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TWO–BODY DECAYS OF KAONS IN MICROSCOPIC FRAMEWORK*

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ABSTRACT

A scheme is introduced in which amplitudes of weak decays of mesons can be analyzed without the use of the current algebra reductions, and with no reference to any of the hadronic models. The interaction of hadrons and their constituents is described by an effective field theory, with two kinds of valence vertices—regular and anomalous—the latter having a "wrong" flavor structure. A suitable parametrization in nonleptonic two-body decays is achieved, and a possibility for the analysis of bound-state effects in rare K decays is opened. The major difference from the standard approaches is that the explanation of the $\Delta I = 1/2$ rule is sought within the long and not the short distance dynamics.

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1. Introduction

The methods of analyses of nonleptonic weak decays were devised in the early days of development of the current algebra.[1] Its modern form was triggered by works of Gaillard and Lee, and Altarelli and Maiani,[2] in which weak interactions of quarks were modified by QCD corrections. Soon it became possible to evaluate the matrix elements of the resulting operators in some of the hadronic models (*e.g.,* in BAG[3,4] or harmonic oscillator[5,6] type models). In 1977 a new set of operators (so-called PENGUIN operators) was introduced[7] as a theoretical and phenomenological necessity, and the framework was completed.[8] The basic elements of today's analyses are (a) an effective Hamiltonian (dressed by QCD), (b) the usage of current algebra techniques, or some other procedure (*e.g.,* vacuum insertions) in the process of the reduction of matrix elements, and (c) the use of some hadronic model in actual evaluations of amplitudes. This scheme will be referred to later on as a "standard" one.

Was the question of nonleptonic weak decays in this way settled down? Was there any necessity for further analyses of this subject? A look into the literature shows that nonleptonic decays are still a hot topic. Numerous recent works could be classified in two major categories. In the first one, on the basis of the standard approach (which is assumed to be mainly correct), constraints and restrictions on parameters (e.g. masses or mixing angles) and forms of the underlying theories are set (e.g. restrictions on supersymmetric theories, subconstituent models, left–right models, etc.); or analyses and predictions for processes not yet experimentally clear (some rare K decays, $P-V$ in high–energy scattering, weak decays of heavy mesons, CP violating processes, etc.) are made. It is interesting that basically the same three-step procedure, (a)–(c), is in another context used in analyses of possible decay modes of protons.[9]

In spite of this broadly accepted confidence in standard methods,[10] there exists another class of papers more critically inclined. In these works authors are looking for alternative approaches,[11,12] or for omitted but perhaps important contributions.[13] Some steps, or even the entire (a)–(c) procedure is challenged,[14,15] and the bounds on amplitudes rather than their absolute values are calculated.[16] The present situation is to a certain extent confusing, in the sense that both classes of works (*i.e.,* critical of and, on the contrary, satisfied with the standard framework) coexist in literature, although—if the criticism is for example justified—the predictions and restrictions obtained by standard methods may have no value.

Indeed, the objections are all but insignificant. Here are just some of them: macroscopic methods (such as current algebra reductions[1]) are used side-by-side with microscopic concepts (*e.g.,* with an effective quark-quark interaction), despite the possibility of double counting; the factorization on the soft and hard QCD contribution is questionable;[17] only leading (and more recently next to leading[18]) logarithmic corrections are taken into account in a dynamical situation where the other, ignored terms might be of the same importance. While the origin of the crucial PENGUIN operators[7] is easily understood, the way they become phenomenologically important is somewhat obscure and unnatural (basically, some small masses appear in certain denominators), *etc.* If one tries to cope with such issues, remaining still within the standard framework, most usually a proliferation of assumptions is needed and a complicated and inconvenient description results.

In this work I'll try to elucidate some of these problem, not from within the standard procedure, but devising a new simple framework for description of weak decays of mesons.[19] The simplicity of the new approach might even tempt one to consider it a step backward instead of a step forward. Particularly shocking will be the apparent negligence of gluonic corrections. However, in the next sections this and similar stages will be thoroughly explained and put in a logical context. The goal was to build a procedure not subjected to macroscopic techniques that are used in standard approaches. For that purpose a kind of effective field theory with direct hadron-constituents couplings was introduced. As a consequence a diagrammatic analyses at the level of constituents was made possible, and results free of ambiguities related to hadronic models and to the reduction procedure were obtained. The new, just-described framework will often be referred to as a "microscopic". The main purpose of this introductory paper is to establish the new method and to analyze some of the simplest processes (including two-body hadronic decays of pseudoscalars). Similarities (or differences) as related to earlier results and previously used methods will be clearly explained. In a future work a few more exciting processes are to be treated in the same way.

After this introduction, two sections follow in which basic assumptions of the new model are described and discussed. In Sec. 4 the weak decay constants and form factors are introduced in terms of finite loop integrals. The generalization of the Fierz rearrangement rule to the bound state formalism is described in Sec. 5. The contributions of "regular," and "anomalous" vertices to the $K \to \pi\pi$ decays are calculated in Secs. 6 and 7, respectively, and obtained results are discussed and compared to experimental values in Sect. 8. In the last section the new and old approaches are confronted, and some concluding remarks on advantages (and imperfections) of the microscopic picture are given. Throughout the work the four-flavor version of SU(3) $\times$ SU(2) $\times$ U(1) model is used. The electroweak part of the model is described in the renormalizable gauge. In this gauge in addition to weak bosons, W, the Higgs–ghost particles, φ, are also carriers of the weak force. However, since only the leading terms in $1/M_W^2$ expansions will be interesting in analyses, the φ fields contributions will be whenever possible ignored.

2. Microscopic Approach and Related Assumptions

To introduce the new model, let me start with a consideration of $M_{\ell 2}$ decays (M stands for a pseudoscalar meson), which are probably the most simple hadronic processes in nature. At macroscopic scale at which our detectors work, only a meson, a lepton and its neutrino are observed (Fig. 1(a)). However, one can proceed a step further. The standard model teaches us that the lepton pair is created by a weak boson, which also interacts with hadronic constituents (Figs. 1(b),1(c)). With even stronger "resolution" one would observe gluons forming an effective $(\bar{q}q)M$ vertex (Fig. 2). Unfortunately, although one knows the way constituents (gluons and quarks) interact mutually, the precise mechanism by which a meson is formed is not known. Therefore, as long as the problem of confinement is unsolved, Fig. 1(c) with an unknown function in the $(\bar{q}q)M$ vertex seems to be the most one can achieve in a microscopic description of the process. Fortunately, it is not exactly so.

Although one can't theoretically determine the vertex function, its form is not completely arbitrary. The most important constraint is related to diagram 1(c). Since the probability for $M \to \ell\nu$ decay is finite, the momentum dependence of the vertex function must be such that the quark loop contribution in Fig. 1(c) is convergent. Another important source of restrictions on vertex functions are various symmetries at the level of hadrons and/or constituents. One could proceed and impose further constraints (*e.g.,* to require a form which doesn't allow the emission of on-shell quarks from mesons), but that wouldn't be in the spirit of this work. Rather than to construct an "ideal" model, I'll try to concentrate on relations that can be reached even without the precise form of the $(\bar{q}q)M$ vertex. Such type of analysis will for brevity be referred to as "model-independent," although the more appropriate name would be "independent of the details of the model."

Let me now present the assumptions related to the framework in which in subsequent sections the weak decays of K meson will be analyzed. (In the next section some of these assumptions will be discussed more thoroughly.) It will be supposed that:

(*i*) The interactions of macroscopic particles (mesons) and their constituents (quarks and gluons) can be described by an effective field theory into which all confining effects are integrated, leading to nonlocal meson-constituent vertices in momentum space. (The counterparts of these vertices in the coordinate space are wave-functions of mesons, in terms of various Fock states.)

(*ii*) Only the effect of $(\bar{q}q)M$ vertices will be investigated, and higher Fock states such as $G(\bar{q}q)M$ (where G stands for gluon), $GG(\bar{q}q)M$, *etc.,* will be ignored. In one type of vertices $(\bar{q}q)$ will correspond to the regular valence pair of quarks, so that the resulting picture will be similar to the valence approximation used widely in hadronic weak processes. In addition, another type of $(\bar{q}q)M$ vertices ("anomalous") will be introduced in Sec. 7.

(*iii*) It will be assumed that interactions responsible for the formation of a regular $(\bar{q}q)M$ vertex (a) do not spoil Lorentz invariance, (b) are $SU(2)_{flavor}$ and $SU(3)_{color}$ invariant[20]; (c) conserve C, P and T symmetries. Since the four-flavor model is used, no CP–violating effects will be considered in this introductory work. (The generalization to the six-flavor theory is straightforward.)

The assumptions (i) to (iii) lead to the most general form of effective (regular) vertices in the microscopic model. The appropriate vertices in π and K systems are denoted in Fig. 3. The vertex functions Γ_π and Γ_K appearing in the figure can be defined as

$$\Gamma_M(k,Q) = \left\{ P_1 + \frac{1}{m_M} \left[P_2\, Q^\mu + \frac{kQ}{m_M^2}\, P_3 k^\mu \right] \gamma_\mu \right.$$
$$\left. + \frac{1}{m_M^2}\, P_4\, Q^\mu\, k^\nu\, i\sigma_{\mu\nu} \right\} \gamma_5 \quad . \tag{2.1}$$

Momenta Q, $k+Q/2$ and $k-Q/2$ correspond to a meson, outgoing and ingoing quark, respectively. P_1 to P_4 are some unknown functions. For us it is important that they must insure the finiteness of diagram 1(c). As a consequence of the assumption (iii) the functions P_i can be further subdivided into the parts that are either even or odd on the replacements $k \to -k$ and $Q \to -Q$. Note that vertex (2.1) has pseudoscalar, axial-vector and tensor looking terms, but no scalar and vector terms.

With vertices in Fig. 3 and with the definition (2.1), one may start with the test of the model in various exclusive weak decays. However, let me first analyze in more detail the logic of the assumption (ii).

3. Higher Fock States and Why to Neglect Them

There is no doubt that the assumption (ii) in the previous section looks to a certain extent unnatural. No presently known mechanisms suppress the influence of higher Fock states on low energy processes (the situation might be different at higher energies[21]). Still, I'll insist on the valence picture. In order to explain better the logic behind this assumption, let me consider the situation in which gluons do appear in effective vertices. Then not only the $(\bar{q}q)M$ vertex but also the $G(\bar{q}q)M$, $GG(\bar{q}q)M$, ..., vertices have to be defined, too. Two important features of such a complete microscopic picture are: (1) there will be an ***infinite*** number of new vertices with gluons; (2) once the $(\bar{q}q)M$ vertex is defined, new vertices are ***not arbitrary*** but are related to $(\bar{q}q)M$ vertex function. Both of these features are a consequence of the gauge invariance of the underlying theory. The situation is explained in Fig. 4. Imagine that the effective $(\bar{q}q)M$ coupling in the coordinate space is described by

$$F(\partial,\partial')\bar{q}q\,\Phi_M \quad , \tag{3.1}$$

where ∂ (and ∂') are derivatives of outgoing (ingoing) fermion field operators, and F is some functional of ∂ and ∂'. The gauge invariance requires (even in an effective field theory!) that ∂_μ is replaced in a complete model by a covariant derivative

$D_\mu = \partial_\mu - i(g/2)\,\vec{\lambda}\,\vec{G}_\mu$. Equation (3.1) therefore becomes

$$F(D,D')\;\bar{q}q\;\Phi_M \quad . \tag{3.2}$$

thus generating an entire class of new vertices. Since Eq. (3.1) must be nonlocal (in order to ensure the finiteness of $M \to \ell\nu$), expression (3.2) contains generally an infinite number of terms with all possible numbers of gluon field operators, in addition to meson and fermion fields. It is clear that such a scenario leads to an unavoidable catastrophe: if the higher Fock states are important, then one must take into account in any calculation an infinite number of terms describing the complete wave function. Most probably this cannot be done in a satisfactory way, and one must abandon the hope that anything could be understood in hadronic decays. Alternatively, the contribution of higher states could be suppressed by some unknown mechanism (smallness of the strong coupling constant?); but then the main contribution comes from Eq. (3.1), and in the first approximation higher states are negligible. In other words, while the picture in which a meson contains basically a pair of quarks is not natural, it is preferable and necessary if one wants to calculate anything. One could say that by testing the valence picture, our ability to deal with hadronic decays is tested also.

Imagine now that the lowest order results are known, and that one wants to calculate (hopefully small) first order corrections. In the microscopic framework it immediately becomes clear that one cannot disentangle the contributions of the ordinary QCD corrections to the process from the contributions of higher Fock states. The situation is explained in Figs. 5 and 6. The diagram in Fig. 5 looks like a typical "hard gluon" correction to the weak process described in a valence approximation. However, this diagram is by itself gauge-noninvariant and represents only a part of corrections. In Fig. 6 the complete, gauge-invariant form, which comprises also the diagram in Fig. 5, is shown. One sees that the proper way for description of one-gluon corrections is the introduction of higher Fock states, and not only some one-gluon exchanges.

Note that in the standard approach, when four-quark operators are dressed by QCD, one basically uses a type of corrections from Fig. 5. Although it was just mentioned that the diagram in Fig. 5 leads to an incomplete description, that is not an obvious source of problems as long as only "leading log" approximation is used. Namely, it is most probable that leading logarithmic terms ($g^2 \ln M_W$) do not spoil the gauge invariance. The real problem for the standard approach lies somewhere else. In a microscopic model, at the one-gluon level, one can easily demonstrate the appearance of nonlogarithmic terms (*e.g.,* some ratios of masses and momenta). The mass M_W is simply not large enough to insure the dominance of the $\ln M_W$ term in an expression in which various other large nonlogarithms appear. There is no reason to believe that in diagrams with many gluons, the situation will be different. The above objections suggest the following conclusions: (a) if one decides to include gluonic corrections, it is not enough to take into account only $\ln M_W$ terms, because the other terms might be equally important; (b) if other than leading log terms are to be included, than that should be done by diagrams of the type presented in Fig. 6, since otherwise the gauge invariance is lost. I shall return to these questions again in the concluding section.

As explained earlier, in this work only the lowest order description, with valence quarks and with no gluons in vertices will be considered. Since higher order Fock states are neglected, it would be inconsistent to pay any attention to a class of QCD corrections depicted in Fig. 5. Therefore, in the simplest version of the microscopic model one has to deal with diagrams in which no gluon line is attached directly to the valence quarks.[22] Still it doesn't mean that this model drags back to pre–QCD days. Note that gluons ("soft gluons") do participate in the formation of bound states from quarks. Though not explicitly appearing in diagrams, their presence has left a profound trace in the nonlocal character of the vertex functions. In addition, a class of QCD corrections is perfectly allowed; e.g., self-energy and vertex corrections in parts of a diagram not related to the valence quark propagators (see Fig. 7).

Microscopic analyses with valence vertices are not a novelty. In a different framework they were already used in the literature. Two most recent serious attempts are by Chernyak and Zhitnitsky, and by Gilmour (see Ref. 23). In both of these works a particular choice for wave functions was done, so that the results are model-dependent. In addition, higher order QCD corrections were considered without a parallel introduction of higher Fock states, which might—in the light of the above discussion—lead to incomplete and gauge-noninvariant conclusions.

4. Elementary Application

In this section a few elementary processes will be considered. It was already mentioned that $K_{\ell 2}$ decay is the most simple hadronic decay, so let's analyze it first. In a macroscopic notation $K \to \ell\nu$ is characterized by a matrix element of the weak current, $\langle 0|J^\mu(\Delta S = 1)|K\rangle = i f_K P^\mu$. In the microscopic description the invariant amplitude is given by a loop integral (see Fig. 8).

$$\begin{aligned}\mathcal{M} = i\,\frac{G\sin\vartheta}{\sqrt{2}}\,(-3)\int\frac{d^4k}{(2\pi)^4}\,Tr\left\{\frac{1}{\not k - \not P - m}\gamma^\mu(1-\gamma_5)\,\frac{1}{\not k - m_s}\right.\\ \left.\times\,\Gamma_K\left(k-\frac{P}{2}\,,P\right)\right\}\,\bar u_\ell\,\gamma_\mu(1-\gamma_5)v_\nu\,\left[1+O(M_W^{-2})\right]\quad .\end{aligned}\tag{4.1}$$

The minus sign, and the factor $3(=N)$ in front of the integral appear as a consequence of the close fermion loop and the trace over unit matrix in a color space.

Only $\gamma^\mu\gamma_5$ part of the W–boson vertex gives a nonvanishing contribution to the integral. The kaon decay constant is then in microscopic notation given by

$$if_{K^-}\, P^2 = 3 \int \frac{d^4k}{(2\pi)^4}\ Tr\left\{ \frac{1}{\not k - \not P - m}\ \not P\gamma_5\, \frac{1}{\not k - m_s}\, \Gamma_K(k - \frac{P}{2}\ ,\ P) \right\} . \tag{4.2}$$

Since f_K must be finite, the component of Γ_K contributing to the integral (4.2) must for large momentum, $k \to \infty$, behave at least as $1/k^3$. (This important constraint was mentioned in Sec. 2.). f_{K^+} can be defined in a similar way. Due to properties of Γ_K, the relation $f_{K^+} = f_{K^-} \equiv f_K$ is valid. When in Eq. (4.2) m_s is replaced by m,[20] and Γ_K by $\Gamma\pi$, one obtains the definition of f_π.

It is clear that only the lowest Fock state can contribute to the $K_{\ell 2}$ decay. Furthermore, since an effective vertex is used in Eq. (4.1), there is no need for additional radiative corrections (see Fig. 9), and expressions (4.1) and (4.2) would remain unchanged even in an approach not restricted to valence vertices only. Thus (4.2) can be directly related to the experimentally measured values.

Another simple process is $K \to \pi\ell\nu$ decay. This process will help to define weak form factors in terms of microscopic functions. Consider *e.g.*, $K^- \to \pi^\circ\ell\bar\nu$ decay (Fig. 10). The reduced amplitude is

$$\begin{aligned} \mathcal{M} = &- \frac{G\sin\vartheta}{\sqrt{2}}\ \bar u_\ell\, \gamma_\alpha(1-\gamma_5)v_\nu\ (-3) \int \frac{d^4k}{(2\pi)^4} \\ &\times\ Tr\left\{ S_u\,\gamma^\alpha(1-\gamma_5)\ S_s\,[\Gamma_K]\ S_u\left[\frac{1}{\sqrt{2}}\Gamma_\pi\right]\right\}\left[1 + O(M_W^{-2})\right] \quad . \end{aligned} \tag{4.3}$$

S_q stands for the propagator of a q-quark. One can now easily recognize the form factors. With $Q = P_K - P_\pi$, and $m_1 = m_3 = m$, $m_2 = m_s$, it follows

$$iF_1^{K\pi}(Q^2)\,[P_K+P_\pi]^\alpha \;+iF_2^{K\pi}(Q^2)\,[P_K-P_\pi]^\alpha$$

$$=3\int \frac{d^4k}{(2\pi)^4}\; Tr\left\{\left(\not k-\frac{\not Q}{2}-m_1\right)^{-1}\gamma^\alpha\left(\not k+\frac{\not Q}{2}-m_2\right)^{-1}\right.$$

$$\left.\times\,\Gamma_K\left(k-\frac{P_\pi}{2},\,P_K\right)\,\left(\not k-\frac{\not P_\pi+\not P_K}{2}-m_3\right)^{-1}\Gamma_\pi\left(k-\frac{P_K}{2},\,-P_\pi\right)\right\}\quad . \tag{4.4}$$

Note that only the vector part of the W–boson vertex gives a nonvanishing contribution to form factors. To the leading order in $1/M_W^2$, Eq. (4.3) thus becomes

$$\mathcal{M}=i\,\frac{G\sin\vartheta}{\sqrt{2}}\,\frac{1}{\sqrt{2}}\,\left\{F_1^{K\pi}(P_K+P_\pi)^\alpha+F_2^{K\pi}\,Q^\alpha\right\}\times\text{leptonic part}\ . \tag{4.5}$$

In a similar way one can describe all $K_{\ell 3}$ decays in terms of the same form factors $F_1^{K\pi}$ and $F_2^{K\pi}$ defined in Eq. (4.4). For example, in $K_S\to\pi^-\bar{\ell}\nu$ decay

$$\mathcal{M}=i\,\frac{G\sin\vartheta}{\sqrt{2}}\,\left(-\frac{1}{\sqrt{2}}\right)\,\left\{F_1^{K\pi}(P_K+P_\pi)^\alpha+F_2^{K\pi}\,Q^\alpha\right\}\times\text{leptonic part}\ , \tag{4.6}$$

and so on. One can demonstrate that when $m_2\to m_1$ in Eq. (4.4), the second form factor disappears, $F_2(Q^2)\to 0$. That is clearly related to the conserved vector current (CVC) hypothesis known from macroscopic description,[1] and leaves us with only one form factor in the pion system. Namely, when m_s is replaced by m, and K by π, expression (4.4) can serve as a definition of $F_i^{\pi\pi}$ form factors. However, as just mentioned, due to the isospin symmetry, the part of the integral which defines $F_2^{\pi\pi}$ gives zero contribution, and one has ($Q=P_{\pi 1}-P_{\pi 2}$),

$$iF_1^{\pi\pi}(Q^2)\ [P_{\pi 1}+P_{\pi 2}]^\alpha$$

$$= 3 \int \frac{d^4k}{(2\pi)^4} \times Tr\left\{ \left(\not k - \frac{\not Q}{2} - m \right)^{-1} \gamma^\alpha \left(\not k + \frac{\not Q}{2} - m \right)^{-1} \right. \tag{4.7}$$

$$\left. \times\, \Gamma_\pi \left(\not k - \frac{\not P_{\pi 1} + \not P_{\pi 2}}{2} - m \right)^{-1} \Gamma_\pi \right\} \quad .$$

Since weak (charged) and electromagnetic currents in the standard SU(2) $\times$ U(1) model are members of the same isomultiplet, it is evident that $F_1^{\pi\pi}$ in Eq. (4.7) describes not only weak but also electromagnetic form factors. Therefore one obtains an additional constraint on the form of the vertex functions: Γ_M must be such that $F_1^{MM}(Q^2 \to 0) = +1$, which is the usual normalization of the electromagnetic form factors.

These elementary definitions of weak decay constants and form factors, will find their full application in the analysis of nonleptonic decays. However, it will be instructive to introduce first another valuable tool in microscopic considerations, namely the generalized Fierz transformation.

5. Two-Loop Integrals and Generalized Fierz Transformation

In the previous section some basic functions were defined in terms of one-loop integrals. In the analysis of nonleptonic decays we shall mostly have to deal with two-loop integrals. In this section a very useful technique, by which in weak processes two-loop integrals can be reduced to a product of one-loop integrals, will be described. This technique will be illustrated in an analysis of the box diagram contribution to the $K_L \to \gamma\gamma$ decay.

As shown by Gaillard and Lee,[24,25] in the free field limit of the four flavor theory, it is the box diagram which gives the most important contribution to the decay amplitude. According to Ref. 24,

$$\mathcal{M}^{\text{free}f}_{\text{box}} = \frac{G}{\sqrt{2}} \sin\vartheta \, \cos\vartheta \, \left(\frac{16}{9}\right) \times \frac{\alpha}{\pi} \cdot \frac{4}{\sqrt{2}} \, f_K \left[H(m_u^2) - H(m_c^2)\right] \, \epsilon_{\alpha\beta\gamma\delta} \, \partial^\alpha A^\beta(q_1) \, \partial^\gamma A^\delta(q_2) \, , \tag{5.1}$$

where $H(m)$ functions correspond to some calculable integrals. In the microscopic approach the strange and down quarks are not free; They are off-shell particles which can assume all possible momenta in a loop integral. Amazingly enough, the final result of the analysis coincides with the expression (5.1) The mechanism which will be referred to as the generalized Fierz transformation (GFT) is responsible for this coincidence.

Here is how it works. Consider the diagram in Fig. 11. The related matrix element is proportional to ($R = q_1 - q_2$),

$$\mathcal{M} \sim (-3) \int d^4k \; Tr \left\{ S_s\left(k + \frac{P}{2}\right) \frac{1}{\sqrt{2}} \Gamma_K \, S_d\left(k - \frac{P}{2}\right) \int d^4\ell \, \frac{1}{(\ell - k)^2 - M_W^2} \, \gamma_\mu(1 - \gamma_5) \left[S_u\left(\ell - \frac{P}{2}\right) \gamma^\beta S_u\left(\ell - \frac{R}{2}\right) \gamma^\alpha S_u\left(\ell + \frac{P}{2}\right) - (u \to c)\right] \gamma^\mu(1 - \gamma_5) \right\} . \tag{5.2}$$

This expression is quite nontransparent, but it can be simplified. In the first step the W–boson propagator should be expanded in a series:

$$\frac{1}{(\ell - k)^2 - M_W^2} = \frac{1}{\ell^2 - (M_W^2 - k^2)} \left[1 + \frac{2\ell k}{\ell^2 - (M_W^2 - k^2)} + \ldots \right] \quad . \tag{5.3}$$

Note that an expansion in which only M_W^2 (instead of $M_W^2 - k^2$) is used as a large parameter wouldn't do. Higher order terms would then produce an infinite number of UV divergences in the integral over d^4k! With the expansion (5.3) one can evaluate the integral over $d^4\ell$. It is easy to demonstrate that the second and higher terms in the expansion are of the order $O[k/(M_W^2 - k^2)^2]$. Such terms are UV safe in the d^4k space, and suppressed due to large denominator, and could be neglected. Consider now the first term in the r.h.s. of Eq. (5.3). As a consequence of the GIM mechanism, even when the propagator $[\ell^2 - (M_W^2 - k^2)]^{-1}$ is replaced by $[-(M_W^2 - k^2)]^{-1}$, the integral over $d^4\ell$ in Eq. (5.2) is still convergent. But that means that there are no logarithmic corrections of the type $\ell n(M_W^2 - k^2)$ in the leading part,[26] and that one can write the integral over $d^4\ell$ as

$$\begin{aligned} \int d^4\ell \to \frac{1}{k^2 - M_W^2} \int d^4\ell\, \gamma_\mu (1-\gamma_5) \Big[S_u^{'} \gamma^\beta S_u^{''} \gamma^\alpha S_u^{'''} \\ -(u \to c)\Big]\ \gamma^\mu (1-\gamma_5) + O\left(\frac{k}{(k^2 - M_W^2)^2}\right) . \end{aligned} \tag{5.4}$$

At this stage it is helpful to use the relation

$$\gamma_\mu(1-\gamma_5)\ \widehat{T}\ \gamma^\mu(1-\gamma_5) = -\gamma_\mu(1-\gamma_5)\ Tr\Big\{\ \widehat{T}\ \gamma^\mu(1-\gamma_5)\ \Big\}\ , \tag{5.5}$$

which is valid for any combination $\widehat{T}$ of γ matrices. The leading part of Eq. (5.2) can now be rewritten as

$$\begin{aligned} \mathcal{M}\ \sim\ -(-3) \int \frac{d^4k}{k^2 - M_W^2}\ Tr\left\{ S_s\ \frac{1}{\sqrt{2}}\,\Gamma_K\ S_d\ \gamma_\mu(1-\gamma_5)\right\} \\ \int d^4\ell\ Tr\left\{ \Big[S_u^{'} \gamma^\beta S_u^{''} \gamma^\alpha S_u^{'''} - (u \to c)\Big]\ \gamma^\mu (1-\gamma_5)\right\} + \dots\ . \end{aligned} \tag{5.6}$$

Dots in Eq. (5.6) denote terms of order $O(1/M_W^4)$. One can use once again the same trick, this time in d^4k space, and stretch the $1/(k^2 - M_W^2)$ "propagator" to

$1/(-M_W^2)$, since the remaining integral is finite due to the characteristics of Γ_K vertex. The result is (to the leading order in $1/M_W^2$)

$$\mathcal{M} \sim (-3)\,\frac{1}{M_W^2}\int d^4k\; Tr\{\,\dots\}\times\int d^4\ell\;\mathrm{Tr}\{\,\dots\}+\dots$$

$$= \frac{1}{3}\left[(-3)\int d^4k\; Tr\{\,\dots\}\right]\frac{1}{P^2-M_W^2}\left[(-3)\int d^4\ell\;\mathrm{Tr}\{\,\dots\}\right]+\dots\,, \tag{5.7}$$

where the missing expression in curly brackets are the same as in Eq. (5.6). The relation (5.7) is illustrated in Fig. 12. It says the following: if no divergences appear in a two-loop integral when the W–boson propagator is stretched to a point, the integral can be to the leading order factorized in a product of two one-loop integrals. The advantage of the procedure is clear from the second equation in (5.7). The integral $\int d^4k$ doesn't have to be calculated; the comparison with Eq. (4.2) shows that it is proportional to f_K. In addition, the second integral in Eq. (5.7), $\int d^4\ell$, is a convergent integral which can be shown to be identical to $[\,H(m_u^2)-H(m_c^2)\,]$ in Eq. (5.1). It is not difficult to see that the proportionality factors in Eq. (5.7) are identical to those appearing in Eq. (5.1). Thus the equivalence of the free field[24] and the microscopic analysis becomes evident.

Although the transformation in Fig. 12 and in Eq. (5.7) reminds us of the Fierz transformation, two important differences must be pointed out. While the original transformation deals with *free* spinors, here a generalization to the *off shell* fermions was presented. Furthermore, an exact relation for free spinors is replaced here by the relation valid only to leading order in $1/M_W^2$. Note that GFT is possible basically because a $(\bar{q}q)M$ vertex improves a convergence of quark loop integrals.

There are a large number of weak decays of mesons in which the generalized Fierz rearrangement greatly simplifies the analyses, and GFT will be particularly useful as we proceed with a study of $K \rightarrow \pi\pi$ processes. However, this is the proper place to mention that in some decays (mainly in some rare $K-$ decays) the transformation cannot be applied. In such cases the free field results will not coincide with the microscopic analysis, and the microscopic approach will reveal additional (probably important) bound state effects. The detailed analyses of that issue will be presented elsewhere. In a similar way, when higher Fock states (and QCD corrections) are introduced, GFT can be applied only partially, which means that the true factorization occurs only if the higher states are negligible.

6. $K \rightarrow \pi\pi$ Decays

Armed with definitions are techniques from previous sections, we are ready to analyze two-body hadronic decays of K mesons. The six possible decay modes are

$$K^{\pm} \rightarrow \pi^{\pm}\pi^{\circ}, \qquad K_{S,L} \rightarrow \pi^{-}\pi^{+}, \qquad K_{S,L} \rightarrow \pi^{\circ}\pi^{\circ} \quad . \tag{6.1}$$

In the microscopic framework one has first to construct the relevant diagrams. Some of them are presented in Figs. 13–15. There are all together 96 diagrams contributing to processes (6.1), however one shouldn't be too impressed with this number. Many of the diagrams in which the Higgs–ghost particle, φ, propagates are suppressed, and other diagrams group together in just a few distinctive classes. Still, it might look as a surprise that to the leading order in $1/M_W^2$, amplitudes of all processes in Eq. (6.1) are proportional to just one single function.

This effect is a consequence of three different mechanisms. The first one is related to *GFT*. Two typical examples are presented in Figs. 16 and 17. The same mechanism that gave rise to the factorization in the $K \to \gamma\gamma$ box diagram breaks here the two-loop integrals (l.h.s. in Figs. 16, 17) into the product of one-loop integrals "joined" by a W-boson propagator (r.h.s. in quoted figures). The factor $1/3(=1/N)$ is related to the number of colors in the model. Indicated relations are exact in the leading order. The use of *GFT* in such a way reduces the number of independent diagrams, relating *e.g.,* the first and the second diagram in Fig. 13. In addition the evaluation of required diagrams is extremely simplified. The r.h.s. in Fig. 16 is namely proportional to the product of previously defined functions, and one obtains

$$\mathcal{M}_{\text{Fig.16}} \sim \frac{1}{3} \left[F_1^{K\pi}(Q^2)\,(P+R)^\alpha + F_2^{K\pi}(Q^2)\,(P-R)^\alpha \right] Q_\alpha f_\pi \,. \tag{6.2}$$

In a similar way the r.h.s. in Fig. 17 gives the contribution

$$\mathcal{M}_{\text{Fig.17}} \sim \frac{1}{3} P_\alpha f_K \left[F_1^{\pi\pi}(P^2)\,(Q-R)^\alpha + F_2^{\pi\pi}(P^2)\,(Q+R)^\alpha \right] . \tag{6.3}$$

Expression (6.3) can serve as an illustration of the second reason that leads to the simplification of the final result. As discussed in Sec. 4, one has $F_2^{\pi\pi}(P^2) = 0$, and in addition, since the pions are on mass shell,

$$P_\alpha (Q-R)^\alpha \equiv Q^2 - R^2 \to 0 \quad . \tag{6.4}$$

Therefore, the "annihilation" diagrams, and those related to them by the *GFT* do not contribute to amplitudes of $K \to \pi\pi$ decays.[27]

In such a way the most of diagrams in Figs. 13–15 are either proportional to (6.2) or, being of the form (6.3), unimportant. It remains to consider the last two diagrams in each of Figs. 13–15. All these diagrams are characterized by a direct $s \to d$ (or $d \to s$) transition through a self-energy subdiagram. Such a transition is presented in more detail in Fig. 18. Here one is not allowed to ignore the Higgs–ghost exchange: Diagram 18(b) is divergent! Therefore only when the properly chosen counterterms are introduced can the values of diagrams with self-energy insertions be obtained. However, when such counterterms are added, the subdiagrams become not only convergent, but lose their dominant $1/M_W^2$ part. Thus each diagram with a direct $s \to d$ ($d \to s$) transition becomes of the order $1/M_W^4$ in the valence model, and does not contribute significantly to the amplitudes.

The net result of the above analysis can be summarized as follows. From the starting 96 diagrams, 64 are suppressed by the presence of either Higgs–ghosts or the direct $s \to d$ transitions, 16 diagrams are eliminated by the isospin symmetry, while the remaining 16 are all proportional to Eq. (6.2). It is not difficult to write the complete amplitudes now. When all factors 1/3 (from *GFT*) and $\pm 1/\sqrt{2}$ (from some vertex functions) are collected, one obtains in the leading order

$$
\begin{aligned}
A^+ &\equiv \mathcal{M}(K^+ \to \pi^+\pi^\circ) = \frac{4}{3}\,X\,, \qquad & A^- &\equiv \mathcal{M}(K^- \to \pi^-\pi^\circ) = -\frac{4}{3}\,X\,, \\
A^S_{+-} &\equiv \mathcal{M}(K_S \to \pi^+\pi^-) = \frac{6}{3}\,X\,, \qquad & A^L_{+-} &\equiv \mathcal{M}(K_L \to \pi^+\pi^-) = 0\,, \\
A^S_{00} &\equiv \mathcal{M}(K_S \to \pi^\circ\pi^\circ) = -\frac{2}{3}\,X\,, \qquad & A^L_{00} &\equiv \mathcal{M}(K_L \to \pi^\circ\pi^\circ) = 0\,.
\end{aligned}
\tag{6.5}
$$

In an SU(N) theory, the factor 4/3 would be replaced by $(N+1)/N$, the factor $-2/3$ would become $-2/N$, while the amplitude A_{+-}^{S} would remain unchanged.[28] The unknown function X in Eq. (6.5) is given by

$$X = \frac{G}{\sqrt{2}} \sin\vartheta \, \cos\vartheta \, \frac{1}{\sqrt{2}} \, f_\pi \left[\, (m_K^2 - m_\pi^2) \, F_1^{K\pi}(m_\pi^2) + m_\pi^2 \, F_2^{K\pi}(m_\pi^2) \, \right] \, . \quad (6.6)$$

It can be seen from Eq. (6.5) that $A^- = -A^+$, and that $K_L \rightarrow \pi\pi$ decays are forbidden as they should be, but this is just a natural consequence of the starting assumptions. (Remember that the CP conservation was built into the model.)

The real challenge for any model of K decays are the remaining three amplitudes. Form Eq. (6.5) one finds that

$$A^+ = \frac{2}{3} \, A_{+-}^{S} = -2A_{00}^{S} \quad . \quad (6.7)$$

While the function X in Eq. (6.6) depends on the form of the vertex function used in calculation, relations indicated in Eq. (6.7) are valid for any particular choice of Γ. However, if something weren't missing in the previous analysis, Eq. (6.7) would be bad news for the microscopic framework. Namely, the experimentally measured amplitudes (found from the widths in Ref. 29) are

$$|A^+| = 18.3 \text{ eV} \quad ,$$

$$|A_{+-}^{S}| = 389.1 \text{ eV} \qquad \text{and} \qquad |A_{00}^{S}| = 372.2 \text{ eV} \; . \quad (6.8)$$

So, while Eq. (6.7) predicts the A^+ amplitude of the order of A^S amplitudes, experiments show that the latter are enhanced by a factor ~ 20. Does it mean the end of the valence model? No. In the next section it will be shown that an important contribution was missing from expression (6.5), and that its addition can dramatically change relations (6.7).

7. Anomalous Vertices

Relations (6.7) are not a novelty. One gets the same ratios of amplitudes in macroscopic analyses when QCD is turned off. (Compare *e.g.,* with results obtained with vacuum insertions in Ref. 30, and by current algebra in Ref. 31. The coefficients of operators should be set to the free field values.) To change this situation, in the standard approach one introduces operators dressed by hard gluons. The same solution cannot be applied in this work. Since I want to use just soft gluons which never appear explicitly in diagrams, and no hard gluon corrections, it seems that there is no room for a change of relations (6.5) and (6.7). In fact, it is not so. Let me explain that more thoroughly.

The mechanism by which gluons form the regular vertices is schematically presented in Fig. 2. Imagine now that still within the confinement radius a direct $s \to d$ transition has taken place in a valence quark line. This possibility is presented in Fig. 19. As a result, an effective vertex with a "wrong" flavor structure would appear. For the K^- meson, it would be *e.g.,* an anomalous $(\bar{d}u)K^-$ vertex in addition to the regular $(\bar{s}u)K^-$ vertex. It is important to realize that such an anomalous vertex is as natural as regular valence vertices once all the soft gluons within the confinement radius are integrated in. The probability for its appearance might be very low, but it is certainly different from zero. So, let us concentrate on this probability.

If anomalous vertices are to play any role in $K \to \pi\pi$ decays, their vertex function has to be at least of the order $1/M_W^2$. Otherwise, their contribution would be negligible as compared to contributions of regular diagrams. On the other hand, as it was discussed in Sec. 6, the rate for a direct $s \to d$ transition is of the order $1/M_W^4$. However, there is a big difference between diagrams in

Fig. 18 and the relevant subdiagram in the l.h.s. of Fig. 19. While diagrams in Fig. 18—as a result of *GIM* mechanism and renormalization—are really suppressed, the situation is drastically changed when one (or any number) of gluons is emitted from the quark line within the self-energy loop. The *GIM* mechanism then breaks, and *e.g.,* the diagram in Fig. 20 has a nonvanishing $1/M_W^2$ part. (This fact was discovered and used in another context by ITEP group[7].) So, it appears that the general strength of new vertex functions is proportional to $(m_K^2/M_W^2)\sin\vartheta\ \cos\vartheta$, and not to $(m_K^2/M_W^2)^2\sin\vartheta\ cos\vartheta$ as one would naïvely expect. Therefore anomalous vertices on equal footing with normal vertices take part in descriptions of $K \to \pi\pi$ decays and might provide us with a mechanism (within a valence picture) that could bring ratios of amplitudes closer to measured values.

What is the precise form of new vertex functions? Since weak interactions were involved in a creations of anomalous vertices, C and P invariance are broken. However, a combined CP invariance must still be preserved. In a similar way to that by which the form (2.1) was deduced, one can now find the most general Lorentz and momentum structure of the new vertex. The true functional dependence remains, of course, unknown. For simplicity I will not write down explicitly the full Lorentz structure of the anomalous vertex function $\tilde{\Gamma}_K$ appearing in Fig. 21. It will be enough to mention that it has in addition to pseudoscalar, axial-vector and tensor structure, two novel terms with scalar and vector structure.

It is clear that in a complete microscopic picture, due to a gauge invariance, one should also introduce anomalous vertices with gluons in addition to quarks. Once again, for reasons mentioned in Sec. 3, such structures will be ignored with hope that some unknown mechanism enables one to deal only with $(\bar{q}q)M$ (both regular and anomalous) vertices in weak processes, and to disregard higher states. In Fig. 22 new vertices in a pion system are represented. They also have to be taken into account when a new set of diagrams corresponding to $K \rightarrow \pi\pi$ decays are drawn.

So far only a possibility for an $s \rightarrow d$ transition was considered. One may think of a similar $u \rightarrow c$ transition through a self-energy loop, which would create another set of anomalous vertices (*e.g.,* for K^- meson, a new $(\bar{s}c)K^-$ vertex, *etc.*) again of the order $(m_K^2/M_W^2)\sin\vartheta\cos\vartheta$. However, it is easy to see that such vertices cannot contribute to two-body K decays. Furthermore, they are probably much less interesting: while the magnitude of $s \rightarrow d$ (and $d \rightarrow s$) transitions is proportional to the degree of $\mathrm{SU}(4)_{\mathrm{flavor}}$ symmetry breaking, transitions $u \rightarrow c (c \rightarrow u)$ are proportional to the degree of violation of $\mathrm{SU}(3)_{\mathrm{flavor}}$ symmetry. It is known that the latter is small as compared to the former. Therefore even in processes in which diagrams with $u \rightarrow c$ do appear (*e.g.,* in Cabibbo suppressed D decays), the new vertices shouldn't be that important as anomalous vertices contributing to K decays.

There is one point I want to make particularly clear. The anomalous vertices described in this section are not an artifact introduced to meet demands posed by experiments. Although their exact strength is dictated by phenomenology, their mere existence fits as naturally to a confinement scheme and the valence picture as the existence of regular vertices does.

8. Results and Directions for Future Work

If one now adds contributions of diagrams with anomalous vertices (Fig. 23) to the results (6.5), one obtains the ultimate set of amplitudes,

$$\begin{aligned}
\tilde{A}^{+} &= \frac{4}{3} X \quad , & \tilde{A}^{-} &= -\tilde{A}^{+} \\
\tilde{A}^{S}_{+-} &= \frac{6}{3} X + 2Y \quad , & \tilde{A}^{L}_{+-} &= 0 \\
\tilde{A}^{S}_{00} &= -\frac{2}{3} X + 2Y \quad , & \tilde{A}^{L}_{00} &= 0 \quad .
\end{aligned} \tag{8.1}$$

The function X was defined in Eq. (6.6), while Y is given by

$$Y = -i \int \frac{d^4\ell}{(2\pi)^4} \, Tr \left[\left(a\tilde{\Gamma}_K S_d \Gamma_\pi + b\Gamma_K S_s \tilde{\Gamma}_\pi \right) S_{u(d)} \frac{1}{\sqrt{2}} \Gamma_\pi S_d \right] \quad . \tag{8.2}$$

(a and b are defined in Figs. 19 and 22 respectively.) One immediately realizes that new diagrams do not contribute to K^+ (K^-) decays, basically due to the cancellation of diagrams 23(a) and (b). On the other hand, both A^S amplitudes can gain a significant contribution provided Y function is large enough. The simplicity of relations (6.5) and (6.7) is to a certain extent lost,but still three amplitudes are described by only two functions, and from Eq. (8.1) one can form the sum rule

$$2\tilde{A}^{+} = \tilde{A}^{S}_{+-} - \tilde{A}^{S}_{00} \quad . \tag{8.3}$$

This sum rule was for the first time derived many years ago in a macroscopic analysis.[32]. Here its simple derivation is presented in a microscopic valence model, with no reference to the current algebra. While incomplete relations (6.7) were in a heavy conflict with the experiment, Eq. (8.3) reasonably agrees with measured amplitudes [see Eq. (6.8)]. Not only the sum rule, but also the absolute values, Eq. (8.1), are in agreement with experiment, provided

$$X \sim 10 \text{ eV} \qquad , \qquad Y \sim 190 \text{ eV} \quad . \tag{8.4}$$

The approach presented in this work doesn't tell us why the function Y is so much greater than the function X. One could argue that the integral in Y has a part similar to one appearing in strong $V \to PP$ (*e.g.*, $\rho \to \pi\pi$) decays, and that therefore it is so large. However, no firm conclusion can be made in a model-independent way, *i.e.*, before some explicit choices for Γ and $\tilde{\Gamma}$ were made. Although the magnitude of Y cannot be predicted, one can accept the values of Eq. (8.4) and look for consequences in other processes. Large Y suggests, for example, that decays of a short-lived kaon are enhanced as compared to the decays governed only by standard vertices. In this second category there are not only $K^{\pm} \to \pi^{\pm}\pi^{\circ}$ decays, but also many other leptonic and semileptonic decays of kaons, in which anomalous vertices are suppressed. In addition, Cabibbo allowed $D \to K\pi$ decays, in which anomalous vertices cannot contribute in the leading order, should have amplitudes comparable to $\tilde{A}^{\pm}$ rather than to $\tilde{A}^{S}$. What is the experimental situation? Unfortunately, the momentum transfer Q^2 in semileptonic decays is not the same one which appears in nonleptonic $K \to \pi\pi$ decays, where $Q^2 = m_{\pi}^2$. Therefore, without an additional assumption on the behavior of form factors $F_{1,2}^{K\pi}(Q^2)$ for various Q^2, one cannot have a conclusive test. In D decays the situation is slightly different. The experimental values are displayed in Table I for both D and K decays. In the second column the various amplitudes are written with Cabibbo angles factorized. Still, the amplitudes do have a dimension of a mass, and since the relevant scales are not expected to be the same in D and K decays, the values in this column cannot serve for a comparison. In the third column the "reduced" amplitudes are listed. These numbers were obtained when the "natural" mass-scales—M_W^2 in denominators, and masses of mesons (with an appropriate power) in numerators—were extracted. The reduced amplitudes clearly show similarities in K^- and D decays, and a strong enhancement in K^S decays. It remains of course to see whether this conclusion survives in a more quantitative treatment.[33]

So far, several processes (semileptonic K decays, D decays) were mentioned in which anomalous vertices do not contribute to the leading order. And when, in addition to $K^S \to \pi\pi$ decays, could new vertices be important? They can certainly influence radiative and rare decays of kaons. For example, significant $(\bar{d}d)K^L$ and $(\bar{s}s)K^L$ vertices could dramatically change the free-field predictions for $K^L \to \gamma\gamma$ decay (see Fig. 24) and eventually explain the disagreement between a theory and experiments even without an application of a pole model (see, *e.g.,* Ma and Pramudita in Ref. 25). In a similar way new vertices will make important contribution to rare decays. Since both anomalous vertices and bound-state effects (see discussion at the end of Sec. 5) certainly affect the existing analyses of $K \to \mu\mu$, $K \to \nu\bar{\nu}\pi$, and other rare decays, the restrictions on parameters (such as masses, *etc.,*) obtained in the standard fashion from these decays might need a complete reexamination. Finally, another class of processes whose re-analysis can be worthwhile are CP–violating processes in a six-flavor theory. Since the CP violation can be introduced through the self-energy type diagrams, one could expect a simple and model-independent (within a valence model, of course) parameterization, that might shed more light on this important issue. In this introductory work there was no space to treat all these interesting problems, but with all the main steps and techniques thoroughly described in previous section, one has enough elements for further analyses.

9. Discussion and Conclusions

As mentioned in the introduction, many terms, techniques and results in this work might look familiar to the reader. Yet a closer consideration shows some important differences between this work and previous analyses. It was already explained in Sec. 3 how the valence model used here differs from the usual one. Basically, gluons are excluded not only from vertices, but also from diagrams. Consider now some results from Sec. 8. The least original expression is the sum rule, Eq. (8.3). In fact this relation was introduced more than twenty years ago,[32] and has survived through many dramatic changes in our views on nonleptonic decays. In the microscopic approach, even when higher order Fock states and all possible QCD corrections are included, the same sum rule can be relatively easily demonstrated [for the related works, see Ref. 34]. In macroscopic approaches it is the consequence of the lack of the $\Delta I = 5/2$ terms in the effective Hamiltonian. Note that this sum rule is determined by the underlying electroweak theory, has nothing to do with QCD, and therefore is one of the rare results in hadronic physics not depending on details of hadronization of constituents.

In view of the previous discussion, the question "Why $\Delta I = 1/2$ rule is valid?" could be restated as "Why are A^S_{+-} and A^S_{00} amplitudes so close one to another?". (The sum rule then ensures the smallness of A^+.) Let us first consider the explanation in standard approaches. The amplitudes are given in terms of matrix elements of various operators. Without QCD the required ratio is the same as in Eq. (6.7),

$$\frac{A^S_{+-}}{A^S_{00}} = -3 \quad . \tag{9.1}$$

When the leading logarithmic corrections in the form $\sum_n a_n \, (g^2 \, \ell n \, M_W)^n$ are introduced, the absolute value of the ratio changes to,[2]

$$\left| \frac{A^S_{+-}}{A^S_{00}} \right| = \frac{c_- + 2c_+}{c_- - 2c_+} \quad , \tag{9.2}$$

which is still not close enough to one. In Ref. 7 it was noticed that the inclusion of terms $\sum_n b_n \, (g^2 \, \ell n \, m_c)^n$ gives a small but very important change in the effective Hamiltonian. In the original work[7] (with vacuum insertions), the new terms were expected to have large matrix elements due to appearance of some small numbers in denominators of relevant expressions. Even in approaches based on the current algebra, the new terms (PENGUIN operators) are welcome; the matrix elements of standard terms seem to be suppressed in various hadronic models, while such "helicity suppression"[35] doesn't affect PENGUIN operators. In addition, new terms have only $\Delta = 1/2$ part. Anyhow, the idea is to make regular operators that predict "incorrect" ratios (9.1) and (9.2) as negligible as possible, either by making the matrix elements of new operators very large (when vacuum insertions are used), or by using helicity suppression as big as possible (in approaches based on CA). In both cases, one remains basically with the ratio $(A^S_{+-})_{\text{PENGUIN}}/(A^S_{00})_{\text{PENGUIN}}$, which is approximately equal to one, and the $\Delta I = 1/2$ rule is satisfied. The popularity and praises to the standard approach can be related to the fact that one can always choose hadronic model parameters, the "QCD coefficients," and other parameters (masses of quarks, *etc.*) in such a way that regular operators are suppressed, and the PENGUIN operators have just the right magnitude to describe experimental values.[36]

A different picture is suggested in this work: the microscopic framework has quite a small predictive power. It provides a handy (and correct, if the valence model is correct) parametrization of observed results, but not much more. Too many unknown facts on the confining mechanism, wave functions and higher order QCD corrections, force us to use only such a quantitative description of meson decays. The question arises: how is it possible that the standard approach is free of these same problems? The answer is simple. By using standard approaches, one can easily lose control over various steps in the procedure. Particularly the role of QCD becomes unclear once various macroscopic reductions (soft pion techniques, insertions of poles, *etc.,*) are used parallel to microscopic analyses (*e.g.,* effective quark scattering amplitudes). So, the suggested answer is that the standard approach is not free of mentioned difficulties. They are just hidden. The property of the microscopic picture to expose the problems clearly should be considered as an advantage, not a drawback. In the microscopic framework for example the factorization of amplitudes [such as one denoted in Eq. (5.7)] is possible only when all QCD corrections are turned off. More precisely, though some leading $\ell n\, M_W$ terms might be factorizable, the nonlogarithmic terms (which can be of the same magnitude, and shouldn't be neglected) do not factorize (see Secs. 3 and 5). Therefore, the message learned in the microscopic analyses is: QCD corrections (apart from those responsible for the formation of bound states), are either unimportant or uncalculable. In the standard approaches, due to overlaps of various methods and techniques, it is very difficult to present this point clearly. Still, no matter how hidden, this fact must affect the standard analyses.

The major novelty in the microscopic picture is the way in which the direct $s \to d$ transitions are treated. In the standard framework these transitions are built into the crucial PENGUIN operators. The coefficients of such operators are then calculated in the perturbative analysis. In the concept proposed in this work, the anomalous vertices, created also by $s \to d$ transitions, take over the role of PENGUIN operators. However, the chiral structure of anomalous vertices doesn't seem to play any role, while in standard analyses, just the chiral structure was responsible for enhancements. Furthermore, in the microscopic model the strength of anomalous vertices can be only determined by comparison with experiments, while in the standard analyses the relevant coefficients are said to be calculable. But the most significant difference between the two approaches is the environment in which the direct transitions are placed. In the standard approaches the PENGUIN operators are controlled by a short distance expansion and hard gluons. On the contrary, anomalous vertices emerge as a long distance effect within the confinement radius. Why do I believe that the soft and not the hard corrections play such an important role? Let us consider once again the $s \to d$ transitions (Figs. 18 and 19). When no gluons are presented, such transitions are not likely to happen (the probability for a transition is proportional to $1/M_W^4$). So, one needs gluons as catalysts. However, hard gluons are very rare (compare with the smallness of coefficients of PENGUIN operators). On the other hand, the soft radiation is unrestricted, and a natural explanation within the context of soft corrections follows immediately.

In conclusion, the scheme is described in which amplitudes of the weak decays of mesons can be analyzed at the quark level, with no use of hadronic models or CA.[37] Two types of relativistically invariant vertices appear in the model:

the regular and anomalous (with "wrong" flavor content). Both arise as a result of QCD confinement. The phenomenology suggests relatively big importance of anomalous vertices, and that could affect considerably the analyses of rare K decays. The assumption that higher Fock states and QCD corrections are unimportant isn't so far in contradiction with experiment, and is on the other hand a theoretical necessity, but more work is needed to make firmer conclusions.

The approach has presently small predictive power and leads only to a suitable parametrization. (However, it is suggested that the same is true for standard approaches.) Still, in many cases even such a parametrization might bring interesting and experimentally confirmable results. The $\Delta I = 1/2$ rule is qualitatively explained as a consequence of soft and not hard corrections. The radical change in analyses of nonleptonic decays is expected not from a refinement in the treatment of hard gluons (*e.g.,* caclulations of some two- or three-loop corrections) but, on the contrary, from better understanding of confinement mechanisms, and hadronic wave functions. Only the advance along these lines would enable the calculation of integrals in Eqs. (4.2), (4.4), (6.6) and (8.2), and turn the qualitative picture presently available into a powerful quantitative method.

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10. In fact, very often the authors working within standard schemes do add some nonstandard ("long distance") contributions by hand. Such works will also be classified as standard. Note that by such an implantation no big progress is made. It is not very probable that the long- and short-distance contributions disentangle in a simple way.

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19. It would be much more complicated to describe the baryon decays in a similar way. Note that in standard approaches the same technique is used in descriptions of decays of both mesons and baryons.

20. Masses of *up* and *down* quarks are then equal, and will be denoted thereafter with the same symbol: $m_u = m_d \equiv m$.

21. The role and importance of higher Fock states in meson (and baryon) wavefunctions is discussed in S. J. Brodsky and G. P. Lepage, Phys. Scripta 23, 945 (1981); S. J. Brodsky, T. Huang and G. P. Lepage in *Particles and Fields 2,* eds. A. Z. Capri and A. N. Kamal (Plenum, New York, 1983), p. 143.

22. The idea that hard gluons are unimportant in weak decays has circulated in recent years in the literature. In another framework it was used in, *e.g.,* G. Eilam, M. D. Scadron and B. H. J. McKellar, Fresh Look at $D \to K\pi$ Decays, University of Arizona Report (1983), unpublished.

23. V. L. Chernyak and A. R. Zhitnitsky, Nucl. Phys. B201, 492 (1982); C. J. Gilmour, Z. Phys. C18, 163 (1983).

24. M. K. Gaillard and B. W. Lee, Phys. Rev. D10, 897 (1974).

25. See also T. Inami and C. S. Lim, Prog. Theor. Phys. 65, 297 (1981); E. Ma and A. Pramudita, Phys. Rev. D24, 2476 (1981).

26. E. Witten, Nucl. Phys. B122, 109 (1977).

27. In a model with broken isospin symmetry, some small and probably negligible contribution would survive.

28. As a curiosity, let me mention that with $N = -1$, the $\Delta I = 1/2$ rule follows automatically.

29. Particle Data Group, Rev. Mod. Phys. 56, S1 (1984). Only widths are listed in this review. The absolute values of amplitudes can be deduced using standard expressions (see, *e.g.,* formulae A-7 in Ref. 30).

30. M. Milošević, D. Tadić and J. Trampetić, Nucl. Phys. B187, 514 (1981).

31. J. F. Donoghue, E. Golowich, W. A. Ponce and B. R. Holstein, Phys. Rev. D21, 186 (1980).

32. E. C. G. Sudarshan, Nuovo Cimento 41, 283 (1966); See also Ref. 1, p. 601.

33. Cabibbo allowed D decays are in the microscopic picture described by three sets of (uncalculable) functions, U, V and Z. All are produced just by regular vertices. More precisely, one obtains, $B^{+} \equiv \mathcal{M}(D^{+} \to \pi^{+}\bar{K}^{\circ}) \sim U+V/3$, $B^{\circ}_{+-} \equiv \mathcal{M}(D^{\circ} \to \pi^{+}K^{-}) \sim U-Z/3$, and $B^{\circ}_{00} \equiv \mathcal{M}(D^{\circ} \to \pi^{\circ}\bar{K}^{\circ}) \sim (V+Z)/3\sqrt{2}$. U, V, and Z are some combinations of weak decay constants and weak form factors. The resulting sum rule, $B^{+} - B^{\circ}_{+-} = \sqrt{2}\, B^{\circ}_{00}$ agrees with experiments (see Table I).

34. M. Gorn, Nucl. Phys. B191, 269 (1981).

35. See *e.g.,* Ref. 31.

36. One can easily find some other reasonable parameters which however do not lead to a satisfactory explanation of the $\Delta I = 1/2$ rule (see *e.g.,* Tables 2–4 in Ref. 30). Such parameters are then said to be "excluded by experiments."

37. An interesting attempt of a direct calculation (with no use of CA or $PCAC$ — however, within the standard scheme with an effective Hamiltonian) is described in a recent paper by J. F. Donoghue and B. R. Holstein, Phys. Rev. D29, 489 (1984).

Table I.

The experimental results[29] on two-body decays of pseudoscalar mesons. The reduced amplitudes (see the explanation in the text) clearly show the dominance of decays in which anomalous vertices are expected to be important. (G is the Fermi constant; $s \equiv \sin\vartheta$ and $c \equiv \cos\vartheta$ are Cabibbo factors).

Decay Modes	Amplitudes (in KeV)		Reduced Amplitudes	
$K^+ \to \pi^+\pi^\circ$	0.083	sc	0.0589	$m_K^3\ G\ sc$
$K^S \to \pi^+\pi^-$	1.75	sc	1.2494	$m_K^3\ G\ sc$
$K^S \to \pi^\circ\pi^\circ$	1.68	sc	1.1953	$m_K^3\ G\ sc$
$D^+ \to \pi^+\bar{K}^\circ$	(1.20 ± 0.19)	c^2	(0.0158 ± 0.0025)	$m_D^3\ G\ c^2$
$D^0 \to \pi^+K^-$	(2.01 ± 0.23)	c^2	(0.0265 ± 0.0030)	$m_D^3\ G\ c^2$
$D^0 \to \pi^\circ\bar{K}^\circ$	(1.93 ± 0.51)	c^2	(0.0254 ± 0.0067)	$m_D^3\ G\ c^2$
$D^+ \to K^+\bar{K}^\circ$	(2.70 ± 1.85)	sc	(0.035 ± 0.024)	$m_D^3\ G\ sc$
$D^0 \to K^+K^-$	(3.01 ± 1.01)	sc	(0.040 ± 0.013)	$m_D^3\ G\ sc$
$D^+ \to \pi^+\pi^\circ$	< 2.6	sc	< 0.034	$m_D^3\ G\ sc$
$D^0 \to \pi^+\pi^-$	(1.51 ± 0.76)	sc	(0.02 ± 0.01)	$m_D^3\ G\ sc$

Figure Captions

Fig. 1. Various stages in a description of the $M^- \to \ell\bar{\nu}$ decay. The double line denotes a meson, the single line a quark or lepton, and the wavy line a weak boson.

Fig. 2. The vertex in Fig. 1(c) has an additional substructure. It is believed that the nonperturbative QCD leads to a confinement. Dashed lines represent gluons.

Fig. 3. Regular vertices in π and K systems. The vertex functions Γ_π and Γ_K are defined in Eq. (2.1).

Fig. 4. The gauge invariance relates $(\bar{q}q)M$ vertex to vertices containing gluons (G) in addition to fermions.

Fig. 5. An incomplete and gauge dependent correction to a weak two-body decay.

Fig. 6. The gauge invariant description of "one gluon" corrections. Higher Fock states are required.

Fig. 7. A type of "allowed" QCD corrections in the $K^\circ \to \gamma\gamma$ decay. In this figure the valence quarks are denoted with heavy lines.

Fig. 8. Diagram describing $K^- \to \ell\bar{\nu}$ decay in the microscopic framework. Types of particles and their momenta are denoted.

Fig. 9. All radiative corrections are already included in the effective vertex. Therefore, diagrams of that type do not appear in the microscopic model.

Fig. 10. The microscopic description of $K^- \to \pi^\circ \ell\bar{\nu}$ decay.

Fig. 11. One of the box diagrams in $K^L \to \gamma\gamma$.

Fig. 12. The generalized Fierz transformation illustrated in one of the diagrams contributing to $K^L \to \gamma\gamma$ decay. N denotes the number of colors. To the leading order GFT provides a simple relation between different types of diagrams.

Fig. 13. Two-body decays of the charged kaon (K^+) with regular vertices. The heavy line here denotes the strange quark. φ stands for Higgs ghost. The similar set of diagrams corresponds to $K^- \to \pi^-\pi^\circ$ decay.

Fig. 14. $K^S \to \pi^\circ\pi^\circ$ decay. Cross terms (with pions exchanged) should be added. $K^L \to \pi^\circ\pi^\circ$ is described by a similar set of diagrams.

Fig. 15. $K^S \rightarrow \pi^-\pi^+$ with regular valence vertices. The decay $K^L \rightarrow \pi^-\pi^+$ can be described in a similar way.

Fig. 16. GFT in $K \rightarrow \pi\pi$ decays. The indicated relation is exact when terms of the order $1/M_W^4$ are neglected. R and Q are momenta of outgoing pions.

Fig. 17. As a consequence of GFT and assumed isospin symmetry, the entire class of diagrams in Figs. 13–15 give no leading contributions to decay amplitudes. Namely, the amplitude of the diagram on the r.h.s. is exactly zero in an SU(2) symmetric world.

Fig. 18. The direct $s \rightarrow d$ transition through a "self-energy" loop. In spite of GIM mechanism, diagrams are divergent and need a renormalization.

Fig. 19. An $s \rightarrow d$ transition takes place within the confinement radius, which gives rise to an anomalous $(\bar{d}u)K^-$ vertex. The strength of the new vertex is unknown, the order of its magnitude is $a = (m_K/M_W)^2 \sin\vartheta\cos\vartheta$.

Fig. 20. Gluons emitted from a self-energy loop serve as a catalyst and make $s \rightarrow d$ transitions more probable.

Fig. 21. Anomalous vertices in K system, relevent for $K \rightarrow \pi\pi$. a was defined previously in Fig. 19.

Fig. 22. Anomalous vertices in π system. $b = (m_\pi/M_W)^2 \sin\vartheta\cos\vartheta$.

Fig. 23. The diagrams with anomalous vertices contributing to $K^+ \rightarrow \pi^+\pi^\circ$ decays. Similar sets of diagrams can be constructed for other decay modes.

Fig. 24. $K^\circ \rightarrow \gamma\gamma$ with the anomalous vertices.

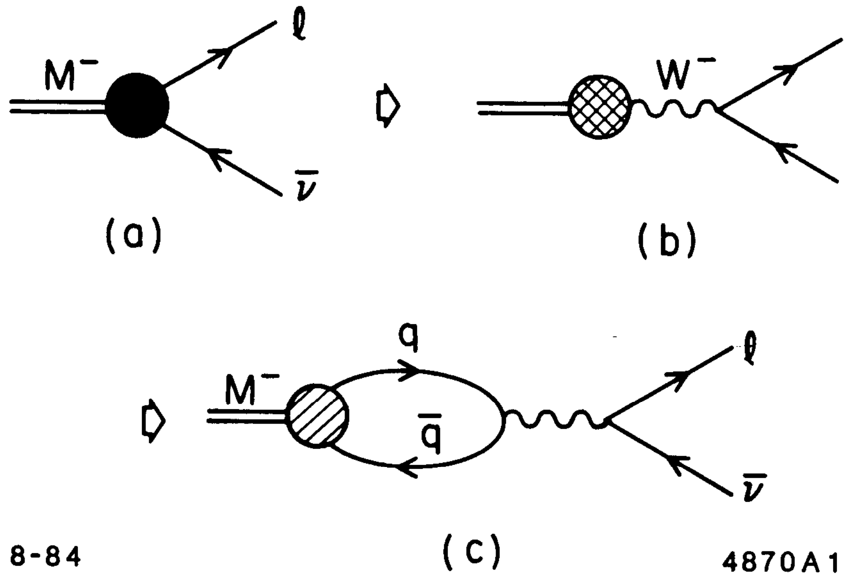


Fig. 1

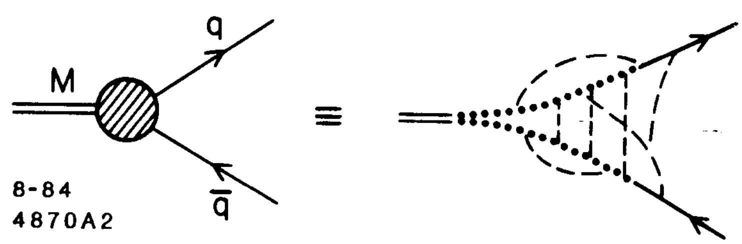
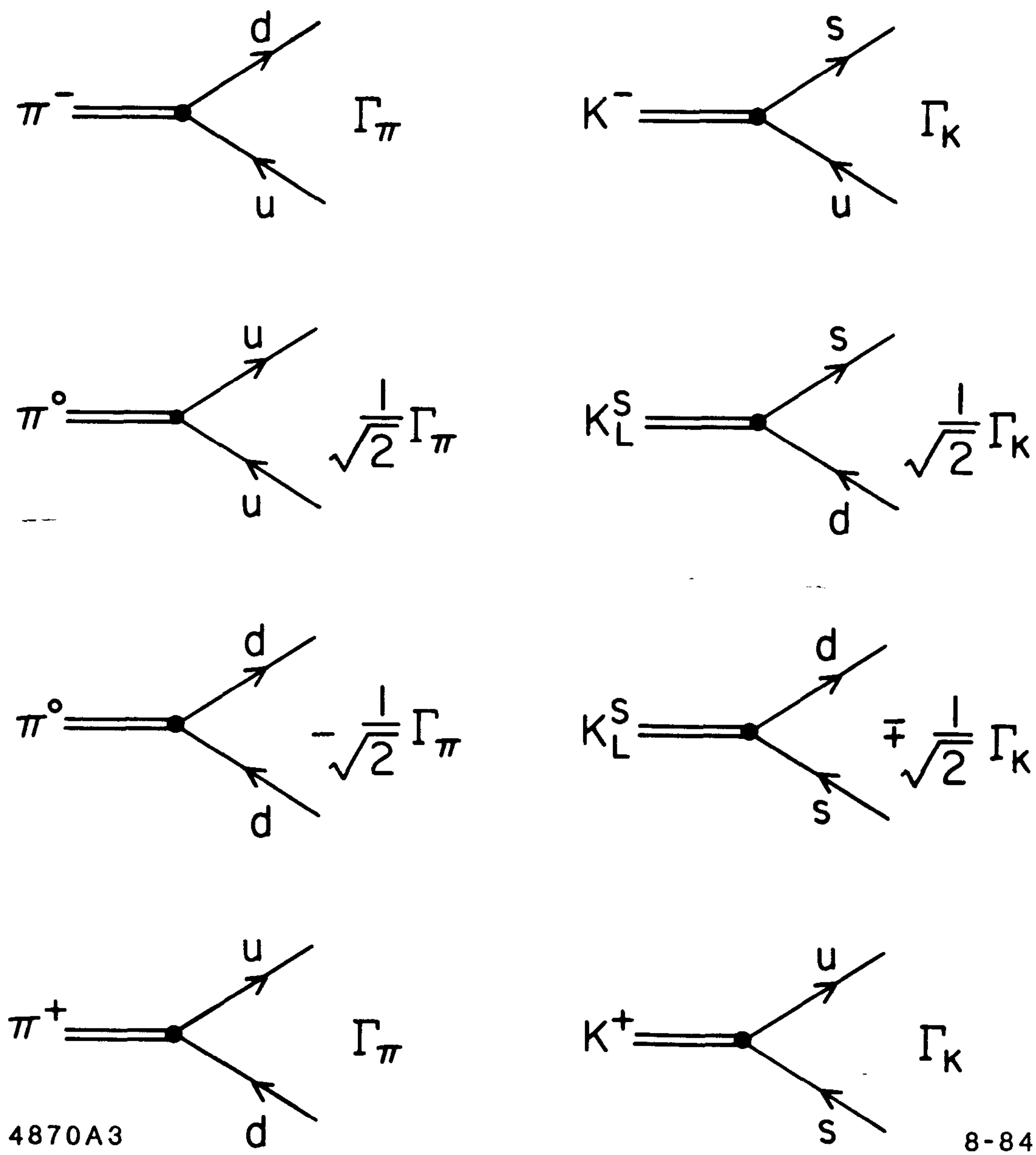
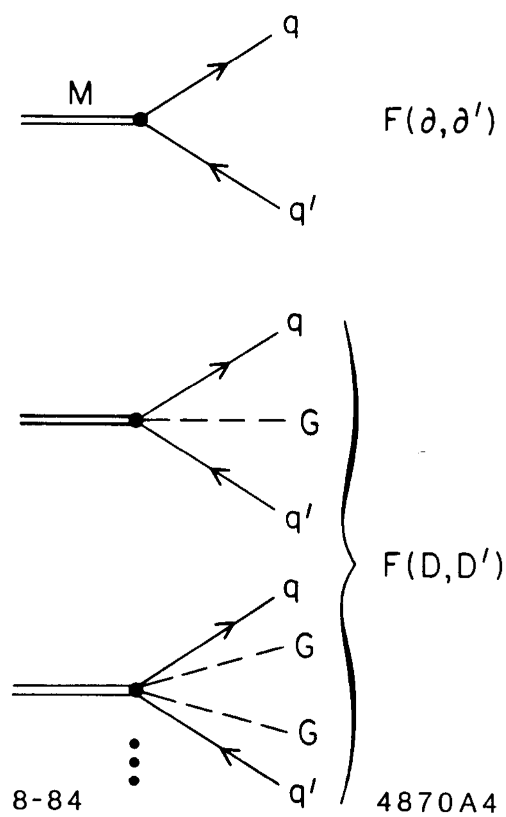


Fig. 2



π^-
d
u
Γ_π
K^-
s
u
Γ_K
π°
u
u
$\frac{1}{\sqrt{2}}\Gamma_\pi$
K_L^S
s
d
$\frac{1}{\sqrt{2}}\Gamma_K$
π°
d
d
$-\frac{1}{\sqrt{2}}\Gamma_\pi$
K_L^S
d
s
$\mp\frac{1}{\sqrt{2}}\Gamma_K$
π^+
u
d
Γ_π
K^+
u
s
Γ_K
4870A3
8-84

Fig. 3



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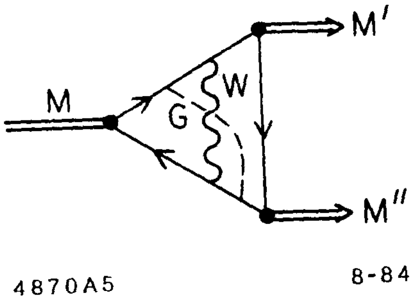


Fig. 5

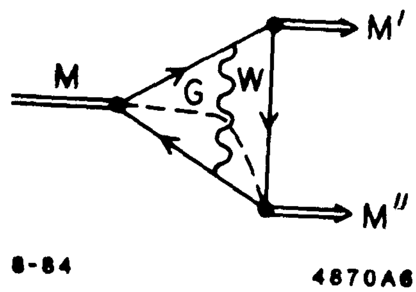
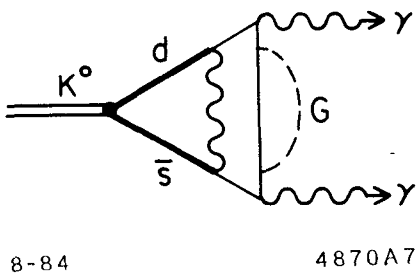


Fig. 6



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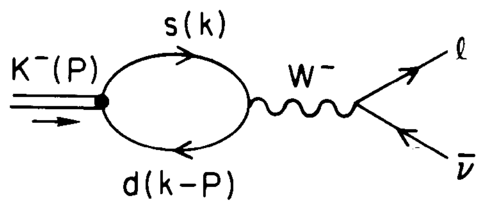


Fig. 8

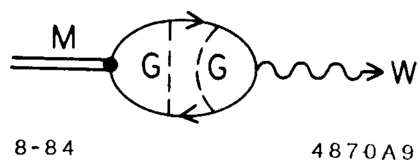


Fig. 9

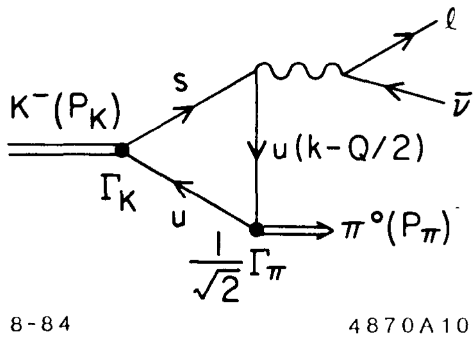


Fig. 10

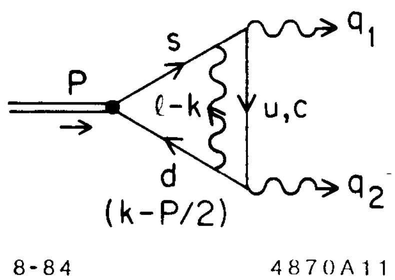
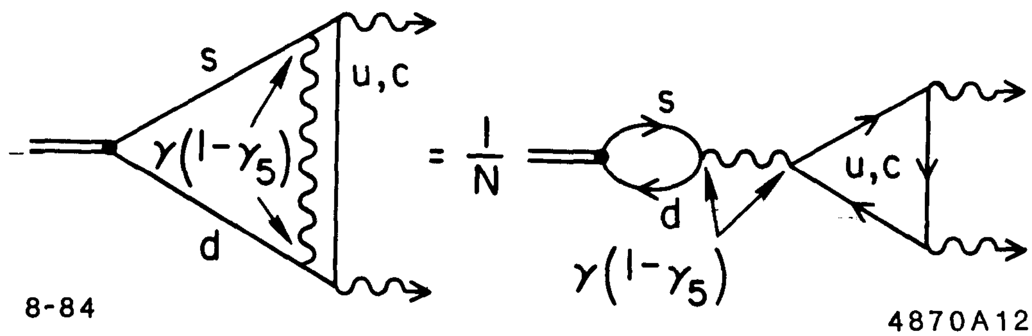


Fig. 11



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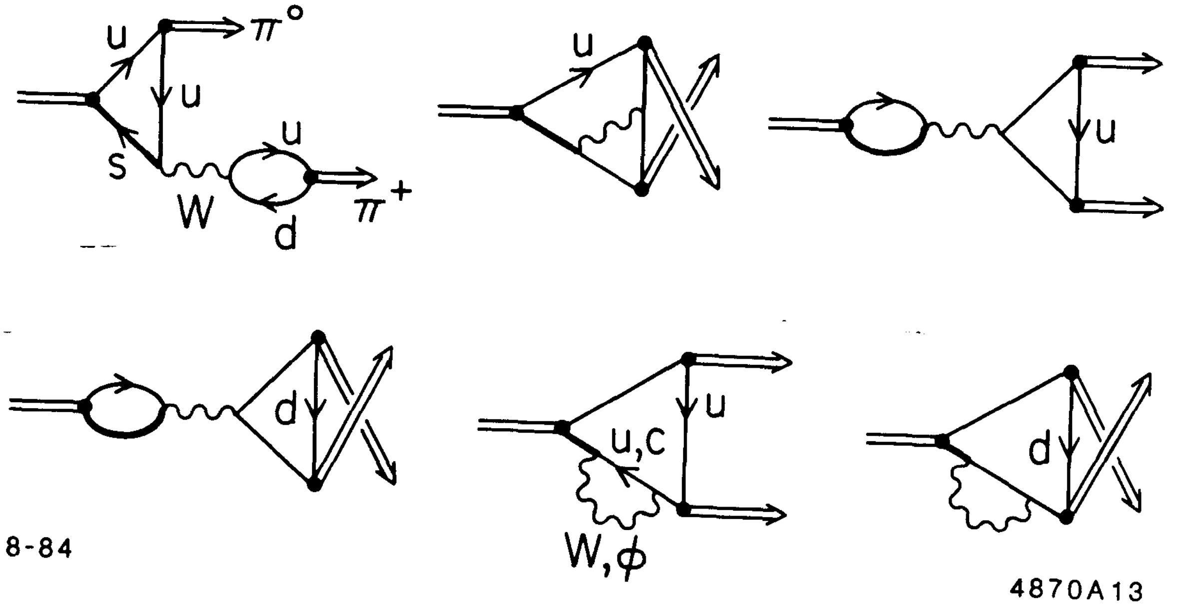
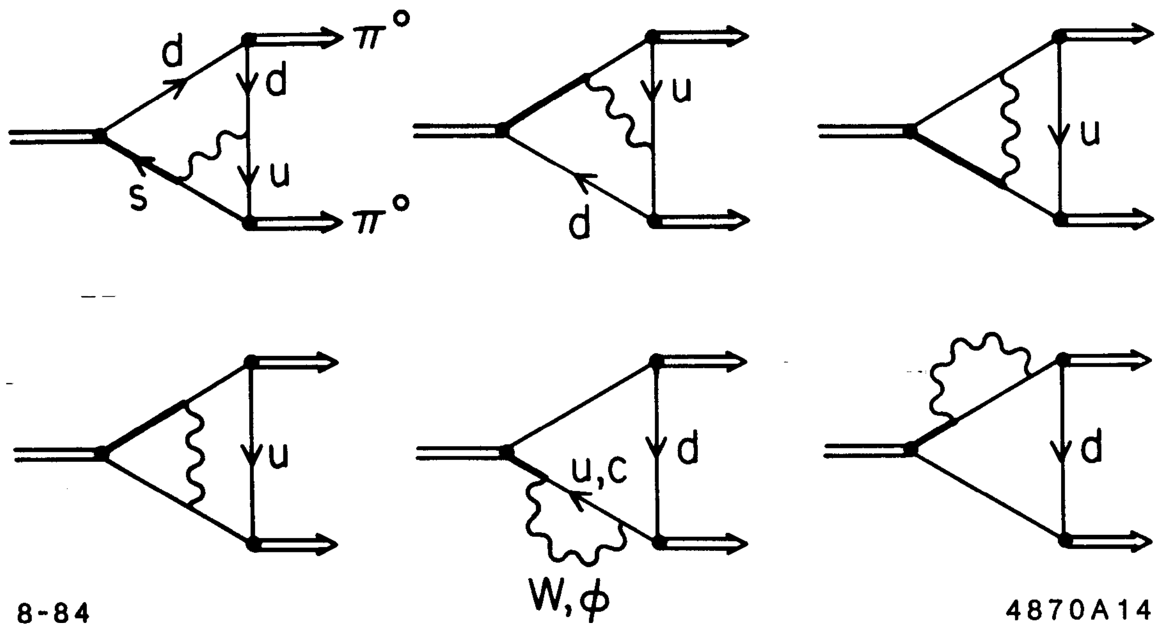


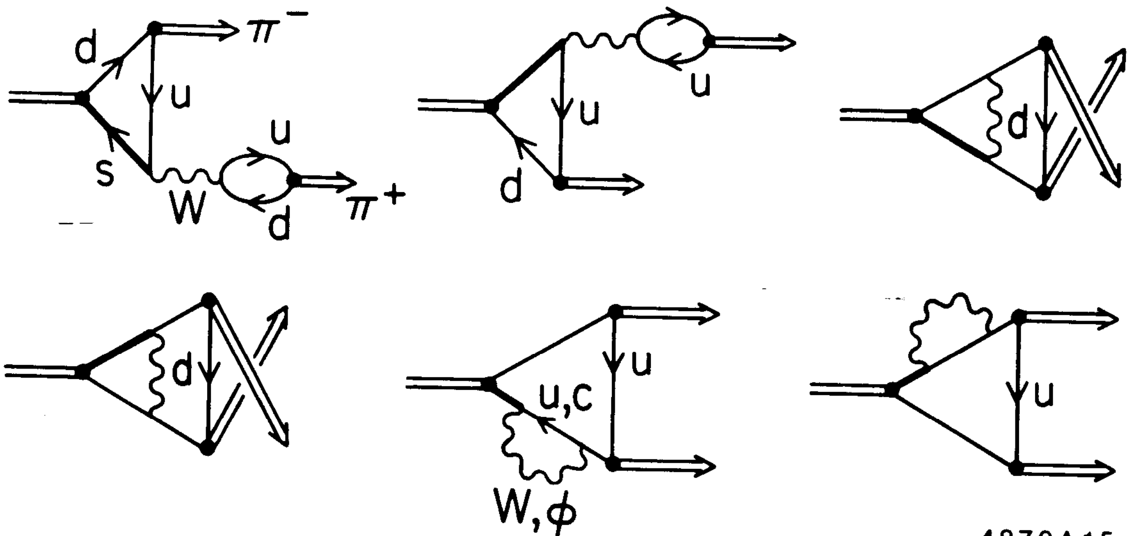
Fig. 13



8-84

4870A14

Fig. 14



8-84

4870A15

Fig. 15

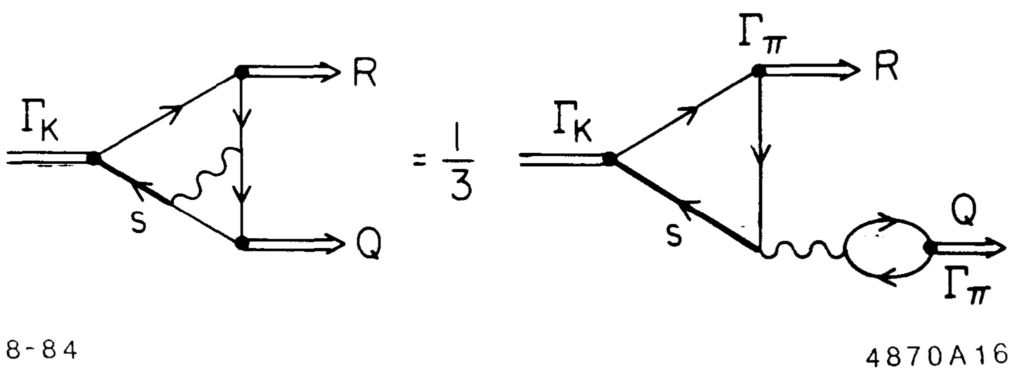
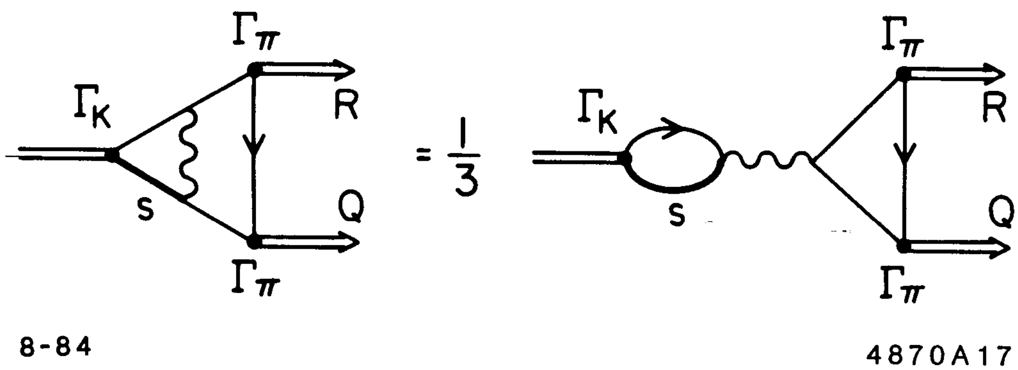


Fig. 16



Γ_K
Γ_π
R
Q
s
Γ_π
$= \frac{1}{3}$
Γ_K
s
Γ_π
R
Q
Γ_π
8-84
4870A17

Fig. 17

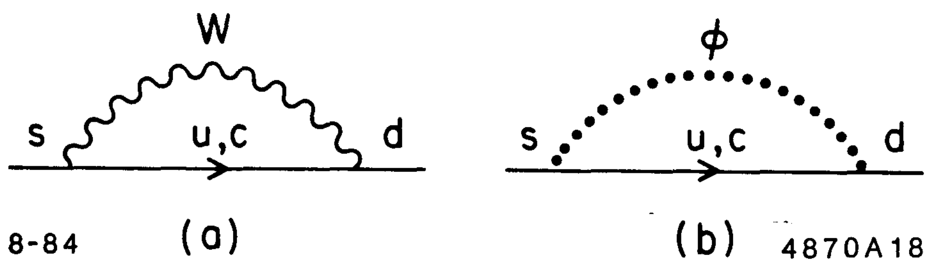


Fig. 18

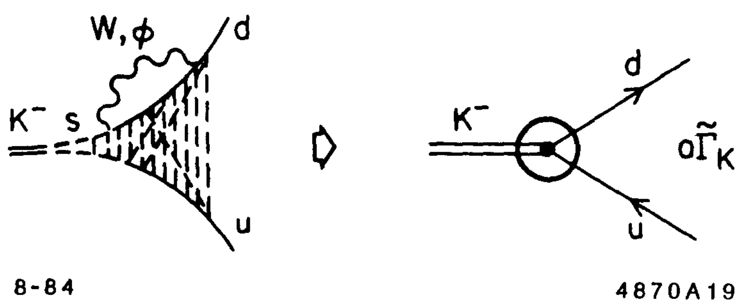


Fig. 19

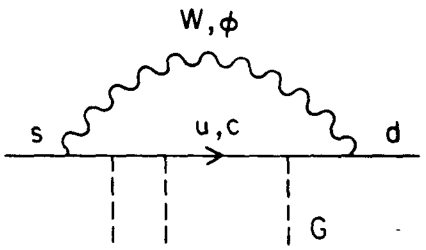
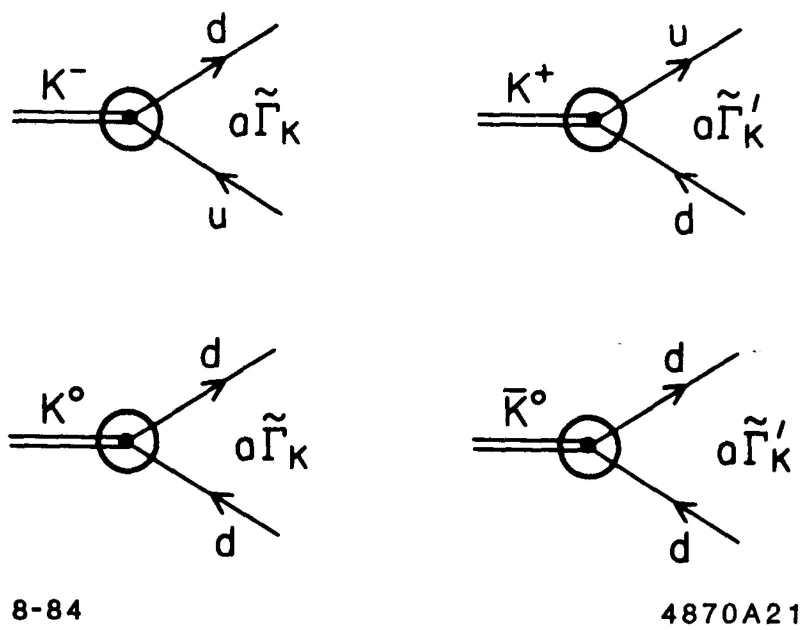


Fig. 20



K⁻
d
u
$a\tilde{\Gamma}_K$
K⁺
u
d
$a\tilde{\Gamma}'_K$
K°
d
d
$a\tilde{\Gamma}_K$
$\bar{K}$°
d
d
$a\tilde{\Gamma}'_K$
8-84
4870A21

Fig. 21

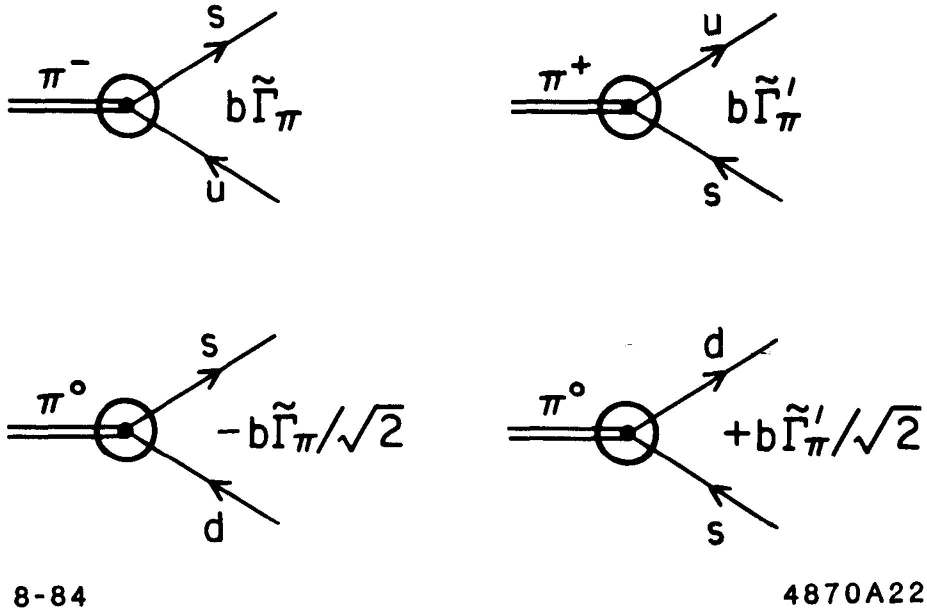
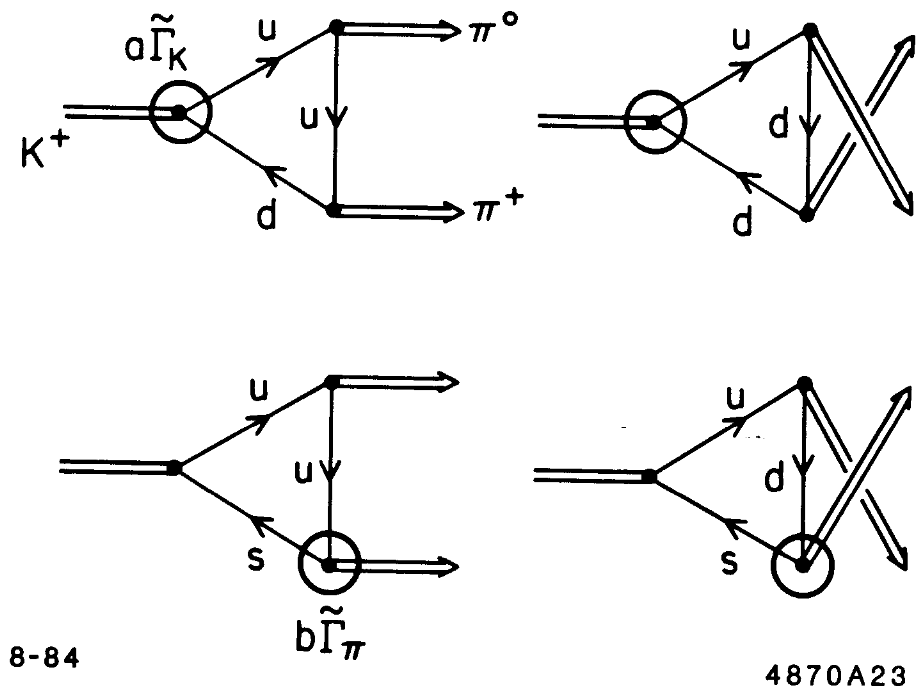

π^-
s
u
$b\tilde{\Gamma}_\pi$
π^+
u
s
$b\tilde{\Gamma}'_\pi$
π°
s
d
$-b\tilde{\Gamma}_\pi/\sqrt{2}$
π°
d
s
$+b\tilde{\Gamma}'_\pi/\sqrt{2}$
8-84
4870A22

Fig. 22



$a\tilde{\Gamma}_K$
u
π°
u
K^+
d
π^+
u
d
d
u
u
s
$b\tilde{\Gamma}_\pi$
u
d
s
8-84
4870A23

Fig. 23

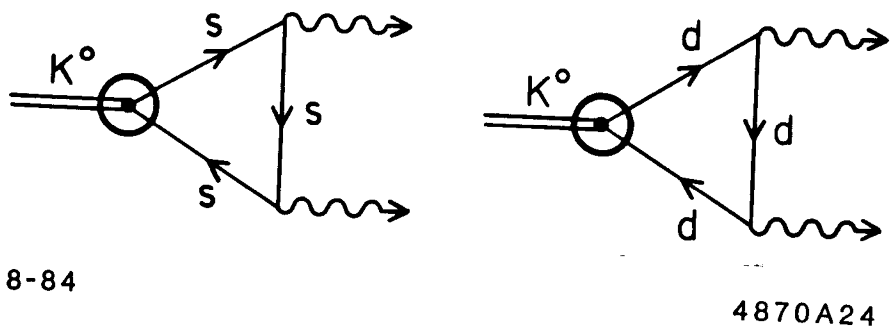


Fig. 24