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# A Fast Monte Carlo Generator for $ee \to eeX$ Untagged Experiments*

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## ABSTRACT

We describe for untagged $\gamma\gamma$ experiments a specific and very fast Monte Carlo based on the Double-Equivalent-Photon Approximation. This generator takes into account the experimental constraints in order to perform approximations involving experimentally unobservable effects and to generate events only within the experimental acceptance. It allows a very fast simulation of the events of any cross section in any invariant-mass range and, in particular, a simultaneous fit to the data of the parameters in any model of the $\gamma\gamma$ hadronic cross section.

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We have previously shown[1,2] that for high energy machines untagged experiments of the two-photon process can be accurately studied using the Double-Equivalent-Photon Approximation of the Williams-Weiszäcker spectrum. For ultrarelativistic two-body production we derived analytic expressions of the differential cross section for various parameters (invariant mass, transverse momentum, visible energy), taking into account the experimental constraints.[2] The expressions are simple and very fast to compute. Most of them are expressed in a beam-energy invariant manner so that data taken at different energies can be combined and simultaneously analyzed using parameters and cuts which are invariant with respect to energy.Results were checked with measurements of lepton pair ($e^+e^-$ or $\mu^+\mu^-$) production with the highest statistics currently available.[3,4]The agreement with the data is as good as that obtained from a simulation based on the QED Vermaseren program[5] (see figures 1, 2, and 3).

However, Monte-Carlo simulation is needed when one wants to take into account specific inefficiencies and resolution effects inherent in the apparatus. It is also needed for the study of multibody and nonrelativistic two-body production. The goal of this paper is to describe a very fast generator based on the Double-Equivalent-Photon Approximation. This program takes into account the experimental constraints in order to perform approximations involving unobservable effects and to avoid the generation of large numbers of events outside the detector acceptance. It is suitable for all experiments which require that only the prongs produced in the $\gamma\gamma$ interaction be observed in the central detector, with the following constraints:

$$|\cos\theta| \leq U_0 < 1, \qquad |\sum \vec{p}_t| \leq \epsilon W/2 < W/2,$$

$$W_{\text{min.}} \leq W \leq W_{\text{max.}} \ll 2E,$$

where $E$ is the beam energy, $\theta$ and $p_t$ are the polar angle and transverse momentum of the prongs with respect to the beam axis, and $W$ is the invariant mass observed.

These constraints are generally imposed in untagged experiments either by the apparatus itself or in order to avoid backgrounds. Notice that actually such experiments are "anti-tagged" because events are rejected if a scattered electron is detected and, more importantly, because the cuts on $|\sum \vec{p_t}|$ limits the scattering angles of the electrons and hence also the masses of the virtual photons. Such a cut allows only the study of exclusive $\gamma\gamma \to X$ channels, with all particles detected within the central detector.

We shall use the following notation. $E = \sqrt{s}/2$ is the energy of each incident electron, $m$ is the mass of the electron, and $E'$, $\vec{p}'$, $\vec{p}_t'$, $\theta'$, and $\phi'$ are the energy, momentum, transverse momentum, and polar and azimuthal angles of the scattered electrons with respect to the incident ones. $E_\gamma$ and $-q^2$ are the energy and mass of the corresponding virtual photons. $W$, $\vec{p}$, $\vec{p_t}$, and $\vec{\beta}$ are the invariant mass, momentum, transverse momentum, and velocity of the $\gamma\gamma$ system with respect to the beam axis. $\vec{p}''$, $\vec{p}_t''$, $\theta''$, and $\phi''$ are the momentum, transverse momentum, and polar and azimuthal angles of the prongs produced from the $\gamma\gamma$ interaction with respect to the beam axis. All of these quantities are defined in the laboratory frame, while the polar and azimuthal angles of the prongs in the center of mass of the $\gamma\gamma$ system are represented by $\theta^*$ and $\phi^*$. Indices $i = 1, 2$ or $j = 1, n$ will be added for the scattered electrons and interacting photons or the produced particles when needed for clarity, but they are usually omitted.

Actually, it is most convenient to use the dimensionless quantities

$$u_j = \cos\theta_j^*, \qquad U_j = \cos\theta_j''$$
$$t_0 = E/m, \qquad U_0 = \cos\theta_0,$$

$$V_0 = \sqrt{\frac{1+U_0}{1-U_0}} \equiv \tan(\theta_0/2),$$

where $\theta_0$ is the limit of the detector's polar acceptance with respect to the beam axis. Also, we define

$$\vec{P} = \vec{p}/E, \qquad \vec{P}_t = \vec{p}_t/E,$$

$$x = E_\gamma/E, \qquad z = W/2E, \text{ and } \quad t^2 = q^2/q^2_{\text{min.}},$$

where $q^2_{\text{min.}} = x^2m^2/(1-x)$ is the lower limit on $q^2$ due to the finite mass of the electron.

Neglecting the electron mass gives $q^2 = 2(1-x)E^2\sin^2\theta'/2$. Since $x$ is strictly limited within the range $0 < x < 1$, then

$$2\sin(\theta'/2) = \frac{x}{1-x}\frac{m}{E}\cdot t \quad \text{and} \quad q^2 = \frac{1-x}{x}\left[\frac{E}{m}\right]^2 \cdot t^2.$$

The errors introduced in these relations by the zero mass approximation always remain experimentally insignificant. They are maximum as $\theta'$ tends to zero ($q^2 \to q^2_{\text{min.}}$, $t \to 1$), where $\Delta p'_t = (1-x)E\sin\theta_e \approx xmt$, so that the error in the transverse momentum, which is actually the parameter measured, always remains smaller than the experimental resolution by orders of magnitude. Then the kinematic parameters of the interacting photons and scattered electrons can be defined by the relations

$$E_\gamma = xE, \qquad q^2 = \frac{1-x}{x}\left[\frac{t}{t_0}\right]^2$$

$$E' = (1-x)E, \qquad \sin(\theta'/2) = \frac{1}{2}\frac{x}{1-x}t_0 t.$$

Now, if we define $\hat{v}_i$ to be the unit vector corresponding to the angles $(\theta'_i, \phi'_i)$, where $\theta'_1$ and $\theta'_2$ are defined with respect to axes in opposite directions, then the kinematic parameters of the $\gamma\gamma$ system are given by the relations

$$\begin{aligned} \vec{P} &= \vec{P}'_1 - \vec{P}'_2 \\ Z^2 &= x_1x_2 - \frac{1}{2}(1-x_1)(1-x_2)(1-\hat{v}_1\cdot\hat{v}_2) \\ \beta^2 &= 1 - \frac{Z^2}{(x_1+x_2)^2}. \end{aligned}$$

Recall that the limitations of phase space require that $0 < Z, x, \beta < 1$ and $t \geq 1$, while the experimental constraints imply that

$$\left|U_j\right| \le U_0 < 1, \quad |\textstyle\sum_j \vec{P}_t''|/Z \le \epsilon < 1, \quad \text{and}$$

$$0 < Z_{\text{min.}} \le Z \le Z_{\text{max.}} \ll 1,$$

where $Z_{\text{max.}} = W_{\text{max.}}/2E$ and $Z_{\text{min.}} = W_{\text{min.}}/2E$. We shall now show that the experimental constraints also lead to strict limitations on $x$, $\beta$, $t$, and $U^*$.

The $\gamma\gamma$ processes, as is well known and will be shown below, are strongly dominated by the interaction of quasi-real photons, which implies that the electrons are scattered through a very small angle. As long as both $\theta_i'$ are close to zero degrees, the velocity of the $\gamma\gamma$ system remains along the beam and one has

$$Z^2 = x_1 x_2, \quad \text{and} \qquad |\beta| = \frac{|x_2 - x_1|}{x_2 + x_1}.$$

Moreover, the detection of all the prongs produced in the $\gamma\gamma$ interaction requires for all of them that $\left|U_j^*\right| \le U_0$ and $|\beta| < U_0$. The limitations for the production of multibody states or non-relativistic two-body states are even more restrictive that these relations derived for ultrarelativistic two-body states. Therefore, for all cases the following consequences must result:

*i*) The polar acceptance of all the prongs produced in the center of mass of the $\gamma\gamma$ interaction is always at least limited by the polar acceptance of the detector in the laboratory.

*ii*) From the relations $|x_2 - x_1|/(x_2 + x_1) \le U_0$ with $Z^2 = x_1 x_2$, it follows that, for a given value of $Z$, $V_0 Z \le x_i \le Z/V_0$. Thus the experimental cuts $Z_{\text{min.}} \le Z \le Z_{\text{max.}}$ require the energy of both photons to be within the range $V_0 Z_{\text{min.}} \le x_i \le Z_{\text{max.}}/V_0$.[6]

On the other hand, assuming $|\vec{P}_t| = |\sum \vec{P}_t'|$ to be essentially due to one scattered electron, where the other is scattered close to zero degrees, one derives from the constraint $\left|\vec{P}_t\right|/Z = |\sum P_t''|/Z \leq \epsilon \ll 1$ that $\sin\theta' \approx \epsilon Z/(1-x) \ll 1$ and also that $t \approx \epsilon\frac{E}{m} \ll t_0$ and $q^2/W^2 \approx \epsilon^2 \ll 1$. Appendix 1 shows in some detail that these limitations remain justified, independent of the assumptions used, over the whole experimental range.

Now, when $q^2/W^2 \ll 1$ one can neglect the longitudinal component of the quasi-real photon. Since the scattered electrons, and in particular their $\phi$ distributions, are not observed, one can also ignore the transverse-transverse interference between the photons. Then the whole process can be factorized as a product of a differential $\gamma\gamma$ luminosity of transverse photons (from $ee\gamma$ vertices) and a $\gamma\gamma \to x$ cross section of quasi-real photons. The $\gamma\gamma$ luminosity itself can be factorized as the product of that of two equivalent transverse photons integrated over the azimuthal angle, which in the following is simply considered to have an isotropic distribution. The E.P.S. formula to be used is somewhat controversial. There are various expressions[7] which differ only in the way the approximation of quasi-real photons is made and how it may be extrapolated to account for more-or-less virtual photons. The differences are in practice purely academic for untagged experiments in which $q^2$ is limited. We shall use the William-Weiszäcker formula as proposed by Budnev and Kessler, substituting the parameters $t^2$ and $x$ for $Q^2$ and $E_\gamma$.

The distributions to be considered are

$$\begin{aligned} \mathrm{d}N(\phi_e) &= \frac{1}{2\pi}\mathrm{d}\phi_e \\ \mathrm{d}^2N(x,t^2) &= \frac{\alpha}{\pi}\left[(1-x+x^2/2)-(1-x)/t^2\right]\frac{\mathrm{d}t^2}{t^2}\frac{\mathrm{d}x}{x}. \end{aligned}$$

In general, the $\gamma\gamma$ cross section includes a unitarity factor $1/W^2 \approx 4E^2/(x_1x_2)$. Such a sharp dependence on $W$ would lead to a very large fraction of the $\gamma\gamma$ systems generated according to the previous distributions to be rejected when including the cross section for $\gamma\gamma \to X$. The solution is to account for the $1/W^2$ factor when generating the $\gamma\gamma$ system by including an additional factor of

$2E/x_i$ in the equivalent spectra of both photons. Then the scattered electrons are generated according to

$$\begin{aligned} dN(\phi_e) &= \frac{1}{2\pi} d\phi_e \\ d^2N(x,t^2) &= \frac{2\alpha E}{\pi} \left[ (1 - x + x^2/2) - (1-x)/t^2 \right] \frac{dt^2}{t^2} \frac{dx}{x^2}, \end{aligned}$$

within the limits $0 \le \phi_e \le 2\pi$, $1 \le t \le E/m$, and $Z_{\text{min.}} V_0 < x < Z_{\text{max.}}/V_0$.

Note that the limitation on $x$ avoids the generation of a very large fraction of photons, mainly at low energy, which would not produce detectable events.[8] On the other hand, the experimental constraints give only an upper limit on $t$ which is not so sensitive. We have chosen the very conservative and constant limit of $E/m$, which leads later to a rejection of only a small number of events. However, this upper limit can be sometimes too large, corresponding to unphysical values of $\theta'$. Therefore, values of $t$ leading to a determination of $\sin\theta'/2$ larger than $\sin\theta_0/2$ will be rejected in the generation not only because they would anyway be eliminated in the analysis, but also to avoid highly improbable and unphysical values which can cause numerical problems in the program.

Integrating $dN(x,t^2)$ over $1 \le t \le t_0$ gives

$$dN(x,t_0) = \frac{2\alpha E}{\pi} \left[ 2(1 - x + x^2/2) \ln t_0 - (1-x)(1 - 1/t_0^2) \right] \frac{dx}{x^2},$$

so the $x$ spectrum integrated over the whole range of $t$ with $t_0 = E/m \gg 1$ is given by

$$dN(x) = \frac{2\alpha E}{\pi} \left[ 2(1 - x + x^2/2) \ln(E/m) - (1-x) \right] \frac{dx}{x^2}.$$

This expression is strongly dominated and slightly overestimated by $dN'(x) = (2\alpha E/\pi) \ln t_0 \, dx/x^2$ which, when integrated over $x_{\text{min.}} \le x \le x'$, gives $N'(x') = 1/x_{\text{min.}} - 1/x'$. By choosing a random number, $R_1$, between 0 and 1 the $x$ values can be analytically generated according to $1/x_{\text{min.}} - 1/x = R_1 \cdot (1/x_{\text{min.}} - 1/x_{\text{max.}})$

and then corrected for the true probability by rejecting the few events for which $dN'(x) \cdot R_2 > dN(x)$.

Once $x$ is given, and with the definition $\eta = \frac{1}{2}(1-x)/(1-x+x^2/2)$ (where $\eta$ always remains less than 1/2), the $t$ distribution, integrated over $1 \le t \le t'$, gives $N(t') = \ln t' - \eta(1-1/t'^2)$, which tends toward simply $\ln t' - \eta$ as $t'$ becomes much larger than 1. Thus $t$ values may be analytically generated according to $\ln t - \eta = R_3 \cdot (\ln t_0 - \eta)$. No corrections of these values are needed because the approximation only causes an error in the determination of $t$ which has no significance on the scale of the experimental resolution. Then, with $t$ given, one can derive $\sin\theta'/2$, reject the few events for which $\sin\theta'/2 > \sin\theta_0/2$, and simply generate $\phi'$ uniformly between 0 and $2\pi$.

Once both sets of $(x, \theta', \phi')_i$ have been accepted, one derives the values $z$ and $\vec{P}$ of the $\gamma\gamma$ system in the laboratory frame. In theory a weight of $x_1 x_2/Z^2$ should be given to each event in order to correct for the $Z^2 = x_1 x_2$ approximation which was made when introducing the unitarity factor into the photon spectra, but in practice that is unnecessary. We have checked that such corrections are completely negligible compared with the statistical errors inherent in an analysis of a data sample of a realistic size.

In summary, with a set of random numbers $R_i$ $(0 \le R_i \le 1)$, both scattered electrons are generated in a loop with $i = 1, 2$ as follows:

1. Select $x = x_{\text{min.}} / \left[1 - R_1(1/x_{\text{min.}} - 1/x_{\text{max.}})\right]$.
2. Compute $s = 1 - x + x^2/2$ and $\eta = (1-x)/2s$.
3. Reject events for which $R_2 \ln t_0 > s(\ln t_0 - \eta)$.
4. Select $t = \exp\left[\eta + R_3(\ln t_0 - \eta)\right]$.
5. Compute $\sin(\theta'/2) = \left[(1-x)/x\right] \cdot t_0 t/2$.
6. Reject events for which $\sin\theta'/2 > \sin\theta_0/2$.
7. Select $\phi' = (2\pi) \cdot R_4$.

Once both electrons have been accepted with a direction $\vec{v}_i$, defined by $(\theta'_i, \phi'_i)$, where the two $\theta'_i$ are defined with respect to axes in opposite directions, we then proceed as follows:

8. Compute $Z^2 = x_1 x_2 - \frac{1}{2}(1 - x_1)(1 - x_2)(1 - \hat{v}_1 \cdot \hat{v}_2)$.
9. Compute $\vec{P} = (1 - x_1)\hat{v}_1 - (1 - x_2)\hat{v}_2$.

Although the generation of either one of the two electrons is independent of the other, rejecting one value of $x$ or $\theta'$ corresponds to rejecting the entire $\gamma\gamma$ event. It is more efficient to increment the count, $N$, of the number of iterations by one each time one generates a new electron with a change in the loop index and by two each time an electron is rejected and a new one is tried within the same loop on $i$. Then to generate the proper number of events corresponding to a given luminosity, $\mathcal{L}_{ee}$, the number of iterations which must be made is

$$N = \frac{2\alpha E^2}{\pi^2}(\ln t_0)^2(1/x_{\text{min.}} - 1/x_{\text{max.}})^2 \cdot N_\sigma \cdot \mathcal{L}_{ee},$$

where $N_\sigma$ is the normalization of the $\gamma\gamma$ cross section as used later in the generation of the events from the $\gamma\gamma$ interaction. One could at this point, before generating the $\gamma\gamma$ cross section, reject events for which the values of $Z$ or $\left|\vec{P}_t\right|$ still remain outside the experimental constraints, because of the choice of independent $x$ and $t$ limits in the generation of the $\gamma\gamma$ system. However, any such rejection must take into account the experimental resolution.[8]

A $\gamma\gamma$ generator as described can be used for simulating within a detector apparatus any exclusive channel of $\gamma\gamma \rightarrow X$ for which the $\gamma\gamma$ cross section is known or may be assumed. The steps to follow are

1. Generate the $\gamma\gamma$ system as above.
2. Generate in the center of mass of the $\gamma\gamma$ system the prongs produced in the $\gamma\gamma$ interaction, according to their $\gamma\gamma$ cross section and within the same polar acceptance as defined by the detector in the laboratory frame (for examples, see Appendix 2).

3. Boost all prongs into the laboratory frame according to the velocity of the $\gamma\gamma$ system generated in that frame.

4. Account for the experimental efficiencies and resolutions. Because a full simulation of the apparatus for each prong requires a much greater amount of computer time than the event generation itself, it is most efficient to perform the detector simulation only once for an isotropic distribution of prongs. The results may be stored in a matrix and later used in the Monte Carlo by smearing the kinematic parameters appropriately to simulate the resolution and by rejecting the appropriate proportion of events in each region to account for inefficiencies. The result is a very fast program which may be economically iterated many times, as required when making a fit to data.

5. Analyze the simulated events and compare with data which has been analyzed by the same analysis program.

In practice, when one wants to check some model and fit various parameters, it is often most efficient to generate events from the largest of the cross sections, within the acceptance, which result from the set of values of the parameters desired to be tried in the fit. After making the analysis cuts, the $\gamma\gamma$ cross section for each set of values of the parameters is calculated and the event is accepted or rejected in each case as is appropriate to give the correct cross section. Then the distributions for all sets of parameters may be compared with the experimental data, allowing the simultaneous determination of the $\chi^2$ for all cases.

The results of such a program are fully normalized, so one can derive from the number of events the *ee* luminosity when one know the $\gamma\gamma$ cross section or vice-versa. In particular, when lepton pairs can be selected, they give a very convenient normalization of the *ee* luminosity, which may be used for the study and determination of the $\gamma\gamma$ cross section of any other process. We would also like to point out that such a program allows the generation of equal numbers of events with equal amounts of computing time, independent of what invariant mass range is considered or how small the cross section is.

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# APPENDIX 1

The $x_{\text{min.}}$ and $x_{\text{max.}}$ values have been determined by considering the cases where $W = W_{\text{min.}}$ and $W = W_{\text{max.}}$ with $Z^2 = x_1 x_2$ and $\beta = U_0$, $\vec{\beta}$ being directed along the beam. This range of $x$ is generally overestimated for a given value of $W$ within the allowed range and becomes more and more so, over the whole $W$ range, as $\beta$ approaches zero (see figure 4). For the case where both $\beta$ and $W$ are near the limits of acceptance and at the same time $\vec{\beta}$ is not exactly along the beam axis, due to the small but nonzero electron scattering angles allowed by the cut on $|\vec{P}_t|$, the $x$ limits are still valid. That is because, first, the acceptance in the case where $\beta$ approaches $U_0$ is drastically decreasing, and, second, $Z^2$ becomes more and more less than $x_1 x_2$ as the angle of $\beta$ with respect to the beam axis increases. In any case, the values of $U_0$, $W_{\text{min.}}$, and $W_{\text{max.}}$ used in the generation will always be set a little less drastic than the experimental cuts used in the analysis in order to allow room for some experimental smearing.

Actually, the $W$ limits in the generation can often be very close to the experimental limits because the corresponding $x$ limits are strongly overestimated for the very dominant configurations. However, the smearing must carefully be taken into account if one rejects generated $\gamma\gamma$ systems before generating the $\gamma\gamma$ interaction. Then any cuts on $W$ must be made conservatively outside the limits used in generation. In the same way, rejection according to $|\vec{P}_t|$ must be conservative if done immediately after the $\gamma\gamma$ system generation. However, it is generally convenient and most efficient to make such cuts at that stage.

A way to generate the $\gamma\gamma$ system even more closely within the acceptance is to choose limits fixed not in $x_i$ but in $Z' = x_1 x_2$ and $y = \tanh^{-1}(\beta') = \tanh^{-1}\left(|x_2 - x_1| / (x_2 + x_1)\right)$. Then one generates $Z'$ and $y$ according to the probabilities $\mathrm{d}Z'/Z'^3$ and $\mathrm{d}y$ and derives the $x$ values from $x_i = Ze^{\pm y}$. Both distributions of $x_i$ must be corrected according to the true probabilities, and values of $\theta'_i$ and $\phi'_i$ must be selected in the same way as in the previous procedure. However, limits on $Z'$ and $\beta'$ must be chosen outside the $Z$ and $\beta \leq U_0$ limits, especially when experimental smearing must be allow for. Such a method does not produce a very significant gain in time over the previous one.

# APPENDIX 2

## Examples of $\gamma\gamma$ Cross Sections

1. For a relativistic pair of fermions with an invariant mass $W$ and produced within a polar acceptance $|\cos\theta^*| = |u| \le U_0 < 1$, the cross section is

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{W^2}\frac{1+u^2}{1-u^2} \le \frac{\alpha^2}{W^2}\frac{1+U_0^2}{1-U_0^2}.$$

   Therefore, with $L_0 = \ln[(1+U_0)/(1-U_0)]$ and the normalization factor $N_\sigma = 4\pi\alpha^2(1+U_0^2)L_0$ already calculated at the beginning of the program, the relativistic fermion pairs are generated from uniform random numbers, $0 \le R_i \le 1$, as follows:

   (a) Select $u = \tanh(R_5 L_0/2)$.

   (b) Reject events for which $(1+U_0^2)R_6 > 1+u^2$.

   (c) Select $\phi^* = (2\pi)\cdot R_7$.

2. For nonrelativistic fermions of mass $m_x$ produced within a polar acceptance $u \le U_0 < 1$ and with an invariant mass $W$, we have

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{W^2}\left[N(\beta,u) - 2\frac{(1-\beta^2)^2}{1-\beta^2u^2}\right]\cdot\frac{\beta}{1-\beta^2u^2} \le \frac{\alpha^2}{W^2}\frac{N(\beta,u)}{1-\beta^2u^2}\cdot\beta,$$

   with $\beta^2 = 1 - 4m_x^2/W^2$ and $N(\beta,u) \equiv 2(2-\beta^2) - (1-\beta^2u^2)$. With $L_0$ and $N_\sigma = 4\pi\alpha^2 L_0$ determined at the beginning of the program, such that $L_0 \ge N(\beta,U_0)\ln[(1+\beta U_0)/(1-\beta U_0)]$ within the entire experimental $W$ acceptance, a nonrelativistic fermion pair for a given value of $W$ is generated as follows:

   (a) Reject events for which $R_5 L_0 > N(\beta,U_0)$.

   (b) Select $u$ according to $u = (1/\beta)\tanh(R_5 L_0/e)$.

   (c) Reject events for which $N(\beta,U_0) > N(\beta,u) - (1-\beta^2)/(1-\beta^2u^2)$.

(d) Select $\phi^* = (2\pi) \cdot R_8$.

3. Finally, consider the case of hadron pairs produced within the polar acceptance $|u| \leq U_0 < 1$ with an invariant mass $W$.

   (a) If we assume the Born approximation, then

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2 \beta}{2W^2}\left[1 - 2N(\beta, u)\right] \leq \frac{\alpha^2 \beta}{W^2},$$

   with $\beta^2 = 1 - 4m_\pi^2/W^2$ and $N(\beta, u) \equiv \beta^2(1-\beta^2)(1-u^2)/(1-\beta^2 u^2)^2$. With the normalization $N_\sigma = 2\pi\alpha^2 U_0^2$ determined at the beginning of the program, the hadron pair of invariant mass $W$ will be generated as follows:

   (i) Select $u = R_5 U_0$.

   (ii) Reject events for which $R_6 > \beta \cdot [1 - 2N(\beta, u)]$.

   (iii) Select $\phi^* = (2\pi) \cdot R_7$.

   (b) For a more general model where the hadronic cross section is defined by

$$\frac{d\sigma}{d\Omega} = \frac{C}{W^2} F(W, \lambda_i, u),$$

   where the $\lambda_i$ are parameters to be determined from the data, we calculate at the beginning of the program $L_0$ and $N_\sigma = 2CL_0$, such that $L_0 \geq F(W, \lambda_i, u)$ over the whole $W$, $u$, $\lambda_i$ acceptance, and then generate events as follows:

   (i) Select $u = R_5 U_0$.

   (ii) Select $\phi^* = (2\pi) \cdot R_6$.

   (iii) The event is then boosted into the laboratory frame, corrected for inefficiency and resolution, and analyzed as is the data. The results are stored in arrays according to the various values of the parameters $\lambda_i$ which we are interested in only if $R_7 L_0 \leq F(W, \lambda_i, u)$.

Thus at the end of the program, comparing the experimental distributions with the ones generated for all values of the $\lambda_i$ allows the computation of a $\chi^2$ for the fit as a function of the $\lambda_i$.

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6. These relations implicitly assume that $Z_{\text{max.}} < 2E\tan(\theta_0/2)$. That is generally the case. As an example, for $W_{\text{max.}} = E/2$, the assumption is justified within a polar acceptance for the central detector of $14^0 < \theta < 166^0$. Moreove r, as long as $W_{\text{max.}} \ll 2E$, one need not worry about this limitation over the entire $W$ range as long as care is taken to set $x_{\text{max.}} = \text{MIN}\,[Z_{\text{max.}}/\tan(\theta_0/2), 1]$.

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8. See Appendix 1

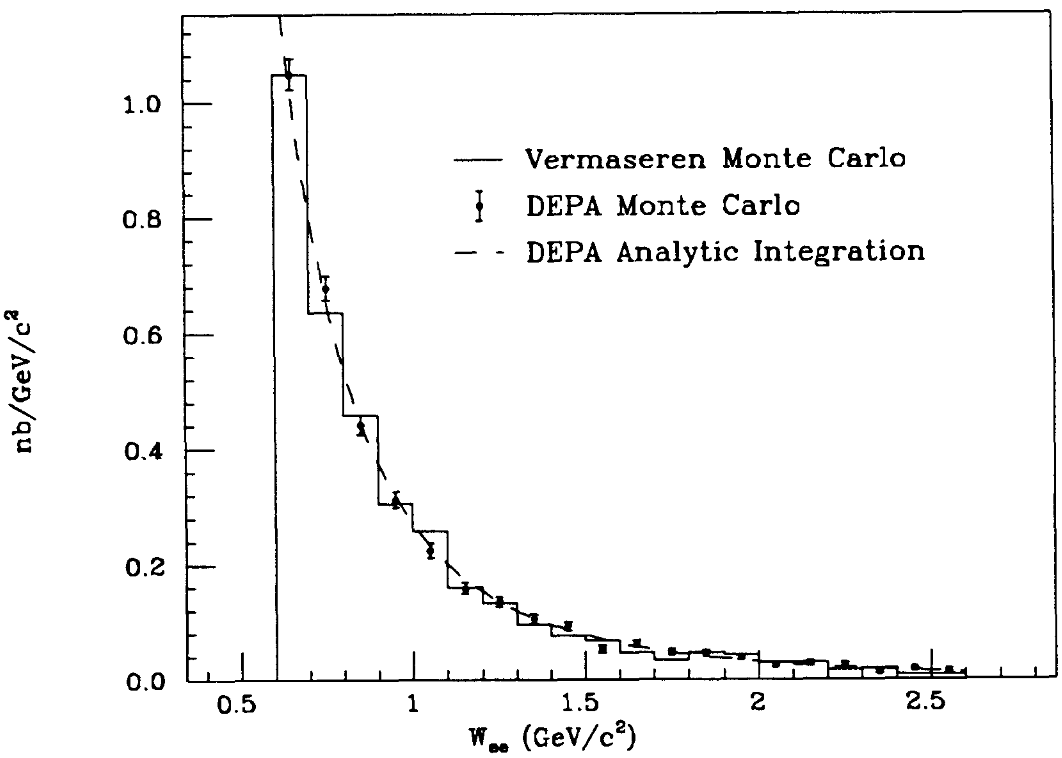
# FIGURE CAPTIONS

1. A comparison of three calculations of the invariant-mass spectrum of electron pairs from $\gamma\gamma$ interactions. The acceptance of the Delco detector is used in the calculations.

2. A comparison of the DEPA calculations of the electron-pair invariant-mass spectrum with data from the Delco experiment.

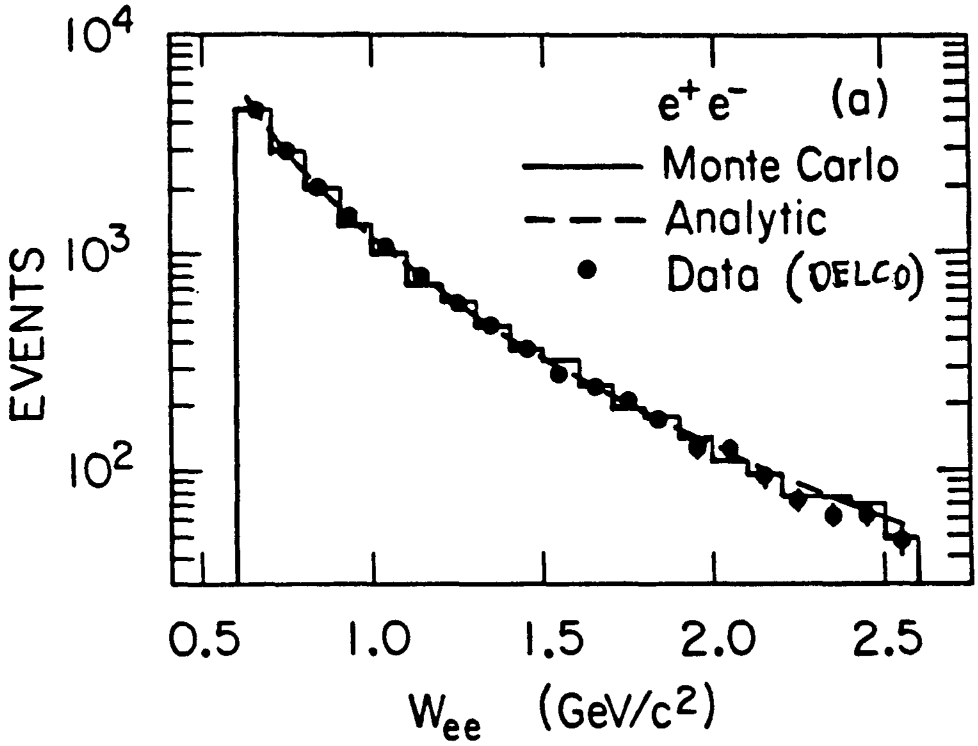
3. A comparison of the DEPA calculations of the electron-pair transverse-momentum spectrum with data from the MARK J experiment.

4. A comparison of the acceptance used for generation with the acceptance allowed by the experiment. *a.*) The acceptance for the photon energies. *b.*) The acceptance used in the center-of-mass frame for generation of the angular distribution from the $\gamma\gamma$



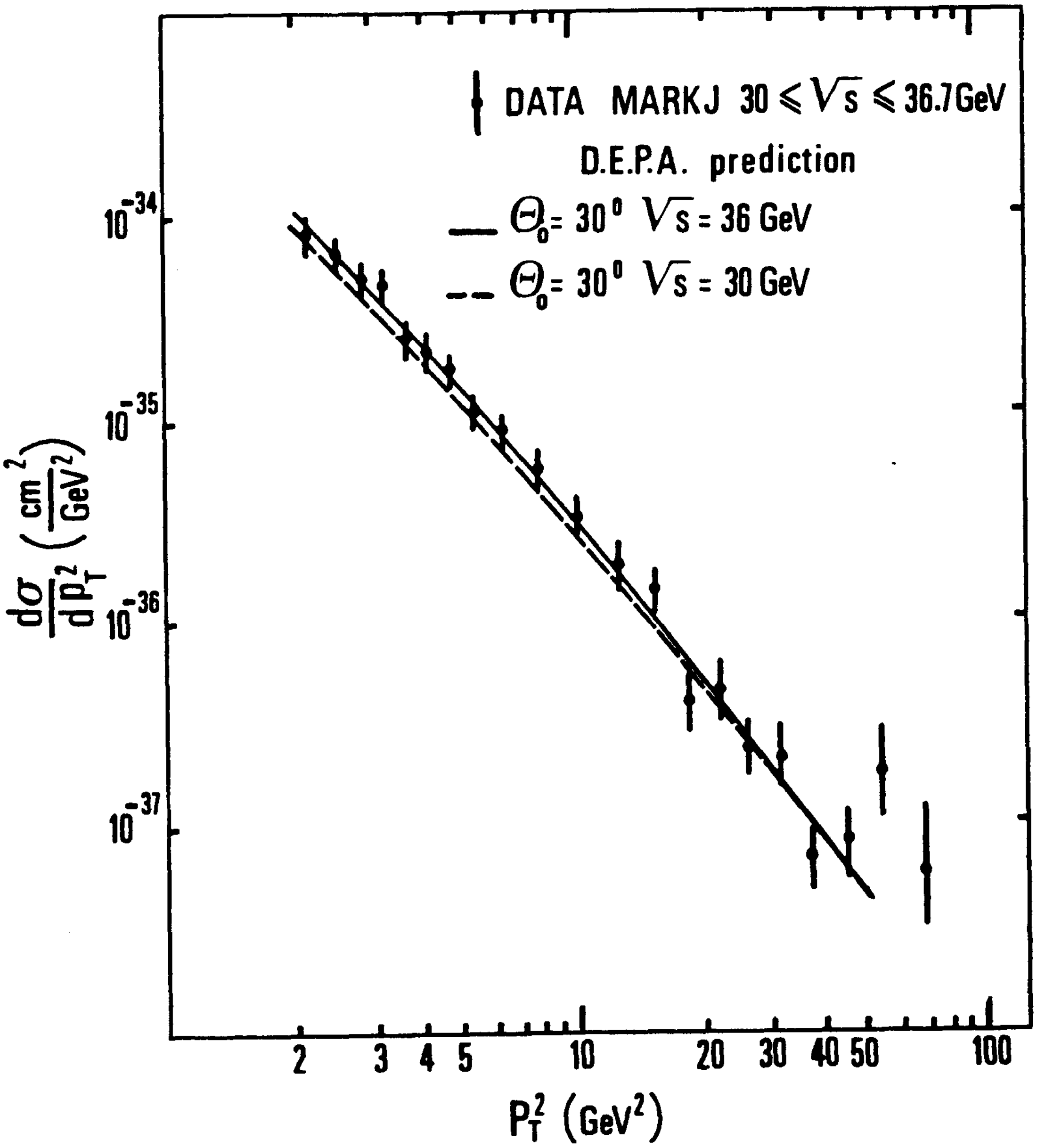
Vermaseren Monte Carlo
DEPA Monte Carlo
DEPA Analytic Integration
nb/GeV/c²
1.0
0.8
0.6
0.4
0.2
0.0
0.5
1
1.5
2
2.5
W_ee (GeV/c²)


Fig. 1



10⁴
10³
10²
EVENTS
e⁺e⁻ (a)
Monte Carlo
Analytic
Data (DELCO)
0.5
1.0
1.5
2.0
2.5
Wee (GeV/c²)


Fig. 2



DATA MARKJ $30 \leqslant \sqrt{s} \leqslant 36.7$ GeV
D.E.P.A. prediction
$\Theta_0 = 30^0$ $\sqrt{s} = 36$ GeV
$\Theta_0 = 30^0$ $\sqrt{s} = 30$ GeV
$10^{-34}$
$10^{-35}$
$10^{-36}$
$10^{-37}$
$\frac{d\sigma}{dp_T^2} \left(\frac{cm^2}{GeV^2}\right)$
2
3
4
5
10
20
30
40
50
100
$P_T^2$ $(GeV^2)$


Fig. 3

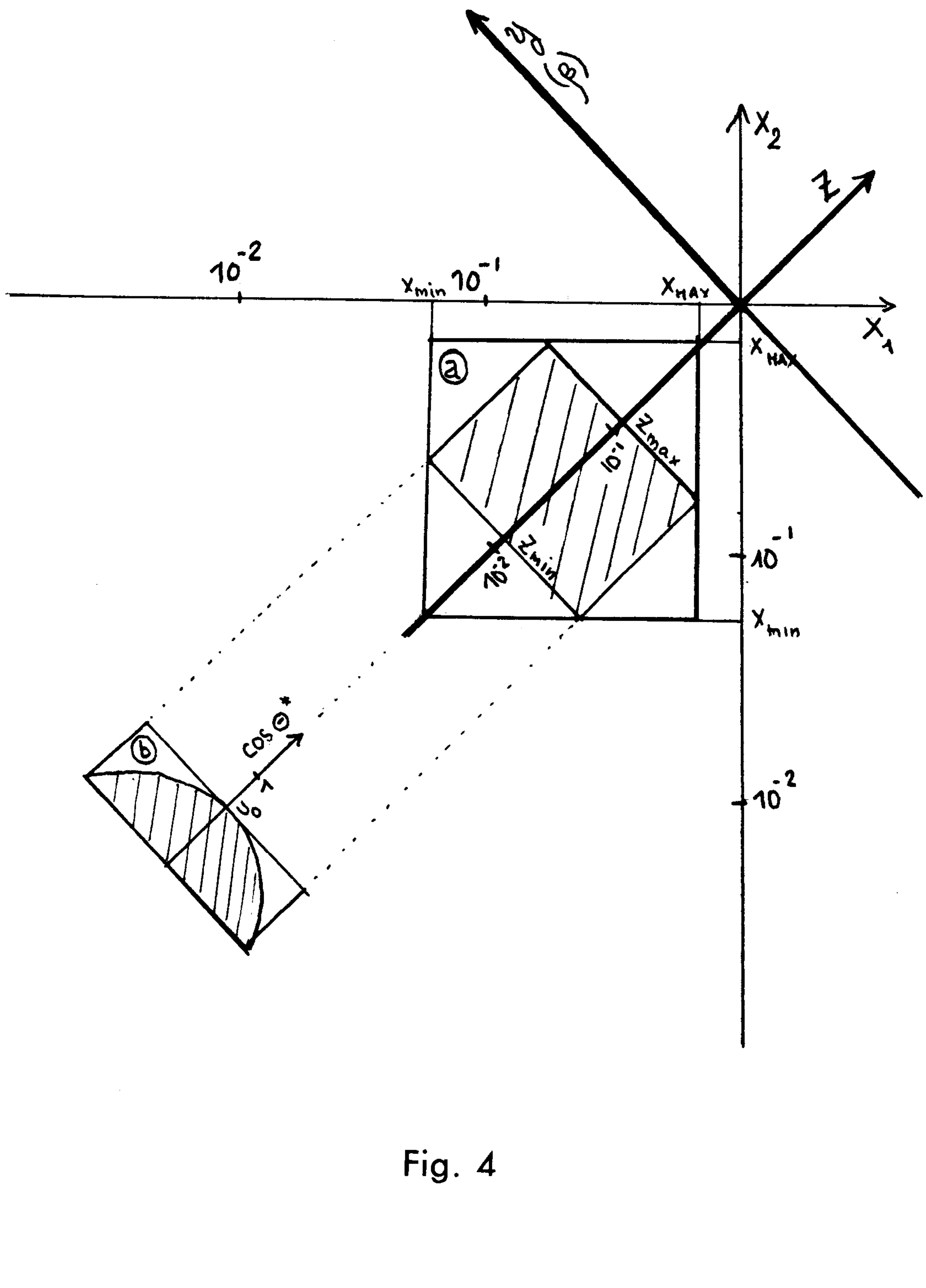



Fig. 4