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PHASE DISPLACEMENT ACCELERATION IN THE SSC*

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1. Introduction

The SSC (Superconducting Super Collider) is supposed to operate with bunched beams and the energy radiated by synchrotron radiation is replaced by the rf-system having a synchronous phase angle slightly different from 180°. Here the case of operating the SSC with a coasting beam is considered. In particular the possibility of replacing the radiated energy by phase displacement acceleration is investigated.

2. Basic Theory of Phase Displacement Acceleration[1,2]

In phase displacement acceleration an empty rf-bucket is moved through the beam from the high to the low energy side. This displaces the beam to higher energies by an amount which is equal to the area A_b of the bucket (in $\Delta E, \phi$) divided by 2π. The energy gain per sweep is therefore

$$\Delta U = \frac{A_b}{2\pi} = \frac{16}{2\pi}\sqrt{\frac{e\ V\ E}{2\pi\alpha h}}\ \alpha(\Gamma) = \Delta U_0\ \alpha(\Gamma) \qquad (1)$$

Here V =rf voltage, E = beam energy, α = momentum compaction factor, h = harmonic number and $\Gamma = \sin\ \phi_s$ where ϕ_s is the synchronous phase angle. The function $\alpha(\Gamma)$ is the ratio between of area of moving and the stationary ($\Gamma = 0$) bucket shown in Fig. 1. The energy gain per sweep ΔU_0 for vanishing Γ is determined completely by the rf hardware and the choice of V

$$\Delta U_0 = \frac{16}{2\pi}\sqrt{\frac{e\ V\ E}{2\pi\alpha h}}$$

while $\alpha(\Gamma)$ can be changed by the choice of the synchronous phase angle ϕ_s. For the derivation of (1) the ultra-relativistic case was assumed ($\beta \approx 1$).

The energy half-height of the bucket is

$$\Delta E = \sqrt{2}\sqrt{\frac{e\ V\ E}{2\pi\alpha h}}\ Y(\Gamma) \ = \ \frac{2\pi\sqrt{2}}{16}\ \Delta U_0\ Y(\Gamma) \qquad (2)$$

where $Y(\Gamma)$ is a function plotted in Fig. 1.

In order to replace the energy loss U_s per turn due to synchrotron radiation by phase displacement acceleration a number $\dot{n}$ of sweeps per second is necessary

$$\dot{n} \ = \ \frac{f_0\ U_s}{\Delta U} \ = \ \frac{U_s\ f_0}{\Delta U_0 \alpha(\Gamma)} \qquad (3)$$

where f_0 is the revolution frequency.

Sweeping an empty bucket through the beam leads to an increase of the energy spread.

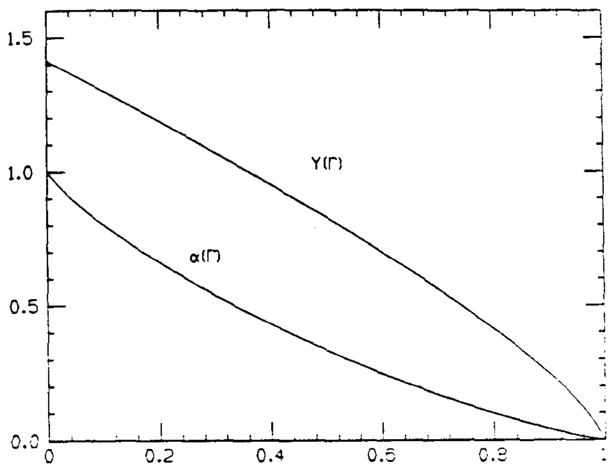

Fig. 1. $\alpha(\Gamma)$ and $Y(\Gamma)$ as a function of $\Gamma = \sin(\phi_s)$, (Ref. 3)

If $(\sigma_E/E)_i$ is the relative rms energy spread before the sweep its value $(\sigma_E/E)_{i+1}$ thereafter is given by

$$\left(\frac{\sigma_E}{E}\right)^2_{i+1} \ = \ \left(\frac{\sigma_E}{E}\right)^2_i \ + \ \left(\frac{\Gamma\ \Delta U_0}{E}\right)^2$$

Since a number $\dot{n}$ of sweeps per second are necessary the energy spread will increase with time t from its initial value $(\sigma_E/E)_0$

$$\left(\frac{\sigma_E}{E}\right)^2 \ = \ \left(\frac{\sigma_E}{E}\right)^2_0 \ + \ \frac{f_0\ U_s\ \Delta U_0\ t}{E^2}\ \frac{\Gamma^2}{\alpha(\Gamma)} \qquad (4)$$

The total frequency excursion Δf_{rf} of the sweeping rf-bucket has to be sufficiently large to separate the beam from the bucket including its height ΔE and assuming a half-spread of the energy in the beam of $N\sigma_E$

$$\Delta f_{rf} \approx \ 2\left(N\frac{\sigma_E}{E} \ + \ \frac{\Delta E}{E}\right)\ \alpha f_{rf}$$

$$= \ 2\left(N\frac{\sigma_E}{E} \ + \ \frac{2\pi\sqrt{2}}{16E}\ Y(\Gamma)\Delta U_0\right)\alpha f_{rf}$$

The rate of change of the rf-frequency is

$$\frac{d\ f_{rf}}{dt} \ = \ \frac{h\ f_0^2\,\alpha\, e\ V\ \Gamma}{E}$$

The time τ necessary for one sweep is therefore

$$\tau \ = \ \frac{\Delta f_{rf}}{df_{rf}/dt} \ = \ \frac{2\left(N\ \sigma_E \ + \ \frac{2\pi\sqrt{2}}{16}\ Y(\Gamma)\Delta U_0\right)}{f_0\ e\ V\ \Gamma}$$

The fraction r of time spent to sweep buckets through the beam

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is

$$r = \tau \dot{n} = \frac{2\,U_s\left(N\sigma_E + \frac{2\pi\sqrt{2}}{16}\,Y(\Gamma)\Delta U_o\right)}{\Delta U_0\, eV\,\Gamma \cdot \alpha(\Gamma)} \qquad (5)$$

Obviously r has to be smaller than 1. This limits the maximum energy spread of a beam which can still be accelerated by phase displacement with a given rf-system

$$\left(\frac{\sigma_E}{E}\right)_{max} = \frac{\Delta U_0}{NE_0}\left(r\frac{eV}{2U_s}\,\Gamma \cdot \alpha(\Gamma) - \frac{2\pi\sqrt{2}}{16}\,Y(\Gamma)\right) \qquad (6)$$

3. Application to the SSC

The above equations are now applied to the compensation of the energy loss due to synchrotron radiation by phase displacement acceleration in the SSC. The following parameters for this machine are assumed:

$E = 20$ TeV, $f_o = 3.3$ kHz, $\alpha = 1.28 \times 10^{-4}$

$U_s = 1.22 \times 10^5$ eV, $f_{rf} = 356$ MHz, $h = 1.08 \times 10^5$

$V = 20$ MV, $(\sigma_E/E)_0 = 5 \times 10^{-5}$

This gives for the maximum energy gain ΔU_0 per sweep (1), $\Delta U_0 = 5.46$ GeV.

The energy acceptance of the machine is of the order of $\pm 3 \times 10^{-4}$. Assuming that 4 rms widths have to fit into this we get

$$\left(\frac{\sigma_E}{E}\right)_{machine} \leq 7.5 \times 10^{-4}$$

For the maximum energy spread in the beam (6) which can still be accelerated by phase displacement with the available rf-voltage one finds

$$\left(\frac{\sigma_E}{E}\right)_{phase\ displacement} = r \times \frac{3.9 \times 10^{-3}}{N} = r\,9.6 \times 10^{-3}$$

During the actual phase displacement the background is usually large and data taking could be difficult for many experiments. We will in the following assume at least 50% efficiency for physics and demand $r \leq 0.5$. This gives for the maximum energy spread

$$\left(\frac{\sigma_E}{E}\right) \leq 4.8 \times 10^{-3}$$

obtained with $\Gamma = 0.41$. Equation (4) indicates that the energy blow-up is fast for large values of Γ. It is therefore desirable to operate with a Γ which is as small as possible but still allows acceleration with $r = 0.5$. As time goes on the energy spread increases and Γ has to be increased to satisfy (6). Using such a program one finds that for the available rf-parameters the phase displacement acceleration can only be maintained for $\sim$ 40 min. This is obviously not interesting except for some trials.

The situation can be improved by increasing the rf-voltage beyond the 20 MV foreseen for the SSC. Choosing rather arbitrarily $V_{rf} = 80$ MV and keeping $\Gamma = 0.05$ fixed one can maintain acceleration for about 5 hours while the fractional time r used for acceleration goes from $r = 0.05$ at the beginning and $r = 0.38$ at the end of the run. By varying Γ during the run one could easily maintain acceleration for over 10 hours.

4. Conclusion

With the present rf-system the energy of a beam of 20 TeV in the SSC could only be kept constant by phase displacement acceleration for less than one hour. Increasing the rf-voltage by a factor 4 could increase this time to over 10 hours. The energy spread in the beam will increase slowly due to phase displacement acceleration. The time spend for acceleration is sizable and can probably not be used for physics data taking. A problem not treated here is the life time due to quantum jumps into the rf-bucket. It is probably long but should be checked. In general the compensation of the energy loss due to synchrotron radiation by phase displacement acceleration does not look attractive. It might be better to decrease the magnetic field in the magnets to follow the decreasing beam energy.

References

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3. C. Bovet et al., CERN/MPS-SI/Int. DL/70/4 (1970).