SLAC – PUB – 3395
August 1984
(T)

$\Delta I = 1/2$ RULE AS A LONG – DISTANCE EFFECT?*

H. GALIĆ†

Stanford Linear Accelerator Center
Stanford University, Stanford, California, 94305

ABSTRACT

A model based on an effective field theory is described. The model allows a simple treatment of nonleptonic decays of mesons. The origin of the octet rule is placed in a framework of long–distance dynamics.

Submitted to *Physics Letters B*

---

* Work supported in part by the Department of Energy, contract DE-AC03-76SF00515, and by the US National Science Foundation and the SIZ-I of S.R. Croatia, grant YOR82/051.

† On leave of absence from the Rudjer Bošković Institute, Zagreb, Croatia, Yugoslavia.

1. Once the PENGUIN operators were introduced [1], the $\Delta I = 1/2$ rule (or "octet" rule) in $K \to \pi\pi$ decays seemed to be explainable as a short–distance phenomenon. Some problems remained unresolved (*e.g.,* whether the current algebra (CA) contribution contains the so called separable contribution or not, should the coefficients of PENGUIN operators be taken as given by the renormalization group analyses or rather as free parameters, etc.), but with reasonable assumptions on such open questions one could get a very good agreement with experiments [2,3]. However, somewhat unexpected failure of the simple standard scenario in D decays forced theoreticians to look for a refinement of analyses. Since then various other possible mechanisms influencing the weak amplitudes were considered. Particularly, the long–distance effects attracted a lot of attention. While it is now accepted by many authors that these effects should be considered more seriously, the disagreement still exists in the estimates of the degree of their importance. The entire spectrum of opinions can be found, ranging from beliefs that the long–distance contributions are incalculable but probably small, to the views that these contributions are calculable and moreover dominant. The latter conclusion [4 - 7], if it comes out to be correct, turns upside down all previous analyses of nonleptonic decays based on the short–distance dominance, and places the explanation of $\Delta I = 1/2$ rule in a completely different context. So far, however, there are no convincing arguments for any of these different opinions, and both standard and nonstandard approaches coexist in the literature.

2. Consider for a moment one of the papers, [6], representing the nonstandard stream. (Similar conclusions follow from considerations of other papers [4 - 7].) The arguments supporting the long–distance dominance in ref. 6 rely on

PCAC, dispersion relations and the asymptotic Regge behavior. Though based on well established facts of hadronic physics, these arguments might look not too impressive to an audience accustomed to the simplicity and elegance of the standard procedure. Furthermore, the short– and long–distance physics are in ref. 6 separated (by means of a cutoff) although such separations are questionable.[♯1] It would be interesting therefore to exhibit the importance of long distances in a simpler and less diverse way. In this work some steps in this direction are described.

The simplest scenario in which the long–distance physics determines the properties of weak K decays is one in which the only role of gluons is to form bound quark–antiquark states. In other words, in such a framework gluons would be allowed to create effective meson–quark–antiquark vertices, but not to participate in the radiative corrections. Imagine thus a situation in which by some unknown mechanism all gluons other than those implicitly taken into account in (generally unknown) vertex functions, are suppressed. Can one in such a simple framework at least qualitatively understand and describe decays of kaons in a proper way? The answer suggested in what follows is affirmative.

3. The main tool in the analysis will be an effective field theory in which mesons are coupled to quark – antiquark states. It will be assumed that once the valence vertices are formed, all QCD corrections as well as higher Fock states are negligible. In such a model $K \to \pi\pi$ decays can be described directly (in terms of diagrams), and with no use of CA reductions or similar tools of the standard analysis. Still, the scheme proposed is not the usual valence–quark scheme. The

♯1 Some indications that a factorization on hard and soft contributions is not possible can be found in ref. 8.

first difference is the absence of explicit gluons; while one usually implants hard QCD corrections into the pure valence–quark picture, here I shall argue that once the gluons in diagrams are allowed it becomes inconsistent to neglect higher Fock states. The consistent procedure, which preserves the gauge invariance, should be: either consider radiative corrections, but than take also higher Fock states into account, or neglect gluons altogether. The second important difference is related to types of vertices: in addition to the regular vertice (fig. 1) formed by interactions of gluons and constituent quarks, another types of "anomalous" vertices are a genuine part of the model. They appear whenever, due to the W exchange, a direct $s \to d$ (or $d \to s$) transition happens within the confinement radius (fig. 2). An object with a "wrong" flavor is formed in such a way. It is easy to see that in the scale in which regular vertices are of order 1 (one), the anomalous vertices are[♮2] of order $(m_M/M_W)^2 \sin\vartheta \cos\vartheta$, m_M being the mass of a meson. Therefore new vertices are not important *e.g.,* in $K_{\ell 2}$ decays (where they are largely suppressed), but on the other hand contribute on equal footing with regular vertices in $K \to \pi\pi$ and some other processes (*e.g.,* in rare K decays). Regular and anomalous vertices relevant to the analysis of the $\Delta I = 1/2$ rule are presented in figs. 3 and 4, and the set of diagrams contributing to $K \to \pi\pi$ decays in the leading $1/M_W^2$ order are given in figs. 5 and 6. Although the exact form of vertex functions is unknown, on the basis of various symmetries, one can determine their Lorentz structure. Furthermore the asymptotic behavior of vertex functions is determined also, by the finiteness of $M_{\ell 2}$ decays. Finally, effective vertices used here are relativistic objects (as opposed to vertices in some

♮2 For simplicity, I am using the four–flavor model with the isospin symmetry. Masses of lightest quarks are assumed to be equal, $m_u = m_d$.

other similar but nonrelativistic descriptions; see *e.g.,* ref. 9), and introduce dynamics into the scheme.

4. Consider the first diagram in fig. 5. If the momenta of the π^{o} (π^{+}) are $P_1(P_2)$, and quark propagators are denoted by S, then the matrix element from this diagram is

$$
\begin{aligned}
X = & \frac{G}{\sqrt{2}} \sin\vartheta \; \cos\vartheta \; 9 \int \frac{d^4k}{(2\pi)^4} \operatorname{Tr}\left[S_u\left(k - \frac{P_1 - P_2}{2}\right)\right. \\
& \times \frac{1}{\sqrt{2}} \Gamma_\pi \; S_u \left(k + \frac{P_1 + P_2}{2}\right) \Gamma_K \; S_s \; \left(k - \frac{P_1 + P_2}{2}\right) \gamma_\mu \; (1 - \gamma_5)\Big] \\
& \times \int \frac{d^4k'}{(2\pi)^4} \operatorname{Tr}\left[S_d(k' - P_2) \; \Gamma_\pi \; S_u(k')\gamma^\mu(1 - \gamma_5)\right] \\
& + O(1/M_W^4) \quad .
\end{aligned}
\tag{1}
$$

Rather surprisingly, it comes out that <u>all</u> diagrams with regular vertices (fig. 5) are either proportional to (1) or give zero contribution.[♯3] In a similar way, one can easily demonstrate that diagrams with anomalous vertices (fig. 6) are either proportional to

$$
\begin{aligned}
Y = (-i) \int \frac{d^4k}{(2\pi)^4} \operatorname{Tr}\Big[& (a \; \tilde{\Gamma}_K \; S_d \; \Gamma_\pi + \\
& + \Gamma_K \; S_s \; b\tilde{\Gamma}_\pi) \; S_{u(d)} \; \frac{1}{\sqrt{2}} \; \Gamma_\pi \; S_{u(d)}\Big] \; ,
\end{aligned}
\tag{2}
$$

or do not contribute at all.[♯3] With some careful bookkeeping one obtains that

---

♯3 The statement is true in the leading order in $1/M_W^2$. More detail can be found in ref. 10.

the amplitudes in $K \to \pi\pi$ decays are to the leading order given by

$$\begin{aligned}
A(K^+ \to \pi^+\pi^\circ) &= \frac{4}{3}X \quad &;& \quad A(K^- \to \pi^-\pi^\circ) = -\frac{4}{3}X \\
A(K^S \to \pi^+\pi^-) &= 2X + 2Y \quad &;& \quad A(K^L \to \pi^+\pi^-) = 0 \\
A(K^S \to \pi^\circ\pi^\circ) &= -\frac{2}{3}X + 2Y \quad &;& \quad A(K^L \to \pi^\circ\pi^\circ) = 0 \ .
\end{aligned} \tag{3}$$

(Note that due to a cancellation, diagrams with anomalous vertices do not contribute to $K^\pm$ decays.) From eq. (3) follows the known [11] sum rule, namely $A^S_{+-} - A^S_{\circ\circ} = 2A^+$. The sum rule and results (3) are valid irrespectively of the exact form of vertices. In addition, with

$$X \sim 10 \text{ eV} \quad , \quad Y \sim 190 \text{ eV} \ , \tag{4}$$

the absolute values of $K \to \pi\pi$ amplitudes, and therefore the $\Delta I = 1/2$ rule can be reproduced. Expression (4) suggests that if the explanation of the octet rule lies in the long–distance QCD, then it must be brought by a relatively large contribution of diagrams with anomalous vertices.

5. It follows from the last paragraph that in a scenario in which anomalous vertices dominate, the $\Delta I = 1/2$ rule can be described even without hard gluons. It remains "only" to understand why the diagrams in fig. 6 would dominate. The question "Why $\Delta I = 1/2$ rule?" reappears now in the form "Why is Y so large?". Unfortunately, it is impossible to give any definitive answer to that question. One must first understand the confining mechanism before any quantitative explanation is produced. Still, the described diagrammatic analysis leads to an interesting and general conclusion on the origins of the octet rule.

Once again it comes out that $s \rightarrow d$ transitions are playing the crucial role in $K \rightarrow \pi\pi$ physics. In the standard approaches such transitions lead to PENGUIN operators [1]. In this letter they participate in the construction of anomalous vertices. So, there is no doubt that without "fast" $s \rightarrow d$ transitions the simple explanation of the octet rule could hardly be achieved. The question is just whether these transitions fits better the short–distance ("PENGUIN") or the long–distance (anomalous vertices) environment. We shall see now that an intuitive argument favors the long distances as a framework in which an enhancement of $s \rightarrow d$ transitions occurs.

When no gluons are presented (fig. 7a), direct transitions are of the order $1/M_W^4$ due to the GIM mechanism and the renormalization procedure, and thus highly suppressed. However, gluons "catalyze" the transitions (fig. 7b), and the probability for $s \rightarrow d + \ldots$ becomes of the order $1/M_W^2$. Hence, the presence of gluons is a necessity if $s \rightarrow d$ transitions are to play any role. However, the hard gluons are relatively rare (this fact is reflected in a smallness of coefficients of PENGUIN operators!). On the other hand, there are no similar restrictions on soft gluons. Consequently, long– rather than short–distances provide an environment in which $s \rightarrow d$ transitions can occur with high probability, enabling ultimately the dominance of $\Delta I = 1/2$ amplitudes.♯4

7. No clear theoretical argument supporting the short–distance dominance in K–decays and in other low energy processes was presented in the literature so far. (Note that a confidence in the standard description is based on its successfulness in a phenomenology.) As a sort of a counterexample, in this letter a scenario is

♯4 The similar idea – the dominance of so–called Tadpole diagrams – was advocated (with different arguments) in ref. 5

described in which it is the long–distance dynamics that provides the explanation of the $\Delta I = 1/2$ rule. Some intuitive arguments tell us that not only possible but even more preferred environment for an enhancement of $s \to d$ transitions (that play a crucial role in both long– and short–distances based approaches) is within soft, and not hard QCD. In table 1 the standard technique and the new approach to the octet rule are briefly described. The intention of this paper is not to prove that the standard approach is incorrect. The goal is rather to stress that a completely different scenario still might come out to be the right one. Much more work is needed before an answer is found on whether the most reliable path is in the left, or in the right column of table 1, or maybe – somewhere in between these two extremes.

REFERENCES

1. M. A. Shifman, A. I. Vainshtein and V. I. Zakharov, Nucl. Phys. B120 (1977), 316.
2. J. F. Donoghue, E. Golowich, W. A. Ponce and B. R. Holstein, Phys. Rev. D 21 (1980), 186.
3. M. Milošević, D. Tadić and J. Trampetić, Nucl. Phys. B187 (1981), 514.
4. C. Schmid, Phys. Lett. 66B (1977), 353.
5. M. D. Scadron, Rep. Prog. Phys. 44 (1981), 213, Section 6.8.
6. G. Nardulli, G. Preparata and D. Rotondi, Phys. Rev. D 27 (1983), 557.
7. G. Eilam, M. D. Scadron and B. H. J. McKellar, Fresh Look at $D \to K\pi$ Decays, University of Arizona Report (1983), unpublished.
8. H. Galić, Phys. Rev. D 24 (1981), 3000.
9. C. J. Gilmour, Z. Phys. C 18 (1983), 163.

10. H. Galić, Two–body Decays of Kaons in Microscopic Framework, SLAC-PUB-3383 (1984), unpublished.

11. R. E. Marshak, Riazuddin and C. P. Ryan, *Theory of Weak Interactions in Particle Physics* (Wiley, New York, 1969), p. 601.

TABLE 1

The main steps in the standard approach and in the approach used in this letter.

SCHEME	STANDARD	NEW
Dynamical Background	Short Distances	Long Distances
Main Tool	Operators (in Effective Hamiltonian)	Vertices (in Effective Field Theory)
Assumptions	Dominance of PENGUIN Operators	Dominance of Anomalous Vertices
Comment	Consequences Believed to Be Calculable	Only Parameterization Possible

FIGURE CAPTIONS

Figure 1 Gluons (dashed lines) form an effective $K^-(\bar{s}u)$ vertex.

Figure 2 A direct $s \to d$ transition within the confinement radius produces an anomalous vertex. q denotes u, c,… quarks propagating in a "self–energy" loop. ϕ is a Higgs ghost particle.

Figure 3 Regular vertices in K and π systems. Γ_π and Γ_K are (generally unknown) functions of momenta and masses. (Isospin symmetry is assumed).

Figure 4 Anomalous vertices contributing to $K \to \pi\pi$ decays. $a = \sin\vartheta\cos\vartheta(m_K/M_W)^2$, $b = \sin\vartheta\cos\vartheta(m_\pi/M_W)^2$.

Figure 5 Diagrams with regular vertices describing $K^+ \to \pi^+\pi^\circ$. Similar sets of diagrams can be constructed for other two–body K decays.

Figure 6 Diagrams with anomalous vertices in $K^+ \to \pi^+\pi^\circ$. Similar diagrams can be constructed for other K decays. Only the combination of diagrams in figs. 5 and 6 can properly describe the octet rule.

Figure 7 Gluons serve as catalysts in $s \to d$ transitions. When no gluons are present (diagram (a)), the probability for the transition is proportional to $1/M_W^4$. When gluons are emitted from the loop (diagram (b)), the probability is dramatically enhanced.

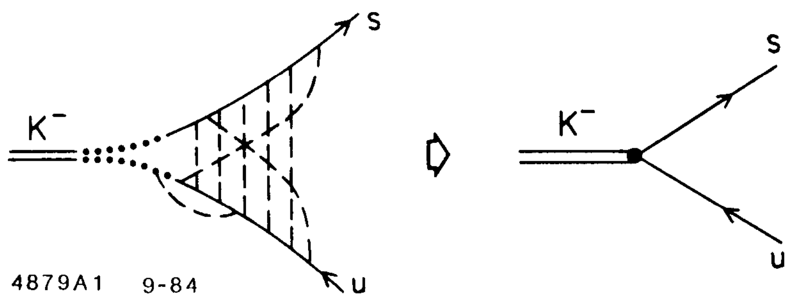


Fig. 1

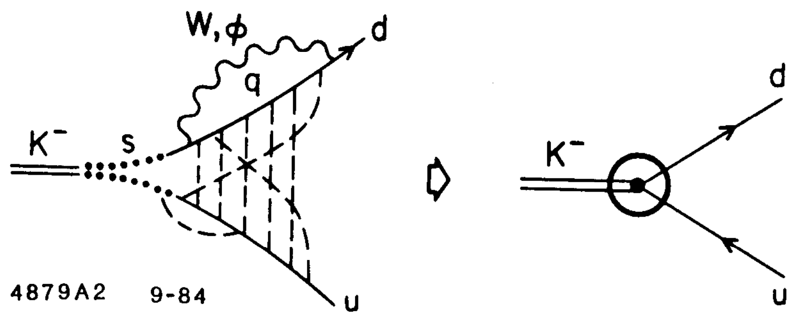
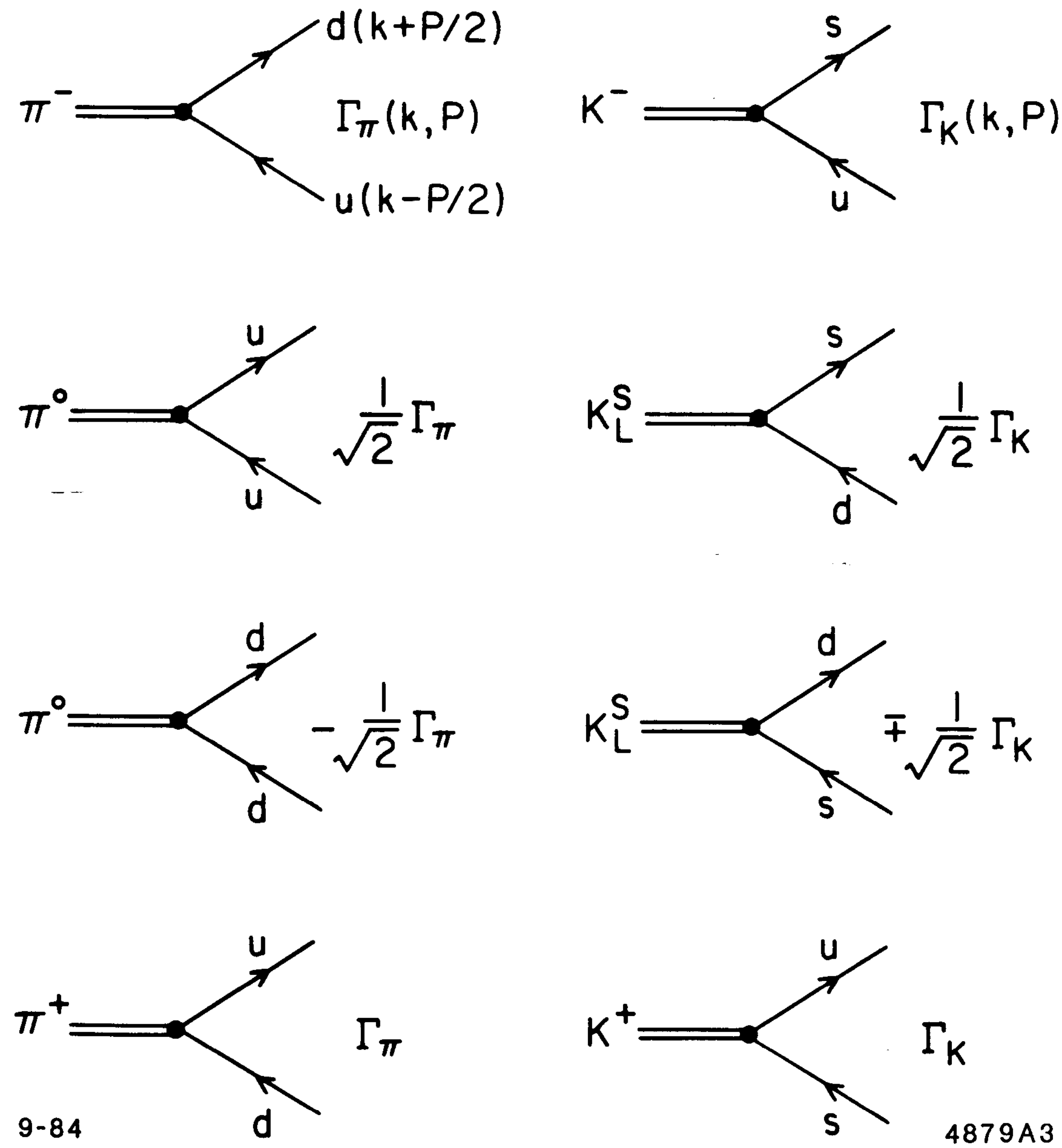


Fig. 2



π^-
d(k+P/2)
$\Gamma_\pi(k,P)$
u(k−P/2)
K^-
s
$\Gamma_K(k,P)$
u
π^0
u
$\frac{1}{\sqrt{2}}\Gamma_\pi$
u
K_L^S
s
$\frac{1}{\sqrt{2}}\Gamma_K$
d
π^0
d
$-\frac{1}{\sqrt{2}}\Gamma_\pi$
d
K_L^S
d
$\mp\frac{1}{\sqrt{2}}\Gamma_K$
s
π^+
u
Γ_π
d
K^+
u
Γ_K
s
9-84
4879A3

Fig. 3

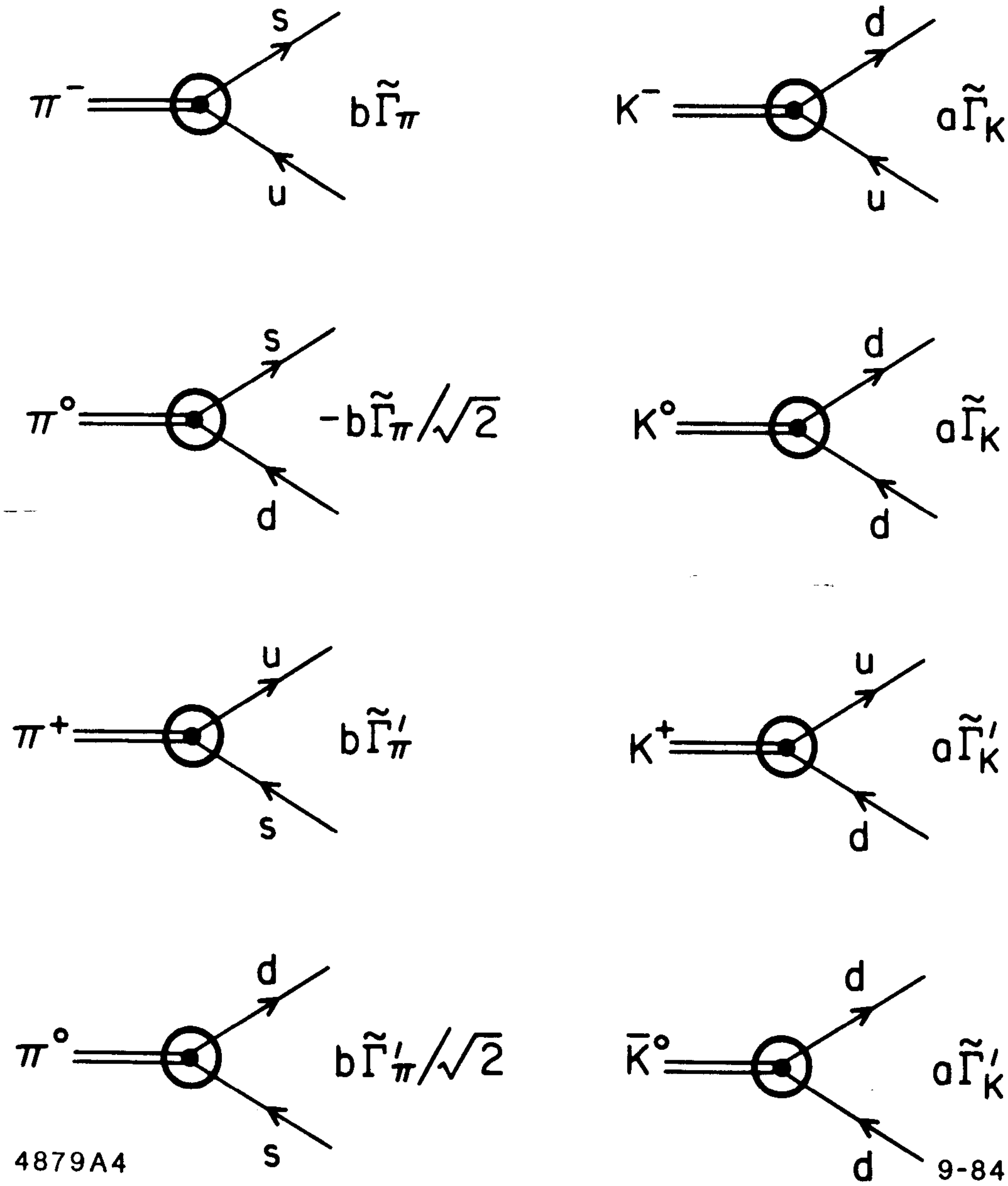

s
π⁻
b$\tilde{\Gamma}_\pi$
u
d
K⁻
a$\tilde{\Gamma}_K$
u
s
π°
$-b\tilde{\Gamma}_\pi/\sqrt{2}$
d
d
K°
a$\tilde{\Gamma}_K$
d
u
π⁺
b$\tilde{\Gamma}'_\pi$
s
u
K⁺
a$\tilde{\Gamma}'_K$
d
d
π°
$b\tilde{\Gamma}'_\pi/\sqrt{2}$
s
d
$\bar{K}$°
a$\tilde{\Gamma}'_K$
d
4879A4
9-84

Fig. 4

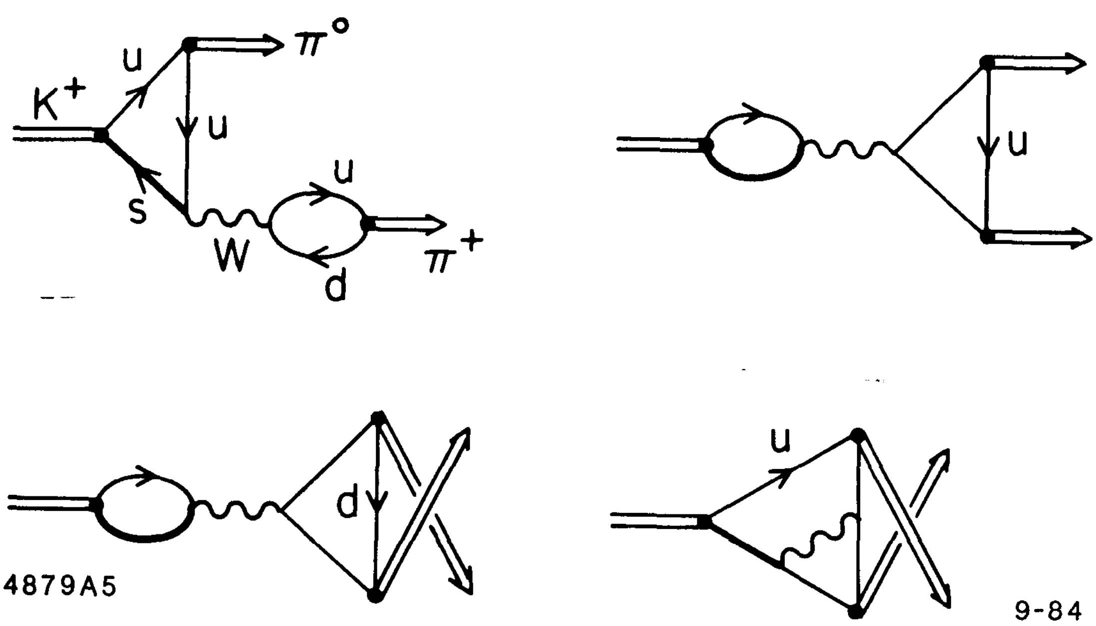
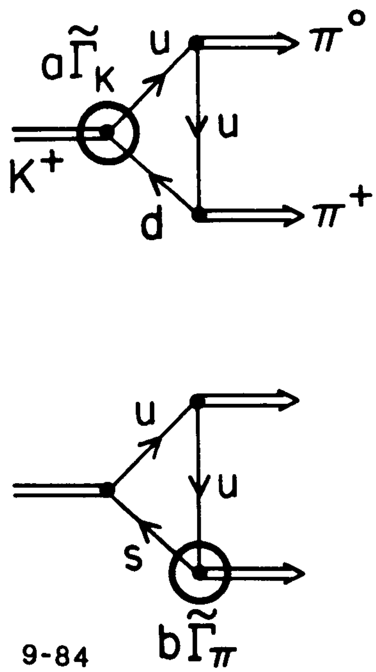
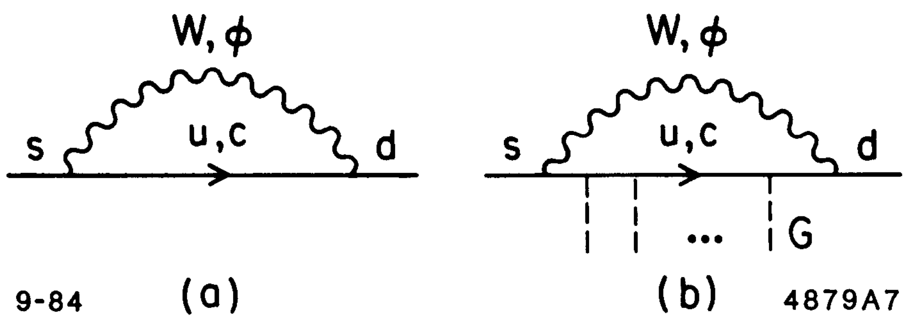


Fig. 5



$a\tilde{\Gamma}_K$
u
π°
u
K^+
d
π^+
u
d
d
u
u
s
u
d
s
9-84
$b\tilde{\Gamma}_\pi$
4879A6

Fig. 6



W, ϕ
s
u,c
d
W, ϕ
s
u,c
d
...
G
9-84
(a)
(b)
4879A7

Fig. 7