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RADIATIVE CORRECTIONS TO MUON LIFETIME, $W^{\pm}$ MASS, AND LONGITUDINAL POLARIZATION ASYMMETRY IN $e^+e^- \to \mu^+\mu^-$ IN $N = 1$ SUSY $SU_3 \times SU_2 \times U_1$ AND $N = 1$ SUGRA I: SLIDING SINGLET MODELS*

B. W. LYNN

Stanford Linear Accelerator Center
Stanford University, Stanford, California 94305

and

University of Oxford, Clarendon Laboratory†
Parks Road OX1 3PU, England

ABSTRACT

We have calculated the one-loop radiative corrections to the muon decay lifetime in the most general version of the minimal global $N = 1$ SUSY $SU_3 \times SU_2 \times U_1$ theory with soft SUSY and R-invariance breaking terms thus giving predictions for M_w and s_θ^2 given M_Z. We also calculate the longitudinal polarization asymmetry A_{pol} on Z^o resonance ($q^2 = -M_Z^2$) in $e^+e^- \to \mu^+\mu^-$ and the forward-backward asymmetry A_{FB}. We study the shifts in these quantities from their values in the standard model when the parameters appearing in the SUSY theory are motivated by coupling to $N = 1$ supergravity where the gauge symmetry is broken with a sliding singlet. The shifts are largest ($\delta M_w \sim 700$ MeV to 1 GeV and $\delta A_{pol} \sim .045$) for large top quark mass ($M_t \sim 230$ GeV) and small gravitino mass ($m_{3/2} \lesssim 100$ GeV) and M_w and A_{pol} are larger than in the standard model in this case. The shifts can also be substantial for smaller M_{top} and M_w and A_{pol} can even be *smaller* than in the standard model in certain cases. These effects can be tested at the SLC and/or the LEP1 indirectly via a careful comparison of the precise Z^o mass with the polarization asymmetry in $e^+e^- \to \mu^+\mu^-$ on Z^o resonance and might be visible in the neutrino-anti-neutrino on electrons asymmetry $R^e_{\nu\bar{\nu}}$.

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† Permanent address.

I. Introduction

One of the most important objectives of physics today must be to answer the question; what is the effective low-energy gauge theory of electro-weak interactions? The standard model [1] (GSW) is so successful phenomenologically that any successful theory must give at least approximate $SU_3 \times SU_2 \times U_1$ at low energies. There are, of course, many possible generalizations; $SU_3 \times SU_2 \times U_1$ with more particles, larger gauge groups such as $SU_3 \times SU_{2L} \times SU_{2R} \times U_1$ and supersymmetric (SUSY) extensions.

In general, these more general theories introduce many new particles which must be heavier than $\sim$ 20 GeV in order to have escaped detection. There are then two ways to test these theories; discover the extra particles with new accelerators or see the effects of the new particles via radiative corrections to low-energy phenomena in experiments of accuracy better than $\sim 1\%$. It must be kept in mind that one cannot really draw ***positive*** conclusions (e. g. SUSY is right) from comparison of model dependent radiative corrections with experiment but rather ***negative*** conclusions (e.g. SUSY is wrong). Thus, we view radiative corrections as having the capacity to eliminate models from consideration rather than to vindicate any particular model.

We will examine in this paper the effects of radiative corrections of a class of global $N = 1$ SUSY models whose parameters are motivated by considerations of $N = 1$ local SUSY (supergravity) or SUGRA where the gauge symmetry is broken by introduction of a 'sliding singlet'. We will examine the so-called 'renormalization group equation' (RGE) models in a later paper.

The purpose of this work is clearly to provide a test of the $N = 1$ SUSY standard model at the one-loop level. Any test of the one loop structure of electro-weak theories must be free of theoretical uncertainties due to strong interactions. This suggests that we limit ourselves to purely leptonic processes or the measurements of the masses of electro-weak particles themselves or certain asymmetries in which strong interaction effects cancel in the ratio of cross sections. Another criteria must be that the experimental precision must be better than $\sim 1\%$. Both of these criteria eliminate neutrino-hadron scattering as a precise test of the theory. For example, the ratio of charged to neutral neutrino-hadron scattering suffers from theoretical uncertainties [2] which are liable to be much larger than the one-loop SUSY corrections. Thus, the ρ parameter, which is usually defined in terms of ν-hadron scattering may not provide a good test of the $N = 1$ SUSY theory and this is why we have not considered it [3]. Instead we have used a renormalization prescription where $\rho = 1$ to all orders. We will show that deviations from GSW due to the SUSY radiative corrections could be detected at the SLC and/or LEP1 by a careful comparison of the precise Z^o mass to the longitudinal polarization asymmetry A_{pol} on Z^o resonance [4].

This paper will be organized as follows. In Section II we will give the $N = 1$ SUSY model for minimal $SU_3 \times SU_2 \times U_1$. We will also calculate all one-loop corrections due to the SUSY part of the model to τ_μ M_w, s_θ^2 and $A_{pol}(e^+e^- \to \mu^+\mu^-)$. The complete one loop standard model predictions for s_θ^2 M_w and A_{pol} may be found in Tables I, II, III in reference [4]. In Section III, we give motivation for the many parameters in the N = 1 global SUSY model by coupling it to an $N = 1$ local SUSY model which uses a sliding singlet to break gauge symmetry [5]. Section IV has the numerical results for the shifts in M_w s_θ^2 and A_{pol} due to the one-loop effects of SUSY partners.

II. The Model, Renormalization, Muon Lifetime, Polarization Asymmetry

2.1 Minimal $N = 1$ SUSY $SU_3 \times SU_2 \times U_1$

We consider the $N = 1$ SUSY version of minimal $SU_3 \times SU_2 \times U_1$ with the following superfields (we use a ˆ to denote superfields) [6].

$$\begin{gathered} \hat{W}_\mu \ (3) \,, \qquad \hat{B}_\mu \ (1) \,, \\ \hat{\ell}_n \ (2) = \begin{pmatrix} \hat{\nu}_n \\ \hat{e}_n \end{pmatrix} \,, \ \hat{e}^c_n \,, \hat{q}_n(2) = \begin{pmatrix} \hat{u}_n \\ \hat{d}_n \end{pmatrix} \,, \ \hat{u}^c_n(1) \,, \hat{d}^c_n(1) \\ \hat{H} \ (2) = \begin{pmatrix} \hat{H}_0 \\ \hat{H}_- \end{pmatrix} \qquad \hat{H}' \ (2) = \begin{pmatrix} \hat{H}'_+ \\ \hat{H}'_0 \end{pmatrix} \end{gathered} \tag{1}$$

where we have indicated the representation of SU_2 and $n = 1, 2, 3$ is a generation index. We also implicitly assume $N_c = 3$ colors for quarks (and squarks) and an octet of gluons (and gluinos) transforming under SU_{3color}. This last, however, will not be explicitly needed and we will not refer to the details of the strong interaction sector further.

The coupling constants for SU_2 and U_1 are g_2 and g_1 respectively and the scalar potential is presumed engineered so that this breaks spontaneously to U_1 with $\langle H \rangle = v$, $\langle H' \rangle = v'$ real and $\langle \hat{\ell}_n \rangle = \langle \hat{q}_n \rangle = 0$. Further, we add the $\hat{H}\,\hat{H}\,'$ mixing term and soft SUSY and R-invariance breaking terms.

$$L_{extra} = \mu \int d^2\theta \; \hat{H} \; \hat{H}' - \left[\frac{1}{2} \; M_2 \; \tilde{\vec{W}} \cdot \tilde{\vec{W}} + \frac{1}{2} \; M_1 \; \tilde{B}\tilde{B} \; + \; c.c.\right] \tag{2}$$

as well as SUSY breaking mass terms for the scalars.

There are then two 4-component charged fermions and four 2-component neutral fermions (winos,) besides the quarks and leptons. Explicit diagonalization of the wino mass matrices (given below) depends on M_1, M_2, μ and $\cos\beta = c_\beta = v/(v^2 + v'^2)^{\frac{1}{2}}$ and is long winded and not particularly interesting and has been done numerically. The model also has squarks $\tilde{q}_n$ and sleptons $\tilde{\ell}_n$ which we take to be approximate mass eigenstates *except* for the top squarks with weak eigenstates $\tilde{U}_3^\dagger = (\tilde{u}_3^\dagger, \tilde{u}_3^{c\,\dagger})$. These have a 2×2 mass-squared mixing matrix $M_{\tilde{t}}^2$ diagonalized by a matrix T.

$$L_{\tilde{t}\,mass} = \tilde{U}_3^\dagger \; M_{\tilde{t}}^2 \; \tilde{U}_3 \quad ; \quad T\,M_{\tilde{t}}^2\,T^\dagger = \; diag\,(M_{\tilde{t}_1}^2 \; , \; M_{\tilde{t}_2}^2) \tag{3}$$

where T depends on an angle $\cos\alpha = c_\alpha$. Finally we add in the rest of the standard model. In the Higgs' sector the combinations $h'_\pm = c_\beta H_\pm + s_\beta H'_\pm$ and $h'_0 = \sqrt{2}\; Im\{c_\beta H_0 + s_\beta H'_0\}$ are Goldstones and are eaten by the $W^\pm$ and Z. Then

$$M_w^2 \; = \; \frac{g_2^2}{2} \; (v^2 + v'^2) \quad ; \quad M_Z^2 = (g_2^2 + g_1^2)\;(v^2 + v'^2)/2 \tag{4}$$

and we have the weak mixing angle $\cos\theta_w = c_\theta = M_w/M_Z$.

For simplicity, we will assume that the combinations h_1^0, $h_2^0 = Re h_0, h_3^0 = Im h_0, h^{\pm}$ are mass eigenstates with

$$
\begin{aligned}
h_1^0 &= \sqrt{2}\ Re\ \{c_\beta H_0 \quad + \quad s_\beta H_0'\} \\
h_0 &= -s_\beta H_0 \quad + \quad c_\beta H_0' \\
h_\pm &= -s_\beta H_\pm \quad + \quad c_\beta H_\pm'
\end{aligned}
\tag{5}
$$

We identify h_1^0 with the GSW Higgs'. These, of course, are statements about the scalar potential.

2.2 One–Loop Muon Lifetime

The one-loop radiative corrections to the muon life-time τ_μ were first calculated by Sirlin [7] in GSW. Following his example we calculate τ_μ in the $N = 1$ SUSY $SU_3 \times SU_2 \times U_1$ theory above using a renormalization prescription with the masses M_w, M_Z, M_s (set of scalars), M_f (set of fermions) and the electric charge α_{em} (0) as renormalized input data. The counter-term sector is used to enforce the definition.

$$
\frac{g_2}{(g_2^2 + g_1^2)^{\frac{1}{2}}} = \cos\theta_w = c_\theta = M_w/M_Z \tag{6}
$$

and relations (4) to *all* orders. This is possible because only scalar doublets got v.e.v.'s.

The precise definition of the muon decay constant G is

$$
\begin{aligned}
\tau_\mu^{-1} = \frac{G_\mu^2\ M_\mu^5}{192\pi^3} &\left[1 \ + \ \frac{\alpha}{2\pi}\left(\frac{25}{4} - \pi^2\right)\left(1 + \frac{2\alpha}{3\pi}\ell_n\frac{M_\mu}{M_e}\right)\right] \\
&\times\left[1 - \frac{8M_e^2}{M_\mu^2}\right]\left[1 + \frac{3}{5}\frac{M_\mu^2}{M_w^2}\right]
\end{aligned}
\tag{7}
$$

We note that *part* of the one-loop QED corrections with virtual internal photons and the bremstrahlung diagrams have been included in this definition – the 'traditional photonic' corrections – and we must be careful not to overcount. Then

$G_\mu = (1.16634 \pm .00002) \times 10^{-5}\ GeV^{-2}$. A comparsion of this definition with the one-loop matrix element yields

$$\frac{G_\mu}{\sqrt{2}} = \frac{g_2^2}{8M_w^2}\ (\ 1\ +\ \Delta r) \tag{8}$$

where Δr is a complicated $0(\alpha_{em})$ function of the input data. Using $g_2 s_\theta = g_1 c_\theta = e$ and eq. (6) we may manipulate this into a relation between the $W^\pm$ and Z masses

$$c_\theta^2 \equiv \frac{M_w^2}{M_Z^2} = \frac{1}{2} + \frac{1}{2}\left(1 - \frac{4A_0^2}{M_Z^2(1-\Delta r)}\right)^{\frac{1}{2}} \tag{9}$$

with $A_0 = \frac{\alpha_{em}\,\pi}{\sqrt{2}G_\mu} = (37.281 GeV)^2$. Thus if we know M_Z and Δr we have one-loop *predictions* for M_w and s_θ^2. Δr is the one-loop contribution to τ_μ and is the object of our calculation. Because we evaluate α_{em} and τ_μ at $q^2 = 0$ while M_w and M_Z are obviously evaluated at high q^2 there will be large logs $\Delta r \sim \alpha_{em}\ell n(M_Z^2/M_f^2)$ with M_f a quark or lepton mass. Marciano and Sirlin [7] have shown that one sums in τ_μ all terms in higher loop orders $\alpha_{em}^{N+1}\ \ell n^N(M_Z^2/M_f^2)$ by renormalization group equation (RGE) methods by writing the mass relation as in (9) where the quantity $(1 + \Delta r\ )$ has been replace by
$(1 - \Delta r\ )^{-1}$.

We now calculate Δr in the $N = 1$ SUSY model above. We divide Δr into two parts

$$\Delta r = \Delta r_{GSW}\ +\ \Delta r_{SUSY} \tag{10}$$

where Δr_{GSW} is the result of Sirlin [7] for the standard model and Δr_{SUSY} *is due to the rest of the model (excluding the GSW contribution).*

We have displayed in Tables II and I of reference [4] the numerical results of Sirlin's formulae for M_w and s_θ^2 for various values of M_Z, $M_{h_1^0}$ (Higgs') and M_t (top quark). These are given for reference so that we may study the results of adding all the extra SUSY partners, extra Higgs', etc.

Explicit formulae for Δr_{SUSY} are given below in terms of the (huge number of) free input parameters of the theory. These are ('d_i' for 'data')

$$\{d_i\} = \{\alpha_{em}(0),\ \tau_\mu,\ M_Z,\ c_\beta,\ M_1,\ M_2, \mu,\ M_{f_n},\ M_{\tilde{f}_n},\ M_{h^0_j}, M_{h_\pm}\} \tag{11}$$

Here $M_{fn}(M_{\tilde{f}n})$ are lepton and quark (slepton and squark) masses for $n = 1$, 2, 3 generations while $M^0_{h_j}(j = 1,2,3)$ and $M_h \pm$ denote the 3 neutral scalars and charged scalar masses. M_1 and M_2 are gaugino masses from the soft SUSY breaking terms and μ is the $\hat{H}\hat{H}'$ mixing. To the above parameters must be added mixing angles from the squark, slepton and quark mass matrices. *All parameters are taken to be real* for simplicity. Note that the only free parameter (since α_{em} and τ_μ are known) in tree-level weak processes involving the known particles is M_Z and variations of M_Z within the allowed $UA1/2$ range 90 GeV $\leq M_Z \leq$ 98 GeV affect many processes dramatically [4, 9]. In particular, we have shown elsewhere that the polarization and forward – backward asymmetries A_{pol} and A_{FB} in $e^+e^- \rightarrow \mu^+\mu^-$ on the Z resonance are very sensitive to small changes in M_Z and will serve as very precise tests of the standard model [9, 4].

2.3 Renormalization Of $N = 1$ SUSY $SU_3 \times SU_2 \times U_1$ With Higgs' Doublets And Δr_{SUSY}

We now outline the renormalization of the theory where only Higgs' doublets get v.e.v.'s so that $\rho = \frac{M_w}{M_Z \cos\theta_w} = 1$ exactly [10]. We also calculate Δr_{SUSY}, the effect of the SUSY partners on muon decay.

The physical content of electric charge renormalization is the Ward identity for the U_1 hypercharge group

$$Z_B^{\frac{1}{2}}\, g_1^0 \;=\; g_1 \tag{12}$$

with Z_B the B_μ (hypercharge) field wave function renormalization and $g_1^0(g_1)$ the bare (renormalized) U_1 coupling constant.

Equation (12) may be re-written to one loop.

$$\delta_+ \;+\; (2s_\theta^2 - 1)\,\delta_- \;+\; \frac{1}{2}\pi'_{AA}\,(0) - \frac{s_\theta}{c_\theta M_Z^2}\pi_{ZA}\,(0) = 0 \tag{13}$$

after we have used the counter-term sector to force the ZA mixing to vanish and the photon (A) residue to one at $q^2 = 0$. Here

$$\delta_{\pm} = \frac{1}{2}\left[\frac{\delta g_2}{g_2} \pm \frac{\delta g_1}{g_1}\right] \tag{14}$$

with $g_1^0 = g_1 + \delta g_1, g_2^0 = g_2 + \delta g_2$. The π's are sums of vector boson one-loop self-energy 1 PI graphs (no counter-terms) which in the Euclidean metric of tHooft and Veltman [11] ($q^2 = \underset{\sim}{q}^2 - q_0^2$, $\{\gamma_\mu, \gamma_\nu\} = 2\delta_{\mu\nu}$ and $\gamma_{\pm} = \frac{1}{2}(1 \pm \gamma_5)$, $\gamma_5 = \gamma_1\gamma_2\gamma_3\gamma_4$) are given by

$$\pi_{\mu v;ab} = \delta_{\mu v}\pi_{ab}(q^2) - q_\mu q_v \pi'_{ab}(q^2) \tag{15}$$

With (13), $g_2 s_\theta = g_1 c_\theta = e$ is now the electric charge of the positron. We may now force $c_\theta = M_w/M_Z$ with M_w and M_Z the physical masses and thus determine δ_- uniquely

$$\delta_- = \frac{1}{4s_\theta^2}\ Re\left[\frac{\pi_{ww}(-M_w^2)}{M_w^2} - \frac{\pi_{ZZ}(-M_Z^2)}{M_Z^2}\right] \tag{16}$$

Armed with these definitions we have

$$\Delta r = \Delta r_{vac.pol} + \Delta r_{vertices} + \Delta r_{boxes} \tag{17}$$

$$\Delta r_{vac.pol} = \frac{\delta g_2}{g_2} + \frac{1}{M_w^2}\left[\pi_{ww}(0) - Re\ \pi_{ww}(-M_w^2)\right] \tag{18a}$$

$$= \frac{c_\theta^2}{s_\theta^2} Re\left[\frac{\pi_{ww}(-M_w^2)}{M_w^2} - \frac{\pi_{zz}(-M_Z^2)}{M_Z^2}\right] - \pi'_{AA}(0)$$

$$+ \frac{2s_\theta}{c_\theta M_Z^2}\pi_{ZA}(0) + \frac{1}{M_w^2}\left[\pi_{ww}(0) - Re\ \pi_{ww}(-M_w^2)\right] \tag{18b}$$

The vertex contribution is gotten by using the counter-term sector so that external muon and electron lines are uncorrected; note that we must now correct external neutrino lines. With definitions of the muon one-loop 1PI self-energy parts (no counter-terms)

$$\Sigma^\mu(p) = i\not p\ \gamma_+ A_+^\mu + i\not p \gamma_- A_-^\mu + M_\mu C^\mu \tag{19}$$

and the 1PI muon-muon neutrino - W vertex part (no counter-terms)

$$\Gamma_{\lambda}^{\nu_\mu-\mu-w} = \frac{ig_2}{\sqrt{2}}\,(\gamma_\lambda\gamma_+\,)\,\Gamma^{\nu_\mu-\mu-w} \tag{20}$$

We have the vertex contribution

$$\Delta r_{vertices} = \frac{1}{2}\Big[A_+^{\mu} + 2M_\mu \frac{\partial}{dp^2}C^{\mu}\Big]_{p^2=-M_\mu^2} + \frac{1}{2}A_+^{\nu_\mu}(0) + \Gamma^{\nu_\mu-\mu-w} \tag{21}$$

+ similar term for electron-electron neutrino - W vertex.

The expression Δr is UV and IR finite with quadratic divergences (in dimensional regularization with $d \to 4$) $\Gamma(1-d/2)$ cancelling within vector self-energies and logarithmic divergences $\Gamma(2-d/2)$ cancelling among 1PI parts. As a further check on gauge invariance we note that all dependence on wavefunction renormalizations Z have cancelled in Δr . *We have included all one-loop vertex, self-energy and box diagrams in the calculation of Δr both in τ_μ and the renormalization of the parameters (like α_{em} (0)) entering τ_μ.* Explicit expressions for Δr_{GSW} have been given in the standard model by Sirlin [7].

We now give the results for Δr_{SUSY} with $\Delta r = \Delta r_{GSW} + \Delta r_{SUSY}$. With the soft SUSY breaking terms and $\hat{H}\hat{H}'$ mixing terms given in (2) the charged wino mass matrix is

$$m_+ = \begin{array}{c} \begin{array}{cc} \tilde{W}_+ & \tilde{H}'_+ \end{array} \\ \begin{bmatrix} M_2 & g_2 v \\ g_2 v' & \mu \end{bmatrix} \end{array} \begin{array}{l} \tilde{W}_- \\ \tilde{H}_- \end{array} \tag{22a}$$

where $\langle H\rangle = v, \langle H'\rangle = v'$ are taken to be real as are M_1, M_2 and μ. This is diagonalized by the matrixes $\mathcal{U}$ and $\mathcal{V}$.

$$\mathcal{U}\,\sigma_3\, m_+\,\sigma_3\,\mathcal{V}^\dagger \;=\; diag(m_{+1}, m_{+2}) = m_{+\ell} \quad (\ell = 1,2) \tag{22b}$$

For convenience we define the 4×4 matrix

$$C = \frac{1}{\sqrt{2}} \begin{bmatrix} -i\sigma_2 \mathcal{U} & i\sigma_2 \mathcal{V} \\ \mathcal{U} & \mathcal{V} \end{bmatrix}; \tag{22c}$$

$$C \begin{bmatrix} 0 & \sigma_3\, m_+\, \sigma_3 \\ \sigma_3\, m_+\, \sigma_3 & 0 \end{bmatrix} C^\dagger = diag(-m_{+2}, -m_{+1}, m_{+1}, m_{+2})$$
$$= m_{+k}\ (k = 1, 4) \tag{22d}$$

The neutral wino mass matrix is

$$m_0 = \begin{array}{c} \\ \end{array} \begin{array}{cccc} \tilde{W}_3 & \tilde{B} & \tilde{H}_0 & \tilde{H}'_0 \end{array}$$
$$m_0 = \begin{bmatrix} M_2 & 0 & \frac{-g_2 v}{\sqrt{2}} & \frac{g_2 v'}{\sqrt{2}} \\ & M_1 & \frac{g_1 v}{\sqrt{2}} & \frac{-g_1 v'}{\sqrt{2}} \\ & & 0 & -\mu \\ & symmetric & & 0 \end{bmatrix} \begin{array}{c} \tilde{W}_3 \\ \tilde{B} \\ \tilde{H}_0 \\ \tilde{H}'_0 \end{array} \tag{23a}$$

which is diagonalized by $N\, m_0\, N^\dagger = diag\,(m_{01},\ m_{02},\ m_{03},\ m_{04}) = m_{0j}; j = 1, 4$. With these definitions, the real matrices $C^\dagger$ and $N^\dagger$ are returned by the standard numerical matrix-diagonalization subroutines such as (NAG) F02ABF or (CERN) EISRS1.

Now form some coefficients

$$
\begin{aligned}
(4\times 1):\quad & A_j = \tfrac{1}{2}N_{j1} - \tfrac{g_1}{2g_2}N_{j2} && E_j = -\tfrac{1}{2}N_{j1} - \tfrac{g_1}{2g_2}N_{j2} \\
& F_j = -\tfrac{1}{\sqrt{2}}C_{j1} && H_j = -\tfrac{1}{\sqrt{2}}C_{j3} \\
& P_j = -g_1/g_2\, N_{j2} && \\
(2\times 2):\quad & \lambda^+_{\ell m} = -c_\theta \delta_{\ell m} + \tfrac{1}{2c_\theta}\mathcal{U}_{\ell 2}\mathcal{U}_{m2} && \lambda^-_{\ell m} = -c_\theta \delta_{\ell m} + \tfrac{1}{2c_\theta}\mathcal{V}_{\ell 2}\mathcal{V}_{m2} \\
(2\times 4):\quad & Z_{\ell k} = \tfrac{1}{\sqrt{2}}\mathcal{V}_{\ell 2}N_{k3} + \mathcal{V}_{\ell 1}N_{k1} && X_{\ell k} = -\tfrac{1}{\sqrt{2}}\mathcal{U}_{\ell 2}N_{k4} + \mathcal{U}_{\ell 1}N_{k1} \\
(4\times 4):\quad & Y^+_{jk} = \tfrac{1}{2c_\theta}N_{j4}N_{k4} && Y^-_{jk} = \tfrac{1}{2c_\theta}N_{j3}N_{k3} \\
& \tilde{Z}_{jk} = \tfrac{1}{\sqrt{2}}C_{j4}N_{k3} + C_{j3}N_{k1} && \tilde{X}_{jk} = -\tfrac{1}{\sqrt{2}}N_{j4}C_{k2} + N_{j1}C_{k1} \\
& Q_{jk} = C_{j3}C_{k3} && \\
& \ell, m = 1,2 && \\
& j,k = 1,4 &&
\end{aligned}
\tag{24}
$$

We assume that the mass eigenstrates in the Higgs' sector are

$$
\begin{aligned}
h_\pm &= -s_\beta H_\pm + c_\beta H'_\pm \\
h_0 &= -s_\beta H_0 + c_\beta H'_0
\end{aligned}
\tag{25a}
$$

with $h_2^0 = Re h_0$ and $h_3^0 = Im\ h_0$ having, in principle, different masses. Here $cos_\beta = c_\beta = v/(v^2 + v'^2)^{\frac{1}{2}}$ and we identify the combination

$$
h_1^0 = \sqrt{2}\ Re\Big[c_\beta H_0 + s_\beta H'_0\Big]
\tag{25b}
$$

with the standard-model Higgs' scalar; thus it doesn't enter our calculation of the SUSY -partner of the theory. The orthogonal combinations to the above were the Goldstone bosons and were eaten by the $W^\pm$ and Z. We identify the combinations of coupling constants λ_1, λ_2, λ_3, λ_4 and a 2×2 matrix T

$$\lambda_1 = \frac{2 - v_\theta}{6c_\theta} \qquad \lambda_2 = -\frac{2\,s_\theta^2}{3\,c_\theta}$$

$$\lambda_3 = \frac{v_\theta - 5}{6c_\theta} \qquad \lambda_4 = \frac{s_\theta^2 - c_\theta^2}{c_\theta} \tag{26}$$

where $v_\theta = 4s_\theta^2 - 1$. The matrix T diagonalizes the (2×2) top squark mass matrix $M_{\tilde{t}}^2$

$$\begin{aligned} L_{\tilde{t}mass} &= (\tilde{u}_3^\dagger, \tilde{u}_3^{c\dagger}) M_{\tilde{t}}^2 \begin{pmatrix} \tilde{u}_3 \\ \tilde{u}_3^c \end{pmatrix} \;;\; T = \begin{pmatrix} c_\alpha & s_\alpha \\ -s_\alpha & c_\alpha \end{pmatrix} \\ T M_{\tilde{t}}^2 T^\dagger &= diag(M_{\tilde{t}_1}^2, M_{\tilde{t}_2}^2) \end{aligned} \tag{27}$$

We will write expressions where all *other* squarks and sleptons in $n = 1$, 2, 3 generations are mass eigenstrates but it is a trivial matter to include possible mixings there as well. The doublets are

$$\tilde{q}_n = \begin{pmatrix} \tilde{u}_n \\ \tilde{d}_n \end{pmatrix} \quad ; \quad \tilde{\ell}_n = \begin{pmatrix} \tilde{\nu}_n \\ \tilde{e}_n \end{pmatrix} \tag{28}$$

with singlets $\tilde{u}_n^c$, $\tilde{d}_n^c$, $\tilde{e}_n^c$ (and $\tilde{\nu}_n^c$ decouples completely).

The contributions from the *extra* particles (SUSY partners plus *extra* Higgs') appearing in the $N = 1$ SUSY version of the theory may then be written down. We use $N_c = 3$ the number of squark colors. All form factors used in this section are carefully defined in the Appendix.

$$\pi_{ZA}^{SUSY} = \frac{g_2^2}{16\pi^2} q^2 \Big\{ 2s_\theta \lambda_4 B_{13}(h_+, h_+) - 4s_\theta (W_{\ell\ell}^+ + W_{\ell\ell}^-) B_3(m_{+\ell}, m_{+\ell})$$

$$+ N_c \Big[\frac{2s_\theta}{3} \lambda_3 B_{13}(\tilde{d}_n , \tilde{d}_n) + \lambda_2^2 \frac{c_\theta}{s_\theta} B_{13}(\tilde{d}_n^c , \tilde{d}_n^c)$$

$$- \frac{8s_\theta}{3} \lambda_1 B_{13}(\tilde{u}_i , \tilde{u}_i) + 4\lambda_2^2 \frac{c_\theta}{s_\theta} B_{13}(\tilde{u}_i^c , \tilde{u}_i^c)$$

$$- \frac{8s_\theta}{3} \lambda_1 B_{13}(\tilde{t}_1 , \tilde{t}_1) + 4\lambda_2^2 \frac{c_\theta}{s_\theta} B_{13}(\tilde{t}_2 , \tilde{t}_2) \Big]$$

$$+ 2s_\theta \lambda_4 B_{13}(\tilde{e}_n , \tilde{e}_n) + \frac{4s_\theta^3}{c_\theta} B_{13}(\tilde{e}_n^c , \tilde{e}_n^c) \Big\} \tag{29a}$$

$$\pi_{AA}^{\prime SUSY} = \frac{e^2}{4\pi^2} \Big\{ -B_{13}(h_+, h_+) + 2B_3(m_{+\ell}, m_{+\ell})$$

$$- N_c \Big[\frac{1}{9} B_{13}(\tilde{d}_n^c , \tilde{d}_n^c) + \frac{1}{9} B_{13}(\tilde{d}_n , \tilde{d}_n) + \frac{4}{9} B_{13}(\tilde{u}_n , \tilde{u}_n)$$

$$+ \frac{4}{9} B_{13}(\tilde{u}_n^c , \tilde{u}_n^c) \Big]$$

$$- B_{13}(\tilde{e}_n , \tilde{e}_n) - B_{13}(\tilde{e}_n^c , \tilde{e}_n^c) \Big\} \tag{29b}$$

$$\pi_{ww}^{SUSY} = \frac{g_2^2}{16\pi^2} \Big\{ B_7(h_2^0, h_+) + B_7(h_3^0, h_+) + [Z_{\ell k}^2 + X_{\ell k}^2] B_{14}(m_{+\ell}, \; m_{0k})$$

$$+ Z_{\ell k} X_{\ell k} m_{+\ell} m_{0k} B_0(m_{+\ell}, m_{0k}) + 2N_c [B_7(\tilde{u}_i , \tilde{d}_i) + c_\alpha^2 B_7(\tilde{t}_1 , \tilde{d}_3)$$

$$+ s_\alpha^2 B_7(\tilde{t}_2 , \tilde{d}_3)] + 2B_7(\tilde{\nu}_n , \tilde{e}_n) \Big\} \tag{29c}$$

$$
\begin{aligned}
\pi_{ZZ}^{SUSY} = \frac{g_2^2}{16\pi^2}\Big\{ & \lambda_4^2 B_7(h_+, h_+) + \frac{1}{c_\theta^2} B_7(h_2^0, h_3^0) + [Y_{jk}^{+^2} + Y_{jk}^{-^2}] B_{14}(m_{0j}, m_{0k}) \\
& + Y_{jk}^{+} Y_{kj}^{-} m_{0j} m_{0k} B_0(m_{0j}, m_{0k}) + [\lambda_{\ell k}^{+^2} + \lambda_{\ell k}^{-^2}]\, B_{14}\,(m_{+\ell}, m_{0k}) \\
& + \lambda_{\ell k}^{+}\lambda_{\ell k}^{-} m_{+\ell} m_{0k} B_0(m_{+\ell}, m_{0k}) + N_c\Big[\lambda_3^2 B_7(\tilde{d}_n, \tilde{d}_n) + \lambda_2^2 B_7(\tilde{d}_n^c, \tilde{d}_n^c) \\
& + 4\lambda_1^2 B_7(\tilde{u}_i, \tilde{u}_i) + 4\lambda_2^2 B_7(\tilde{u}_i^c, \tilde{u}_i^c) + 4(\lambda_1 c_\alpha^2 + \lambda_2 s_\alpha^2)^2 B_7(\tilde{t}_1, \tilde{t}_1) \\
& + 4(\lambda_2 - \lambda_1)^2 s_\alpha^2\, c_\alpha^2 B_7(\tilde{t}_1, \tilde{t}_2) + 4(\lambda_1 s_\alpha^2 + \lambda_2 c_\alpha^2)^2 B_7(\tilde{t}_2, \tilde{t}_2)\Big] \\
& + \lambda_4^2 B_7(\tilde{e}_n, \tilde{e}_n) + 9\lambda_2^2 B_7(\tilde{e}_n^c, \tilde{e}_n^c) + \frac{1}{c_\theta^2} B_7(\tilde{\nu}_n, \tilde{\nu}_n)\Big\}
\end{aligned}
\tag{29d}
$$

The contribution to vertices and boxes are evaluated at $q^2 = 0$. We ***assume $\mu - e$ universality*** and so have

$$
\begin{aligned}
\Delta r_{vertices}^{SUSY} = \frac{g_2^2}{16\pi^2}\Big\{ & -F_j^2 B_4(\tilde{e}_1, m_{+j}) \; - A_j^2 B_4(\tilde{\nu}_1, m_{0j}) \; - \; H_j^2 B_4(\tilde{\nu}_1, m_{+j}) \\
& - E_j^2 B_4(\tilde{e}_1, m_{0j}) + 4A_j E_j C_{24}^0(m_{0j}, \tilde{e}_1, \tilde{\nu}_1) - 4F_j H_j C_{24}^0(m_{+j}, \tilde{\nu}_1, \tilde{e}_1) \\
& + 4\sqrt{2} F_j \tilde{X}_{jk} E_k C_{24}^0(\tilde{\nu}_1, m_{0k}, m_{+j}) - 4\sqrt{2} A_j \tilde{Z}_{jk} H_k \\
& \times C_{24}^0(\tilde{e}_1, m_{+k}, m_{0j})\Big\}\Big|_{q^2=0}
\end{aligned}
\tag{30}
$$

The boxes are also evaluated at $q^2 = 0$ and give

$$
\begin{aligned}
\Delta r_{boxes}^{SUSY} = \frac{-g_2^2}{8\pi^2} M_w^2 \Big\{ & F_k^2\, E_j^2\, D_{27}^0\,(m_{0j}, m_{+k}, \tilde{e}_1, \tilde{e}_2) \\
& + A_j^2 H_k^2 D_{27}^0\,(m_{0j}, m_{+k}, \tilde{\nu}_1, \tilde{\nu}_2)\Big\}\Big|_{q^2=0}
\end{aligned}
\tag{31}
$$

In the above we have used $m_{+\ell} = m_{+1}$, m_{+2} and $m_{+k} = -m_{+2}$, $-m_{+1}$, m_{+1}, m_{+2}. Repeated indices are summed over the ranges

$$i, \ell = 1, 2$$

$$j, k = 1, 4$$

$$n = 1, 3$$

We then form the total correction to muon decay.

$$\Delta r_{SUSY} = \Delta r^{SUSY}_{vac.pol} + \Delta r^{SUSY}_{vertices} + \Delta r^{SUSY}_{boxes}$$

2.4 Polarization Assymmetry $e^+ e^-_{pol} \to \mu^+ \mu^-$

The Z^o mass will be measured to great accuracy in the near future [12, 13]. Unfortunately, the $W^{\pm}$ mass will not, so that the shifts δM_w computed in the previous part of this section will probably be of academic interest until W's can be produced in pairs. Other calculations of one-loop effects in $N = 1$ SUSY [3, 12] concentrate on corrections to the ρ parameter or ν-hadron scattering and properly conclude that the corrections are not observable in these processes in the near future. Fortunately, the longitudinal polarization asymmetry A_{pol} in $e^+ e^-_{pol} \to \mu^+ \mu^-$ will be measured to $\pm.01$ or better at the SLC [13] (or in the τ^- polarization in $e^+ e^- \to \tau^+ \tau^-$ at LEP and/or the SLC) and so a test of the model, and in particular the corrections from the SUSY part is forthcoming. A_{pol} is defined for left-handed (e_L) and right-handed (e_R) polarized electrons as

$$\sigma_L = \frac{d\sigma}{d\Omega} \, (e^+ e^-_L \to \mu^+ \mu^-) \tag{32a}$$

$$\sigma_R = \frac{d\sigma}{d\Omega} \, (e^+ e^-_R \to \mu^+ \mu^-) \tag{32b}$$

$$A_{pol}(q^2, x) = \frac{\int_0^{2\pi} d\phi \int_{-x}^{x} dcos\theta (\sigma_L - \sigma_R)}{\int_0^{2\pi} d\phi \int_{-x}^{x} dcos\theta (\sigma_R + \sigma_L)} \tag{32c}$$

This will be measured at the Z^o resonance ($q^2 = -M_Z^2$) where statistics are good. The radiative corrections to A_{pol} in the standard model were calculated in a previous paper [4] and the one-loop GSW results are presented in Table III of reference [4]. We will now calculate the one-loop deviations from the results of Table III ref. [4] due to the $N=1$ SUSY $SU_3 \times SU_2 \times U_1$ theory given above. We define the shift from GSW as [9].

$$\delta A_{pol}^{SUSY} = A_{pol}(q^2=-M_Z^2,1)|_{N=1\ SUSY} - A_{pol}(q^2=-M_Z^2,1)|_{GSW} \quad (33)$$

There will be three sources of radiative corrections in $A_{pol}(q^2=-M_Z^2)$ Having eliminated M_w in favor of τ_μ as an input parameter, we have A_{pol} as a function of the data (11). At tree level (on Z^o resonance)

$$A_{pol}^{tree} = \frac{-2\,\hat{v}_\theta}{1+\hat{v}_\theta^2} \quad (34)$$

with $\hat{v}_\theta = 4\,\hat{s}_\theta^2 - 1$ evaluated using $\hat{s}_\theta^2$ in the $N=1$ global SUSY model calculated in the previous subsection. Thus, there are shifts in A_{pol} due to the shifts in $\hat{s}_\theta^2$

$$\delta A_{pol}^{(1)} = \lambda_{pol} \Delta r_{SUSY} \quad (35a)$$

$$\lambda_{pol} = \frac{-64\,s_\theta^4\,c_\theta^2}{(1+v_\theta^2)^2} \quad (35b)$$

where s_θ^2 may be evaluated in λ_{pol} using the tree-level data (which depends only on M_Z).

The second source of shift due to radiative corrections in A_{pol} is from the $Z-A$ mixing graphs in the asymmetry itself.

$$\delta A_{pol}^{(2)} = \lambda_{pol}\,\frac{(1-2s_\theta^2)}{s_\theta c_\theta}\,Re\Big(\frac{\pi_{ZA}^{SUSY}(-M_Z^2)}{M_Z^2} - \frac{c_\theta}{s_\theta}\,\frac{\pi_{ww}^{SUSY}(-M_w^2)}{M_w^2} + \frac{c_\theta}{s_\theta}\,\frac{\pi_{ZZ}^{SUSY}(-M_Z^2)}{M_Z^2}\Big) \quad (35c)$$

when the last two terms come from the counter-term in $Z-A$ mixing.

The third source of radiative corrections is from corrections to the $e - Z$ vertices. This gives

$$\delta A^{(3)}_{pol} = \left(\frac{2s^2_\theta - 1}{c_\theta}\right)^2 Re\left(\tilde{\Gamma}^{e-Z}_- - \tilde{\Gamma}^{e-Z}_+\right) \tag{35d}$$

with the $\tilde{\Gamma}_\pm$ defined below. Putting this all together yields

$$\delta A^{SUSY}_{pol} = \lambda_{pol}(\Delta r_{SUSY} + \Delta b_{SUSY}) \tag{36a}$$

$$\begin{aligned}\Delta b_{SUSY} = \frac{(1-2s^2_\theta)}{s_\theta c_\theta} Re\Big[\frac{\pi^{SUSY}_{ZA}\, Re(-M^2_Z)}{M^2_Z} - \frac{c_\theta}{s_\theta}\frac{\pi^{SUSY}_{ww}(-M^2_w)}{M^2_w} \\ + \frac{c_\theta}{s_\theta}\frac{\pi^{SUSY}_{ZZ}(-M^2_Z)}{M^2_Z} + \frac{(1-2s^2_\theta)s_\theta}{c_\theta}(\tilde{\Gamma}^{e-Z}_- - \tilde{\Gamma}^{e-Z}_+)\Big]\end{aligned} \tag{36b}$$

Note that we have used the fact that

$$\pi^{SUSY}_{ZA}(0) = 0 \tag{36c}$$

in Δb_{SUSY}. The shift in the asymmetry δA_{pol} may be re-written

$$\begin{aligned}\delta A_{pol} = \lambda_{pol}\, Re\Big[-\frac{\pi^{SUSY}_{ZZ}(-M^2_Z)}{M^2_Z} + \frac{\pi^{SUSY}_{ww}(0)}{M^2_w} + \frac{(1-2s^2_\theta)}{s_\theta c_\theta}\frac{\pi^{SUSY}_{ZA}(-M^2_Z)}{M^2_Z} \\ - \pi'^{SUSY}_{AA}(0) + \Delta r^{SUSY}_{vertices} + \Delta r^{SUSY}_{boxes} + \left(\frac{1-2s^2_\theta}{c_\theta}\right)^2(\tilde{\Gamma}^{eZ}_- - \tilde{\Gamma}^{eZ}_+)\Big]\end{aligned} \tag{37a}$$

where the one-loop 1 PI corrections to the electron-Z^o vertex which enters into the calculation of the asymmetry of itself is defined as (no counter-terms)

$$\Gamma^{e-Z}_\lambda = \frac{g}{c_\theta} i\,\gamma_\lambda\left\{\left(\frac{2s^2_\theta - 1}{2}\right)\Gamma^{e-Z}_+\gamma_+ + s^2_\theta \Gamma^{eZ}_-\gamma_-\right\} \tag{37b}$$

which after inclusion of the $e - Z$ vertex counter-term gives

$$\tilde{\Gamma}^{eZ}_\pm = \Gamma^{e-Z}_\pm + \left\{A^\mu_\pm + 2M^2_\mu \frac{\partial}{dp^2} C_\mu\right\}_{p^2=-M^2_\mu} \tag{37c}$$

with $A_{\pm}^{\mu}$ and C^{μ} coming from the muon self-energy (see eq. (19)). Note that box diagrams in the asymmetry itself do not contribute to A_{pol} on Z^{o} resonance to the required accuracy but that box diagrams in muon decay do. All box diagrams involving exchanges of a Z^{o} and photon have been included in $A_{pol}|_{GSW}$. A thorough discussion of eq. (37a) has been given elsewhere [9].

Expressions for all one-loop quantities due to the SUSY part of the model appearing in δA_{pol} are given earlier in this section except $\tilde{\Gamma}_{\pm}^{eZ}$. These are easily computed.

$$\tilde{\Gamma}_{-}^{eZ} = \frac{g_2^2}{16\pi^2} PjP_j^2 C_6(m_{oj}, \tilde{e}_1^c, \tilde{e}_1^c)|_{q^2=-M_Z{}^2} \tag{38a}$$

$$\tilde{\Gamma}_{+}^{eZ} = \frac{g_2^2}{16\pi^2}\Big\{E_j^2 C_6(m_{0j}, \tilde{e}_1, \tilde{e}_1) + \frac{1}{2s_\theta^2 - 1} H_j^2 C_6(m_{+j}, \tilde{\nu}_1, \tilde{\nu}_1)$$

$$-\Big(\frac{c_\theta^2}{2s_\theta^2 - 1}\Big) 2Q_{jk} H_j H_k C_7(\tilde{\nu}_1, m_{+j}, m_{+k})\Big\}\Big|_{q^2=-M_Z{}^2} \tag{38b}$$

with the form factors defined carefully in the Appendix. This completes the calculation of M_w, s_θ^2 and A_{pol} to one loop in the $N = 1$ SUSY standard model presented above. Equation (18b) and (29) to (38) are the main results of this section. We will evaluate the shifts in M_w, s_θ^2 and δA_{pol} numerically in Section IV.

III. $N = 1$ Local SUSY

Clearly, the number of free parameters (11) in the $N = 1$ global SUSY $SU_3 \times SU_2 \times U_1$ theory above with the soft SUSY and R-invariance breaking terms is much too large; we need some motivation for them. This we take to be given by coupling the gauge theory to $N = 1$ *local* SUSY (SUGRA)[6]. We do this not so much because we believe any particular model but rather to get a feel for the possible size of the effects of such models on low-energy phenomena. We will show that, for at least some models, the radiative corrections calculated in the last section give effects which can certainly be seen experimentally in the next generation of accelerators (LEP1/SLC) and maybe in neutrino scattering experiments in the present generation (CHARM II).

The effect of the breaking of $N = 1$ local SUSY via the super-Higgs' mechanism is to induce in the flat space limit certain soft global $N = 1$ SUSY and R-invariance breaking terms in the effective theory at low energies. We now describe briefly what happens in this scenario [14]. In the simplest case, local $N = 1$ supergravity is coupled to a Yang-Mills gauge theory based on the local semi-simple gauge group G and one chiral $N = 1$ scalar superfield $\hat{Z} = (z, \chi)$. The generalization to the product of groups $SU_3 \times SU_2 \times U_1$ is straight forward.

$\hat{Z}$ is to transform as a singlet under G. The coupling of this to $N = 1$ SUGRA with vierbein $e_{\mu a}$ and massless gravitino ψ_μ (2-components each) and to the vector superfield $\hat{V}^i_\mu = (V^i_\mu, \lambda^i)$ as well as the chiral scalar superfield $\hat{y}_i = (y_i, \chi_i)$ with i an index for G has been given by Cremmer et. al. [14] and depends on the gravitational coupling $k^2 = 8\pi G_N$ with G_N Newton's constant. The super-Higgs' mechanism will cause ψ_μ to eat χ thus acquiring a mass $m_{3/2}$ and breaking local SUSY . Since ψ_μ transforms under R-parity this also breaks R-invariance. The result in the flat space limit $k \to 0$ will be certain soft terms which break the rigid $N = 1$ SUSY and R-invariance. The price of this is the extra scalar z but it will decouple completely as does ψ_μ and $e_{\mu a}$ in the flat space $k \to 0$ limit from the effective low-energy gauge theory with broken global $N = 1$ SUSY .

The classical $N = 1$ SUGRA Lagrangian of Cremmer, et. al. is given as a certain function of two arbitrary functions $\mathcal{G}(z, y_i)$ and $f_{ab}(z, y_i)$ where a, b are G indices. In our $SU_3 \times SU_2 \times U_1$ theory there would be three functions f_{ab}. In order that the scalar kinetic energy terms be of canonical form , we require $\mathcal{G},_{zz^*} = \mathcal{G},_{y_i y_i^*} = -\frac{1}{2}\kappa^2$ where the notation $\mathcal{G},_{zz^*} = \partial^2\mathcal{G}/\partial z \partial z^*$ has been introduced. The potential for the scalars is then

$$V = \frac{1}{k^4}\, e^{-\mathcal{G}}\Big(\frac{2}{k^2}\mathcal{G},_z \mathcal{G},_{z^*} - 3 + \frac{2}{k^2}\mathcal{G},_{y_i}\mathcal{G},_{y_i^*}\Big) + \frac{1}{2}(Ref)^{-1}_{ab}\, D^a D^b \qquad (39a)$$

with the 'D-term'

$$D^a = -\frac{1}{2} g_G y_i^\dagger (T^a)_{ij}\, y_j \qquad (39b)$$

T^a and g_G are the generators and gauge coupling of G. In order that the low-energy gauge theory decouple from z in the $k \to 0$ limit they choose the Kahler potential

$$\mathcal{G} = -\frac{1}{2}k^2\,|z|^2 - \frac{1}{2}k^2\; y_i\, y_i^* - \ell n p_1(z) - \ell n\, p_2\,(y_i) \tag{40}$$

The function $p_1(z)$ is chosen so as to force $V \geq 0$ everywhere

$$\frac{2}{k^2}\; \mathcal{G},_z \mathcal{G},_{z^*} - 3 \geq 0 \tag{41}$$

with the minimum $V_0 = 0$ occurring at the classical values

$$z = 0 \tag{42a}$$

$$D^a = 0 \tag{42b}$$

$$\mathcal{G},_{y_i} = -\frac{1}{2}\,k^2\,y_i^* \; - \; \frac{1}{p_2}\frac{\partial p_2}{\partial y_i} \; = 0 \tag{42c}$$

This last equation is made more transparent by writing

$$p_2(y_i) = exp\;(k^2 h\;(yi)/2m_{3/2}) \tag{43a}$$

so that

$$\mathcal{G}, yi = -\frac{1}{2}\frac{k^2}{m_{3/2}}\Big[\partial h/\partial y_{\,i} \; + \; m_{3/2}\,y_i^*\Big] = 0 \tag{43b}$$

with $h(y_i)$ the super potential for the global $N = 1$ theory in the flat space limit. The graviton mass $m_{3/2}$ is the coefficient in the local SUSY theory (with $e = \sqrt{det\; e_{\mu a}}$)

$$m_{3/2} = \; e^{-\mathcal{G}_0/2} \tag{44}$$

$$e^{-1}\,L_{SUGRA} \; = e^{-\mathcal{G}/2}\;\bar{\psi}_\mu\;\sigma^{\mu\nu}\;\psi_\nu + \bar{\lambda}_a\,M^{ab}\lambda_b \; + \; --- \tag{45}$$

where p_1 is chosen such that $\mathcal{G}_0 = \mathcal{G}(z = D^a = \mathcal{G}, y_i = 0) \neq \infty$ at the minimum $V_0 = 0$. The standard example is

$$p_1(z) = k\,m_{3/2}\left(\frac{k\,z}{\sqrt{2}} + 1\right) exp\Big[-k\,z(1-\sqrt{3})\Big] \tag{46}$$

but beyond the requirements (41) and that $\mathcal{G}_0 \neq \infty$ it is not necessary to specify $p_1(z)$.

The gaugino mass M_G is gotten from

$$M^{ab} = -\frac{1}{6} k^3 e^{\mathcal{G}/2} (Ref)^{-\frac{1}{2}}_{ac} (Ref)^{-\frac{1}{2}}_{bd} D^c D^d - \frac{1}{2k^3} e^{\mathcal{G}/2} \mathcal{G}, {}_z \partial f_{ab}/dz$$
$$- \frac{1}{2k^3} e^{-\mathcal{G}/2} \mathcal{G}, {}_{y_i} \partial f_{ab}/dy_i \tag{47}$$

and since $D^a = 0$ at the minimum we need the derivatives of f_{ab} non-zero there. The simplest choice is (f_{ab} real gives CP conservation in the pure Yang-Mills sector)

$$f_{ab} = \delta_{ab} \, exp\left[\sqrt{\frac{2}{3}} \frac{M_G}{m_{3/2}} \, k \, z\right] \tag{48}$$

Then the $k^2 \to 0$ limit is taken keeping $m_{3/2}$ and M_G fixed. The Lagrangian becomes in the $\chi = 0$ gauge (of local SUSY)

$$L_{SUGRA} \;\to\; L_1(e, \psi_\mu, z) \,+\, L_G \tag{49}$$

where L_1 describes a theory of a massless graviton, a massive gravitino ψ_μ (which has eaten χ) and a massive scalar z whose mass depends on the choice of p_1. These are *completely decoupled* from the rest of the theory in the $k \to 0$ limit

$$L_G = -\frac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu} - \frac{1}{2} \bar{\lambda}^a \not{D} \lambda^a$$
$$-\frac{1}{2} M_G \bar{\lambda}^a \lambda^a + \bar{\chi}^i M^{ij} \chi^j + 2 \bar{\chi}^i M^{ia} \lambda^a$$
$$-V \tag{50a}$$

$$V = \frac{1}{2} \sum_i \left| \frac{\partial h}{\partial y_i} + m_{3/2} y_i^* \right|^2 + \frac{1}{8} g_G^2 \sum_a \left[y_i^+ T^a_{ij} y_i \right]^2 \tag{50b}$$

$$M^{ij} = -\frac{1}{2} \partial^2 h / \partial y_i \partial y_j \tag{50c}$$

$$M^{ia} = \frac{1}{2}\, i g_G\, y_k^+\, T_{ik}^a \tag{50d}$$

with $h(y_i)$ the superpotential of the global $N = 1$ theory. The reader will recognize L_G as the Lagrangian for the global $N = 1$ SUSY gauge theory with gauge-group G and R and SUSY breaking parameters M_G and $m_{3/2}$.

Note that because of the addition of the SUSY breaking terms $m_{3/2}^2$ to all scalars' $(\text{mass})^2$ the gauge group G will no longer obviously break spontaneously to $SU_3 \times U_1$ at low energies. There are two ways to handle this:

i) the addition of a 'sliding singlet' chiral scalar superfield $\hat{N}$ with an F-term in the super potential $h(y_i)$ [5].

ii) to regard the theory L_G as an effective theory at scales M_{Plank} or M_x (unification) whose parameters $d_i(M_{Pl})$ are evaluated at this high scale. Then we use the renormalization group equations (RGE) to show that the parameters of the theory at low energies $di(M_w)$ do indeed break G spontaneously at scale $\sim M_w$. Usually, this involves a heavy top quark [15].

In this paper, we shall examine the radiative corrections to τ_μ and A_{pol} in a large class of sliding singlet models and show that these effects can be large. The radiative corrections in RGE models will be examined in a later paper.

Perhaps the simplest sliding singlet model is due to Cremmer et. al. [5]. They introduce a singlet field $\hat{N}$ (under $SU_3 \times SU_2 \times U_1$). The relevant terms in the super potential are

$$W = 2h\ \hat{H}'\ \hat{H}\ \hat{N}\ +\ \sigma \hat{N}\ +\ --- \tag{51}$$

The potential (50b) for the scalars may then be written (we drop the ^ when referring to the scalar components of a superfield)

$$V(H, H', N) = \frac{1}{2} g_2^2 (|H'|^2 |H|^2 - |H \cdot H'|^2) + \frac{(g_2^2 + g_1^2)}{8} (|H'|^2 - |H|^2)^2$$

$$+ \left| \frac{h}{2} N H + m_{3/2} H' \right|^2 + \left| \frac{h}{2} N^\dagger H' + m_{3/2} H \right|^2 \tag{52}$$

$$+ \frac{1}{2} \left| \sigma + hHH' + m_{3/2} N^\dagger \right|^2 + \text{squarks, sleptons}$$

We are looking for solutions where $\langle q_n \rangle = \langle \ell_n \rangle = 0$. The first two terms are D terms and from (42), force $v = v'$, $s_\beta = c_\beta = \frac{1}{\sqrt{2}}$. We choose $\langle N \rangle$ real for simplicity. Then the 3rd and 4th terms may be re-written

$$\left| m_{3/2} + \frac{1}{2} h \langle N \rangle \right|^2 \left[\frac{H + H'}{\sqrt{2}} \right]^2 + \left| m_{3/2} - \frac{1}{2} h \langle N \rangle \right|^2 \left[\frac{H - H'}{\sqrt{2}} \right]^2 \tag{53}$$

Thus the theory chooses the value $h\langle N \rangle$ such that, say,

$$m_{3/2} + \frac{1}{2} h \langle N \rangle = 0 \tag{54}$$

so that the fields $\frac{1}{\sqrt{2}}(H + H')$ act as Goldstone bosons after spontaneously symmetry breaking and become the longitudinal components of the $W^\pm$ and Z as well as the GSW Higgs'. The orthogonal combinations get masses

$$M_{h_\pm}^2 = M_w^2 + 4 m_{3/2}^2 \tag{55a}$$

$$M_{h_3^0}^2 = M_Z^2 + 4 m_{3/2}^2 \tag{55b}$$

with the mass $M_{h_3^0}$ a free parameter. Note that the net effect of the sliding singlet $\hat{N}$ has been to generate the term $\mu \hat{H} \hat{H}'$ with

$$\mu = -m_{3/2} \tag{56}$$

(the sign a matter of convention). Otherwise it decouples completely from the rest of the theory to the required accuracy.

The squark and slepton mass matrices become

$$m^2_{\tilde{q},\tilde{\ell}} = \begin{bmatrix} m^2_{3/2} + M^2_{q,\ell} & (A-1)m_{3/2}M_{q,\ell} \\ (A-1)m_{3/2}M_{q,\ell} & m^2_{3/2} + M^2_{q,\ell} \end{bmatrix} \tag{57}$$

with $A = 3$ a consequence of the type of local SUSY breaking chosen. The free parameters in the theory are then

$$\alpha_{em}\,(0),\ \tau_\mu,\ M_Z,\ m_{3/2},\ M_{f_n},\ M_{h_1^0},\ M_{h_2^0},\ M_1,\ M_2 \tag{58}$$

where of the fermion masses M_{f_n}, only M_t (top quark mass) is unknown if we limit ourselves to 3 generations. Since we have identified h_1^0 with the GSW Higgs' it doesn't enter our calculation of Δr_{SUSY} or δA_{pol}.

IV. Results

In this section we present the numerical results of the formulae in section II with the sliding singlet model of $N = 1$ local SUSY considered in section III used as motivation for the parameters of the theory. We now briefly summarize the program of calculation.

After elimination of M_w in favor of τ_μ as renormalized input data, the parameters of the theory which enter into our one-loop formulae are

$$M_Z, m_{3/2}, M_t, M_{h_3{}^0}, M_1, M_2 \tag{59}$$

since the values of α_{em}, τ_μ are known. Clearly, only M_Z will enter into tree-level predictions of the theory for interactions of the known particles. The one-loop predictions for s^2_θ, M_w and A_{pol} in the standard GSW model are given in Tables I, II and III of reference [4]. We then have the one-loop shifts due to the SUSY - partner part of the $N = 1$ global SUSY theory given above.

$$\delta s^2_\theta = \hat{s}^2_\theta\,|_{SUSY} - s^2_\theta|^{Table\ I,\ ref.4}_{GSW} = \lambda_\theta \Delta r_{SUSY} \tag{60a}$$

$$\delta M_w = \hat{M}_w\,|_{SUSY} - M_w|^{Table\ II,\ ref.4}_{GSW} = M_Z \lambda_w \Delta r_{SUSY} \tag{60b}$$

$$\delta A_{pol} = \hat{A}_{pol}\,|_{SUSY} - A_{pol}|^{Table\ III,\ ref.4}_{GSW} = \lambda_{pol}(\Delta r_{SUSY} + \Delta b_{SUSY}) \tag{60c}$$

where the coefficients

$$\lambda_\theta = \frac{s_\theta^2 \; c_\theta^2}{1 - 2 \; s_\theta^2} \tag{61a}$$

$$\lambda_w = \frac{s_\theta^2 \; |c_\theta|}{2(2 \; s_\theta^2 - 1)} \tag{61b}$$

$$\lambda_{pol} = \frac{-64 \; s_\theta^4 \; c_\theta^2}{(1 + v_\theta^2)^2} \tag{61c}$$

with $v_\theta = 4s_\theta^2 - 1$ and the one-loop formulae for Δr_{SUSY} and Δb_{SUSY} given in Section II are to be evaluated using the tree-level expression

$$s_\theta = \left[\frac{1}{2} - \frac{1}{2}\left(1 - \frac{4A^2}{M_Z^2}\right)^{\frac{1}{2}}\right]^{\frac{1}{2}} \tag{62a}$$

$$A^2 = \frac{A_o^2}{1 - \Delta r_{GSW}} \simeq (38.66 \; GeV)^2 \tag{62b}$$

with $c_\theta s_\theta \geq 0$. Note that we have included the largest standard model corrections (from lepton and quark contributions to vacuum polarization graphs) in Δr_{GSW} in our 'tree-level' data.

The formulae (17) and (36b) for Δr_{SUSY} and Δb_{SUSY} have been evaluated numerically. We find that the results are essentially independent of $M_{h_3^0}$ the second neutral scalar mass. Thus, the results are displayed as functions of the parameters

$$m_{3/2}, M_{top}, M_1, M_2 \tag{63}$$

only and were evaluated for

$$M_Z = 94 \; GeV \tag{64}$$

The shifts due to SUSY radiative corrections in M_w, s_θ^2 and A_{pol} for the precise value of M_Z within the allowed $UA1/2$ range $90GeV \leq M_Z \leq 98$ GeV [16] must of course be re-evaluated once this is known. They will change by almost 10%

within this range. We display the results for δs_θ^2 and δM_w in Figures 1 to 4 for the sliding singlet model of Cremmer et. al. for various values of $M_1 = M_2, m_{3/2}$ and for $M_{top} = 30$, 130 and 230 GeV. All masses are in GeV throughout this paper. In order to see clearly the effects of the parameters, we try $M_1 = M_2$ initially in Figures 1 to 9 for simplicity. We will explore the region of parameter space $M_1 \neq M_2$ in Figures 10 and 11. In order to avoid fine tuning between the F-term and $m_{3/2}$ SUSY breaking we explore $m_{3/2} \leq 250$ GeV. Further, we explore $M_{top} < 230$ GeV so that the top Yukawa coupling remains perturbative. Some comments on the Figures are in order here.

In order that radiative corrections be large, a remnant global SU_2 symmetry must be broken [4, 10, 8, 9]. An example would be a large top-bottom quark or squark splitting. This means that δs_θ^2, and δM_w and δA_{pol} will be completely insensitive to $M_{h_3^0}$ and the effect of this parameter may be neglected. Radiative corrections will also be small for very large $m_{3/2}$ or M_1 and M_2 because the global SU_2 is restored in that limit. Large M_{top} will break the global SU_2 flavor symmetry $(\tilde{t} \leftrightarrow \tilde{b})$ for small $m_{3/2}$. For small $m_{3/2}$, M_1 and M_t there is a complicated non-linear interplay of effects from winos, sleptons and squarks in one-loop vertices, boxes and self-energies. We note with interest, that for small $M_1 = M_2$ and $m_{3/2}$ the shift in M_w may be large in magnitude and either positive (larger than in GSW, the wrong way for UA1) or negative (smaller than GSW, the right way for UA1).

For large $M_t \gg M_Z$ and small $m_{3/2} \ll M_Z$ (fixed $M_{\tilde{b}} \ll M_Z$) the corrections are huge. This is because

$$\Delta r_{SUSY} \sim - \frac{N_c \alpha}{16\pi\, s_\theta^4} \frac{M_t^2}{M_Z^2} \tag{65a}$$

in this region. Note that this is the same direction (and magnitude) as the quadratic blowup

$$\Delta r_{GSW} \sim - \frac{N_c \alpha}{16\pi\, s_\theta^4} \frac{M_t^2}{M_Z^2} \tag{65b}$$

pointed out by Sirlin [7] so that low-energy experiments should be even more sensitive to very large M_{top} in the SUSY $SU_3 \times SU_2 \times U_1$ model than in the standard model.

The reader may worry that radiative corrections, (or variations in the $N = 1$ SUGRA model) will cause deviation from $v = v'$ and $c_\beta = \frac{1}{\sqrt{2}}$. We therefore include Figure 5 where the shifts for $M_2 = M_1 = -m_{3/2}$ with $v \neq v'$ and $c_\beta = .99$ and $c_\beta = .999$ are displayed to demonstrate that even large changes in v/v' do not affect our results.

We now indicate how these effects might be seen experimentally. The Z^0 mass will be measured to great accuracy at LEP and at the SLC [12] and so this is a very good input parameter. We have given precise predictions for M_w and s_θ^2 through one-loop in this paper by calculating the known muon decay lifetime. Unfortunately, the W^+ mass will not be measured with great precision soon and thus this prediction is really for future reference. We have, however, succeeded in eliminating M_w (and s_θ^2) as a free parameter of the theory. Note that we will then have *predictions* for the tree-level results of all electro-weak processes involving the experimentally known particles once M_Z is known.

A true test of the theory will be given by the polarization and forward - backward assymmetries A_{pol} and A_{FB} in $e^+e^- \rightarrow \mu^+\mu^-$ on Z^0 resonance where the statistics are very good [13]. We have shown elsewhere that these experiments are very sensitive to small changes in the input parameters like M_Z (and s_θ^2) and to one-loop corrections (especially for large M_t and $M_{\mathrm{Higgs'}}$) in the standard model [4, 9]. This is because A_{pol} and the forward-backward asymmetry A_{FB} are proportional to v_θ and v_θ^2 respectively at tree level where $v_\theta = 4s_\theta^2 - 1$ and s_θ^2 is close to $\frac{1}{4}$. Thus, these experiments will also be very sensitive to the SUSY -induced correction δs_θ^2 of this paper. Of course we really need complete one-loop corrections for the asymmetries themselves to make a test of the $N = 1$ SUSY models considered here. The complete one-loop shifts in A_{pol} due to the SUSY - partner part of the $N = 1$ SUSY model considered above are displayed for various values of $m_{3/2}$, $M_1 = M_2$, and $M_t = 30$, 130 and 230 GeV in Figures 6, 7, 8 and 9. Of course our previous comments about global SU_2 breaking apply here as well. The experimental proposal at the SLC [13] indicates that it may be possible to measure A_{pol} to $\pm .01$. This standard deviation from A_{pol} has been indicated in the Figures. We show in Figures 6 to 9 that deviations in A_{pol} due to the SUSY part of the $SU_3 \times SU_2 \times U_1$ theory can be much larger than this. This is especially true for large M_{top} where the radiative corrections in the SUSY part go in the *same* direction as in GSW for large M_{top}.

In fact, for $M_{\tilde{t}} \gg M_Z$, $M_{\tilde{b}} \ll M_Z$ we find

$$\delta A_{pol} \sim \frac{\alpha}{\pi} \frac{4N_c s_\theta^2}{(1+v_\theta^2)^2} \frac{M_{\tilde{t}}^2}{M_Z^2} \tag{66a}$$

This is to be compared with the result of reference [4]

$$A_{pol}^{GSW} \sim \frac{\alpha}{\pi} \frac{4N_c s_\theta^2}{(1+v_\theta^2)^2} \frac{M_t^2}{M_Z^2} \tag{66b}$$

Thus, deviations for large M_t from A_{pol}^{GSW} calculated for $M_t = 30 GeV$ should be twice as much for SUSY theories as for the standard model. We note that the coefficient of quadratic blowup with large $M_{\tilde{t}}^2$ in (66a) is smaller by a factor $s_\theta^2/c_\theta^2 \sim .28$ (for $s_\theta^2 = .22$) than that which would be inferred from (36a) and (65a) by dropping the corrections to the asymmetry itself (dropping the term Δb_{SUSY}). This illustrates the dangers in relying on the result of partial calculations. Note also that of the (dimensionless) parameters δs_θ^2, $\delta M_w/M_w$, δA_{pol} (and even the ρ parameter) which all blow up quadratically with large M_t, δA_{pol} blows up by far (factor 2.5) the fastest. It is therefore the best place to look for large M_t effects [9].

We explore in Figures 10 and 11 the effect of allowing $M_1 \neq M_2$. Note from Figure 10 that it is M_2 which can contribute large radiative corrections in the wino sector, but for $M_2 = -55$ GeV really only in the exceptional region $m_{3/2} \approx 125$ GeV. This may be understood as follows. For $v = v' = M_w/g_2$, the *charged wino* mass matrix (22a) develops a zero eigenvalue when

$$M_2 = M_w^2/\mu \tag{67}$$

Since we have $\mu = -m_{3/2}$ (56) from the sliding singlet model considered here, this means that the photon vacuum polarization $\pi_{AA}^{SUSY}(0)$ in eq. (29b) will develop an infra-red mass singularity as $m_{+1} \to 0$ which feeds into the one-loop corrections δA_{pol} and δM_w as

$$\delta A_{pol} \underset{m_{+1}\to 0}{\longrightarrow} -\frac{\alpha}{3\pi}\frac{64 s_\theta^4 c_\theta^2}{(1+v_\theta^2)^2}\ell n\frac{M_Z^2}{m_{+1}^2} \tag{68a}$$

$$\delta M_w \underset{m_{+1}\to 0}{\longrightarrow} -\frac{\alpha}{3\pi}\frac{M_Z s_\theta^2 |c_\theta|}{2(2s_\theta^2-1)}\ell n\frac{M_Z^2}{m_{+1}^2} \tag{68b}$$

with the light charged wino mass $m_{+1} \to 0$ as $M_2 \to -M_w^2/m_{3/2}$. The complete one-loop radiative corrections to δA_{pol} in this region for various M_2 have been plotted in Figure 11. Of course, lower bounds to charged wino masses can be set phenomenologically from other data, but it is interesting that a lower bound can be gotten in principle from δA_{pol} as well.

The figures cover a huge range of spectra for SUSY partners. The combinations of parameters M_t, $m_{3/2}$, M_1, M_2 and c_β displayed in the figures were chosen in such a way that the radiative corrections for *other* values of these parameters can be easily estimated from combinations of figures. In particular, if $M_t = 40 \pm 10$ GeV as may be indicated by CERN data, the small M_t regime may be explored by use of the figures when we note that *the radiative corrections change little as M_t changes form 30 to 50 GeV*. However, if the top quark has been discovered at CERN, it should be remembered that our results also apply to a 4^{th} sequential generation quark doublet or (with a factor $\frac{1}{3}$) lepton doublet in which the mass matrix breaks global SU_2 very badly; e.g. $M_{b'} \gg M_Z \gg M_{t'}$.

It is interesting to see what spectrum might be indicated if large radiative corrections δA_{pol} were observed. We therefore include here in Table I the spectra for models 1 to 5 for the points indicated in Figures 6 to 11. Note that it is possible to have a SUSY spectrum well above anything observable in the next generation of accelerators and still have observable δA_{pol}. This means that it is possible to set limits (even upper limits) on various parameters appearing in the

$N = 1$ SUSY model we have been studying by a precise measurement of A_{pol}. It therefore can serve as a powerful constraint on model building.

Any substantial improvement over $\sigma(A_{pol}) = \pm.01$ by, say, a factor two, would obviously be highly desirable. $\sigma(A_{pol})$ is liable to be limited by systematic effects, notably the calibration of the electron polarization, rather than statistics since SLC expects roughly 30,000 muon pair events per year [13].

Another possible test of these effects could be in the forward-backward or charge asymmetry [4, 12, 13] in $e^+e^- \to \mu^+\mu^-$. This is defined in terms of θ, the angle between incoming electron and outgoing muon as

$$A_{FB}(q^2, x) = \frac{\int_0^{2\pi} d\phi [\int_0^x - \int_{-x}^0] dcos\theta (\sigma_L + \sigma_R)}{\int_0^{2\pi} d\phi [\int_0^x + \int_{-x}^0] dcos\theta (\sigma_L + \sigma_R)} \tag{69a}$$

The one-loop GSW prediction on Z^o resonance was calculated in ref. [4] and is displayed in Table V there. Deviation from these results for the $N = 1$ SUSY model above are given by the formula

$$\begin{aligned} \delta A_{FB} &= \hat{A}_{FB}(q^2 = -M_Z^2, 1)\Big|_{\substack{N=1 \\ SUSY}} - A_{FB}(q^2 = M_Z^2, 1)\Big|_{GSW}^{Table\ V, ref.4} \\ &= \frac{3}{2} A_{pol}^{GSW} \delta A_{pol} \end{aligned} \tag{69b}$$

Since A_{pol} is itself suppressed by a factor of $v_\theta = 4s_\theta^2 - 1$, these deviations δA_{FB} are liable to be smaller in absolute magnitude than in the polarization asymmetry.

It is also possible that an experiment in neutrino-electron scattering at CERN by the CHARM II collaboration might see these effects [19]. They claim to be able to give an effective measurement of s_θ^2 to $\pm.005$ via the neutrino anti-neutrino on electron asymmetry $R^e_{\nu\bar{\nu}}$. Since it involves comparison of purely leptonic processes, this asymmetry will be free of strong interaction uncertainties. One difficulty, however, is that the ratio of neutrino to anti-neutrino fluxes must be known to great accuracy. Nevertheless, we have shown that changes in s_θ^2 due to the pure SUSY part of the $N = 1$ SUSY standard model could also be detected within the standard deviation quoted in $R^e_{v\bar{v}}$ and this has been indicated in Figures 1 to 5. Of course we need a *complete* calculation of all one-loop corrections in $R^e_{v\bar{v}}$ itself in order to make direct comparison with experiment.

A more detailed comment on why we have considered only purely leptonic processes or direct measurements of M_w and M_Z as tests of electro-weak theories at the one-loop level is in order here. We concentrate on the determination of $\sin^2\theta_w$ by comparison of charged to neutral neutrino-hadron scattering at low q^2 by way of illustration [2].

It has been shown that ν scattering on non-isoscalar hadronic targets suffers from irreducible theoretical errors $\sim$ 10% from higher-twist effects within the framework of QCD. C. H. Llewelyn-Smith has shown that this can be remedied to a great extent by turning to isoscalar targets but there are then other sources of theoretical uncertainty to contend with. In the Kobayashi-Maskawa mixing matrix (which enters into tree-level charged scattering) uncertainties in $|U_{cs}|$ can give an uncertainty in s^2_θ of $\Delta s^2_\theta(1) = \pm.008$. Further, uncertainties in $|U_{dc}|$ and $|U_{du}|$ can give an additional $\Delta s^2_\theta(2) = \pm.004$. These might be improved with a good measurement of the b lifetime. There are also uncertainities due to strong interaction dynamics; notably a chiraly symmetry breaking parameter $\tilde{\epsilon}$ which can give $\Delta s^2_\theta(3) \leq \pm.002$ and isospin-breaking one loop electro-weak corrections which combined with strong interactions might give $\Delta s^2_\theta(4) \lesssim \pm.002$. Further, S. Gupta and H. Quinn [17] have given arguments why perturbative QCD itself may break down for massive quarks at the few percent level due to non-perturbative effects $\sim m^2_q/\Lambda^2_{QCD}$ which do not fall off as inverse powers of q^2. These effects might be minimized in *total* cross sections. These sources of theoretical uncertainty in s^2_θ are small but may add up. The present quoted experimental error in s^2_θ is from neutrino scattering is $\pm.015$ which is quite large for our purposes. However, with both theoretical and experimental work, it may (or may not) be possible to deal with all of these errors and extract accurate information about electro-weak physics from semi-leptonic experiments. These arguments might eliminate the ρ parameter from consideration though unless it is re-defined in terms of purely leptonic processes such as $\nu_\mu + e \to \nu_\mu + e$,

$\bar{\nu}_\mu + \mu \rightarrow \bar{\nu}_\mu + \mu$ or $\bar{\nu}_\mu + e \rightarrow \bar{\nu}_\mu + e$. Radiative corrections to the ρ parameter in $N = 1$ *SUSY* have appeared in the literature [3].

On the other hand, the tree-level predictions for τ_μ, M_w, s_θ^2, and A_{pol} are completely free of strong interactions and Kobayashi-Mashawa angles and so they will, in principle, be theoretically clean. There *is* a one-loop strong interaction uncertainty even for purely leptonic processes associated with the hadronic contribution to the photon vacuum polarization $\pi'_{AA}(0)$ see eq. (18b) which appears in τ_μ and A_{pol} via electric charge renormalization. This has been discussed elsewhere. The upshot is an uncertainty (in Δr_{GSW}) which has been estimated a number of ways [18].

$$\delta(\Delta r_{GSW})|_{hadrons} < \pm .002 \tag{70a}$$

This will give strong interaction uncertainties.

$$\delta(\delta M_w)|_{hadrons} < \pm 36 MeV \tag{70b}$$

$$\delta(s_\theta^2)|_{hadrons} < \pm .00066 \tag{70c}$$

$$\delta(\delta A_{pol})|_{hadrons} < \pm .0052 \tag{70d}$$

These theoretical uncertainties must be kept in mind when making comparison between our results and experiments. It is possible that these uncertainties can be improved by a factor of two by using more recent data in $e^+e^- \rightarrow$ hadrons near the ρ threshold.

In conclusion, we have calculated the one-loop corrections to the muon- decay life-time in the most general version of the minimal $N = 1$ global SUSY $SU_3 \times SU_2 \times U_1$ model thus giving a prediction for M_w (and s_θ^2). We then calculated the shifts in the longitudinal polarization asymmetry A_{pol} in $e^+e^- \rightarrow \mu^+\mu-$ on Z^o resonance. Motivation is given to the parameters of the theory by coupling it to $N = 1$ local SUSY (supergravity) using the super-Higgs' mechanism to break SUSY and R-invariance via the gravitino mass $m_{3/2}$ and gaugino masses M_G and a sliding singlet to break the gauge symmetry. We have shown that these radiative corrections can be large compared to the quoted experimental error for

$A_{pol}(q^2 = -M_Z^2)$ and could be detected at the SLC and/or LEP for a large class of models of $N = 1$ SUGRA .

Note: After this was finished, we received a preprint by K. H. G. Schwarzer [19] which examines SUSY corrections to M_w as well as to neutrino-hadron scattering.

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Appendix

We need some form factors from Veltman and Passarino and Consoli [20]. First the two-point functions with arguments e.g., $B_0(q^2, m_1, m_2)$

$$B_0\,;\, q_\mu B_1\,;\, \delta_{\mu\nu}B_{22} + q_\mu q_\nu B_2 = \int \frac{d^dk}{i\pi^2} \frac{1\,;\, k_\mu\,;\, k_\mu k_\nu}{[k^2 + m_1^2 - i\epsilon]\,[(k+q)^2 + m_2^2 - i\epsilon]} \quad (a.1)$$

and the combinations

$$\begin{aligned} B_3 &= B_2 + B_1 \qquad & B_{14} &= q^2 B_3 + \tfrac{1}{2} m_2^2 B_1 - \tfrac{1}{2} m_1^2 B_4 \\ B_4 &= B_0 + B_1 & & \\ B_{13} &= B_3 + \tfrac{1}{4} B_0 \qquad & B_7 &= -q^2 B_{13} + \tfrac{1}{4}(m_1^2 - m_2^2)(2B_1 + B_0) \end{aligned} \quad (a.2)$$

Explicit expressions for the B_0, B_1, B_2 are

$$\begin{aligned} B_n(q^2, m_1, m_2) &= \frac{(-1)^n}{n+1} \\ &\times\Big[\Delta - \ell n(-q^2 - i\epsilon) - \sum_{j=1}^{2}\Big(\ell n(1 - xj) + F(n+1, xj)\Big)\Big] \end{aligned} \quad (a.3)$$

for $n = 0, 1, 2$ with $\Delta = \pi^{\frac{d}{2}-2}\,\Gamma(2 - \frac{d}{2})$. The x_j are the roots of the equation $-q^2x^2 + (q^2 + m^2 - m_1^2)x + m_1^2 - i\epsilon = 0$ and the function $F(n, x)$ is given from

$$(n+1)\int_0^1 dx\, x^n\, \ell n(x - x_1) = \ell n(1 - x_j) + F(n+1,\, x_j) \quad (a.4a)$$

$$\begin{aligned} F(n, x) &= -x^n \ell n\Big(1 - \frac{1}{x}\Big) - x^{n-1} - \frac{1}{2}x^{n-2} - - - \frac{1}{n} \\ &= \frac{1}{n+1}x^{-1} + \frac{1}{n+2}x^{-2} + + + + \end{aligned} \quad (a.4b)$$

We will sometimes use a more streamlined notation in which the argument q^2 is suppressed in the B's. For example

$$B_7\,(q^2,\; M_{\tilde{e}_n},\; M_{\tilde{\nu}_n}) \equiv B_7\,(\tilde{e}_n\,,\; \tilde{\nu}) \quad (a.5)$$

We will also need the 3-point functions with arguments e.g. $C_0(p', q; m_1, m_2, m_3)$

$$C_o;\ \delta_{\mu\nu}\, C_{24} + \text{terms}\ p'_\mu q_\nu \ldots$$

$$= \int \frac{d^d k}{i\pi^2} \frac{1;\qquad k_\mu k_\nu}{[k^2 + m_1^2 - i\epsilon]\,[(k+p')^2 + m_2^2 - i\epsilon]\,[(k+p'+q)^2 + m_3^2 - i\epsilon]} \tag{a.6}$$

with $p = p' + q$. These need be evaluated only for $p'^2 = p^2 = 0$ with $q^2 \neq 0$ for our purposes. Then we find that we can write all of the necessary 3-point form factors in terms of C_0 and B_0.

$$\begin{aligned} C_{24} = \frac{1}{4}\Big\{ & 1 + \alpha\, C_0 + \beta\, B_0(q^2, m_2, m_3) \\ & + \gamma B_0(0, m_1, m_3) + \delta B_0(0, m_1, m_2)\Big\} \end{aligned} \tag{a.7}$$

with coefficients

$$\begin{aligned} &\delta = (m_1^2 - m_2^2)/q^2 && \sigma = (m_2^2 - m_3^2)/q^2 \\ &\gamma = \delta + \sigma && \beta = -2\delta - \sigma + 1 \\ &\alpha = -m_1^2(\beta + 1) - m_2^2\gamma - \delta m_3^2 \end{aligned} \tag{a.8}$$

To evaluate C_0 we need the Spence functions

$$S_p(x) = -\int_0^x \frac{dt}{t}\ \ell n\ (1 - t) \tag{a.9}$$

as well as the coefficients

$$\begin{aligned} &\hat{q}^2 = q^2/m_1^2 && c = a - b + \hat{q}^2 \\ &a = (m_3^2 - m_1^2)/m_1^2 && d = b + 1 \\ &b = (m_2^2 - m_1^2)/m_1^2 && s = b/\,\hat{q}^2 \\ &h = a\ s - 1 && e = 2\ s\,\hat{q}^2 + c \\ &f = -\hat{q}^2 s^2 - cs + d && t = \tfrac{1}{s} + 1 \end{aligned} \tag{a.10}$$

Now define the roots of the equation (with $Re\ y_1 \geq\ Re\ y_2$)

$$-\hat{q}^2\ y^2\ +\ e\,y\ +\ f = 0 \tag{a.11}$$

and the combinations (with $\epsilon > 0$ a small real infinitesimal)

$$\begin{aligned} u_1 &= (1+s-i\epsilon)/y_1 & u_2 &= (s-i\epsilon)/y_1 \\ u_3 &= (1+s+i\epsilon)/y_2 & u_4 &= (s+i\epsilon)/y_2 \\ u_7 &= (a\,s-i\epsilon)/h & u_8 &= (a(1+s)-i\epsilon)/h \end{aligned} \tag{a.12}$$

Then

$$\begin{aligned} C_0 = \; & \frac{1}{q^2}\Big[\Big\{\ell n(-\hat{q}^2-i\epsilon)+\ell n(-y_1-i\epsilon) \\ & + \ell n(-y_2+i\epsilon) - \ell n(-h-i\epsilon)\Big\}\ell n\ t \\ & + \textstyle\sum_{k=1}^{8}(-1)^k\ S_p(u_k)\Big] \end{aligned} \tag{a.13}$$

It is convenient to define

$$\eta(m_1,m_2) = B_0(0,m_1,m_2) + B_1(0,m_1,m_2) \tag{a.14}$$

and the UV finite form factors

$$C_6 = 2C_{24} - \frac{1}{2}\eta(m_2,m_1) - \frac{1}{2}\eta(m_3,m_1) \tag{a.15a}$$

$$\begin{aligned} C_7 = & -m_1^2 C_0 - 2C_{24} + B_0(q^2,m_2,m_3) \\ & - \frac{1}{2}\eta(m_1,m_2) - \frac{1}{2}\eta(m_1,m_3) \end{aligned} \tag{a.15b}$$

The $q^2 = 0$ limits of C_0 and C_{24} are easily written down with $C_0^0 = C_0(q^2=0)$ and $C_{24}^0 = C_{24}(q^2=0)$

$$\begin{aligned} C_0^0(m_1,m_2,m_3) = & (m_1^2-m_2^2)^{-1} \\ & \times \{B_0(0,m_2,m_3) \ - \ B_0(0,m_1,m_3)\} \\ C_{24}^0(m_1,m_2,m_3) = & \frac{1}{4}(m_2^2-m_1^2)^{-1} \\ & \times \{m_2^2 B_0(0,m_2,m_3) - m_1^2 B_0(0,m_1,m_3)\} \end{aligned} \tag{a.16}$$

We will use the more streamlined notation e.g.

$$C_6(p', q, m_{0j}, M_{\tilde{e}_1}, M_{\tilde{e}_1}) = C_6(m_{0j}, \tilde{e}_1, \tilde{e}_1) \tag{a.17}$$

We will need to evaluate box diagrams only for muon decay ($q^2 = 0$) since A_{pol} is evaluated on Z^0 resonance. Therefore, we need 4-point functions only in the limit where all external momenta are zero

$$\delta_{\mu\nu}\, D^0_{27} = \int \frac{d^d k}{i\pi^2}\, [k^2 + m_1^2 - i\epsilon]^{-1}\, [k^2 + m_2^2 - i\epsilon]^{-1}\, [k^2 + m_3^2 - i\epsilon]^{-1}$$
$$\times\ [k^2 + m_4^2 - i\epsilon]^{-1} k_\mu k_\nu$$

$$D^0_{27}(m_1, m_2, m_3, m_4) = \frac{1}{4}(m_2^2 - m_1^2)^{-1} \times \{m_2^2\, C^0_0(m_2, m_3, m_4) - m_1^2\, C^0_0(m_1, m_3, m_4)\} \tag{a.18}$$

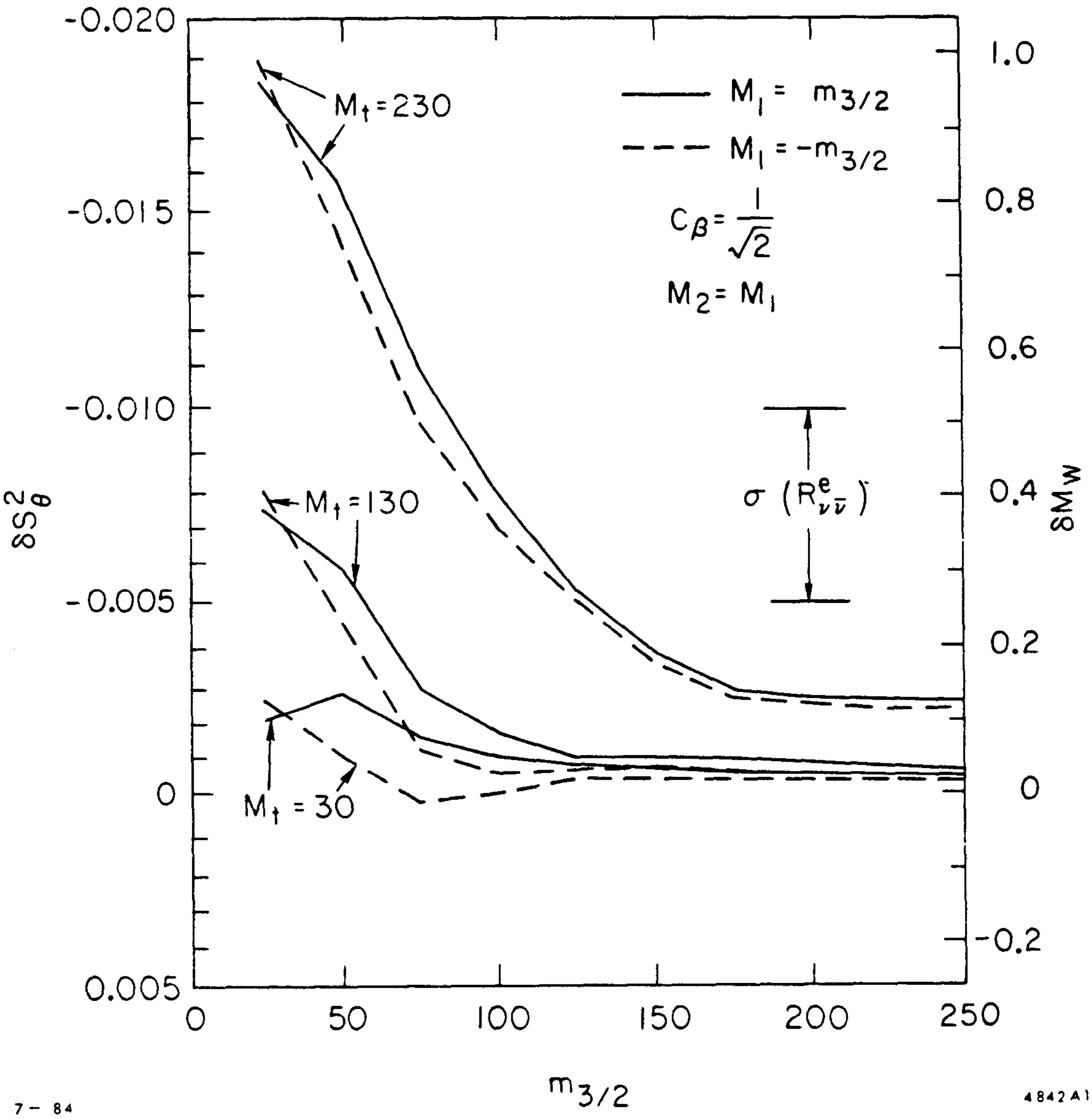
The form factors above are well documented in the literature [8, 10, 20]. We have however included in this Appendix all formulae necessary for evaluating the form factors needed in the calculation of τ_μ and A_{pol} to one-loop in ***any*** gauge theory.

Table I

Spectra of Models labelled (1) to (5) in Figs. 6 to 11

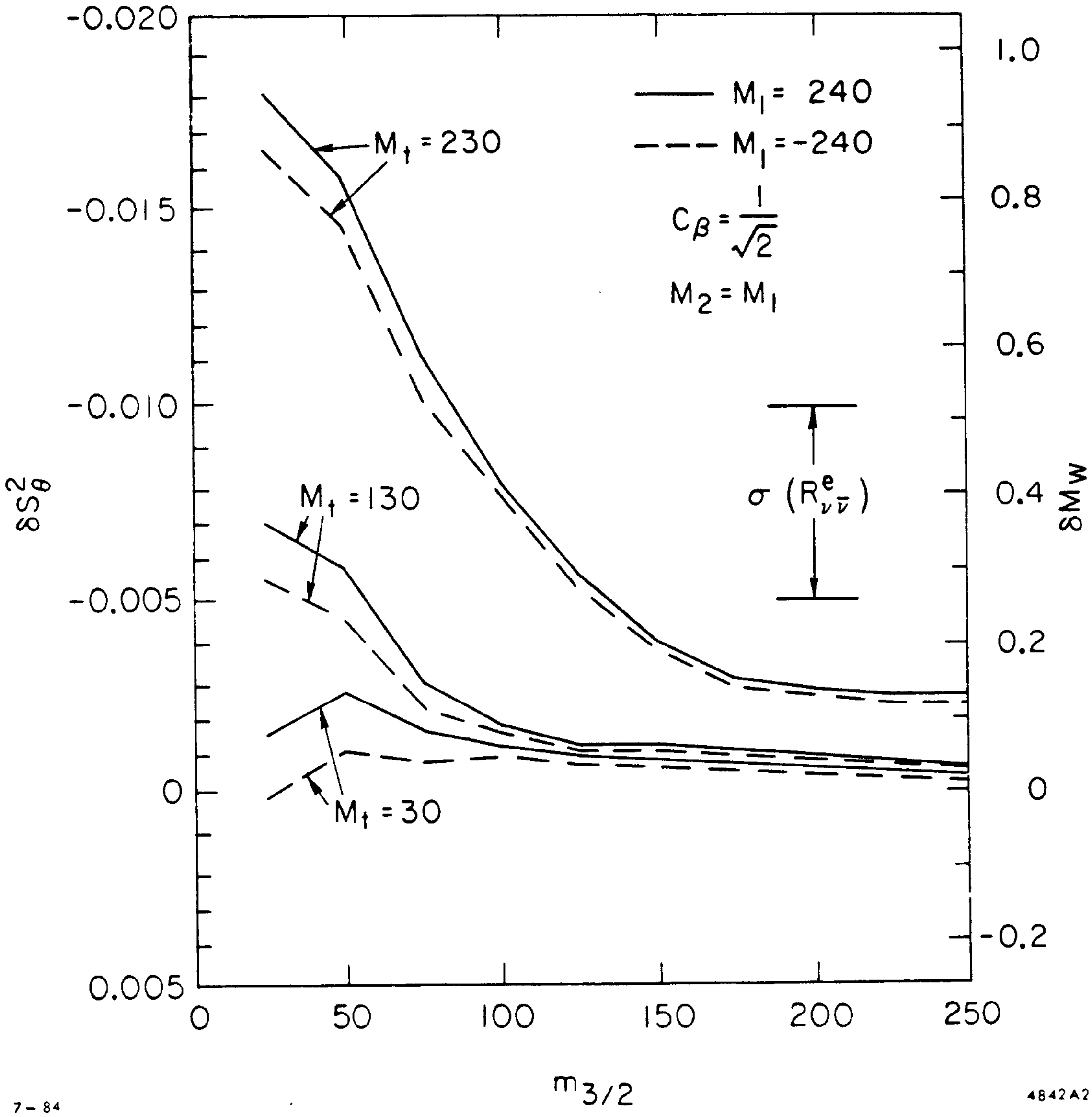
Shifts in s_θ^2, M_w, A_{pol}

	(1)	(2)	(3)	(4)	(5)
$\tilde{t}_1, \tilde{t}_2$	180, 280	80, 180	5, 55	20, 240	205, 255
other squarks sleptons	50	50	25	110	25
charged winos	97, 97	72, 263	66, 101	5, 170	45, 125
neutral winos	50, 50 107, 107	50, 78 240, 268	10, 25 77, 112	15, 55 110, 180	25, 55 55, 135
δs_θ^2	−.016	−.006	−.002	.0008	−.020
δM_w	.836	.307	.112	−.041	1.059
δA_{pol}	.036	.015	.010	−.013	.044



-0.020
-0.015
-0.010
-0.005
0
0.005
δS^2_θ
1.0
0.8
0.6
0.4
0.2
0
-0.2
δM_W
$M_t = 230$
$M_t = 130$
$M_t = 30$
$M_1 = m_{3/2}$
$M_1 = -m_{3/2}$
$C_\beta = \frac{1}{\sqrt{2}}$
$M_2 = M_1$
$\sigma\ (R^e_{\nu\bar{\nu}})$
0
50
100
150
200
250
$m_{3/2}$
7 - 84
4842A1

Fig. 1



-0.020
-0.015
-0.010
-0.005
0
0.005
δS_θ^2
1.0
0.8
0.6
0.4
0.2
0
-0.2
δM_W
0
50
100
150
200
250
$m_{3/2}$
$M_t = 230$
$M_t = 130$
$M_t = 30$
$M_1 = 240$
$M_1 = -240$
$C_\beta = \frac{1}{\sqrt{2}}$
$M_2 = M_1$
$\sigma\ (R^e_{\nu\bar\nu})$
7-84
4842A2

Fig. 2

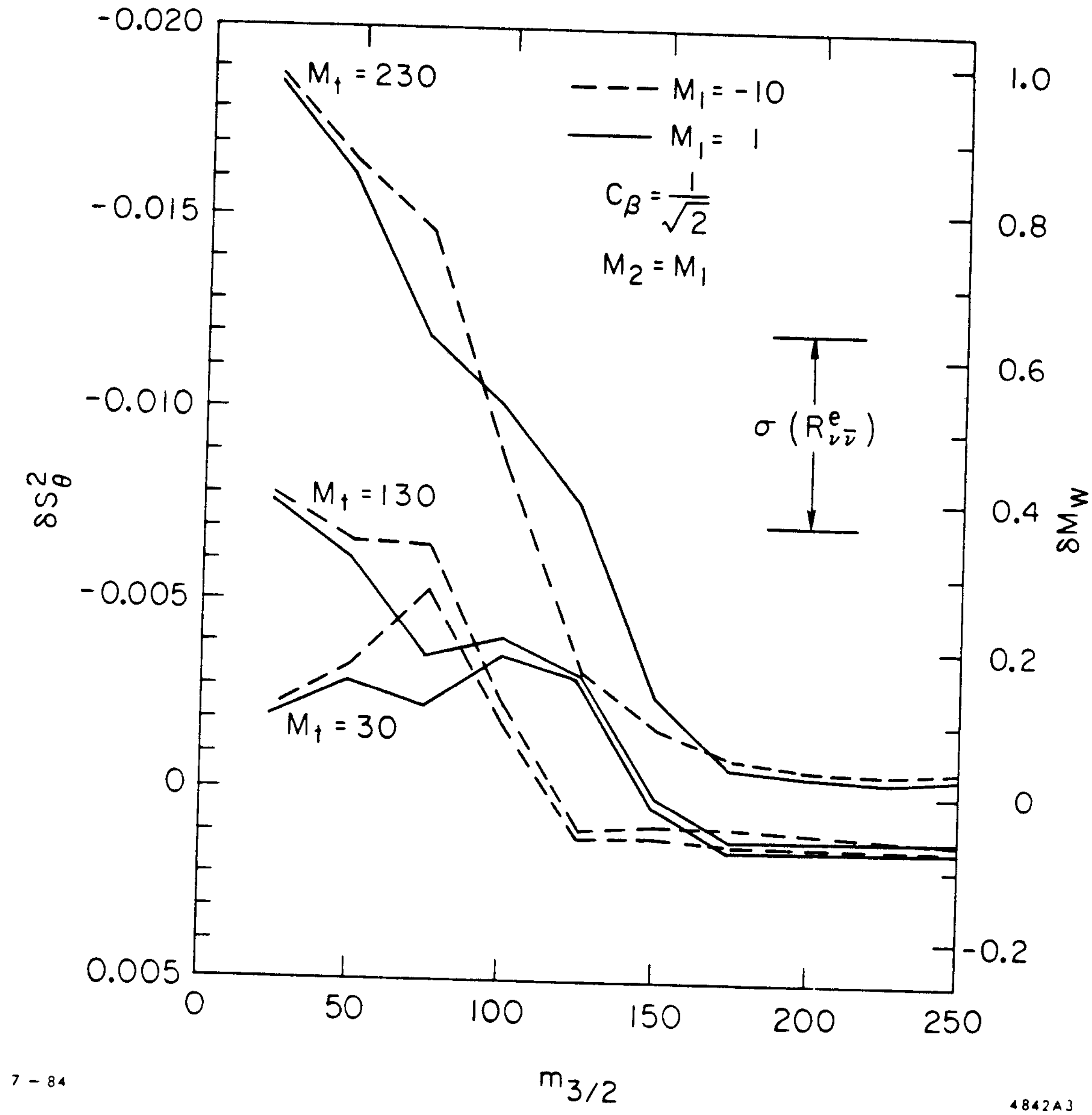
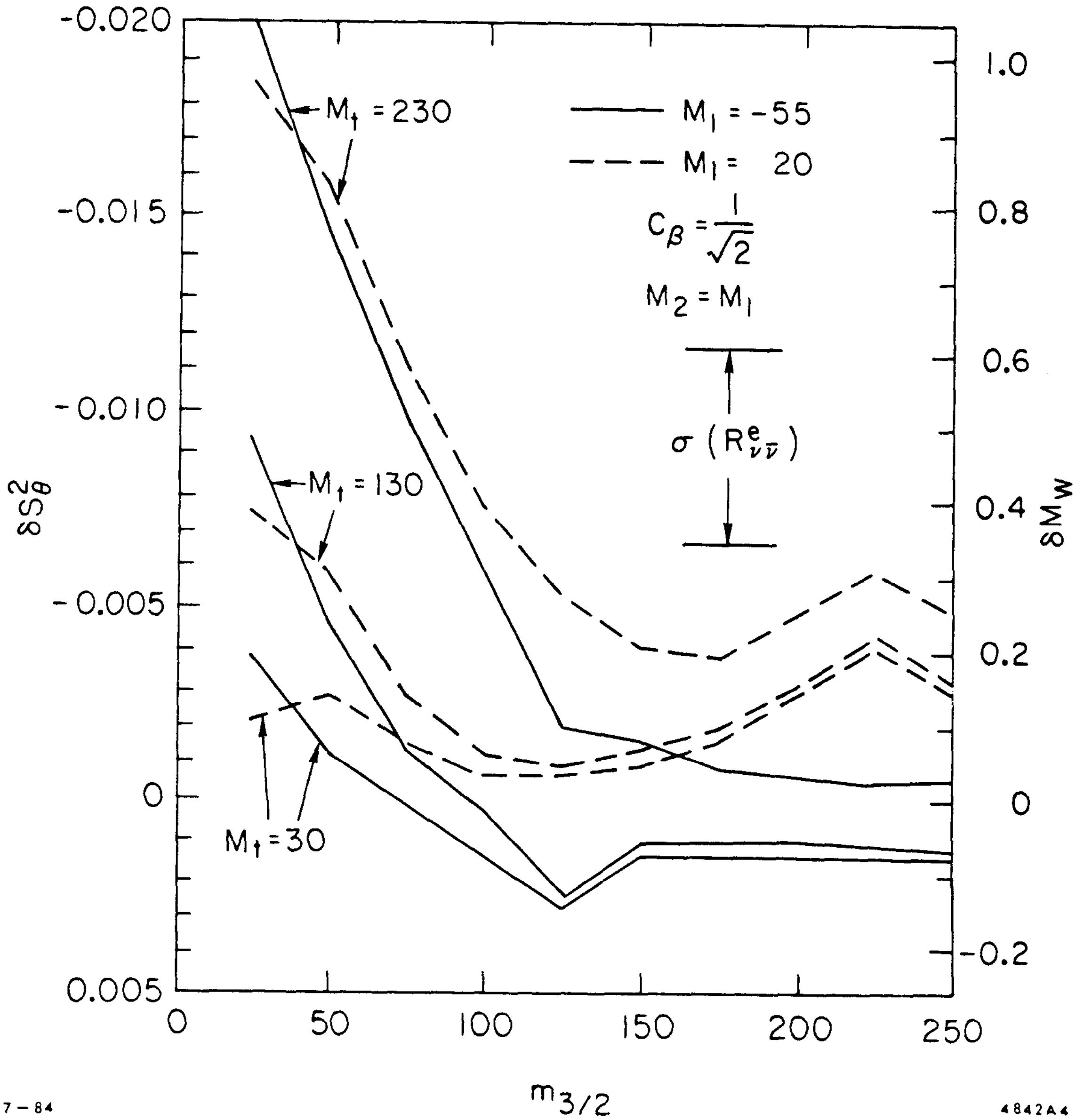
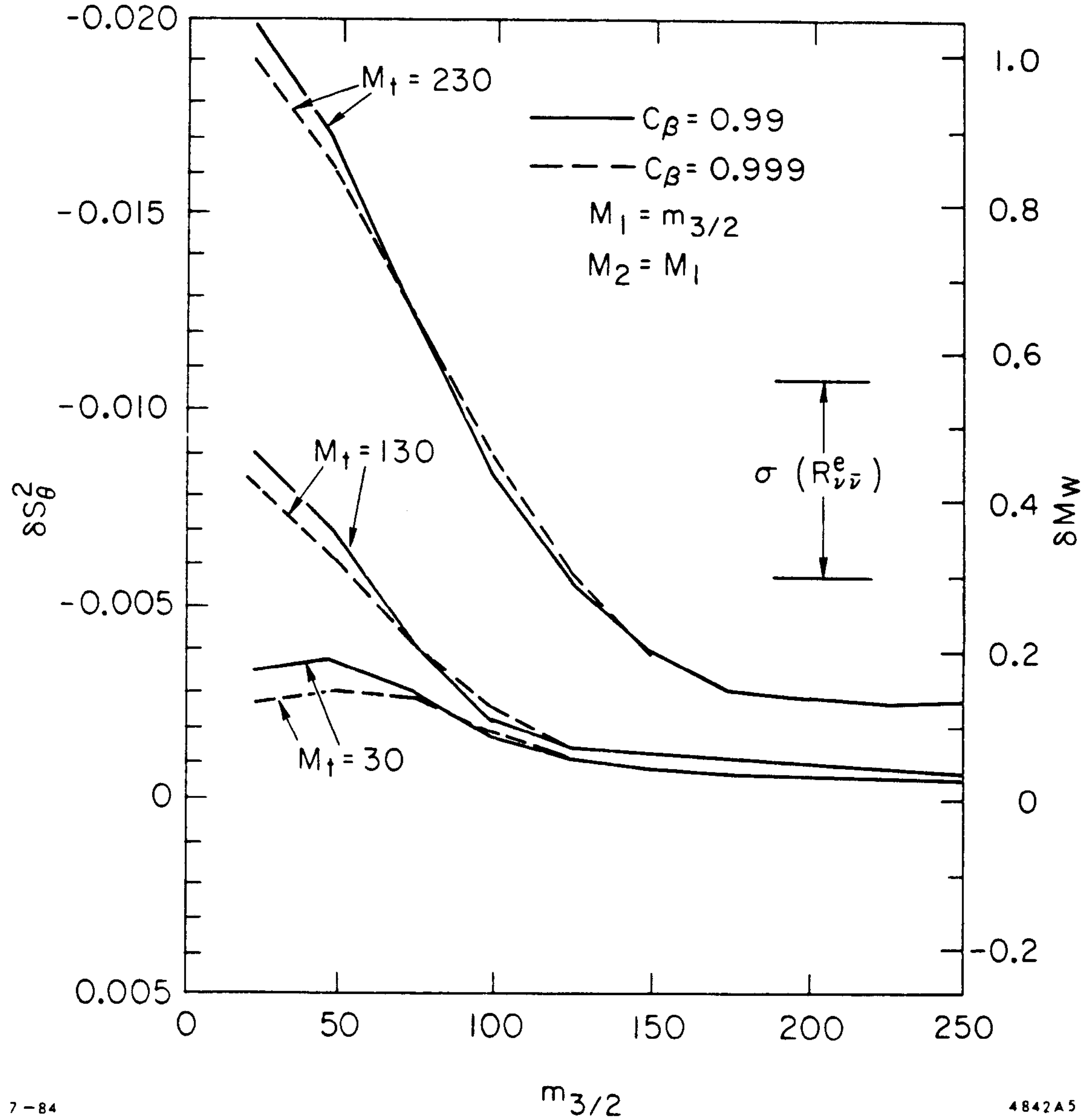

-0.020
-0.015
-0.010
-0.005
0
0.005
δS_θ^2
1.0
0.8
0.6
0.4
0.2
0
-0.2
δM_W
$M_t = 230$
$M_t = 130$
$M_t = 30$
$M_1 = -10$
$M_1 = 1$
$C_\beta = \frac{1}{\sqrt{2}}$
$M_2 = M_1$
$\sigma\,(R^e_{\nu\bar{\nu}})$
0
50
100
150
200
250
$m_{3/2}$
7 - 84
4842A3

Fig. 3



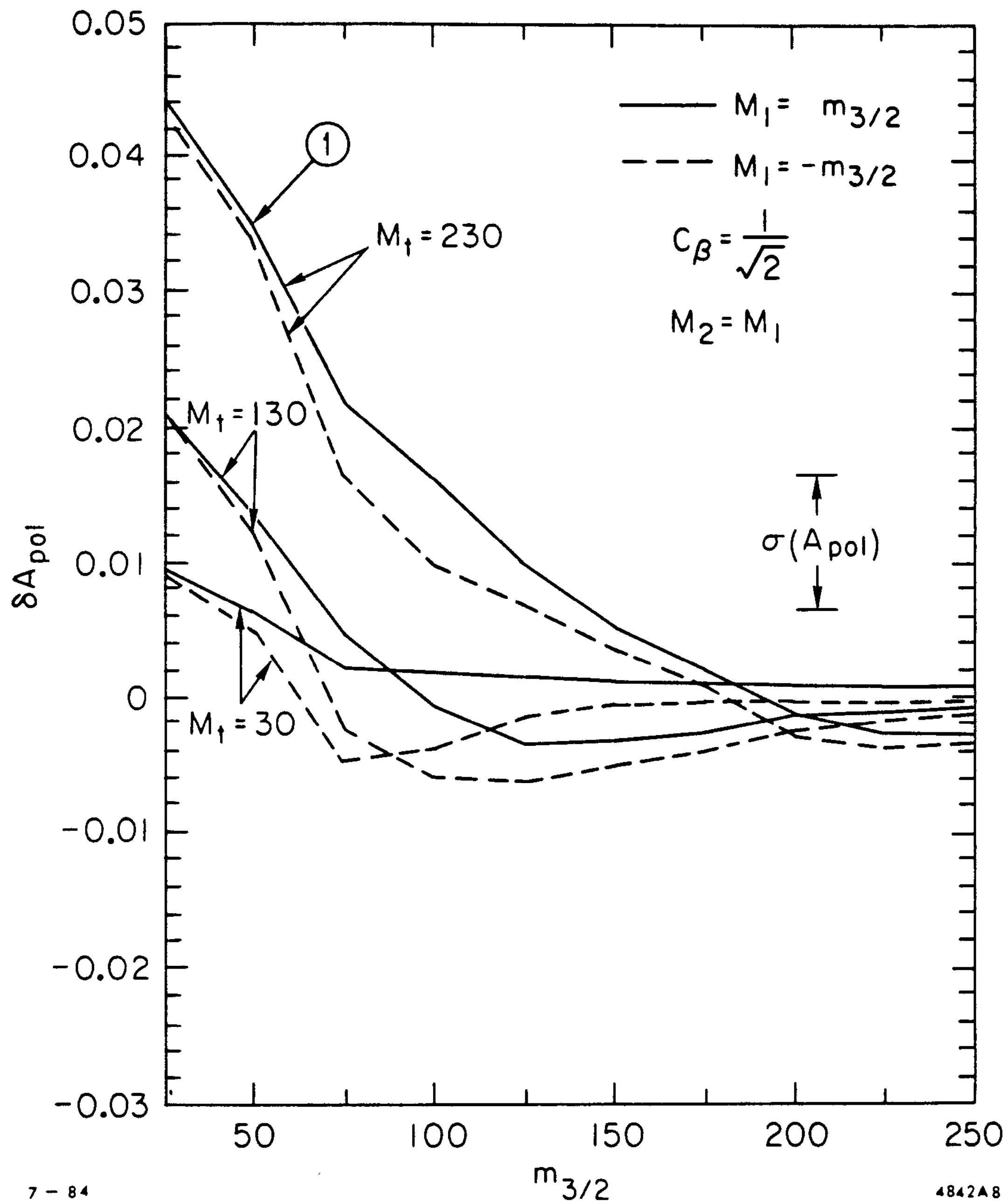
-0.020
-0.015
-0.010
-0.005
0
0.005
δS_θ^2
1.0
0.8
0.6
0.4
0.2
0
-0.2
δM_W
0
50
100
150
200
250
$m_{3/2}$
$M_t = 230$
$M_t = 130$
$M_t = 30$
$M_1 = -55$
$M_1 = 20$
$C_\beta = \frac{1}{\sqrt{2}}$
$M_2 = M_1$
$\sigma\,(R^e_{\nu\bar{\nu}})$
7-84
4842A4

Fig. 4



-0.020
-0.015
-0.010
-0.005
0
0.005
1.0
0.8
0.6
0.4
0.2
0
-0.2
$M_t = 230$
$M_t = 130$
$M_t = 30$
$C_\beta = 0.99$
$C_\beta = 0.999$
$M_1 = m_{3/2}$
$M_2 = M_1$
$\sigma\ (R^e_{\nu\bar{\nu}})$
δS^2_θ
δM_W
0
50
100
150
200
250
$m_{3/2}$
7-84
4842A5

Fig. 5



0.05
0.04
0.03
0.02
0.01
0
-0.01
-0.02
-0.03
δA_{pol}
50
100
150
200
250
$m_{3/2}$
$M_1 = m_{3/2}$
$M_1 = -m_{3/2}$
$C_\beta = \frac{1}{\sqrt{2}}$
$M_2 = M_1$
$\sigma(A_{pol})$
1
$M_t = 230$
$M_t = 130$
$M_t = 30$
7 – 84
4842A8

Fig. 6

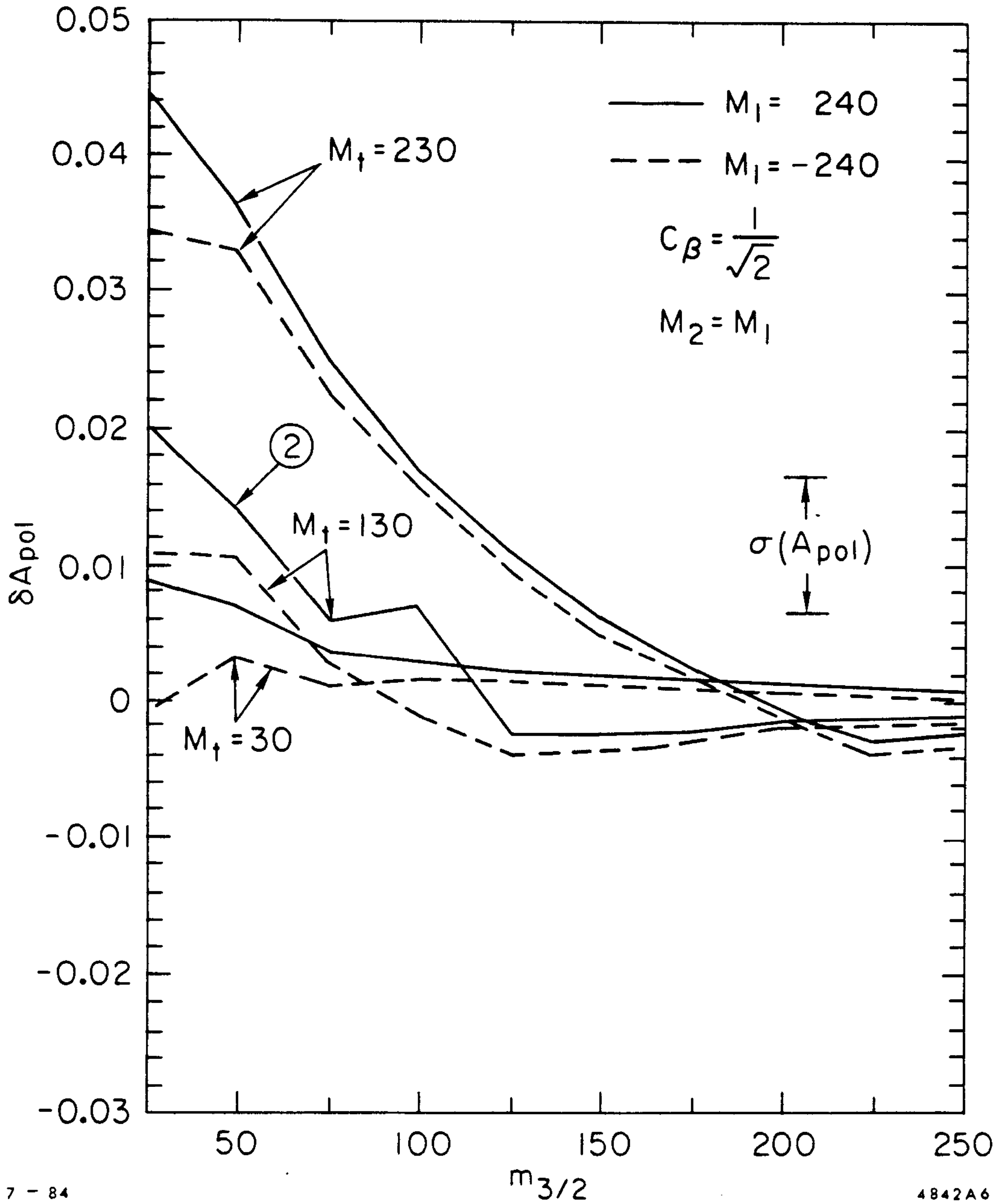
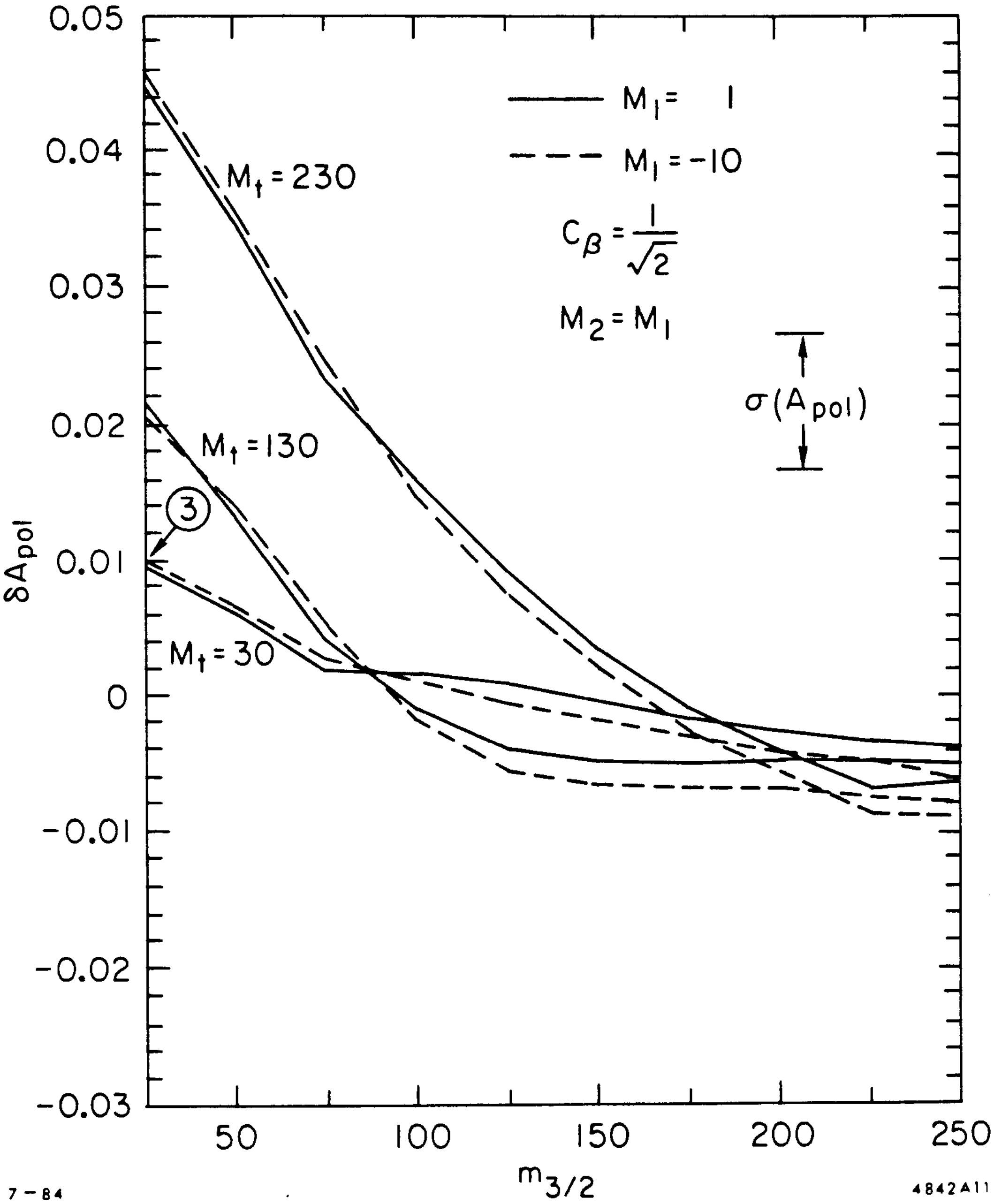

0.05
0.04
0.03
0.02
0.01
0
-0.01
-0.02
-0.03
50
100
150
200
250
$m_{3/2}$
δA_{pol}
$M_1 = 240$
$M_1 = -240$
$C_\beta = \frac{1}{\sqrt{2}}$
$M_2 = M_1$
$M_t = 230$
$M_t = 130$
$M_t = 30$
2
$\sigma(A_{pol})$
7 - 84
4842A6

Fig. 7



0.05
0.04
0.03
0.02
0.01
0
-0.01
-0.02
-0.03
50
100
150
200
250
δA_{pol}
$m_{3/2}$
$M_1 = 1$
$M_1 = -10$
$C_\beta = \frac{1}{\sqrt{2}}$
$M_2 = M_1$
$\sigma(A_{pol})$
$M_t = 230$
$M_t = 130$
$M_t = 30$
3
7-84
4842A11

Fig. 8

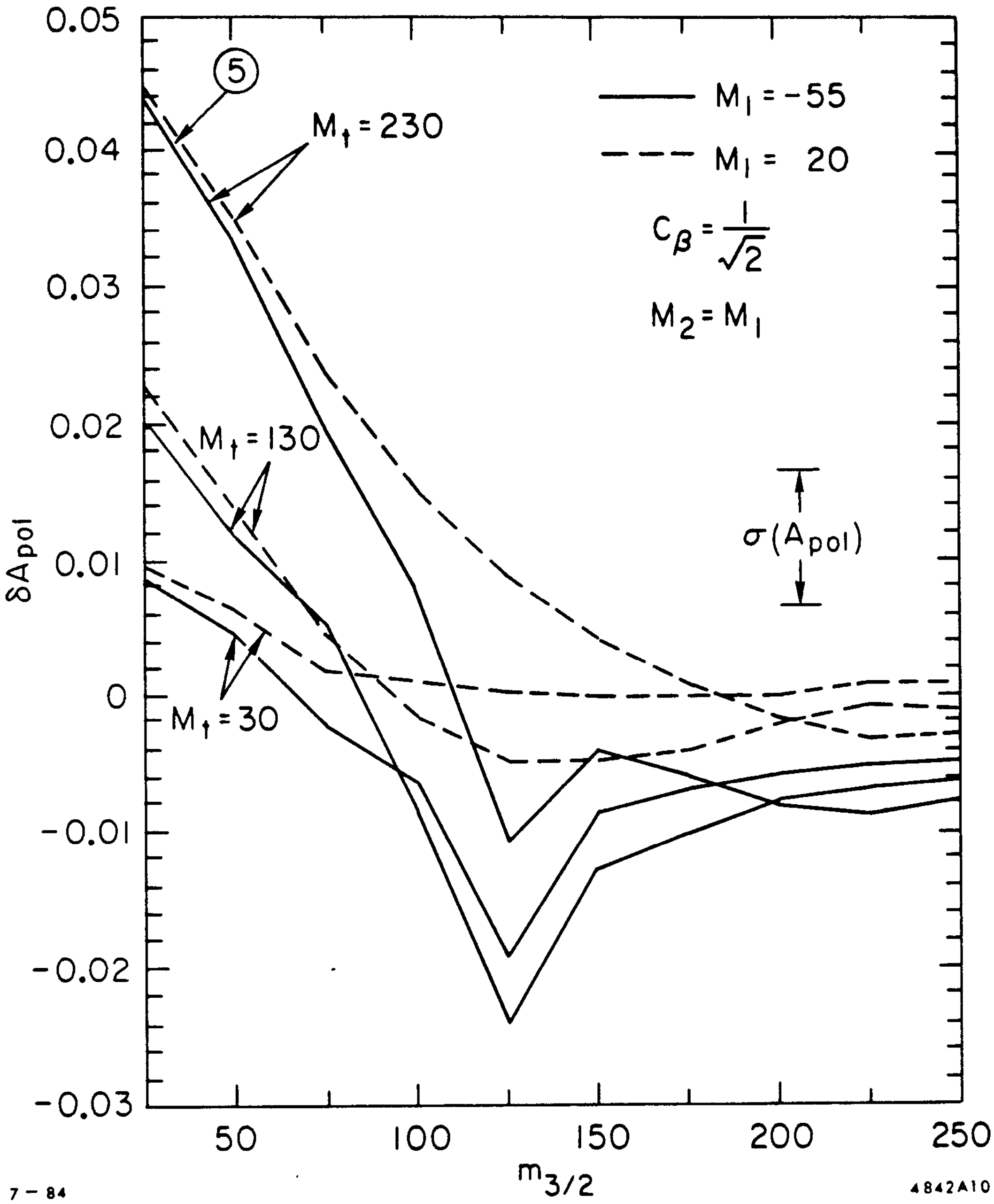
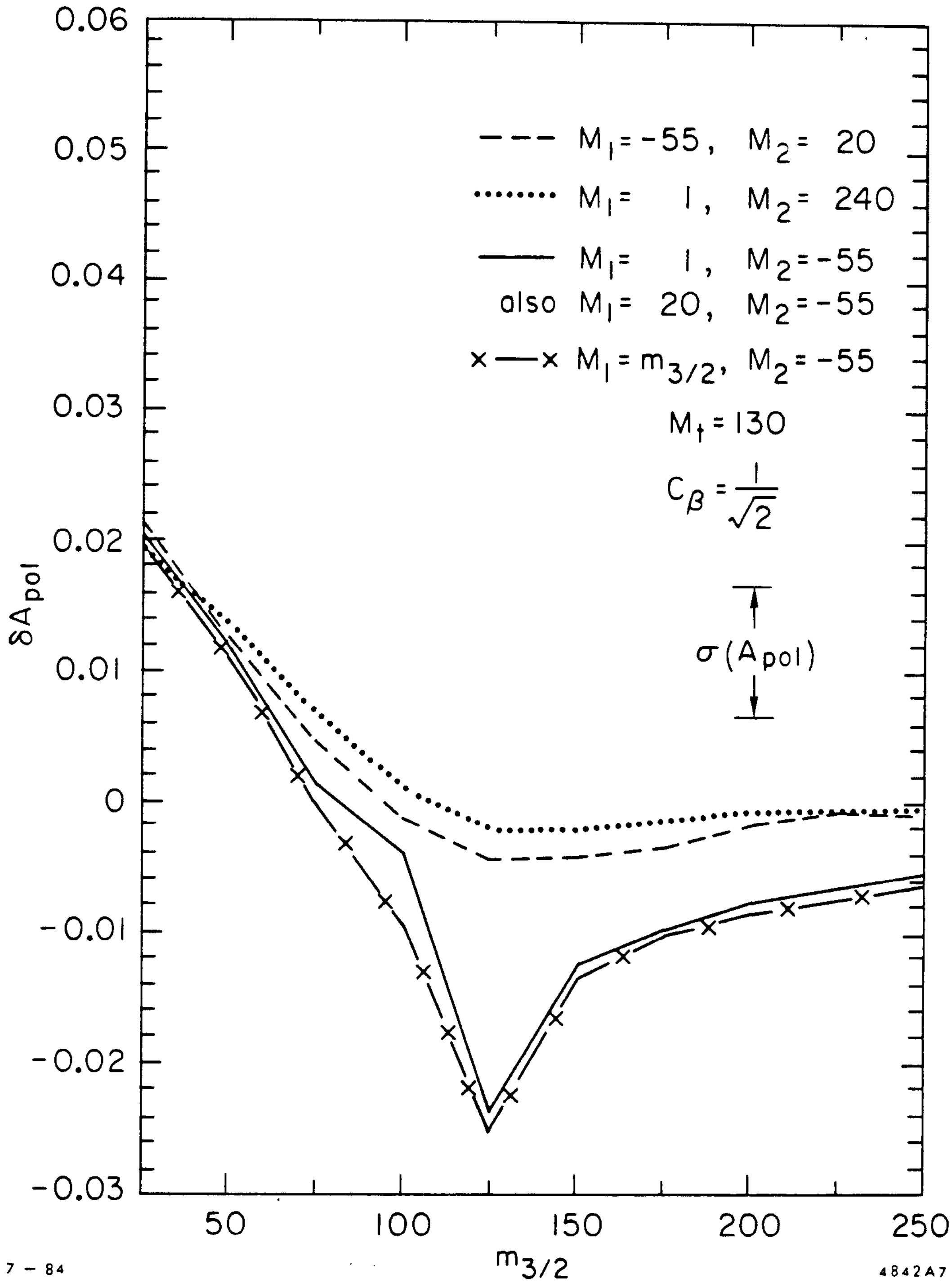
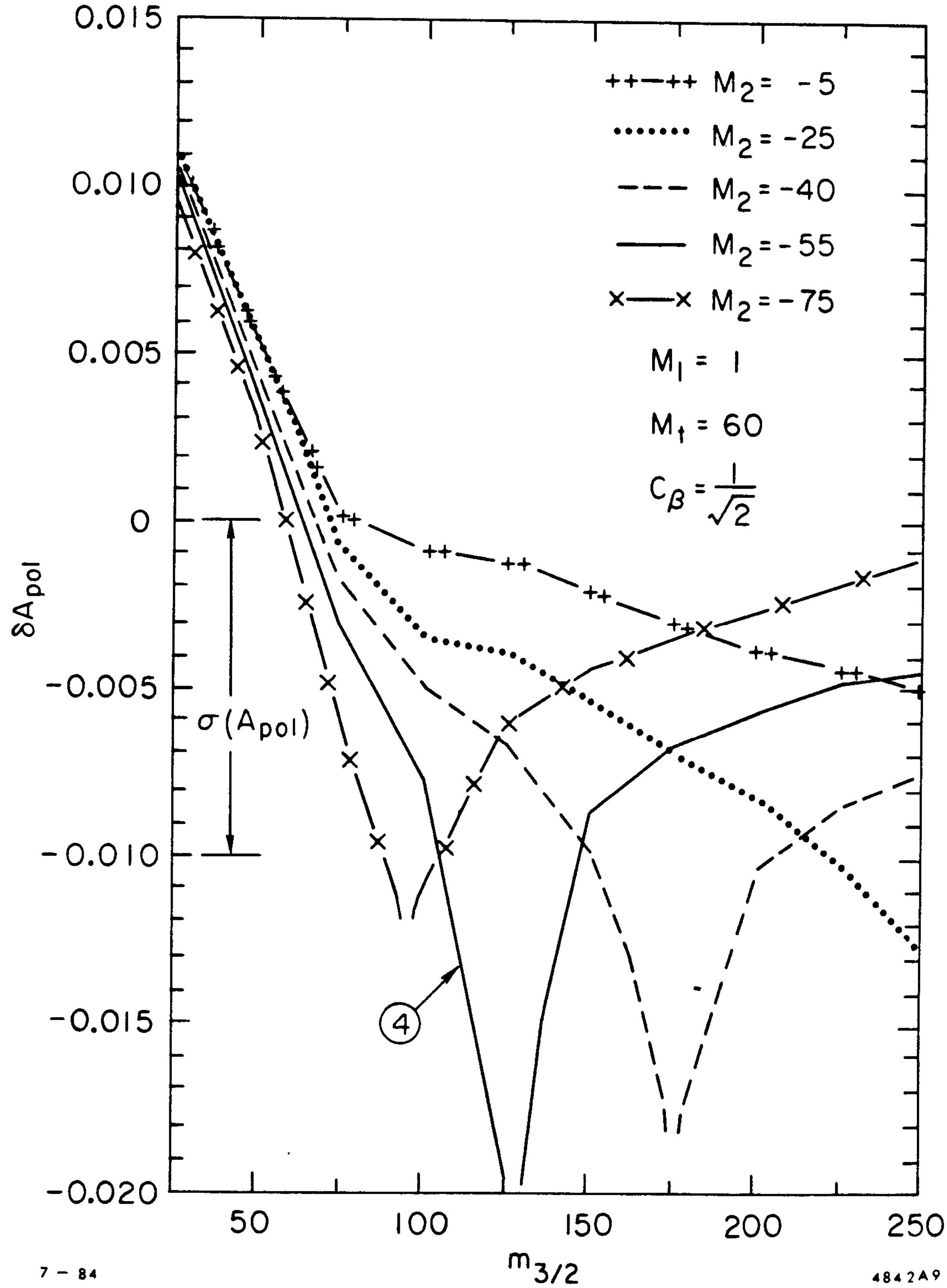

0.05
0.04
0.03
0.02
0.01
0
-0.01
-0.02
-0.03
δApol
5
M1 = -55
M1 = 20
Cβ = 1/√2
M2 = M1
Mt = 230
Mt = 130
Mt = 30
σ(Apol)
50
100
150
200
250
m3/2
7-84
4842A10

Fig. 9



0.06
0.05
0.04
0.03
0.02
0.01
0
-0.01
-0.02
-0.03
50
100
150
200
250
δA_{pol}
$m_{3/2}$
$M_1 = -55, \; M_2 = 20$
$M_1 = 1, \; M_2 = 240$
$M_1 = 1, \; M_2 = -55$
also $M_1 = 20, \; M_2 = -55$
$M_1 = m_{3/2}, \; M_2 = -55$
$M_t = 130$
$C_\beta = \frac{1}{\sqrt{2}}$
$\sigma(A_{pol})$
7 – 84
4842A7

Fig. 10



0.015
0.010
0.005
0
-0.005
-0.010
-0.015
-0.020
δA_pol
σ(A_pol)
4
50
100
150
200
250
m_3/2
++—++ M2 = -5
M2 = -25
M2 = -40
M2 = -55
×—× M2 = -75
M1 = 1
Mt = 60
Cβ = 1/√2
7 - 84
4842A9

Fig. 11