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ANGULAR DISTRIBUTIONS IN ANOMALOUS $\ell^+\ell^-\gamma$ EVENTS*

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ABSTRACT

Among various models proposed so far, attempting to explain the anomalous $\ell^+\ell^-\gamma$ events, four representative types of models, i.e. I) $Z^0 \to S$ or $P + \gamma \to \ell^+\ell^-\gamma$, II) $Z^0 \to Z^0_{virt} + \gamma \to \ell^+\ell^-\gamma$ (anomalous magnetic moment interaction), III) $Z^0 \to \bar{\ell}\ell^* + \ell\bar{\ell}^* \to \ell^+\ell^-\gamma$ (excited lepton) and IV) S or $P \to Z^0_{virt} + \gamma$ are confronted with a test of the distribution in the angle between photon and one of the leptons. All these four models seem to fail to provide the explanation to the angular characteristics of the data. However, more statistics is needed to arrive at firm conclusions with regards to the mechanism of the events, including as well the bremsstrahlung contribution.

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1. Introduction

The anomalous radiative decays $Z^0 \to \ell^+\ell^-\gamma$ ($\ell = e, \mu$) observed recently by UA1 and UA2 collaborations [1,2] at the CERN $\bar{p}$p collider seem to require a new physical picture which might strongly depart from the standard SU(2)xU(1) electroweak theory.[3] The most natural way of going beyond the standard model is perhaps to invoke the underlying substructure within the weak gauge bosons $W^{\pm}, Z^0$ and/or presently "basic" fermions, i.e. leptons and quarks. Indeed from such viewpoint extensive theoretical efforts have already been devoted to understanding the observed anomalous $\ell^+\ell^-\gamma$ events. The explanations suggested for such events in the literature can be classified into the following three types based on the assumption of the compositeness of either the weak bosons [4-9] or fermions:[8] [10]

(I) The radiative Z^0 decay proceeds via $Z^0 \to S$ or $P + \gamma \to \ell^+\ell^-\gamma$, where the intermediate $S(P)$ is a hypothetical scalar (pseudoscalar) partner to the Z^0-boson. Within the composite Z^0 model, such particle can exist as a result of splitting from the Z^0 by hyperfine interactions among the constituents.

(II) The radiative transition goes via anomalous $ZZ\gamma$ couplings due to large anomalous magnetic moment of the Z^0 : $Z^0 \to Z^0_{virt} + \gamma \to \ell^+\ell^-\gamma$.

(III) The radiative transition goes via production of an excited lepton ℓ^* and a normal lepton $\ell(Z^0 \to \ell^*\bar{\ell} + \ell\bar{\ell}^*)$, followed by subsequent electromagnetic de-excitation of the $\ell^*(\ell^* \to \ell\gamma)$.

Among other options considered,[11] we note:

(IV) The attempts to account for the anomalous events due to other particle decays such as $S \to Z^0_{virt} + \gamma \to \ell^+\ell^-\gamma$, where S is the scalar bound state of the composite models (Veltman[11]) and $P \to Z^0_{virt} + \gamma \to \ell^+\ell^-\gamma$, where P is the corresponding pseudoscalar boson (Marciano[11]).

Which model if any can be realistic? To answer the question one has to perform various detailed tests to these models. The aim of the present paper is to perform the systematic study on the angular distribution: $d\Gamma/dE_\gamma d\cos\theta_{\ell-\gamma}$,where $\theta_{\ell-\gamma}$ is the angle between the photon and one of the produced leptons $\ell^- = e^-$ or μ^-,assuming the above four mechanisms and to point out that the angular characteristics of the observed events impose rather strong constraints on the models. The data indicate that[1][2] $\theta^*_{e-\gamma}(\text{in } e^+e^-\gamma) = 14.4^0 \pm 4.0^0$ (UA1), $\theta^*_{e\gamma}(\text{in } e^+e^-\gamma) = 31.8^0$ (UA2), $\theta^*_{\mu-\gamma}(\text{in } \mu^+\mu^-\gamma) = 7.9^0$ (UA1). Note that one of the leptons is always produced in the "forward" direction, or in other words, ℓ^+ and ℓ^- are emitted nearly back to back with each other. In order to compare the theoretically calculated angular distributions with the experimental data,we utilize as a measure the following "forward-backward" asymmetry,

$$\alpha = \frac{\frac{d\Gamma}{dE_\gamma d\cos_{\ell-\gamma}}(0 \le \theta_{\ell-\gamma} \le \theta_0, \theta_1 \le \theta_{\ell-\gamma} \le \pi) - \frac{d\Gamma}{dE_\gamma d\cos\theta_{\ell-\gamma}}(\theta_0 \le \theta_{\ell-\gamma} \le \theta_1)}{\frac{d\Gamma}{dE_\gamma d\cos_{\ell-\gamma}}(0 \le \theta_{\ell-\gamma} \le \theta_0, \theta_1 \le \theta_{\ell-\gamma} \le \pi) + \frac{d\Gamma}{dE_\gamma d\cos\theta_{\ell-\gamma}}(\theta_0 \le \theta_{\ell-\gamma} \le \theta_1)}, \tag{1}$$

where "forward" means a direction in which an angle between one of the leptons and photon is smaller than θ_0, i.e. $0 \le \theta_{\ell-\gamma} \le \theta_0$ or $0 \le \theta_{\ell+\gamma} \le \theta_0$ (equivalently $\theta_1 \le \theta_{\ell-\gamma} \le \pi$) and "backward" means a direction in which the angles between a photon and both of the leptons are larger than θ_0, i.e. $\theta_0 \le \theta_{\ell-\gamma} \le \theta_1$ (equivalently $\theta_0 \le \theta_{\ell+\gamma} \le \theta_1$). Taking $\theta_0 = 45^0(60^0)$ and $E_\gamma = 30$ GeV, we have $\theta_1 = 164.4^0(158.0^0)$. For the three observed events we have obviously $\alpha_{exp} = 1.$[◇] We show that all of the four models find it hard to reproduce this value.

We organize the paper as follows: In Section 2 we present the basic formulas and the numerical results of the angular distribution $d\Gamma/dE_\gamma d\cos\theta_{\ell-\gamma}$for each type of models. Section 3 is devoted to discussions with a few concluding remarks.

◇ The above mentioned data for the angles $\theta^*_{\ell-\gamma}$, i.e. $14.4 \pm 4.0^0, 31.8^0$ and 7.9^0, are actually given in the $\bar{p}p$ c.m.s. However, we have checked that in the rest system of Z^0(or P, S) these angles satisfy the condition $\theta_{\ell-\gamma} \le 30^0$ which is within the interval $0^0 - 45^0$ we have used. Thus α_{exp} remains equal to 1.

2. Formulas and Numerical Results

The formula of the angular distribution for the decay products is given as follows:

$$d\Gamma_i(Z^0 \to \ell^+\ell^-\gamma) = \frac{(2\pi)^{-5}}{6M_Z}|\mathcal{M}_i|^2 R_3, (i = I, II, III) \tag{2}$$

where $\mathcal{M}_i$ is the matrix element of $Z^0 \to \ell^+\ell^-\gamma$, and

$$R_3 = \int \frac{d^3q_1}{2q_{10}} \frac{d^3q_2}{2q_{20}} \frac{d^3k}{2k_0} \delta^4(P - q_1 - q_2 - k), \tag{3}$$

with P, k, q_1, and q_2 being momenta of the Z^0, γ, ℓ^-, and ℓ^+, respectively. In the rest system of Z^0-boson, taking the z-axis to be $\vec{k}$ and defining $\theta = \theta_{\ell^-\gamma}$ equal to be the angle between $\vec{k}$ and $\vec{q}_1$, one can rewrite R_3 as

$$R_3 = \frac{\pi^2 M_Z^2 N_I(1 - x_3)x_3 dx_3 d\cos\theta}{8}, \tag{4}$$

$$N_I = \frac{1}{\{1 - \frac{x_3}{2}(1 - \cos\theta)\}^2}.$$

The following dimensionless quantities are used:

$$x_1 = 2q_{10}/M_Z, \quad x_2 = 2q_{20}/M_Z, \quad x_3 = 2k_0/M_Z, \tag{5}$$

for which we have

$$x_1 = \frac{1 - x_3}{1 - \frac{x_3}{2}(1 - \cos\theta)},$$

$$x_2 = 2 - x_1 - x_3.$$

The corresponding formulas for the case IV : S or $P \to Z^0_{virt} + \gamma \to \ell^+\ell^-\gamma$ can be obtained by putting $\frac{1}{3}|\mathcal{M}_i|^2 \to |\mathcal{M}_{IV}|^2$ and M_Z(mass of Z-boson)$\to M_S$ or M_P (mass of S or P) in Eqs.(2),(4), and (5). In the following we calculate the angular distribution $d\Gamma/dE_\gamma d\cos\theta$ and α for each case in order. The values of α will be given in Table 1. The distribution in the angle $\cos\theta_{\ell^+\gamma}$: $d\Gamma/dE_\gamma d\cos\theta_{\ell^+\gamma}$ can be obtained by interchanging $x_1 \leftrightarrow x_2$ in the corresponding formula of $d\Gamma/dE_\gamma d\cos\theta_{\ell^-\gamma}$.

$\underline{I. Z^0 \rightarrow S\, or\, P + \gamma \rightarrow \ell^+\ell^-\gamma}$

The possibility of enhancing the decay $Z^0 \rightarrow \ell^+\ell^-\gamma$ through an intermediate spin-0 partner to the Z^0-boson has been discussed by many authors.[4-8] Assuming for the couplings:

$$SZ(e,P)\gamma(\varepsilon,k) : ig_1\varepsilon^{\mu\nu\rho\sigma}e_\mu\varepsilon_\nu P_\rho k_\sigma, \qquad (6a)$$

$$PZ(e,P)\gamma(\varepsilon,k) : g_2\{(e\varepsilon)(Pk)-(ek)(\varepsilon P)\}, \qquad (6b)$$

where e^μ, ε^μ are the polarization vectors of the Z^0 and γ, respectively, and

$$S(Q)\ell^-(q_1)\ell^+(q_2) : \bar{u}(q_1)a(-ib\gamma_5)v(q_2), \qquad (7)$$

we obtain

$$|\mathcal{M}_I(S\text{-boson})|^2 = g_1^2 a^2 M_Z^2 \frac{(1-x_3)x_3^2}{(1-x_3-M_S^2/M_Z^2)^2 + M_S^2\Gamma_S^2/M_Z^4}. \qquad (8)$$

$|\mathcal{M}_I(P\text{-boson})|^2$ is obtained by replacing $g_1 \rightarrow g_2, a \rightarrow b, \Gamma_S$(total width)$\rightarrow \Gamma_P$, and $M_S \rightarrow M_P$ in Eq.(8). The simple model of the intermediate S- or P-boson encounters a criticizm[12] in that it violates chiral symmetry and that this symmetry breaking unavoidably causes the strong suppression of the process $Z^0 \rightarrow S(P)+\gamma \rightarrow \ell^+\ell^-\gamma$. To avoid such difficulty Peccei has invented a model in which the Z^0 transition goes through a pair of parity doublet spin 0 states thus preserving chiral invariance.[8]

Let us study the angular distribution for the models I. It is essentially determined by the kinematical factor N_I in Eq.(4), since $|\mathcal{M}_I|^2$ does not exhibit any $\cos\theta$ dependence. Thus Fig.1 calculated with $E_\gamma = 30$ GeV and $M_Z = 93$

GeV (we use these values for the models $II \sim IV$ as well) indicates that the models I including Peccei's one seem to be unfavoured by the experimental data.*

$\underline{II. Z^0 \to Z^0_{virt} + \gamma \to \ell^+\ell^-\gamma}$

In composite models a large anomalous moment χ to the Z^0, different from the gauge theory value is possible.[18] The possibility that the interactions induced by such $ZZ\gamma$ anomalous magnetic moments are responsible for the radiative Z decays have been considered in Refs. [8] and [9]. Assuming that electroweak currents are first class only, i.e. $CP = +1$ gauge invariant coupling, we have the couplings of electric and magnetic quadrupole transitions[8]

$$Z(e,P)Z_{virt}(e',Q)\gamma(\varepsilon,k) : ie\chi\{f_1(e'k)\varepsilon^{\mu\nu\rho\sigma}Q_\mu k_\nu \varepsilon_\rho e_\sigma + f_2(eQ)\varepsilon^{\mu\nu\rho\sigma}Q_\mu k_\nu \varepsilon_\rho e'_\sigma\}, \tag{9}$$

and for the couplings,

$$Z^0\ell^-(q_1)\ell^+(q_2) : \frac{ie}{\sin\theta_W \cos\theta_W}\bar{u}(q_1)\gamma_\sigma\{(\sin^2\theta_W - 1/4) + \frac{1}{4}\gamma_5\}v(q_2), \tag{10}$$

as in the standard model, we obtain

$$|\mathcal{M}_{II}|^2 = K[|f_1|^2(1-x_1)(1-x_2) + |f_2|^2(1-x_3)\{(1-x_1)^2 + (1-x_2)^2\}], \tag{11}$$

where

$$K = \frac{(4\pi\alpha)^2\chi^2 x_3^2 M_Z^4}{x_3^2 + \Gamma_Z^2/M_Z^2}\frac{1}{\sin^2\theta_W\cos^2\theta_W}\{(\sin^2\theta_W - \frac{1}{4})^2 + \frac{1}{16}\}.$$

• Lindfors et al. in Ref. 6 have proposed a model in which mixture of Z^0 with a high-mass $J^{PC} = 1^{--}$ toponium state $V(t\bar{t})$ is produced and this $V(t\bar{t})$ subsequently decays into a Higgs particle and a photon: $V(t\bar{t}) \to H + \gamma$ with $H \to \ell^+\ell^-$. They also considered the distribution $W(\cos\theta)$ in the angle between ℓ^- and γ. However, their conclusion regarding the angular distribution is misleading, since it is based on the incorrect formula for $W(\cos\theta)$ corresponding to our formula (4).

The angular dependences of the first and the second terms:

$$N_{II,1} = N_I(1-x_1)(1-x_2), \tag{12a}$$

$$N_{II,2} = N_I\{(1-x_1)^2 + (1-x_2)^2\}, \tag{12b}$$

are separately shown in Fig. 2. Clearly both terms are peaked at θ large and hence no choice of f_1 and f_2 can make the model to be compatible with the present data.

$III. \underline{Z^0 \to \bar{\ell}\ell^* + \ell\bar{\ell}^* \to \ell^+\ell^-\gamma}$

The attempts to describe the anomalous radiative Z-decays by means of an excited lepton[14] as intermediate state are those listed in Refs. [8] and [10]. Taking excited quarks as spin 1/2 objects we assume the following phenomenological effective Lagrangian for the interactions between the excited lepton ℓ^* with mass m_* and the photon and the Z^0:

$$\mathcal{L}_{\ell^*\ell\gamma} = \frac{e}{2m_*}\bar{\psi}_{\ell^*}\sigma^{\mu\nu}F_{\mu\nu}(a_\gamma - \gamma_5 b_\gamma)\psi_\ell + h.c., \tag{13a}$$

$$\mathcal{L}_{Z^0\ell^*\ell} = \frac{e}{2m_*}\bar{\psi}_{\ell^*}\sigma^{\mu\nu}F^{(Z)}_{\mu\nu}(a_Z - \gamma_5 b_Z)\psi_\ell + h.c., \tag{13b}$$

where $F_{\mu\nu}$ and $F^{(Z)}_{\mu\nu}$ are the field strength tensors of photon and Z^0-boson, and a_i and $b_i (i = \gamma, Z)$ are dimensionless coupling constants. We assume the CP invariant couplings, i.e. a_i and b_i are real. Then straightforward calculation leads us to

$$|\mathcal{M}_{III}|^2 = \frac{4(\pi\alpha)^2}{r^2}\{g^2_{\ell^*\ell\gamma}g^2_{Z\ell^*\ell}R + 4a_Z b_Z a_\gamma b_\gamma S - 2r(a_Z^2 - b_Z^2)(a_\gamma^2 - b_\gamma^2)T\}, \tag{14}$$

where

$$R = \frac{1-x_1}{(1-x_1-r)^2}[(1-x_1)x_1(3-2x_1) + r\{2x_1x_3 - (1-x_2)\}] + (x_1 \leftrightarrow x_2), \tag{15a}$$

$$S = \frac{1-x_1}{(1-x_1-r)^2}[(1-x_1)x_1(3-2x_1) - r\{2x_1x_3 - (1-x_2)\}] + (x_1 \leftrightarrow x_2), \quad (15b)$$

$$T = \frac{1}{(1-x_1-r)(1-x_2-r)}[-x_3\{x_1(1-x_1)+x_2(1-x_2)-x_3(1-x_3)\}+(1-x_1)(1-x_2)], \quad (15c)$$

$$r = m_*^2/M_Z^2, \quad g_{\ell^*\ell\gamma}^2 = a_\gamma^2 + b_\gamma^2, \quad g_{Z\ell^*\ell}^2 = a_Z^2 + b_Z^2.$$

In Fig.3 the angular dependences for R, S, and T, multiplied by N_I, are plotted. From $\Gamma(Z^0 \to e^+e^{*-} + e^-e^{*+}) = 15$ MeV and using

$$\Gamma(Z^0 \to \ell\bar{\ell}^* + \bar{\ell}\ell^*) = \frac{2e^2 g_{Z\ell^*\ell}^2 M_Z (1-r)^2(1+2r)}{24\pi r}, \quad (16)$$

one has with $m_* = 80$ GeV:[8]

$$\begin{aligned} \frac{e^2}{m_*^2} g_{Z\ell^*\ell}^2 &\simeq 0.45 \cdot 10^{-5}[GeV^{-2}], \\ \frac{e^2}{m_*^2} g_{\ell^*\ell\gamma}^2 &\simeq 10^{-4}e^2[GeV^{-2}]. \end{aligned} \quad (17)$$

$g_{\ell^*\ell\gamma}^2$ is calculated using the W-dominance and γ-W mixing.[15] These values are consistent with the present experimental limit obtained from $e^+e^- \to e^+e^-, \gamma\gamma$ and $\mu^+\mu^-$.[16] The $(g-2)$ measurement of electron and muon, and also the electric dipole moment of electron imposes the following relation:[7]

$$a_\gamma = b_\gamma, \quad a_Z = b_Z \frac{b_\gamma}{a_\gamma}. \quad (18)$$

Using Eqs. (17) and (18), we have calculated $d\Gamma/dE_\gamma d\cos\theta_{\ell-\gamma}$ for $m_* = 80$ GeV (see Fig. 4). From Figs. 3 and 4, we see again that the models of type III seem to fail in explaining the angular characteristics of the three $\ell^+\ell^-\gamma$ events.

$IV. S or P \to Z^0_{virt} + \gamma \to \ell^+\ell^-\gamma$

Among various other explanations[11] we also consider the mechanism due to S or $P(\sim 93$ GeV$) \to Z^0_{virt} + \gamma \to \ell^+\ell^-\gamma$, suggested by Veltman[11] in the S-boson case and by Marciano[11] in the P-boson case as the source of the anomalous $\ell^+\ell^-\gamma$ events. Assuming for the relevant couplings:

$$S(P)Z^0(e,Q)\gamma(\varepsilon,k) : g_2\{(e\varepsilon)(kP) - (ek)(\varepsilon P)\}, \tag{19a}$$

$$P(P)Z^0(e,Q)\gamma(\varepsilon,k) : ig_1\varepsilon^{\mu\nu\rho\omega}k_\mu\varepsilon^*_\nu P_\rho e^*_\omega, \tag{19b}$$

and Eq.(10) for $Z^0\ell^+\ell^-$, we obtain

$$|\mathcal{M}_{IV}(S\text{-boson})|^2 = C[\mu_S^2 x_3\{(1-\mu_S x_1)x_1 + (1-\mu_S x_2)x_2\} - 2(1-\mu_S x_1)(1-\mu_S x_2)], \tag{20}$$

$$C = \frac{g_2^2 M_S^2}{\sin^2\theta_W \cos^2\theta_W}\{(\sin^2\theta_W - \frac{1}{4})^2 + \frac{1}{16}\}$$

$$\cdot \frac{1}{(1-\mu_S x_3 - \mu_S^2)^2 + \mu_S^4\Gamma_Z^2/M_Z^2},$$

where $\mu_S = M_Z/M_S \simeq 1$. The formula for $|\mathcal{M}_{IV}(P\text{-boson})|^2$ can be obtained from Eq.(20) by replacing $g_2 \to g_1, M_S \to M_P$, and $\mu_S \to \mu_P = M_Z/M_P$. In Fig.5, we display the angular dependence of

$$N_{IV} = N_I[x_3\{(1-x_1)x_1 + (1-x_2)x_2\} - 2(1-x_1)(1-x_2)]. \tag{21}$$

The models IV again cannot be made compatible with the data.

3. Conclusion

In this paper we have performed the study on the angular distribution of the anomalous $\ell^+\ell^-\gamma$ events, employing the representative four types of models. All of the four models predict that leptons are produced prominently with large angle with respect to the photon. On the contrary, the few observed events indicate that one of the leptons is produced rather in the "forward" direction while the other one in the "backward" direction. In Table 1 we list the comparison of the asymmetry α of Eq.(1) with the data. We see that all of the four models seem to fail to provide explanation for the angular characteristics in the observed three events.

Now the following comments are in order:

i) The invariant mass distribution $d\Gamma/dm^2_{\ell\gamma}$ reflects the distribution in the angle $\theta_{\ell\gamma}$ as well, since one has the relation

$$d\Gamma/dm^2_{\ell\gamma} = -(2/M^2_Z x_3 N_I)d\Gamma/d\cos\theta_{\ell\gamma}. \qquad (22)$$

However, the angular distribution together with the "forward- backward" asymmetry α defined in Eq.(1) makes it more convenient in dealing with the scarce experimental data.

ii) Of course, more statistics has to be accumulated to arrive at any firm conclusions. This statement should be true as well to the contributions from the bremsstrahlung process[17] and the possible contributions from the interference between the bremsstrahlung and any one of the four mechanisms.

In concluding, let us emphasize that the study on the distributions in the angle between the decay products in the $\ell^+\ell^-\gamma$ events impose strong constraints on any model attempting to explain the anomalous $\ell^+\ell^-\gamma$ events and hence such study in the future experimentation will provide invaluable informations on the underlying dynamics of the anomalous phenomena.

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angle	experiment	model I	model II^*	model III^-	model IV
$\theta_0 = 45^0$	1	-0.79	-0.91	-0.39	-0.77
			-0.72		
$\theta_0 = 60^0$	1	-0.6	-0.90	-0.05	-0.54
			-0.46		

Table 1. Comparison of the predicted values for the "forward-backward" asymmetry α with the data $\alpha_{exp} = 1$. With $\theta_0 = 45^0(60^0)$, we have $\theta_1 = 164.4^0(158.0^0$, resp.).

*) The upper(lower) values correspond to the ones obtained with $f_1 - (f_2-)$ term only in Eq.(11).

FIGURE CAPTIONS

1. The $\cos\theta$ dependence of $N_I = \{1-(x_3/2)(1-\cos\theta)\}^{-2}$ in Eq.(4), with $E_\gamma = 30$ GeV and $M_Z = 93$ GeV.

2. The $\cos\theta$ dependence of $N_{II,1} = N_I(1-x_1)(1-x_2)$ and $N_{II,2} = N_I\{(1-x_1)^2+(1-x_2)^2\}$. E_γ and M_Z are as in Fig.1.

3. The $\cos\theta$ dependence of R, S, and T in Eq.(15), multiplied by N_I. E_γ and M_Z are as in Fig.1.

4. $d\Gamma_{III}/dx_3 d\cos\theta$ for $Z^0 \to \ell\bar{\ell}^* + \bar{\ell}\ell^* \to \ell^+\ell^-\gamma$, calculated with $m_* = 80$ GeV and $a_\gamma = \pm b_\gamma, a_Z = b_Z b_\gamma/a_\gamma$. E_γ and M_Z are as in Fig.1.

5. The $\cos\theta$ dependence of $N_{IV} = N_I[x_3\{(1-x_1)x_1 + (1-x_2)x_2\} - 2(1-x_1)(1-x_2)]$ in Eq.(20) ($\mu_S = 1$). E_γ and M_Z are as in Fig.1.

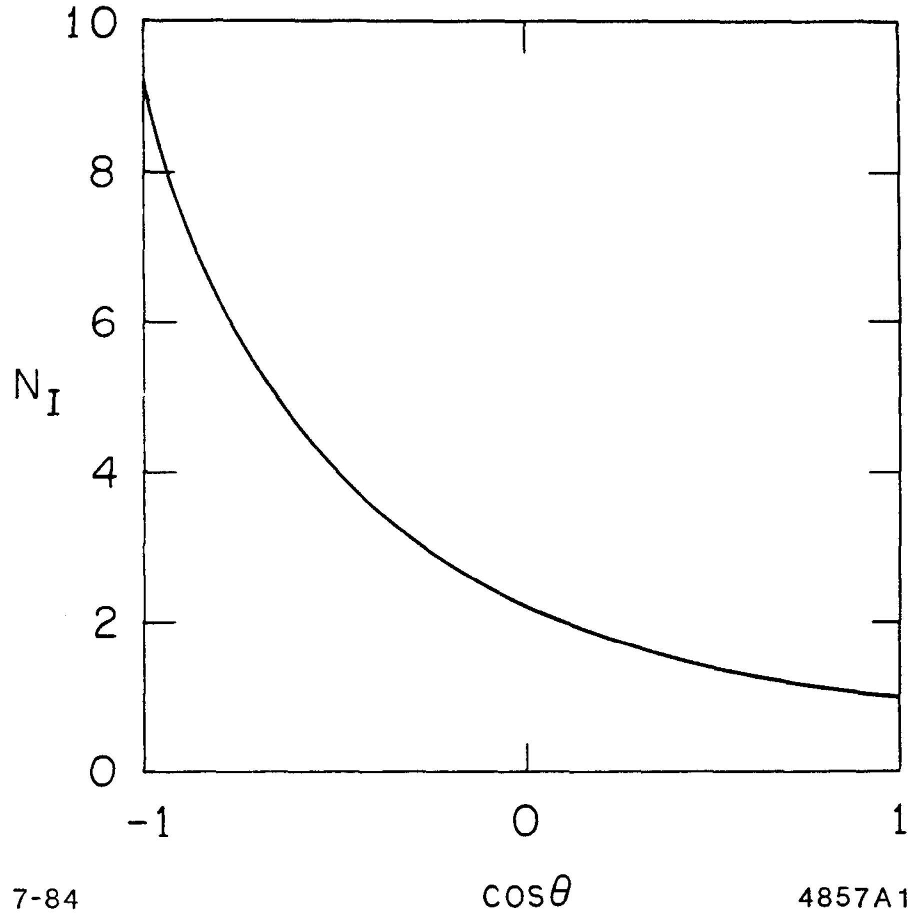

Fig. 1

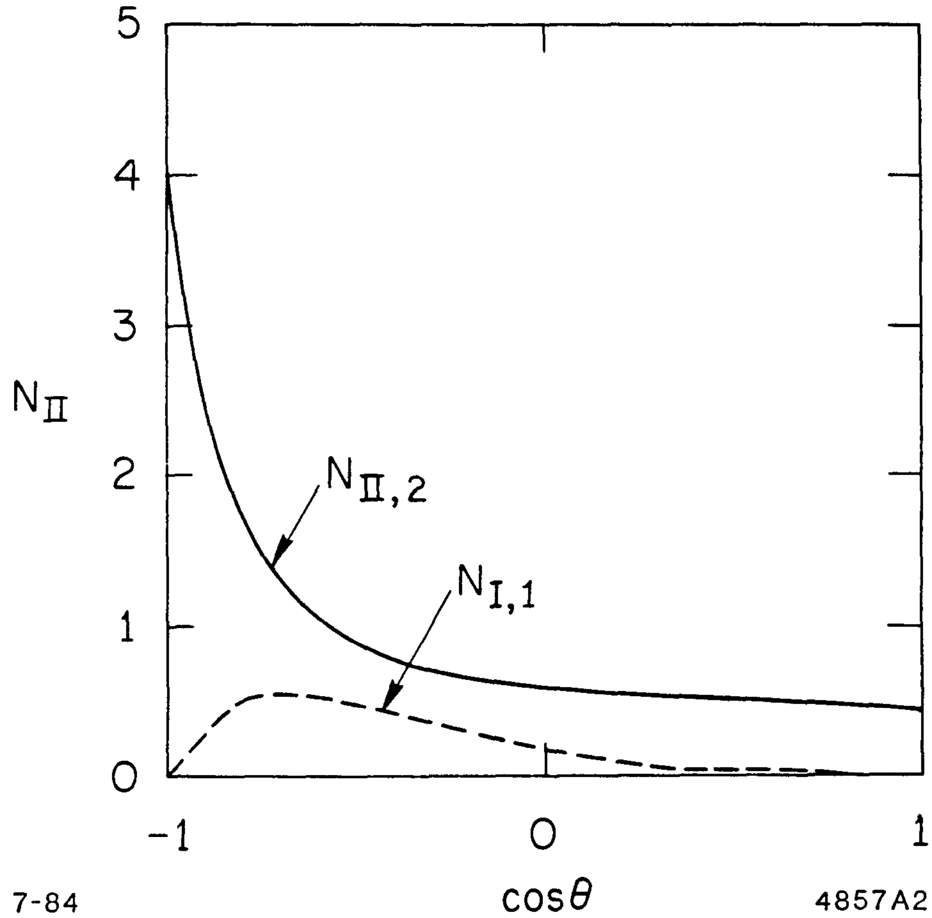

Fig. 2

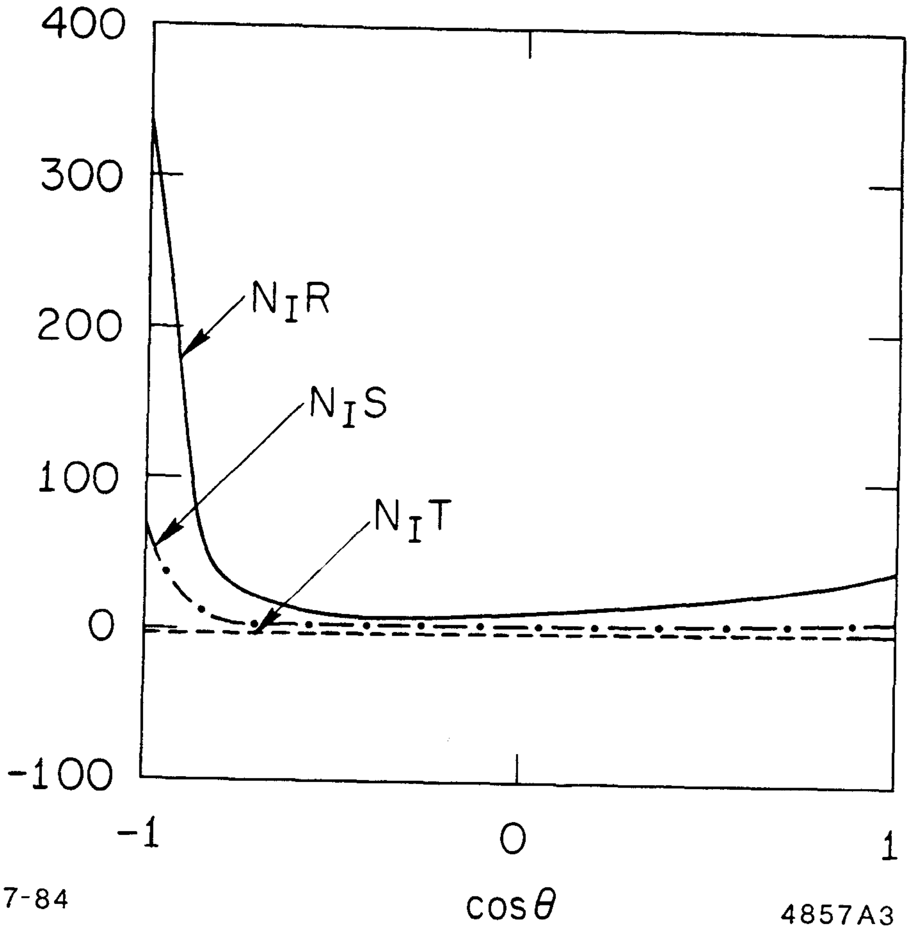

Fig. 3

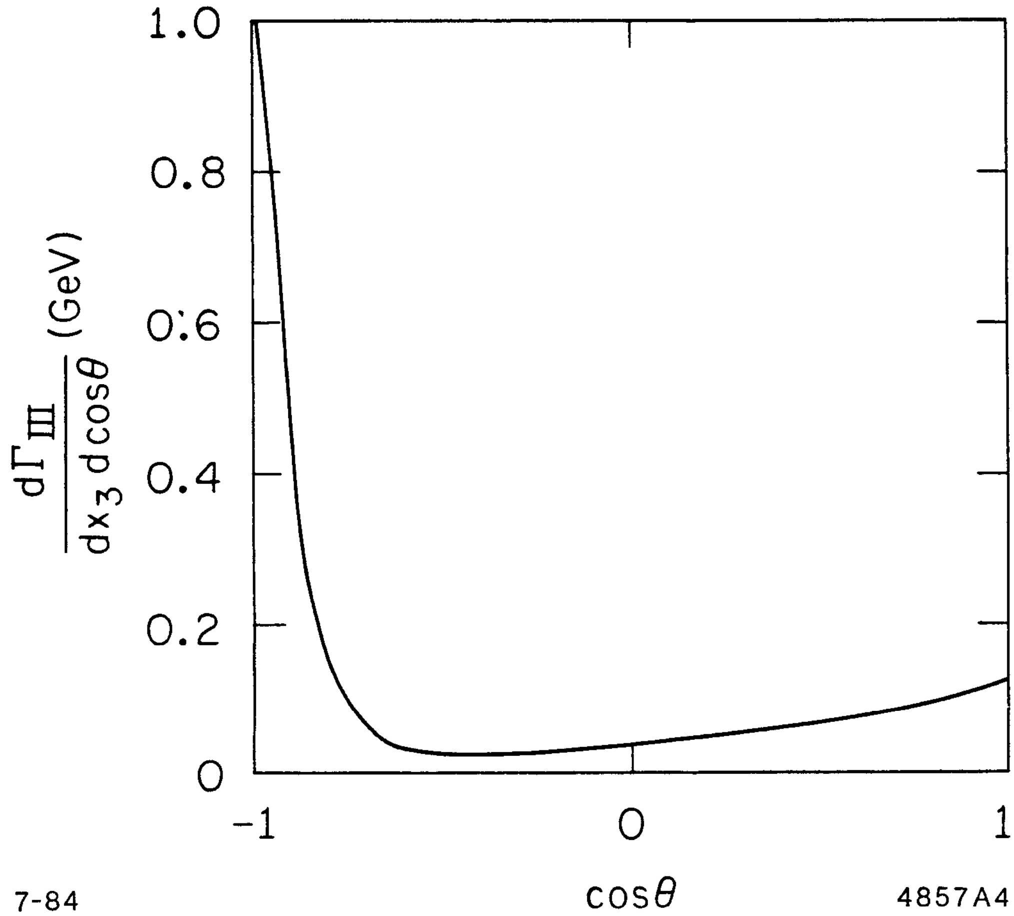
1.0
0.8
0.6
0.4
0.2
0
$\frac{d\Gamma_{III}}{dx_3\, d\cos\theta}$ (GeV)
-1
0
1
7-84
$\cos\theta$
4857A4

Fig. 4

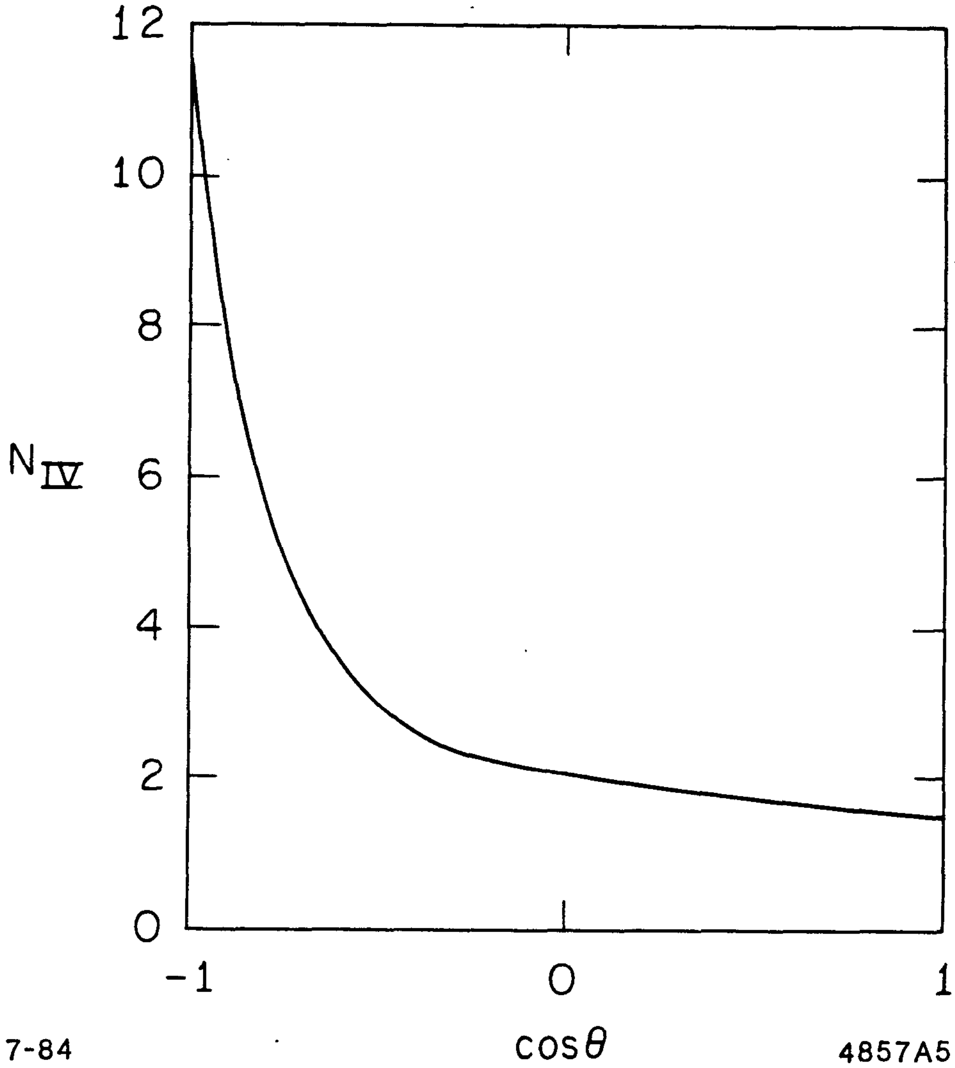

Fig. 5