by a more intricate factor $\mathbf{M}$. For the case of a pseudoscalar Higgs, the photon is radiated via an M1 transition and the b-$\bar{b}$ intermediate state is in the $^1S_0$ channel. Then, if $\vec{\epsilon}$ and $\vec{n}$ are the polarizations of the photon and Υ, respectively, $\widehat{k}$ is the photon direction, and $\mathbf{H}_0$ is the one-body Hamiltonian on $^1S_0$ states (including the relevant spin-dependent forces), and the indicated state denote the spatial part of the wavefunction only, we find

$$\begin{aligned} \mathbf{M}_P &= \frac{1}{2}\sum_{\widehat{k},\vec{\epsilon}} \left| E_\gamma \cdot \langle \vec{r}=0 | \, \frac{1}{2} tr_\sigma \, \frac{1}{m_{\mathrm{H}} - \mathbf{H}_0} (\vec{\sigma} \times \widehat{k} \cdot \vec{\epsilon}) \; (\vec{n} \cdot \vec{\sigma}) | \Upsilon \rangle \right|^2 \\ &= \left| (m_\Upsilon - m_{\mathrm{H}}) \cdot \langle \vec{r}=0 | \frac{1}{m_{\mathrm{H}} - \mathbf{H}_0} | \Upsilon \rangle \right|^2 . \end{aligned} \tag{5}$$

For the case of a scalar Higgs, the photon is radiated via an E1 transition and the b-$\bar{b}$ intermediate state is in the $^3P_0$ channel. If $\mathbf{H}_1$ is the Hamiltonian in this

† We normalize $\psi(r)$ to the convention: $\int_0^\infty 4\pi r^2 dr |\psi(r)|^2 = 1$

channel,

$$
\begin{aligned}
\mathbf{M}_S &= \frac{1}{2}\sum_{\widehat{k},\vec{\epsilon}}\left|E_\gamma \cdot \langle \vec{r}=0|\,\frac{1}{2}\,tr_\sigma\,(\vec{\mathbf{p}}\cdot\vec{\sigma})\frac{1}{m_{\mathrm{H}}-\mathbf{H}_1}(\vec{\mathbf{r}}\cdot\vec{\epsilon})\;(\vec{n}\cdot\vec{\sigma})|\Upsilon\rangle\right|^2 \\
&= \left|\frac{1}{3}\,(m_\Upsilon - m_{\mathrm{H}})\cdot\langle\vec{r}=0|\vec{\mathbf{p}}\cdot\frac{1}{m_{\mathrm{H}}-\mathbf{H}_1}\vec{\mathbf{r}}|\Upsilon\rangle\right|^2 .
\end{aligned}
\tag{6}
$$

Both formulae are evaluated in the simplest nonrelativistic approximation. However, it is remarkable that both (5) and (6) tend to $|\psi(0)|^2$, the correct extreme relativistic limit, as the mass of the Υ is taken to be large compared to the Higgs mass. Thus, (5) and (6) are actually interpolating formulae; it is perhaps more correct to use these formulae for large photon energies than to add in some, but not all, of the relativistic corrections.

It is straightforward to evaluate the formulae (5), (6). One method is to represent the energy denominator using a complete set of intermediate states. Let $|nS\rangle$, $|nP\rangle$, represent the nth state in the relevant channel and $\psi_n(r)$ its radial wavefunction. Then,

$$
\begin{aligned}
\mathbf{M}_P &= \left|(m_\Upsilon - m_{\mathrm{H}})\sum_n \frac{\psi_n(0)\langle nS|\Upsilon\rangle}{m_{\mathrm{H}}-E_n}\right|^2 , \\
\mathbf{M}_S &= \left|\frac{1}{3}\,(m_\Upsilon - m_{\mathrm{H}})\sum_n \frac{\psi_n'(0)\langle nP|r|\Upsilon\rangle}{m_{\mathrm{H}}-E_n}\right|^2 .
\end{aligned}
\tag{7}
$$

Alternatively, one might represent the energy denominator as a Green's function: Let $g_\ell(r)$ be the solution to

$$
\left[\frac{1}{m_b}\left(-\frac{1}{r^2}\frac{\partial}{\partial r}r^2\frac{\partial}{\partial r} + \frac{\ell(\ell+1)}{r^2}\right) + V_\ell(r) + (2m_b - m_{\mathrm{H}})\right]g_\ell(r) = 0 \tag{8}
$$

which is regular at ∞ and tends to $1/r^{\ell+1}$ as $r \to 0$. Let $\Delta = (m_\Upsilon - m_{\mathrm{H}})\cdot m_b$.

Then, after some algebra, one finds:

$$\begin{aligned}\mathbf{M}_P &= \left|\Delta \int_0^\infty dr\ (r g_0(r)) \cdot r \cdot \psi_\Upsilon(r)\right|^2, \\ \mathbf{M}_S &= \left|\frac{1}{3}\Delta \int_0^\infty dr\ (r^2 g_1(r)) \cdot r \cdot \psi_\Upsilon(r)\right|^2.\end{aligned} \tag{9}$$

It is not difficult to predict the general form of $\mathbf{M}$ as a function of the mass of the Higgs. The amplitude for formation of the Higgs vanishes as the photon energy tends to zero, so the total rate, proportional to the product of $\mathbf{M}$ and phase space, tends to zero as E_γ^3. However, in the pseudoscalar case, this behavior applies only to a very small mass region: $\mathbf{M}$ contains a resonance at the mass of the spin-0 hyperfine partner of the Υ state in question, and, below this resonance, the energy denominator very nearly cancels the factor of E_γ in the numerator[16] . A detailed estimate of $\mathbf{M}(m_{\mathrm{H}})$ using the Richardson potential[17] plus a small short-range $(\sim exp(-m_b r))$ hyperfine interaction to adjust the Υ-η_b splitting to 40 MeV, is presented in Fig. 2. Actually, we plot the more useful quantity

$$r(m_{\mathrm{H}}) = (1 - BR(\text{cascade})) \cdot \frac{\mathbf{M}(m_{\mathrm{H}})}{|\psi(0)|^2} \cdot \left(1 - \frac{m_{\mathrm{H}}^2}{m_\Upsilon^2}\right), \tag{10}$$

which is directly proportional to the branching ratio for $\Upsilon \to \gamma + \mathrm{H}$. This figure shows precisely the behavior expected from the argument we have just given. There is no particular suppression of the decay of the Υ'; the ratio of the two curves shown is almost exactly that due to phase space alone.

The behavior of $\mathbf{M}(m_{\mathrm{H}})$ in the scalar case is rather different, however. Since there is no ${}^3P_0$ resonance below the Υ, one should expect the rate of Υ decay to vary as E_γ^3 over a wide region. This effect works in the wrong direction, suppressing the denominator of (1). In the case of the Υ' decay, one should find a resonant enhancement when the mass of the Higgs is near that of the singlet

χ_b[14,15] . One might guess that below this resonance, the rate would tend rapidly to the value expected from phase space. Remarkably, it does not. As Fig. 3 indicates, $\mathbf{M}(m_{\rm H})$ for the Υ' has a zero for $m_{\rm H} = 9.2$ GeV and rises only slowly for smaller Higgs masses. The Υ' decay is then much more strongly suppressed than the Υ decay, just the situation needed to explain the observations. For completeness, we show also, in Fig. 4, the behavior of $\mathbf{M}$ for $\Upsilon'' \to \gamma + {\rm H}$ as a function of $m_{\rm H}$. Note that, up to the effects of spin-orbit forces, which displace the Υ' curve less than 100 MeV to the right, the predictions of Figs. 3 and 4 apply equally well to any hypothetical 1^+ or 2^+ resonance which couples directly to b-$\bar{\rm b}$.

Since the zero of $\mathbf{M}(m_{\rm H})$ found in the scalar case is of such importance to our analysis, it is worth pausing to understand its origin. A crucial piece of information is that, for reasonable quarkonium potentials, the $1P$ radial wavefunction is large in the region beyond the node of the $2S$ wavefunction, so that the E1 matrix element between the $2S$ and $1P$ states is negative; this is shown in Table 1. The E1 matrix elements between the $2S$ and the higher P states are positive. Using this information, one can understand the presence of the zero in either of two ways. In the viewpoint of eq. (7), the $1P$ contribution to the indicated sum dominates near its resonance, but the weight of the contributions from higher states dominates as the Higgs mass becomes small and the various energy denominators become equal. Alternatively, one can take the viewpoint of eq. (9) and consider the way in which the Green's function $g_1(r)$ evolves as the $m_{\rm H}$ changes. For $m_{\rm H}$ near the $1P$ mass, $g_1(r)$ has the shape of the $1P$ state (divided by a small energy denominator), so that its overlap with the $2S$ is negative. But for $m_{\rm H}$ well below the Υ, $g_1(r)$ falls off rapidly with r, so that it samples only the wavefunction at the origin; this gives a positive overlap integral. Either argument implies that $\mathbf{M}$ goes through a zero as $m_{\rm H}$ is decreased from the mass of the $1P$.

One can also understand that the bound state corrections are much larger in the scalar than in the pseudoscalar case by considering the limit opposite to the one we have been studying so far, the limit of large photon energy. In this limit, one may estimate the dependence of the amplitude for $\Upsilon \to \gamma + \mathrm{H}$ on the initial wavefunction by computing the momentum dependence of the amplitude for $b\bar{b} \to \gamma + \mathrm{H}$. One finds for the scalar case that one must replace

$$\langle \vec{r} = 0 | \Upsilon \rangle \to \langle \vec{r} = 0 | \left(1 - \frac{2\mathrm{p}^2}{3m \cdot k_\gamma} + \mathcal{O}\left(\frac{\mathrm{p}^2}{m_b^2}\right)\right) | \Upsilon \rangle, \tag{11}$$

to include the leading corrections, whereas in the pseudoscalar case the corresponding replacement is

$$\langle \vec{r} = 0 | \Upsilon \rangle \to \langle \vec{r} = 0 | \left(1 + \mathcal{O}\left(\frac{\mathrm{p}^2}{m_b^2}\right)\right) | \Upsilon \rangle, \tag{12}$$

One can thus see from the direction of large photon energies that the approximation of the wavefunction at the origin should have a wide range of validity in the case of a pseudoscalar but should be easily spoiled in the case of a scalar. Vysotsky[18] has computed the leading QCD corrections to Wilczek's formula (4) for a scalar Higgs and has found that they are large and negative. Much of this correction can be traced directly to the k_γ^{-1} behavior of the tree amplitude displayed in (11). The presence of such large corrections seems consistent with the fact that the curves of Figs. 3, 4 show such a slow convergence to the Wilczek limit.

Though the strong suppression of the rate for $\Upsilon' \to \gamma + \mathrm{H}$ is model independent, the precise magnitude of this suppression depends on the details of the calculation. The particular estimate shown in Fig. 3 uses the Richardson potential, plus spin-dependent forces of the form suggested by Gupta, Radford, and Repko[19] :

$$\begin{aligned} V_{spin}(r) = (0.03)\Big[&\frac{\vec{s}_1 \cdot \hat{r}\vec{s}_2 \cdot \hat{r} - \frac{1}{3}\vec{s}_1 \cdot \vec{s}_2}{r^3}(1.88 + 0.18\log(r)) \\ &+ \frac{\vec{L}\cdot\vec{S}}{r^3}(1 + 0.17\log(r)) - 0.13\frac{\vec{L}\cdot\vec{S}}{r}\Big] \end{aligned} \tag{13}$$

(where r is in GeV^{-1} and we regulate by writing $\bar{r} = (r^2 + (1/2m_b)^2)^{\frac{1}{2}})$, and the short-ranged hyperfine interaction described above. For these parameters, we find for $Z(m)$ at the ς mass:

$$Z(8.32\ \text{GeV}) = \begin{cases} 0.97 & (0^-\ \text{Higgs}) \\ 0.52 & (0^+\ \text{Higgs}) \end{cases} \tag{14}$$

These values correspond to a ratio of branching fractions (eq. (1)) of 0.74 and 0.40, respectively. The absolute value of $r(m)$ is very sensitive to the particular parameters used; the value of $Z(m)$ is somewhat less so. The value of $Z(m_\varsigma)$ for the scalar case can vary by 10% if one changes to the Martin[20] or to the Cornell[21] potential, or if one chooses to omit the spin-dependent forces* . Despite the fact that the formula (7) indicates an apparent sensitivity to highly excited P states, cutting off the potential at $\text{B}\bar{\text{B}}$ threshold has less than a 1% effect on $Z(m_\varsigma)$; this is sensible because $g_1(r)$ is quite short-ranged at $m_\text{H} = m_\varsigma$. $Z(m_\varsigma)$ does increase significantly (to about 0.6) if one includes the effects of retardation, replacing r by $(k_\gamma/6)^{-1} \cdot j_1(k_\gamma r/2)$. Using the relativistic form $(m_\text{H}^2 - m_{nP}^2)^{-2}$ for the energy denominator gives an upward correction of similar size. However, as we have noted below eq. (6), it does not necessarily make the calculation more accurate to include these effects. We should recall also that there are large (negative) relativistic corrections to the E1 transition rates in the charmonium system; these are expected to be small for the usual E1 transitions in the b-$\bar{\text{b}}$ system, but they grow with energy and could well give important corrections to value of $Z(m_\varsigma)$ for the scalar case. (The simple exercise of the previous paragraph also indicates the possibility of large relativistic corrections in the scalar case.) None of these modifications has any significant effect on $Z(m_\varsigma)$ for the pseudoscalar case; in no case have we found a value for this quantity less than 0.95. We conclude that a value of $Z(m_\varsigma)$ less than 0.3, as required by

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* V. Zambetakis and N. Byers have informed us that they have repeated this calculation using the spin-dependent potentials of McClary and Byers[22] ; they find $Z(m_\varsigma) = 0.48$[23] .

experiment, is possible for a Higgs with scalar coupling to b-$\bar{\text{b}}$ but very unlikely for a pseudoscalar Higgs.

It is worth noting that the suppression we have observed for the rate of $\Upsilon \to \gamma + \mathrm{H}$ in the case of scalar Higgs has an intriguing, and possibly disturbing, consequence. In a model with two Higgs doublets, the branching ratio for Υ decay to an 8.3 GeV Higgs is given by

$$BR(\Upsilon \to \gamma + \mathrm{H}) = (5 \times 10^{-5}) \cdot \frac{\mathrm{M}(m_{\mathrm{H}})}{|\psi(0)|^2} \cdot \left(\frac{v_2}{v_1}\right)^2 , \tag{15}$$

where v_1 and v_2 are the two Higgs vacuum expectation values. Since our value of M for the Υ decay is quite small, we expect $(v_2/v_1) \sim 12$, or, since v_2 must give the bulk of the W boson mass, $v_1 \sim 20$ GeV. The Yukawa coupling of this light Higgs to the b quark would not be so large $(\lambda_b^2/4\pi \sim \frac{1}{100})$. However, if the ς is to have a large branching fraction to hadrons, it must also have an enhanced coupling to c-$\bar{\text{c}}$, and thus also to t-$\bar{\text{t}}$, by the rule of Glashow and Weinberg[24] . In this case, one would have $\lambda_t^2/4\pi \sim 0.6$. (Note that this number depends on our result for the absolute rate of ς production and so is rather poorly determined.)

Let us now turn to the second possible source of contributions to $Z(m)$, the structure of the b-$\bar{\text{b}}$ annihilation amplitude. A concrete model which contains an annihilation amplitude of nontrivial structure is the formation of a bound state of new colored fermions F; this would proceed by the annihilation of the b and $\bar{\text{b}}$ to a photon and two gluons and the subsequent recombination of the two gluons into the new bound state. The amplitude for this process has been extensively studied by Kühn and collaborators[9,25–27] . However, all of this work made use of the approximation that the decay rate of a given Υ state was proportional to the wavefunction at the origin. It is just this approximation that we wish to check. A tractable way of approaching this question is shown in Fig. 5. We specialize to the case of formation of a 0^- bound state, since only this case can give a rate in Υ decays large enough to account for the ς. Label this new state the $\tilde{\eta}$. We treat the photon as soft and, therefore, radiated via an M1 transition. We have just shown that, in this case, the factors arising from the M1 transition amplitude and the subsequent energy denominator cancel almost exactly, so we will ignore both. We will, however, treat in detail the annihilation amplitude, given by the 2-gluon box diagram shown in Fig. 5(b), and the crossed gluon diagram. We are interested in the dependence of this diagram on the 3-momentum of the external fermions: if there is no significant dependence on the b-quark momentum, the decay rate is exactly proportional to the wavefunction at the origin. In order to include variable external 3-momenta, we must take the external momenta of the b, $\bar{\text{b}}$, F, and $\bar{F}$ off shell. We choose to take the b and $\bar{\text{b}}$ momenta equal to $(E, \pm\vec{\mathbf{k}})$ and the F and $\bar{F}$ momenta equal to $(E', \pm\vec{\mathbf{k}}')$, where $\vec{\mathbf{k}}$ and $\vec{\mathbf{k}}'$ are the momenta appearing in the bound-state wavefunctions, and the masses of the internal b and F lines equal to the squares of these four-vectors. In this approximation, the diagram of Fig. 5(b) and the crossed graph are equal for the 0^- channel. When we report numerical values below, we set both E and E' equal to $(m_{\tilde{\eta}}/2)$, their smallest reasonable value, and orient $\vec{\mathbf{k}}$ parallel to $\vec{\mathbf{k}}'$. Clearly, we bias our estimate toward the worst case.

In the approximation we have just described,

$$\begin{aligned}\Gamma(\Upsilon \to \gamma + \tilde{\eta}) = 256\alpha\, \alpha_s^4\, Q_{EM}^2 \frac{m_{\tilde{\eta}}}{m_\Upsilon^2}\left(1 - \frac{m_{\tilde{\eta}}^2}{m_\Upsilon^2}\right) \\ \left|\int \frac{d^3\vec{\mathbf{k}}d^3\vec{\mathbf{k}}'}{(2\pi)^6}\, \psi_{\tilde{\eta}}^*(\vec{\mathbf{k}}')\, \mathcal{I}(\vec{\mathbf{k}}',\vec{\mathbf{k}})\, \psi_\Upsilon(\vec{\mathbf{k}})\right|^2.\end{aligned} \tag{16}$$

where $\mathcal{I}(\vec{\mathbf{k}}',\vec{\mathbf{k}})$ is equal to $(32\pi^2)$ times the diagram of Fig. 5(b). To evaluate this diagram, combine denominators using Feynman parameters x_1–x_4 (clockwise from the bottom); we parametrize these in turn as: $x_1 = xy$, $x_3 = x(1-y)$, $x_2 = (1-x)z$, $x_4 = (1-x)(1-z)$. Then, after some algebra, we find

$$\mathcal{I} = \int_0^1 dx\, x(1-x) \int_0^1 dy\, dz\, \left[\frac{3}{2}\frac{1}{\Delta} + \frac{x^2(y\vec{\mathbf{k}} + (1-y)\vec{\mathbf{k}}')^2}{\Delta^2}\right], \tag{17}$$

where, if k and k' are, respectively, the b and F four-momenta,

$$\Delta = x^2 A^2 - (1-x)^2 B^2, \qquad \text{with}$$
$$A^2 = (yk + (1-y)k')^2, \; B^2 = z(1-z)m_{\tilde{\eta}}^2. \tag{18}$$

Note that A^2 and B^2 are both positive. From this expression, it appears that there might be large cancellations between the regions where Δ is positive and the regions where it is negative, leading to a sensitive dependence on $\vec{\mathbf{k}}$ and $\vec{\mathbf{k}}'$. But no such sensitivity is apparent in the decay of Υ states to an object of the mass of the ς. It is straightforward to perform the integral over x in (17):

$$I = \int_0^1 dy\, dz \left\{ \frac{3}{2}\frac{1}{(A+B)^2}\left[\frac{A^2+B^2}{(A-B)^2}\log\left(\frac{A}{B}\right) - \frac{A+B}{A-B} - i\frac{\pi}{2}\right] \right.$$
$$\left. + \frac{(y\vec{\mathbf{k}} + (1-y)\vec{\mathbf{k}}')^2}{(A+B)^3}\left[\frac{A^2+3B^2}{(A-B)^3}\log\left(\frac{A}{B}\right) - \frac{3A^2+B^2}{2A^2(A-B)^2} - i\frac{\pi}{2A}\right]\right\}. \tag{19}$$

Now I has a relatively smooth integrand, so that the remaining integrals may be done numerically. One finds that the real part of I has a value within 10% of (1.35×10^{-3}) over the whole range $0 < |\vec{\mathbf{k}}|, |\vec{\mathbf{k}}'| < 3$ GeV, a range which covers the bulk of the Υ and $\tilde{\eta}$ wavefunctions. In Fig. 6, we graph the real and imaginary parts of I as a function of $|\vec{\mathbf{k}}|$ for $|\vec{\mathbf{k}}'| = 0$ and 3 GeV. Unless one is prepared to take seriously that the singularity of I on the tail of the wavefunction might make a large contribution, it is difficult to arrange that $Z(8.3 \text{ GeV})$ be less than 0.8. We should note that the radiative decays of c-$\bar{\text{c}}$ states to η' and ι show a substantial suppression of the ψ' rate relative to the ψ [3,8] . This is not inconsistent with our analysis; the singularity evident in Fig. 6 occurs at $|\vec{\mathbf{k}}| = m_{\tilde{\eta}}/2$ and thus does greatly disturb the ratio $Z(m)$ if the mass of the $\tilde{\eta}$ is of order 1 GeV.

We have now assessed the two major effects of the b-$\bar{\text{b}}$ wavefunction on the radiative decay of an Υ state to a new massive resonance. We have found that the conventional picture, in which the decay amplitude is simply proportional to the wavefunction at the origin, should be quite accurate for any 0^- resonances, both those, such as the pseudoscalar Higgs, which couple directly to b-$\bar{\text{b}}$ and those, such as a bound state of new colored fermions, which couple indirectly through a 2-gluon process. For heavy 0^+ resonance, however, there is a substantial suppression of the Υ' decay rate relative to the prediction of this simple picture. We have argued that this suppression is plausibly, though marginally, able to account for observed suppression of the process $\Upsilon' \rightarrow \gamma + \varsigma$. We conclude* that the $\varsigma(8320)$ must have a scalar coupling to b-$\bar{\text{b}}$.

It is worth asking whether the parity of the ς can be determined directly from experiment. With sufficient statistics, the spin of the ς may be determined from the photon angular distribution† , but for the formation of both 0^+ and 0^- states, the angular distribution of the photons is proportional to $(1 + cos^2\theta)$. The photons are distinguished only by their polarization: Photons emitted at 90°to the beam direction, are completely polarized parallel to the beam axis in the 0^- case and perpendicular to the beam axis in the 0^+ case. This polarization is difficult to observe directly, but it may be observed indirectly through study of the Dalitz decay $\Upsilon \rightarrow \varsigma e^+e^-$* . In this process, the plane of a low-mass e^+e^- pair aligns preferentially with the direction of polarization of the parent virtual photon. Let $\mathcal{M}^2$ be the mass of the pair, $\widehat{3}$ be the beam axis, $\widehat{n}$ be a unit vector in the plane of the pair which is perpendicular to the virtual photon 3-momentum, and $\phi(\widehat{n})$ be an azimuthal angle, running from 0 to π, which specifies the direction of $\widehat{n}$. Then[30] ,

* Barring the possibility raised in the model of ref. 4 that the ς is not actually a decay product of the Υ.

† $(1 + a\cos^2\theta)$, with $a = 1, -\frac{1}{3}, \frac{1}{13}$ for $J = 0, 1, 2$

* For other methods of distinguishing the 0^+ and 0^- cases, see refs. 28, 29.

$$\frac{d\Gamma}{d\cos\theta d\phi(\hat{n})}(\Upsilon \to \varsigma e^+e^-) = \Gamma(\Upsilon \to \varsigma\gamma)\cdot\frac{\alpha}{2\pi^2}\int\frac{d\mathcal{M}^2}{\mathcal{M}^2}\Big(\frac{\sqrt{\lambda(m_\Upsilon^2, m_\varsigma^2, \mathcal{M}^2)}}{m_\Upsilon^2 - m_\varsigma^2}\Big)$$

$$\cdot\begin{cases} \frac{3}{2}+\frac{1}{2}\cos^2\theta - (\hat{n}\cdot\hat{3})^2 + \frac{\mathcal{M}^2}{E_\gamma^2}\sin^2\theta & (0^+) \\ (1-\frac{\mathcal{M}^2}{E_\gamma^2})(\frac{1}{2}+\frac{3}{2}\cos^2\theta + (\hat{n}\cdot\hat{3})^2) & (0^-) \end{cases}, \tag{20}$$

The rate for such Dalitz decays is about 10^{-3} of the rate for $\Upsilon \to \varsigma\gamma$, so this experiment would require a data sample of several million Υ's. This seems daunting for the near term, but this experiment should be kept in mind as a long-term goal for ς studies.

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Table 1

E1 transition matrix in the dipole approximation

$4\pi < m^3P_0 \,|\, r \,|\, n^3S_1 >$

	$1S$	$2S$	$3S$
$Pstate$	GeV^{-1}	GeV^{-1}	GeV^{-1}
$1P$	1.127	−1.637	0.013
$2P$	0.235	1.939	−2.656
$3P$	0.107	0.313	2.676
$4P$	0.0637	0.129	0.369
$5P$	0.0430	0.0716	0.144
$6P$	0.0314	0.0457	0.078
$7P$	0.0242	0.0326	0.049
$8P$	0.0193	0.0244	0.034
$\vdots$	$\vdots$	$\vdots$	$\vdots$

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FIGURE CAPTIONS

1. Amplitude for the process $\Upsilon \rightarrow \gamma$ + Higgs (a) in the hard-photon approximation and (b) in the soft-photon approximation

2. Behavior of the quantity $r(m_{\mathrm{H}})$, defined in the text, for the case of a pseudoscalar Higgs. The prediction for the Υ' branching ratio is given by multiplying $r'_{\Upsilon}(m_{\varsigma})$ by $(BR(\Upsilon \rightarrow \gamma + \varsigma)/r_{\Upsilon}(m_{\varsigma}))$. The dotted lines represent the prediction of phase space, or, equivalently, of the Wilczek formula for finite m_{H}. We have taken $BR(\text{cascade}) = 0$ for Υ, 0.45 for Υ'.

3. Behavior of the quantity $r(m_{\mathrm{H}})$ for a scalar Higgs, for decays from the Υ and Υ'. The notation is the same as in Fig. 2.

4. Behavior of the quantity $r(m_{\mathrm{H}})$ for a scalar Higgs, for decays from the Υ''. The notation is the same as in Fig. 2. We have taken $BR(\text{cascade}) = 0.5$ for the Υ''.

5. An approximation to the amplitude for formation of a 0^- bound state $\tilde{\eta}$ of new colored fermions in radiative Υ decays.

6. Behavior of $I(\vec{\mathbf{k}}', \vec{\mathbf{k}})$, computed using the recipe in the text, as a function of $|\vec{\mathbf{k}}|$ for $|\vec{\mathbf{k}}'| = 0$ (slid) and 3 GeV (dashed). The dotted curves show the shapes of $\psi_{\Upsilon}(|\vec{\mathbf{k}}|)$, $\psi_{\Upsilon'}(|\vec{\mathbf{k}}|)$, and $\psi_{\tilde{\eta}}(|\vec{\mathbf{k}}'|)$. The $\tilde{\eta}$ wavefunction was computed as the 1S level of a bound system of color octet fermions of mass 4.91 GeV, bound by a potential equal to $\frac{9}{4}$ of the Richardson potential.

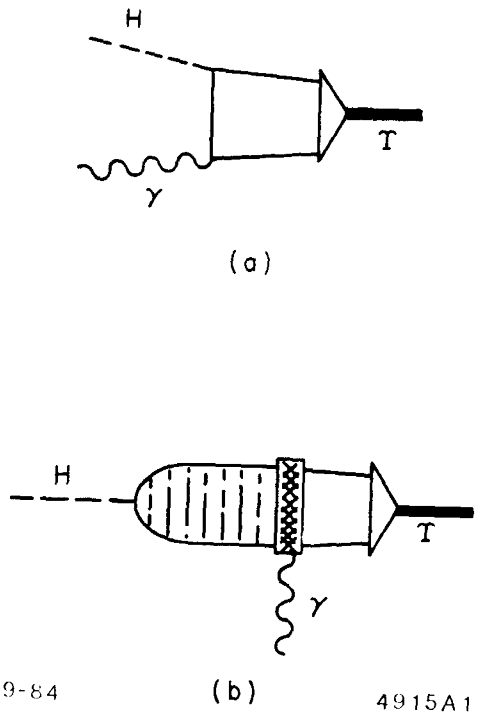
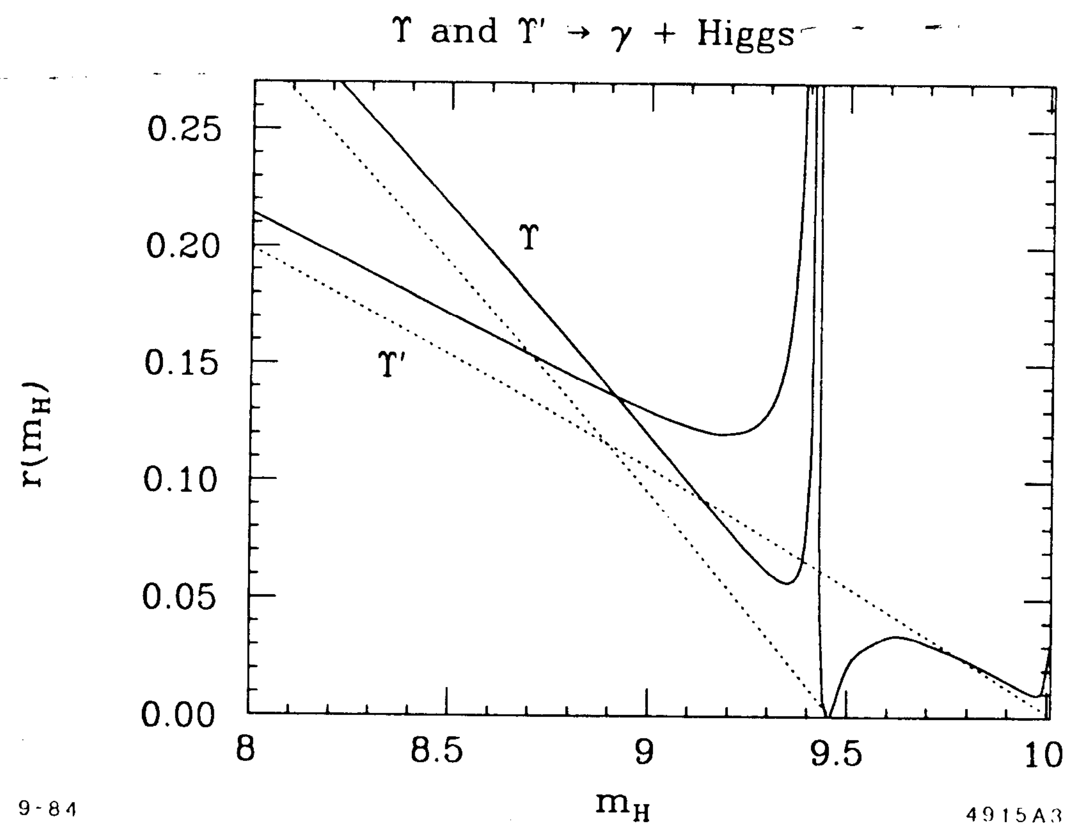
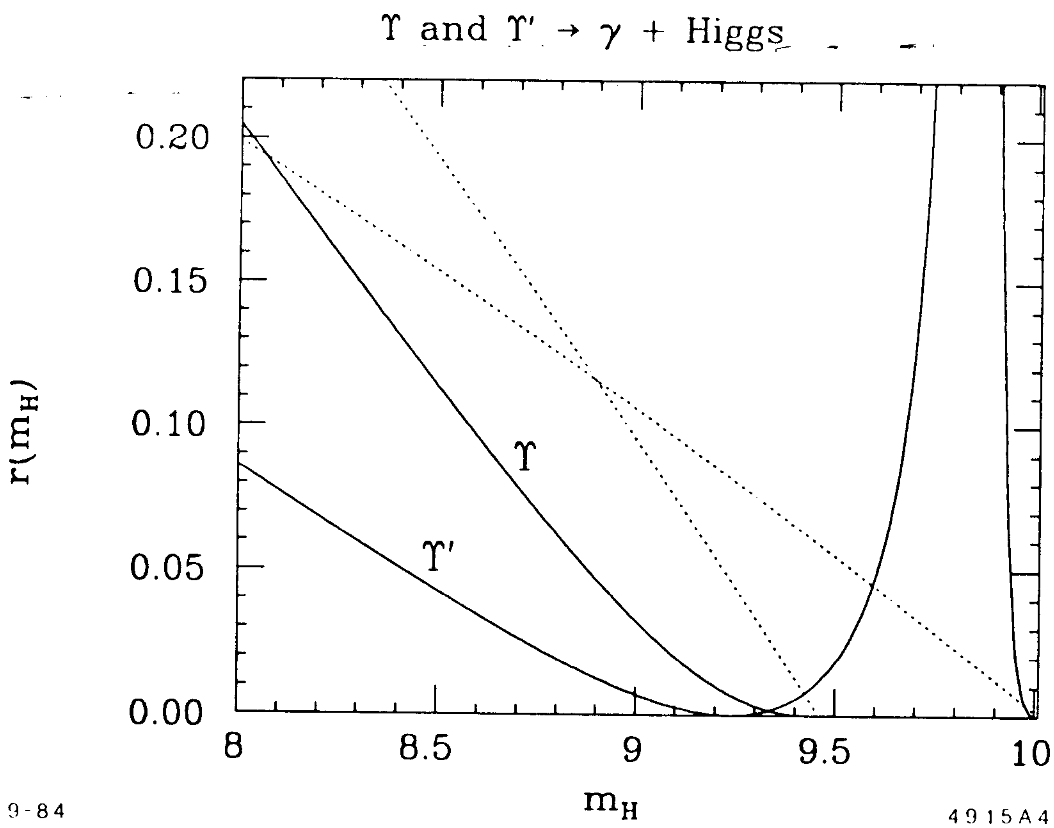


Figure 1



ϒ and ϒ′ → γ + Higgs
0.25
0.20
0.15
0.10
0.05
0.00
8
8.5
9
9.5
10
r(m_H)
m_H
ϒ
ϒ′
9-84
4915A3

Figure 2



Υ and Υ′ → γ + Higgs
0.20
0.15
0.10
0.05
0.00
r(m_H)
Υ
Υ′
8
8.5
9
9.5
10
m_H
9-84
4915A4

Figure 3

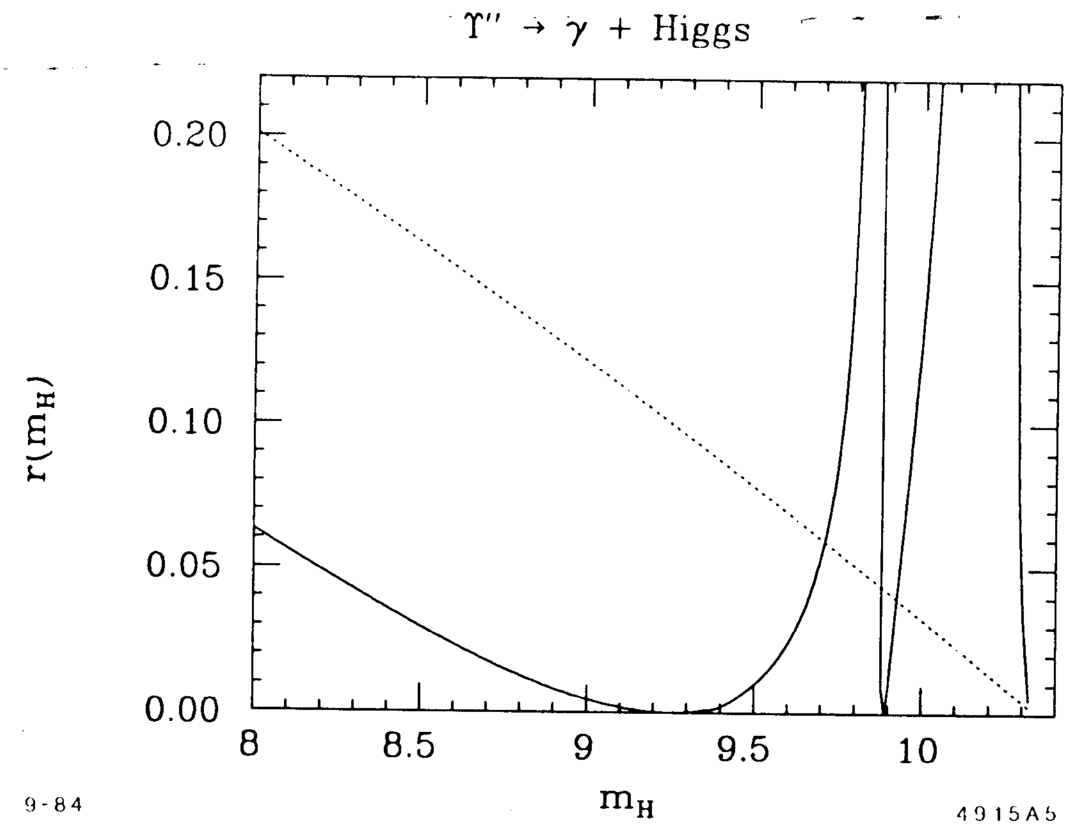


Figure 4

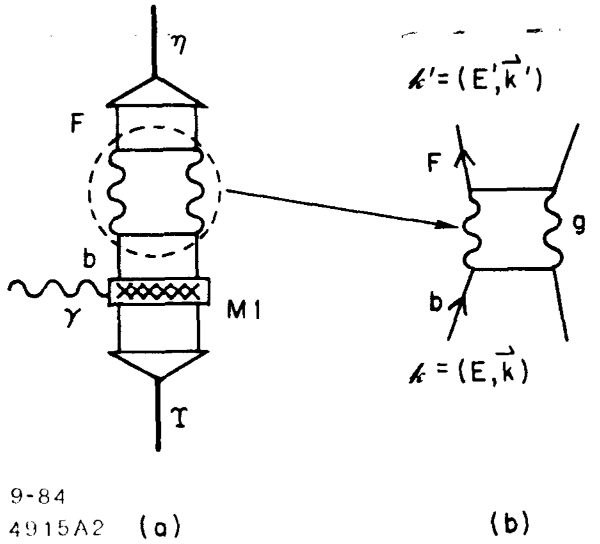


Figure 5

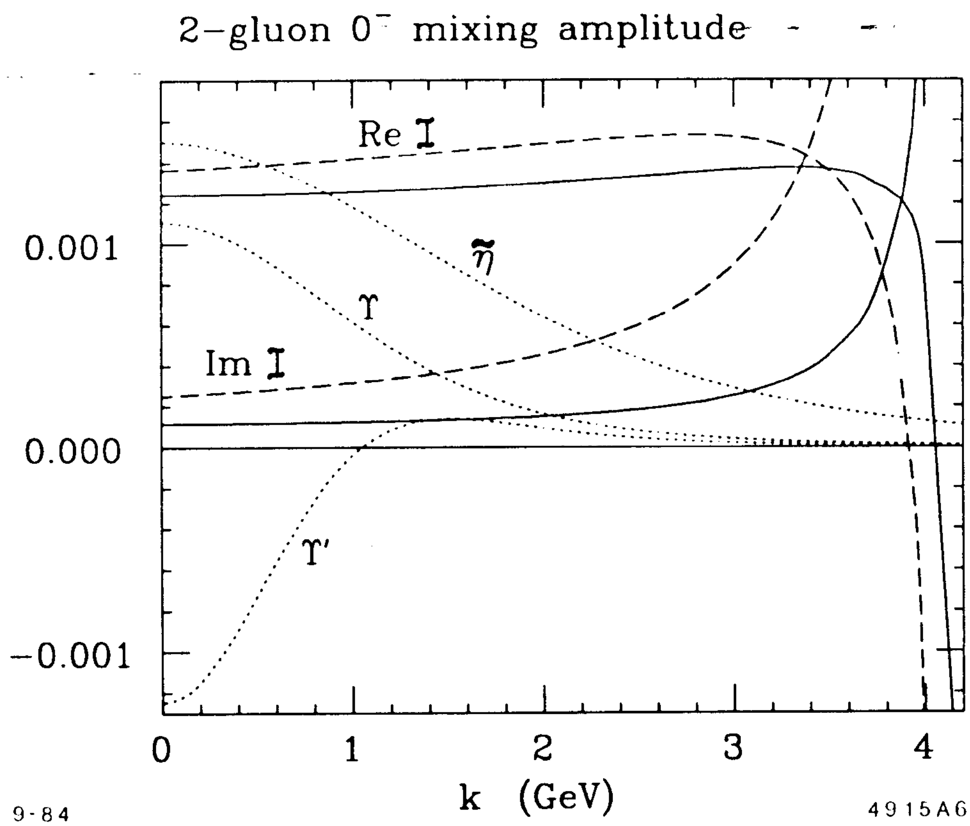

2-gluon 0⁻ mixing amplitude
Re I
$\tilde{\eta}$
Υ
Im I
Υ'
0.001
0.000
−0.001
0
1
2
3
4
k (GeV)
9-84
4915A6

Figure 6