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STATIC SOLUTIONS IN THE VACUUM SECTOR OF THE SKYRME MODEL*

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ABSTRACT

We use topological methods to prove the existence of a nontrivial static solution in the vacuum sector of the gauged Skyrme model. We then search for a purely strong-interaction saddle point by computing the interaction energy of a Skyrmion-anti-Skyrmion pair.

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1. Introduction

Classical solutions to nonlinear field equations can provide important insights into the nonperturbative regimes of quantum field theory. Recently Taubes has developed a powerful new tool for finding static classical solutions.[1] His method, although topological in origin, gives solutions that are distinct from the minimum energy configurations in the topological sectors to which they belong.

Taubes used his technique to prove the existence of a static saddle point solution in the SO(3) Yang-Mills-Higgs model. His solution can be thought of as a monopole-antimonopole pair in unstable equilibrium.[1] Subsequently, Manton used the same method to study the existence of saddle points in Weinberg-Salam theory.[2] The purpose of this report is to call attention to an analogous solution in the chiral soliton, or Skyrme, model.[3]

The quantum mechanical meaning of unstable classical solutions is an open theoretical question. If our vacuum solution is physically significant, it could be relevant to the low energy $\bar{p}p$ system. The Skyrme model opens the intriguing possibility that present-day experiments might shed some light on the question of saddle point quantization in quantum field theory.

This paper is organized as follows. In Sections 2 and 3 we review the gauged Skyrme model and discuss the topology of the vacuum sector. In Section 4 we prove the existence of a nontrivial static solution in the vacuum sector of this model. The existence of our solution relies crucially on the presence of the electromagnetic interactions. In Section 5 we compute the interaction energy of a Skyrmion-anti-Skyrmion pair. We use this result to search without success for a purely strong-interaction saddle point solution.

2. The Skyrme Model

The Skyrme model is a field theoretic realization of the observation that, in the large N_c limit, QCD baryons behave as though they are solitons in a phenomenological meson field theory.[4,5] The model is obtained by augmenting the standard nonlinear sigma model for N_F quark flavors by a particular four-derivative interaction,[3,6]

$$\mathcal{L} \;=\; \frac{F_\pi^2}{16}\,\mathrm{Tr}\,(\partial_\mu U\,\partial^\mu U^\dagger) \;+\; \frac{1}{32a^2}\,\mathrm{Tr}\,[(\partial_\mu U)U^\dagger,\,(\partial_\nu U)U^\dagger]^2\;, \tag{1}$$

where $U(x) \in \mathrm{SU}(N_F)$ and $U(x) \to 1$ as $|\vec{x}| \to \infty$. Since $\pi_3[\mathrm{SU}(N_F)] = Z$, static configurations $U(\vec{x})$ are classified by their topological charge

$$\mathcal{Q} \;=\; \int d^3x\; J^0, \tag{2}$$

where J^μ is the topological number current

$$J^\mu \;=\; \frac{1}{24\pi^2}\,\epsilon^{\mu\nu\rho\sigma}\,\mathrm{Tr}\,[\,U^\dagger\partial_\nu U\;U^\dagger\partial_\rho U\;U^\dagger\partial_\sigma U\,]\;. \tag{3}$$

The topological charge $\mathcal{Q}$ has been recently identified with the baryon number B.[7,8] In what follows we restrict our attention to $N_F = 2$. We reserve a comment on the case $N_F > 2$ for our concluding remarks.

The lowest energy configuration in each nontrivial topological sector is a static soliton solution. The lowest energy solution in the $B = 1$ sector is of the form

$$U_S(\vec{x}) \;=\; \exp(iF_S(r)\,\hat{x}\cdot\vec{\tau}), \quad r \;=\; |\vec{x}|\;, \tag{4}$$

where $F_S(r)$ solves the equations of motion, with $F_S(0) = \pi$ and $F_S(\infty) = 0$. Quantization of this solution, the Skyrmion, gives a reasonably accurate description of the static properties of the nucleon and delta.[9] Small oscillations about this solution can be used to analyze the strong interaction dynamics of a nucleon and pions at low energies.[10]

The Skyrme model can be extended to include electromagnetic interactions by gauging a U(1) subgroup of diagonal $SU(2)_V$.[11] The gauged Skyrme Lagrangian takes the following form,

$$
\begin{aligned}
\mathcal{L} \;=\; & \frac{F_\pi^2}{16}\,\mathrm{Tr}\,(\mathcal{D}_\mu U\,\mathcal{D}^\mu U^\dagger) \;+\; \frac{1}{32a^2}\,\mathrm{Tr}\,[(\mathcal{D}_\mu U)U^\dagger,\,(\mathcal{D}_\nu U)U^\dagger]^2 \;-\; \frac{1}{4}F_{\mu\nu}\,F^{\mu\nu} \\
& +\; \frac{e}{16\pi^2}\,\epsilon^{\mu\nu\rho\sigma}A_\mu\;\mathrm{Tr}\,Q\,(\partial_\nu U U^\dagger\,\partial_\rho U U^\dagger\,\partial_\sigma U U^\dagger + U^\dagger\partial_\nu U\,U^\dagger\partial_\rho U\,U^\dagger\partial_\sigma U) \\
& +\; \frac{ie^2}{8\pi^2}\,\epsilon^{\mu\nu\rho\sigma}(\partial_\mu A_\nu)\,A_\rho\,\mathrm{Tr}\,(Q^2\,\partial_\sigma U U^\dagger + Q^2\,U^\dagger\partial_\sigma U \\
& +\; \frac{1}{2}\,Q\,\partial_\sigma U\,Q\,U^\dagger - \frac{1}{2}\,Q\,U\,Q\,\partial_\sigma U^\dagger)\;,
\end{aligned}
\tag{5}
$$

where $\mathcal{D}_\mu U = \partial_\mu U - ieA_\mu\,[Q,\,U]$ and $Q = \mathrm{diag}(\frac{2}{3}, -\frac{1}{3})$. The last two terms in $\mathcal{L}$ describe anomalous pion-photon interactions.

In the presence of electromagnetism, the baryon number current (3) is not gauge invariant. The unique conserved gauge invariant current is given by

$$
\begin{aligned}
K^\mu \;=\; & \frac{1}{24\pi^2}\,\epsilon^{\mu\nu\rho\sigma}\;\mathrm{Tr}\,[\,U^\dagger\partial_\nu U\;U^\dagger\partial_\rho U\;U^\dagger\partial_\sigma U\,] \\
& +\; \frac{1}{24\pi^2}\,\epsilon^{\mu\nu\rho\sigma}\partial_\nu[\,3ie\,A_\rho\;\mathrm{Tr}\;Q\,(U^\dagger\partial_\sigma U \;+\; \partial_\sigma U U^\dagger)]\;.
\end{aligned}
\tag{6}
$$

Equation (6) differs from (3) by a total derivative, so that the topological charge $\mathcal{Q}$ of an SU(2) configuration is unchanged by the presence of a nonsingular electromagnetic field. We will consider only nonsingular configurations, and once again the space of static configurations breaks up into distinct topological sectors.

Because we are examining nonsingular static configurations, it is convenient to work in the Coulomb gauge, $\vec{\nabla} \cdot \vec{A} = 0$. For such configurations, the Coulomb condition completely fixes the local gauge freedom. Furthermore, the equation of motion for A^0 is a constraint. This allows one to solve for A^0 in terms of the dynamical variables $\vec{A}$ and U. To leading order in e, A^0 is just the Coulomb potential produced by the topological charge density $J^0(\vec{y})$,

$$A^0(\vec{x}) \;=\; \frac{e}{2} \int d^3\vec{y}\, \frac{J^0(\vec{y})}{|\vec{x} \,-\, \vec{y}|} \;. \tag{7}$$

All configurations of topological charge $\mathcal{Q}$ have electric charge $e\mathcal{Q}/2$.

After eliminating A^0, it is easy to find the potential as a function of the unconstrained variables U and $\vec{A}$. The extrema of this potential are static solutions to the full equations of motion, despite the fact that the Hamiltonian and the potential are nonlocal functionals of the unconstrained fields. In the Coulomb gauge, to order e^2, the potential energy is

$$\begin{aligned} E[U, \vec{A}] \;=\; \int d^3\vec{x} \,\Bigg[& \frac{F_\pi^2}{16} \,\mathrm{Tr}\,(\mathcal{D}_i U\, \mathcal{D}_i U^\dagger) \;-\; \frac{1}{32a^2}\,\mathrm{Tr}\,[(\mathcal{D}_i U)U^\dagger,\, (\mathcal{D}_j U)U^\dagger]^2 \\ & + \frac{1}{4} F_{ij}\, F^{ij} \;+\; \frac{e^2}{4}\, J^0(\vec{x}) \int d^3\vec{y}\, \frac{J^0(\vec{y})}{|\vec{x} \,-\, \vec{y}|} \;+\; \mathcal{O}(e^3) \Bigg] \;. \end{aligned} \tag{8}$$

To lowest order in e, the minimum of E in the $B = 1$ sector is just the static Skyrmion solution (4). The radial function F_S is known numerically; its essential features are a rapid falloff beyond a distance of order $(aF_\pi)^{-1}$ from the origin, and the asymptotic behavior[9] $F_S(r) \to 8.6/(aF_\pi r)^2$. The energy of this solution is

$$E[U_S, \vec{A} = 0] \;=\; \frac{F_\pi}{a}\,(G[U_S] \;+\; S[U_S]) \;=\; 36.4\, \frac{F_\pi}{a} \;\equiv\; E_S \;, \tag{9}$$

where

$$G[U] = \frac{aF_\pi}{16} \int d^3x \; \mathrm{Tr}(\partial_i U \, \partial_i U^\dagger)$$
$$S[U] = -\frac{1}{32aF_\pi} \int d^3x \; \mathrm{Tr}[(\partial_i U)U^\dagger, (\partial_j U)U^\dagger]^2 \; . \tag{10}$$

To higher orders in e, the lowest energy solution in the $B = 1$ sector remains essentially the Skyrmion (4). Symmetry considerations require that $\vec{A}_S(\vec{x}) = 0$. Electromagnetic effects give a small Coulomb contribution to the energy (8). The Coulomb contribution slightly deforms $F_S(r)$ and leads to a small change in the self-energy of the Skyrmion. This deformation will not affect our analysis, and we shall neglect it in what follows.

3. Topology of the Vacuum Sector

To find a nontrivial vacuum solution, we shall examine the topology of configuration space in light of the recent work of Taubes and Manton.[1,2] We shall use the term configuration space to denote the set of all finite energy static field configurations $U(\vec{x})$ and $\vec{A}(\vec{x})$. Because its topology does not depend on the nonsingular electromagnetic field $\vec{A}(\vec{x})$, configuration space breaks up into disjoint subspaces labelled by the topological charge associated with $\pi_3(\mathrm{SU}(2))$. We shall restrict our attention to the vacuum, or zero baryon number, sector.

Let us begin our discussion by considering a path in configuration space that begins and ends at the vacuum, and depends continuously on a parameter τ, $0 \leq \tau \leq \pi$:

$$U(\vec{x},\tau): \quad U(\vec{x},0) = U(\vec{x},\pi) = 1, \quad U(\vec{x},\tau) \to 1 \text{ as } |\vec{x}| \to \infty \quad . \tag{11}$$

This set of maps of $S^3 \to S^3$ provides a single map of $S^4 \to S^3$ and falls into one of the homotopy classes of $\pi_4(\mathrm{SU}(2)) = Z_2$. We see that there are two types of loops through configuration space, beginning and ending at the vacuum, and never leaving the zero baryon number sector: trivial, or contractible, loops, which can be continuously deformed to the vacuum; and nontrivial, or noncontractible, loops, which cannot be so deformed.

The significance of the noncontractible loops can be understood with the help of a simple example: the two-dimensional torus. On the torus, there are two classes of loops, as shown in Figure 1. The paths L_0 and L_1 are, respectively, contractible and noncontractible loops, beginning and ending at the point p. Now, standard results of Morse theory relate the critical points of a smooth function to the topology of the manifold over which it is defined. The archetypal example is given by the height function on the torus, with critical points p, q, r and s. The existence of the minimum, at p, and the maximum, at s, depends only on the compactness of the manifold. The saddle points at q and r, however, are contingent on the torroidal topology, and can be inferred from the existence of the noncontractible loops. For example, the height of the saddle point at q can be found by minimizing, over the set of all noncontractible loops, the maximum height attained on each loop.[12]

Returning to the Skyrme model, we define a "height function" – the energy functional $E[U, \vec{A}]$ – on the manifold of the vacuum sector. As we have seen, this space has noncontractible loops, along which we can compute the maximum value attained by the energy functional. By minimizing this maximum E over all such loops, we might expect to find a configuration for which E is stationary. Such a solution would have zero topological charge, yet have $E > 0$.

There is, however, a potential loophole in this procedure. For the infinite-dimensional manifold encountered in field theory, it is not at all clear that the "mini-max" procedure converges. Figure 2 illustrates this possibility for a non-compact two-dimensional manifold; although there are noncontractible loops, no sequence of such loops with decreasing maximum heights converges to a saddle point.

Taubes investigated the difficulties arising from noncompactness in the SO(3) Yang-Mills-Higgs model and managed to rigorously generalize, and prove the convergence of, the mini-max procedure that we have described. Although the details of his proof are technical, Taubes found that the convergence issue has a clear physical interpretation: one must ensure that the mini-max procedure converges to a new static solution, rather than to a soliton-antisoliton pair infinitely far apart. In the next two sections, we will exploit this physical picture to analyze the convergence problem for the Skyrme model.

4. Electromagnetic Saddle Points

In this section we will deduce the existence of an electromagnetic saddle point solution in the gauged Skyrme model. To do this we shall first discuss the convergence of the mini-max procedure from a physical point of view. The mini-max procedure relies on noncontractible loops in the vacuum. These loops are a familiar feature of the SU(2) Skyrme model; they permit the soliton to be quantized as a fermion.[7,13] For this application, $\pi_4(\mathrm{SU}(2))$ distinguishes trivial from nontrivial field configurations in compactified spacetime. Nontrivial field configurations are homotopically equivalent to the following history: from the vacuum, moving forward in time, a soliton-antisoliton pair is extracted and separated; the soliton is then rotated by 2π relative to the antisoliton; finally, the pair is brought together back and annihilated. The field which describes this process obeys $U(\vec{x},t) \to 1$ for either $|\vec{x}| \to \infty$ or $t \to \pm\infty$ and lies in the nontrivial class of $\pi_4(\mathrm{SU}(2))$ by virtue of the 2π rotation. Evidently, a twisted field of this sort is an example of a noncontractible loop, once t is identified with the compact parameter τ.

To determine whether a classical solution must exist, we need only explore the convergence of the mini-max procedure. Convergence, in turn, depends only on the behavior of the energy functional at large soliton-antisoliton separations. If the energy increases with separation for all soliton-antisoliton orientations, the minimax procedure must converge. If the energy decreases for some orientations, one cannot be sure that a new solution does, in fact, exist. In this case it is possible that mini-max procedure gives a sequence of configurations that approach a Skyrmion-anti-Skyrmion pair infinitely far apart – a trivial and uninteresting result. We shall denote this limiting solution by U_∞.

Therefore, we must investigate the behavior of the energy functional in the vicinity of U_∞. Configurations in this neighborhood are accurately parametrized by the distance between the centers of the Skyrmions, their relative isospin orientation, and an overall orientation that is irrelevant to classical physics in the static limit. This leads us to consider fields of the form

$$U_{S\bar{S}}(\vec{x}) \;=\; U_S(\vec{x}-\vec{x}_1)\, B(\vec{\beta})\, U_S^\dagger(\vec{x}-\vec{x}_2)\, B^\dagger(\vec{\beta}) \tag{12}$$

where $B(\vec{\beta}) = \exp(i\vec{\beta}\cdot\vec{\tau}/2)$ and $U_S(\vec{x})$ is the Skyrme solution. For large $d = |\vec{x}_1 - \vec{x}_2|/2$, distortions of the Skyrmion field can be neglected – they give subleading corrections to the energy (8).

In the absence of electromagnetism, the interaction of a well-separated Skyrmion-anti-Skyrmion pair is governed by one pion exchange. For massless pions, we shall show that the potential falls as $1/d^3$. Therefore the Coulombic interaction between the oppositely charged solitons dominates for sufficiently large d. This interaction is attractive and independent of isospin orientation. Since the long-range force is attractive for all isospin orientations, the mini-max procedure must converge to a nontrivial classical solution.

5. Strong Interaction Saddle Points

The existence of the saddle point found in Section 4 relies essentially on electromagnetic interactions. Therefore, one expects the saddle point solution to describe a Skyrmion and an anti-Skyrmion quite far apart in unstable equilibrium, with the weak, attractive Coulomb force balanced by a repulsive force. It is unlikely that such a configuration will be important in $\bar{p}p$ annihilation. A purely strong interaction saddle point would be of greater significance.

The asymptotic strong interaction of a Skyrmion-anti-Skyrmion pair is not attractive for all orientations. As d tends to infinity, $U_{S\bar{S}}$ approaches U_∞, and $E[U_\infty, \vec{A} = 0] = 2E_S$, independent of the orientation $\vec{\beta}$. From the asymptotic falloff of $F_S(r)$, one can work out the leading correction to E for finite d:

$$\overline{E}[U_{S\bar{S}}, \vec{A} = 0] = 2E_S - \frac{4\pi}{3}(8.6)(\kappa_G + \kappa_S)\sin^2\frac{\beta}{2}\,\frac{3(\hat{\beta}\cdot\hat{d})^2 - 1}{(2aF_\pi d)^3} + \mathcal{O}((aF_\pi d)^{-4})\,, \tag{13}$$

where

$$\kappa_G = -aF_\pi^3 \int_0^\infty dr\, r^2 \left(\frac{\sin 2F_S}{r} + \frac{dF_S}{dr}\right) ,$$

$$\kappa_S = -\frac{2F_\pi}{a} \int_0^\infty dr\, r^2 \left[\frac{\sin 2F_S}{r}\left(\frac{dF_S}{dr}\right)^2 + \frac{2\sin^2 F_S}{r^2}\frac{dF_S}{dr} + \frac{\sin 2F_S \sin^2 F_S}{r^3}\right] \tag{14}$$

and $\kappa_G + \kappa_S = 11.2\ (F_\pi/a)$. The dipole form indicates that the expansion (13) recovers one pion exchange. For $\hat{\beta}\cdot\hat{d} > 1/\sqrt{3}$ and $\beta \neq 0$, the long range potential is attractive, and for $\hat{\beta}\cdot\hat{d} < 1/\sqrt{3}$ and $\beta \neq 0$, the long range potential is repulsive. Therefore one cannot infer the existence of a strong-interaction saddle point from asymptotic arguments alone.

The convergence of the mini-max procedure can be established by exhibiting a noncontractible loop on which the maximum energy is less than $2E_S$. For simplicity, we first consider the static configurations of equation (12), with $\hat{\beta} = \hat{d}$. Asymptotically, $\hat{\beta} = \hat{d}$ is the most attractive orientation, Note, however, that the asymptotic interaction vanishes to order $1/d^3$ for $\beta = 0$.

The energy of such static configurations can be determined numerically for all values of β and d. In Figure 3 we have plotted the energy surface for $0 \leq d \leq \infty$ and $0 \leq \beta \leq 2\pi$. As $d \to \infty$, the energy approaches $2E_S = 72.8(F_\pi/a)$, independent of the orientation β. The vacuum, at $d = 0$ and $\beta = 0$, or at $d = 0$ and $\beta = 2\pi$, has energy zero. For large d and $\beta \neq 0$, there is a weak attraction, reflecting one pion exchange. Note that the energy surface is not symmetric about the line $\beta = \pi$, although $\beta = 0$ is equivalent to $\beta = 2\pi$. Noncontractible loops run from $d = 0,\ \beta = 0$ to $d = 0,\ \beta = 2\pi$.

As we see in Figure 3, the energy along $\beta = 0$ decreases monotonically from

$aF_\pi d = 3$ to $aF_\pi d = \infty$. Therefore, the mini-max procedure does not converge within this restricted subspace. This is indicated by the noncontractible loop of Figure 3. Its maximum energy is at $\beta = 0$, $d = \infty$, where $E = 2E_S$. No other noncontactible loop has a lower maximum energy. The configuration approached by applying mini-max within this limited space is that of a Skyrmion and an anti-Skyrmion infinitely far apart, with relative orientation $\beta = 0$.[14]

Although the configuration space we have studied in this section does not contain a loop with maximum energy less than $2E_S$, it is possible that such a loop might exist. If so, it would indicate the existence of a strong interaction saddle point solution that might be related to the broad resonance C(1620) observed at LEAR.[15]

Along the line $d = 0$ it is easy to extremize the energy with respect to variations in $F(r)$. It turns out that the optimal energy is given by a simple rescaling of F_S,

$$\begin{aligned} F(r) &= F_S(r(\sqrt{2}|\sin\frac{\beta}{2}|)^{-1}), \\ E_\beta &= \frac{8}{3}\sqrt{2}\,|\sin\frac{\beta}{2}|^3\,E_S\ . \end{aligned} \tag{15}$$

At $\beta = \pi$, $U(\vec{x})$ takes the following form,

$$U_{S\bar{S}}(\vec{x}) = U_S(\frac{\vec{x}}{\sqrt{2}})\ \hat{\beta}\cdot\vec{\tau}\ U_S^\dagger(\frac{\vec{x}}{\sqrt{2}})\ \hat{\beta}\cdot\vec{\tau}\ . \tag{16}$$

The energy of this configuration is stationary against variations in β, d and F. However, contrary to a recent claim in the literature,[16] (16) is not a solution to the full equations of motion. The configuration (16) satisfies the equations of motion only in the $\hat{\beta}\cdot\hat{x} = 0$ plane.

With the help of Figure 3 one can get a rough idea of the role of electromagnetism in producing the saddle point. To do this one must alter the potential of Figure 3 by including the long-distance Coulomb attraction between the Skyrmion-anti-Skyrmion pair,

$$\Delta E \;=\; -\frac{e^2}{4d}\,. \tag{17}$$

If this attraction is balanced by the repulsive component of the strong interaction, then one expects a saddle point in the vicinity of $\beta = 0$, $2d \gtrsim 16\,(aF_\pi)^{-1} \simeq 5$ fm, with energy on the order of $2E_S \simeq 1.7$ GeV.[17]

In conclusion, we have shown that a classical saddle point solution exists in the vacuum sector of the gauged Skyrme model. The existence of the solution does not depend on which higher-derivative terms one adds to the basic sigma model Lagrangian. Note that our result does not depend on the restriction $N_F = 2$. A solution for $N_F = 2$ can always be embedded in a model with $N_F > 2$ at the expense of possibly introducing extra unstable directions.[18]

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the mini-max procedure still fails to converge when working within this larger configuration space.

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16. K. Fujii, S. Otsuki, F. Toyoda, Kyushu Preprint KYUSHU-84-HE-5 (October, 1984).
17. We have used the values of a and F_π found in Ref. 9.
18. When $N_F = 3$, the strange quark mass should help stabilize a saddlepoint in the additional directions in configuration space.

FIGURE CAPTIONS

1. Critical points p, q, r, and s of the height function on the two-torus. L_0 and L_1 are, respectively, contractible and noncontractible loops.

2. A non-compact manifold with noncontractible loops but no saddle point.

3. Equipotentials of the static energy $E[U]$, with $e^2 = 0$, for the Skyrmion-anti-Skyrmion pair given in equation (12). Energy is in units of F_π/a. The separation $d = |\vec{x}_1 - \vec{x}_2|/2$ is half the distance between the Skyrmion centers and is given in units of $(aF_\pi)^{-1} \cong 0.3$ fm. We have set $\hat{\beta} = (\vec{x}_1 - \vec{x}_2)/|\vec{x}_1 - \vec{x}_2|$ so that β is the relative orientation of the Skyrmions about the axis joining their centers. The broken line represents a noncontractible loop along which the maximum energy attained is a minimum for this restricted space of configurations.

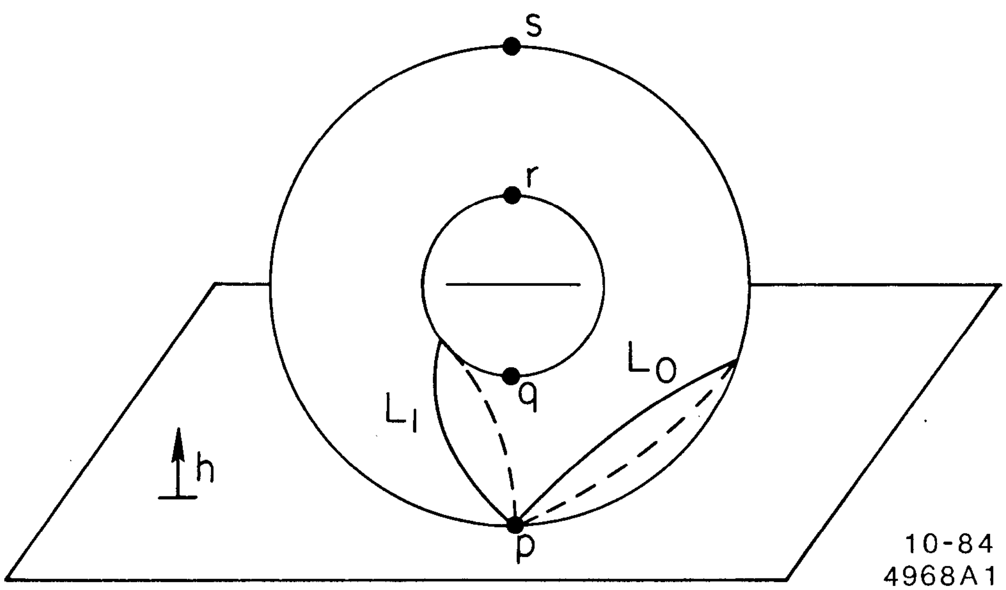


Fig. 1

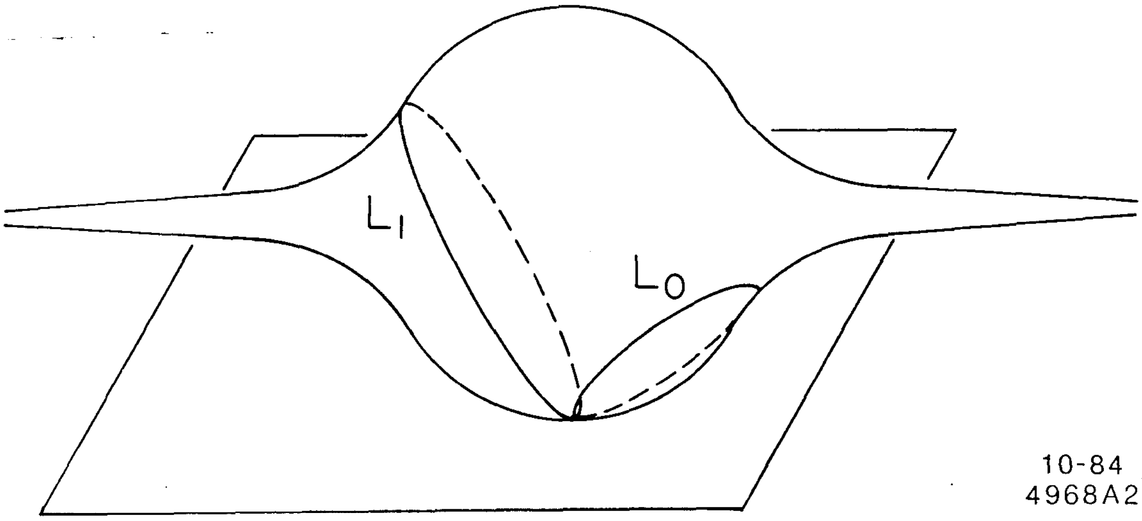
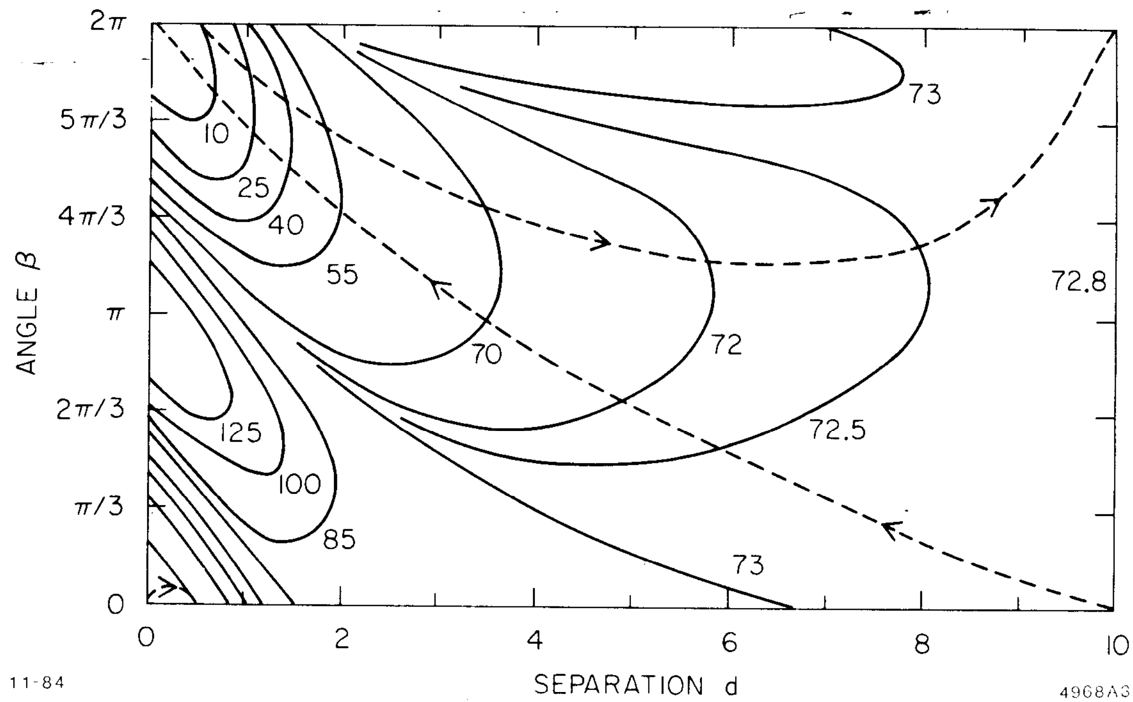


Fig. 2



2π
5π/3
4π/3
ANGLE β
π
2π/3
π/3
0
0
2
4
6
8
10
SEPARATION d
11-84
4968A3
10
25
40
55
70
72
72.5
72.8
73
73
125
100
85

Fig. 3