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A NONMINIMAL SU(5) MODEL*

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ABSTRACT

The existence of the solution of the symmetry-breaking problem

$$SU(5) \rightarrow SU(3)\times SU(2)\times U(1)_Y \rightarrow SU(3)\times U(1)_Q$$

is shown in a nonminimal model where the 75-dimensional Higgs field is used instead of the usual 24-dimensional one. The result is given in an explicit and simple form. The relevance of this solution to existing theories and to problems related to proton decay is discussed.

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I. INTRODUCTION

In the minimal SU(5) model,[1] the 24-dimensional adjoint Higgs tensor was used to break the SU(5) symmetry to the $SU(3)\times SU(2)\times U(1)_Y$ symmetry of the standard model. The fundamental representation together with its complex conjugate is used to perform the next stage of symmetry breaking to $SU(3)\times U(1)_Q$ symmetry. However, recent experiments,[2] combined with a theoretical analysis of implications of these experiments on the mass M_X of heavy vector bosons,[3] are incompatible with the calculated value of M_X from SU(5) renormalization-group equations.[4] At this point one may be tempted to abandon the SU(5) model. The other possibility is to realize that, strictly speaking, the above argument eliminates the minimal version of the SU(5) model. For this reason and because of the simplicity of SU(5), it is worthwhile to explore other nonminimal versions of this model. In this paper we study the following symmetry-breaking pattern of the SU(5) model:

$$SU(5) \xrightarrow{75} SU(3)\times SU(2)\times U(1)_Y \xrightarrow{5+\bar{5}} SU(3)\times U(1)_Q \quad .$$

In other words, we use the 75-dimensional Higgs field instead of the adjoint 24-dimensional representation used in the minimal SU(5) model. As mentioned in Ref. 5, there are several reasons that make such a choice attractive and, for completeness, we repeat them here briefly.

(1) The 75-dimensional Higgs tensor plays a crucial role in constructing hierarchical fermion masses in a model developed by Barbieri, Nanopoulos, and Wyler.[6]

(2) It has been pointed out by several authors[7] that the heavy Higgs field can cause significant uncertainties in evaluations of M_X. This effect is small for the minimal model, but may be significant for other tensors.

(3) This representation is also used in some SUSY SU(5) models.[8] The analysis of its symmetry pattern may thus be also useful for these theories.

(4) Despite the fact that progress has been made[9,10,5,11] in understanding and solving the symmetry-breaking problem in gauge theories, explicit solutions are known only in some simpler cases. This paper provides another explicit example to this list.

The paper is organized as follows. In Sec. I we introduce a form of the potential. This form is considerably simplified in comparison with its form for the same tensor but for a general SU(n) group. This is due to some nontrivial relations between invariants which we use. In Sec. II we present briefly the derivation of the first stage of symmetry breaking. This has already been shown in Ref. 5. However, in this analysis we also include cubic terms in the potential. The analysis is considerably simplified owing to the relations between invariants mentioned above. In Sec. III we present results obtained after the second stage of symmetry breaking. In the end we draw conclusions.

A. Higgs potential

We introduce the following SU(5) multiplets: a fundamental covariant five-dimensional field H^E, $E = 1,\ldots,5$, its complex conjugate partner $\bar{H}_F$, $F = 1,\ldots,5$, and a 75-dimensional multiplet Φ^{AB}_{CD}, $A,B,C,D = 1,\ldots,5$. This tensor is antisymmetric in upper and lower indices separately and all traces vanish. In other words,

$$\Phi^{AB}_{CD} = -\Phi^{AB}_{DC} = -\Phi^{BA}_{CD} = \Phi^{BA}_{DC} \ . \tag{1.1}$$

We have to construct the most general polynomial in these fields up to the fourth order. We can decompose it into three terms,

$$V(\Phi,H,\bar{H}) = V_{75} + V_m + V_5 \ , \tag{1.2}$$

where V_{75} depends only on Φ, V_5 only on H and $\bar{H}$, and V_m contains the interactions between them.

In order to construct V_{75}, we have to know SU(5) invariants up to the fourth order in this tensor. The basis for these invariants was previously introduced[10,11] for the general SU(n) case and we list it here:

the quadratic invariant

$$I^{(2)} = \Phi^{AB}_{CD}\Phi^{CD}_{AB} \ , \tag{1.3}$$

the cubic invariants

$$I^{(3)} = \Phi^{AB}_{MN}\Phi^{MN}_{EF}\Phi^{EF}_{AB} \ , \tag{1.4}$$

$$I^{(3)}_2 = \Phi^{AM}_{FN}\Phi^{NB}_{ME}\Phi^{EF}_{AB} \ , \tag{1.5}$$

and six quartic invariants

$$I_o^{(4)} = I^{(2)\,2} = (\Phi^{AB}_{CD}\Phi^{CD}_{AB})^2, \quad I_1^{(4)} = H^{AB}_{CD}H^{CD}_{AB} ,$$

$$I_2^{(4)} = H^{AB}_{CB}H^{CD}_{AD} , \quad I_3^{(4)} = H^{AB}_{CD}G^{CD}_{AB} , \tag{1.6}$$

$$I_4^{(4)} = G^{AB}_{CD}G^{CD}_{BA} , \quad I_5^{(4)} = G^{AB}_{CD}H^{CD}_{AB} ,$$

where

$$H^{AB}_{CD} = \Phi^{AB}_{EF}\Phi^{EF}_{CD} ,$$

$$G^{AB}_{CD} = \Phi^{AF}_{DE}\Phi^{EB}_{FC} . \tag{1.7}$$

However, nontrivial relations between these invariants can be derived in special cases. One of these is the SU(5) group. For this group, the following relations hold:[12]

$$I_1^{(3)} + 4I_2^{(3)} = 0 , \tag{1.8}$$

$$4I_4^{(4)} = -I_o^{(4)} - I_1^{(4)} + 6I_2^{(4)} , \tag{1.9}$$

$$8I_3^{(4)} = -I_o^{(4)} - 2I_1^{(4)} + 8I_2^{(4)}, \tag{1.10}$$

$$16I_5^{(4)} = -5I_o^{(4)} - 2I_1^{(4)} + 32I_2^{(4)} . \tag{1.11}$$

This means that we can use a smaller set of invariants. We choose

$$I^{(2)}, I^{(3)} = I_1^{(3)}, I_o^{(4)}, I_1^{(4)}, I_2^{(4)} . \tag{1.12}$$

Now we are able to write the potential (1.2) explicitly:

$$V_5 = -\frac{v^2}{2}\,\overline{H}_A H^A + \frac{\lambda}{4}\,(\overline{H}_A H^A)^2 , \tag{1.13}$$

$$V_m = \alpha H_A H^A \Phi^{BC}_{DE}\Phi^{DE}_{BC} + \beta H_A \Phi^{AB}_{CD}\Phi^{CD}_{EB} H^E , \tag{1.14}$$

$$V_{75} = -\frac{\mu^2}{2}\,\Phi^{AB}_{CD}\Phi^{CD}_{AB} + \frac{a}{4}\,(\Phi^{AB}_{CD}\Phi^{CD}_{AB})^2$$
$$+\frac{b_1}{2}\,\Phi^{AB}_{GH}\Phi^{CD}_{AB}\Phi^{EF}_{CD}\Phi^{GH}_{EF}$$
$$+\frac{b_2}{2}\,\Phi^{AB}_{CD}\Phi^{CD}_{AF}\Phi^{EF}_{GH}\Phi^{GH}_{EB}$$
$$+\frac{c}{3}\,\Phi^{AB}_{EF}\Phi^{CD}_{AB}\Phi^{EF}_{CD}\,. \tag{1.15}$$

B. Breaking of SU(5) to $SU(3)\times SU(2)\times U(1)_Y$

This particular part of the problem was solved in Ref. 5 with no cubic terms included. We present here the solution of a more general problem with cubic terms included and with simplifications due to Eqs. (1.8), (1.9), (1.10), and (1.11). We introduce the decomposition of the tensor Φ to its irreducible constituents under $SU(3)\times SU(2)\times U(1)_Y$:

$$75 = (1,1,0) + (8,1,0) + (8,3,0)$$
$$+ \left[(3,2,-\tfrac{5}{6}) + (\overline{6},2,-\tfrac{5}{6}) + (\overline{3},1,-\tfrac{5}{3}) + \text{compl.conj.}\right], \tag{2.1}$$

together with the following explicit form of its subtensors:

$$\Phi^{AB}_{CD}\Big|_{(1,1,0)} = \Phi\{(\delta^{\alpha\beta}_{\gamma\delta}-\delta^{\alpha\beta}_{\delta\gamma})+3(\delta^{ab}_{cd}-\delta^{ab}_{dc})-(\delta^{db}_{\gamma d}-\delta^{db}_{\delta c}+\delta^{\beta a}_{\delta c}-\delta^{\beta a}_{\gamma d})\}\,,$$
$$\Phi^{AB}_{CD}\Big|_{(8,1,0)} = \frac{-1}{2}\{(2\delta^{\alpha}_{\gamma}-\delta^{a}_{c})\tilde{\Phi}^{\beta}_{\delta}-(2\delta^{\alpha}_{\delta}-\delta^{a}_{d})\tilde{\Phi}^{\beta}_{\gamma}+(2\delta^{\beta}_{\delta}-\delta^{b}_{d})\tilde{\Phi}^{\alpha}_{\gamma}$$
$$-(2\delta^{\beta}_{\gamma}-\delta^{b}_{c})\tilde{\Phi}^{d}_{\delta}\}\,,$$
$$\Phi^{AB}_{CD}\Big|_{(8,3,0)} = \{\Phi^{\alpha b}_{\gamma d}+\tilde{\Phi}^{\alpha b}_{c\delta}+\tilde{\Phi}^{a\beta}_{c\delta}+\tilde{\Phi}^{a\beta}_{\gamma d}\}\,, \tag{2.2}$$

$$\Phi^{AB}_{CD}\Big|(3,2,-\tfrac{5}{6}) = \frac{-1}{2}\{(\delta^{\alpha}_{\gamma}-2\delta^{a}_{c})\tilde{\Phi}^{\beta}_{d}-(\delta^{\alpha}_{\delta}-2\delta^{a}_{d})\tilde{\Phi}^{\beta}_{c}$$

$$+ (\delta^{\beta}_{\delta}-2\delta^{b}_{d})\tilde{\Phi}^{\alpha}_{c}-(\delta^{\beta}_{\gamma}-2\delta^{b}_{c})\tilde{\Phi}^{\alpha}_{d}\} ,$$

$$\Phi^{AB}_{CD}\Big|(\bar{6},2,-\tfrac{5}{6}) = \{\tilde{\Phi}^{\alpha\beta}_{\gamma d}+\tilde{\Phi}^{\alpha\beta}_{c\delta}\},$$

$$\Phi^{AB}_{CD}\Big|(\bar{3},1,-\tfrac{5}{3}) = \{\tilde{\Phi}^{\alpha\beta}_{cd}\} ,$$

where we have used the conventional index splitting

$$A,B,C,D \rightarrow (\alpha,a),(\beta,b),(\gamma,c),(\delta,d) ,$$
$$\alpha,\beta,\gamma,\delta = 1,2,3 , \quad a,b,c,d = 4,5 . \tag{2.3}$$

The quantities in (2.2) are related to the original tensor by

$$\Phi = \frac{1}{6}\,\Phi^{\alpha\beta}_{\alpha\beta} = \frac{1}{6}\,\Phi^{ab}_{ab} = -\frac{1}{6}\,\Phi^{\alpha b}_{\alpha b}$$

$$\tilde{\Phi}^{\alpha}_{\gamma} \equiv \Phi^{\alpha f}_{\gamma f} - \frac{1}{3}\delta^{\alpha}_{\gamma}\Phi^{\epsilon f}_{\epsilon f} , \quad \tilde{\Phi}^{\alpha}_{c} \equiv \Phi^{\alpha f}_{cf} , \quad \tilde{\Phi}^{\alpha\beta}_{cd} \equiv \Phi^{\alpha\beta}_{cd} ,$$

$$\tilde{\Phi}^{\alpha b}_{\gamma d} \equiv \Phi^{\alpha b}_{\gamma d} - -\frac{1}{3}\delta^{\alpha}_{\gamma}\Phi^{\epsilon b}_{\epsilon d} - \frac{1}{2}\delta^{b}_{d}\Phi^{\alpha f}_{\gamma f} + \frac{1}{6}\delta^{\alpha b}_{\gamma d}\Phi^{\epsilon f}_{\epsilon f} , \tag{2.4}$$

$$\tilde{\Phi}^{\alpha\beta}_{\gamma d} \equiv \Phi^{\alpha\beta}_{\gamma d} - \frac{1}{2}(\delta^{\alpha}_{\gamma}\Phi^{\beta\phi}_{d\phi}-\delta^{\beta}_{\gamma}\Phi^{\alpha\phi}_{d\phi}) .$$

We shall solve this problem by using the methods of Refs. 10 and 11. Our approach essentially consists in introducing the decomposition (2.2) into Eq. (1.15) for the potential. For the purpose of the problem, it is sufficient to retain terms up to the second order in subtensors. It is useful to separate the vacuum expectation value in the singlet variable:

$$\Phi = s + \theta , \qquad \langle\Phi\rangle = s . \tag{2.5}$$

In the SU(3) SU(2) $U(1)_Y$ invariant point, all subtensors vanish except the singlet. In the new decomposition of the potential, the terms linear in θ produce the equation for stationary points and quadratic terms in the fluctuations provide stability criteria as well as squares of boson masses. To obtain these results, one has to perform straightforward but lengthy calculations. We can however use the results derived previously[11] for the general SU(n) case and specify them for n = 5. Compared with this general case, the solution is simpler owing to smaller number of subtensors and smaller number of invariants. More specifically, four of nine subtensors in Ref. 11 do not appear in the problem considered here. In the notation of Ref. 11, these are X_1, X_4, X_5, and X_7. It is now straightforward to apply the results from Ref. 11, in particular to use Eqs. (37), (38), (39), and (40), together with the tables from Appendix B of the same reference. Of course, the use of the expressions given there can be avoided by performing calculations mentioned above.

We obtain the condition for the stationary point

$$\mu^2 = \frac{2}{3}\,(108a + 60b_1 + 56b_2)s^2 + \frac{8}{3}\,cs \tag{2.6}$$

and the value of the potential in this point

$$V = -\left[12(108a + 60b_1 + 56b_2) + 32\,\frac{c}{s}\right]s^4 + \frac{\lambda v^4}{16}\ . \tag{2.7}$$

Boson masses squared, i.e., stability conditions read as follows:

$$m^2_{(1,1,0)} = 96s^2(108a+60b_1+56b_2+2\tfrac{c}{s}) \geq 0\ ,$$

$$m^2_{(8,1,0)} = -4s^2(24b_1+8b_2+2\,\tfrac{c}{s}) \geq 0\ ,$$

$$m^2_{(8,3,0)} = -\frac{8}{3} s^2 (24b_1+8b_2+10\frac{c}{s}) \geq 0 ,$$

$$m^2_{(\bar{6},2,-\frac{5}{6})} = -\frac{8}{3} s^2 (24b_1+10b_2+2\frac{c}{s}) \geq 0 , \tag{2.8}$$

$$m^3_{(\bar{3},1,-\frac{5}{3})} = -\frac{8}{3} s^2 (-24b_1+2b_2-2\frac{c}{s}) \geq 0 ,$$

$$m^2_{(3,2,-\frac{5}{6})} = 0 \text{ ("Goldstone bosons")} .$$

We find a relation between masses

$$4m^2_{(8,1,o)} + m^2_{(\bar{3},1,-\frac{5}{3})} = 5m^2_{(\bar{6},2,-\frac{5}{6})} . \tag{2.9}$$

If cubic couplings vanish owing to some discrete symmetry, we have another relation

$$m^2_{(8,3,0)} = \frac{2}{3} m^2_{(8,1,0)} . \tag{2.10}$$

For completeness, we add the value for the mass of the vector gauge bosons M_X^2

$$M_X^2 = 48g^2s^2 , \tag{2.11}$$

where g is the SU(5) coupling constant.

C. Breaking of $SU(3)\times SU(2)\times U(1)_Y$ to $SU(3)\times U(1)_Q$

We introduce the five-dimensional multiplet

$$H^A, \quad A = 1,\ldots,5 .$$

We recall the decomposition

$$H^A = H^\alpha + H^a ,$$
$$\bar{H}_A = \bar{H}_\alpha + \bar{H}_a , \tag{3.1}$$

or

$$5 = (3,1,-\tfrac{1}{3}) + (1,2,\tfrac{1}{2}) \ ,$$

$$\bar{5} = (\bar{3},1,\tfrac{1}{3}) + (1,\bar{2},-\tfrac{1}{2}) \ .$$

Each subtensor of tensors Φ and H decomposes further under SU(3) $U(1)_Q$. These decompositions are as follows:

Subtensors of Φ:

$$S = S \quad \text{or} \quad (1,1,0) = (1,0) \ ,$$

$$\tilde{\Phi}^{\alpha}_{\alpha} = \tilde{\Phi}^{\alpha}_{\beta} \ \text{or} \quad (8,1,0) = (8,0) \ ,$$

$$\tilde{\Phi}^{\alpha\beta} = \frac{1}{\sqrt{2}}\,\varepsilon_{cd}\tilde{\Phi}^{\alpha\beta} \quad \text{or} \quad (\bar{3},1,-\tfrac{5}{3}) = (\bar{3},-\tfrac{5}{3}) \ , \qquad (3.2)$$

$$\varepsilon_{45} = -\varepsilon_{54} = 1, \quad \varepsilon_{44} = \varepsilon_{55} = 0 \ ,$$

$$\tilde{\Phi}^{\alpha b}_{\gamma d} = \tilde{\Phi}^{\alpha}_{+\gamma} \dotplus \sqrt{2}\ \tilde{\Phi}^{\alpha}_{o\gamma} \dotplus \tilde{\Phi}^{\alpha}_{-\gamma} \quad \text{or } (8,3,0) = (8,1)+(8,0)+(8,-1),$$

where

$$\tilde{\Phi}^{\alpha}_{+\gamma} = \tilde{\Phi}^{\alpha 4}_{\gamma 5},\ \frac{\tilde{\Phi}^{\alpha}_{o\gamma}}{\sqrt{2}} = \tilde{\Phi}^{\alpha 4}_{\gamma 4} = -\tilde{\Phi}^{\alpha 5}_{\gamma 5},\ \tilde{\Phi}^{\alpha}_{-\gamma} = \tilde{\Phi}^{\alpha 5}_{\gamma 4} \ ;$$

$$\Phi^{\alpha}_{c} = X^{\alpha} + Y^{\prime\alpha} \quad \text{or } (3,2,-\tfrac{5}{6}) = (3,-4)+(3,-\tfrac{1}{3}) \ ,$$

where

$$X^{\alpha} = \tilde{\Phi}^{\alpha}_{4} \ , \qquad Y^{\prime\alpha} = \tilde{\Phi}^{\alpha}_{5} \ ;$$

$$\tilde{\Phi}^{\alpha\beta}_{\gamma d} = \tilde{\Phi}^{\alpha\beta}_{+\gamma} + \tilde{\Phi}^{\alpha\beta}_{-\gamma} \quad \text{or} \quad (\bar{6},2,-\tfrac{5}{6}) = (\bar{6},-\tfrac{4}{3})+(\bar{6},-\tfrac{1}{3}) \ ,$$

where

$$\tilde{\Phi}^{\alpha\beta}_{+\gamma} = \tilde{\Phi}^{\alpha\beta}_{\gamma 5} \ , \qquad \tilde{\Phi}^{\alpha\beta}_{-\gamma} = \tilde{\Phi}^{\alpha\beta}_{\gamma 4} \ ;$$

subtensors of H:

$$H^{\alpha} = H^{\alpha} \quad \text{or } (\bar{3},1,-\tfrac{1}{3}) = (3,-\tfrac{1}{3}) \ ,$$
$$H^{a} = \delta^{a}_{4}\chi + \delta^{a}_{5}\,\frac{1}{\sqrt{2}}\,\chi_5 \quad \text{or} \quad (1,2,\tfrac{1}{2}) = (1,1)+(1,0) \ . \qquad (3.3)$$

Let us separate the vacuum expectation values in the singlet variables:

$$\Phi = s + \theta \ , \qquad \langle\Phi\rangle = s \ ,$$
$$\chi_5 = v + \xi + i\eta \ , \quad \langle\chi_5\rangle = v \ . \tag{3.4}$$

Again we have to repeat the whole procedure. We obtain two conditions for the stationary point, one in θ and the other in ξ. These read explicitly as follows:

$$\mu^2 = \frac{2}{3} s^2\left[108a+60b_1+56b_2+4\frac{c}{s}+\frac{\gamma}{2}\left(\frac{v}{s}\right)^2\right] \ ,$$

$$v^2 = \frac{1}{2}\lambda v^2 + 48\gamma s^2 \ ,$$

γ is an abbreviation for

$$\gamma = 3\alpha + \beta \ .$$

The value of the potential V in this point is

$$V_{min} = -\{\left[12(108a+60b_1+56b_2)+32\frac{c}{s}\right] s^4 + \frac{\lambda v^4}{16} + 12v^2\gamma^2 s^2\} \ .$$

We can now find masses of the multiplets which are left after the breaking. These are two (1,0) fields which are mixtures of ξ and θ:

$$\tau = \frac{1}{(1+R_\tau^2)^{1/2}}\left[\xi+R_\tau\theta\right] \ , \ \zeta = \frac{1}{(1+R_\zeta^2)^{1/2}}\left[\xi+R_\zeta\theta\right] \ , \tag{3.6}$$

$$R_{\tau,\zeta} = \frac{1}{48vs}\{\frac{\lambda v^2}{4} - 48Ps^2 \pm \left[(96Ps^2-\frac{\lambda v^2}{2})^2+4(48)^2v^2s^2\gamma^2\right]^{1/2}\} \ , \tag{3.7}$$

where P is an abbreviation for

$$P = 108a+60b_1+56b_2+2\,\frac{c}{s} \quad . \tag{3.8}$$

Masses of these particles are given by

$$m^2_{\tau,\zeta} = \frac{\lambda v^2}{4} + 48Ps^2 \pm \frac{1}{2}\left[(96Ps^2 - \frac{\lambda v^2}{2})^2 + 4(48)^2 v^2 s^2 \gamma^2\right]^{1/2} . \tag{3.9}$$

The positivity of $m^2_{\tau,\zeta}$ is ensured by

$$\gamma^2 < \frac{\lambda P}{48} \quad . \tag{3.10}$$

We see that one of those masses is heavy of the order $O(s)$ and the other light of the order $O(v)$. The latter is the usual Salam-Weinberg Higgs boson.

There are "Goldstone bosons"

$$\begin{aligned} \eta &= \mathrm{Im}\,\chi^5 , \\ \chi &= H^4 , \\ \chi^\dagger &= H_4^\dagger , \end{aligned} \tag{3.11}$$

with quantum numbers of the Salam-Weinberg vector bosons Z^0, W, and W. There are states which are mixtures of $(3,-\frac{1}{3})$ from Φ and H. These mixtures are given by

$$Y^\alpha = \frac{1}{(1+x^2)^{1/2}}\left[H^\alpha + xY'^\alpha\right], \quad U^\alpha = \frac{x}{(1+x^2)^{1/2}}\left[H^\alpha - \frac{1}{x}\,Y'^\alpha\right], \tag{3.12}$$

where

$$x = 2\sqrt{8}\,\frac{s}{v}\ ,$$

Y^{α} is a "Goldstone particle",

and

$$m_U^2 = -\beta\left(16s^2 + \frac{v^2}{2}\right)\ . \qquad (3.13)$$

The resulting stability condition is

$$\beta < 0\ . \qquad (3.14)$$

Becuase of this mixing, the mass of the Y gauge boson is shifted by an amount of order O(v), in full analogy to the minimal model. Other states do not mix and have the following masses, i.e., stability conditions:

(8,0) from (8,1,0):

$$m^2 = -4s^2\left[24b_1+8b_2+2\,\frac{c}{s} + \frac{3}{8}\beta\,\left(\frac{v}{s}\right)^2\right] > 0;$$

(8,0), (8,1), and (8,-1) from (8,3,0):

$$m^2 = -\frac{8}{3}\,s^2\left[24b_1+8b_2+10\,\frac{c}{s} + \frac{\beta}{8}\left(\frac{v}{s}\right)^2\right] > 0;$$

$(\bar{6}, -\frac{4}{3})$:

$$m^2 = -\frac{8}{3}\,s^2\left[24b_1+10b_2+2\,\frac{c}{s} + \frac{\beta}{4}\left(\frac{v}{s}\right)^2\right] > 0;$$

$(\bar{6}, -\frac{1}{3})$:

$$m^2 = -\frac{8}{3}\,s^2\left[24b_1+10b_2+2\,\frac{c}{s} + \frac{1}{16}\,\beta\left(\frac{v}{s}\right)^2\right] > 0;$$

$(\bar{3}, -\frac{5}{3})$:

$$m^2 = -\frac{8}{3}\,s^2\left[-24b_1+2b_2-2\,\frac{c}{s} + \frac{1}{32}\,\beta\left(\frac{v}{s}\right)^2\right] > 0\ .$$

We see that it is possible to perform the desired breaking. Again as in the minimal model, the physical requirement $M_W << M_X$ becomes $v << s$. We comment on other properties of this symmetry-breaking pattern and its analogies to and differences from the minimal model.[1] Salam-Weinberg singlet states from Φ and H mix. However, this is a simpler picture than in the minimal model where an additional Salam-Weinberg singlet state exists in the (1,3,0) subtensor of the adjoint Higgs and a more complicated mixing scheme arises. This is also the reason that the conditions (3.5) for the stationary point are simpler than the analogous conditions for the minimal model. In other words, we have two uncoupled equations, while the minimal model requires three coupled equations.
Our next comment refers to the Salam-Weinberg "Goldstone" bosons η and χ which, in contrast to the minimal model, do not mix with the states from ϕ. As a consequence[13] the Salam-Weinberg relation

$$M_W = M_Z \cos\Theta \qquad (3.15)$$

is unchanged contrary to the minimal model where corrections of the order $O(v^2/s^2)$ have to be included. Let us now consider the states with the $(3,-\frac{1}{3})$ quantum numbers from H and ϕ.

These mix completely analogously as in the minimal case and produce one "Goldstone" boson which can be "eaten up" to give mass to the Y vector boson and produce just one massive Higgs state. We end this section with Figs. 1, 2, and 3, which may serve as a guide through various subtensors mentioned in the paper.

IV. CONCLUSION

We have shown that the 75-dimensional Higgs field can be used to break the SU(5) symmetry to the standard model. In this analysis we have included the cubic term and also used nontrivial relations between invariants to simplify the potential. In fact, the potential contains only one more parameter than the potential for the adjoint representation.

As mentioned in Sec. I, investigation of nonminimal models may be useful in avoiding the conflict of the minimal model with experiment. In fact, there are several possibilities of saving the SU(5) model mentioned in the literature.

(a) Introduction of heavy Higgs fields different from the adjoint tensor may be associated with uncertainties in evaluations of M_X from renormalization-group equations. For example, in the model by Barbieri et al.[6] there is an uncertainty by a factor of 6 in the evaluation of M_X. For the 45-dimensional scalar, a factor of 2.8 was calculated by Cook et al.[7] (see also Ref. 14).

(b) If we combine 75 and 45 scalars, an important effect may arise from the fact that there is no obvious connection between Kobayashi-Maskawa mixing angles and mixing angles relevant to proton decay (see, e.g., Ref. 14).

(c) Another effect may arise from introduction of non-renormalizable terms in the potential (15).

(d) There are also proposals which do not change the Higgs sector, but change the fermion sector (16) instead.

We also hope that this analysis may be helpful in developing the hierarchical mass model as proposed in Ref. 6, in analyzing SUSY GUT's, and in developing a non-minimal SU(5) model. For this last case, we have to break the symmetry down to $SU(3)\times U(1)_Q$. One has several possibilities of which the simplest one is to take a five-dimensional tensor plus its complex conjugate or the 45-dimensional representation instead and again its complex conjugate. Both possibilities are of interest. The latter choice may be more desirable because of its success with fermion masses;[17] however, in this paper we have presented the simpler version. It turns out that this step is simpler than in the minimal model, as explained in Sec. III. For completeness, we have calculated the relative volume of the domain (2.8) to be $(1.77 \pm 0.07)\%$, corresponding to the physical standard minimum.

After completion of this work we received a preprint by Ruegg et al.[18] which is in part overlapping and in part complementary to the first part of this paper treating the potential with the 75-dimensional scalar.

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FIGURE CAPTIONS

FIG. 1. Splitting of the fundamental representation H^A under the chain of subgroups $SU(5) \to SU(3)\times SU(2)\times U(1)_Y \to SU(3)\times U(1)_Q$. The name used in the text is indicated under the subgroups content of the subtensor.

FIG. 2. Splitting of the adjoint representation T^A_B under the chain of subgroups $SU(5) \to SU(3)\times SU(2)\times U(1)_Y \to SU(3)\times U(1)_Q$. This tensor is not used in this paper, but we mention it for comparison of this work with the minimal model.

FIG. 3. Splitting of the 75-dimensional tensor Φ^{AB}_{CD} under the chain of subgroups $SU(5) \to SU(3)\times SU(2)\times U(1)_Y \to SU(3)_Q\times U(1)_Q$. The name used in the text is indicated under the subgroups content of the subtensor.

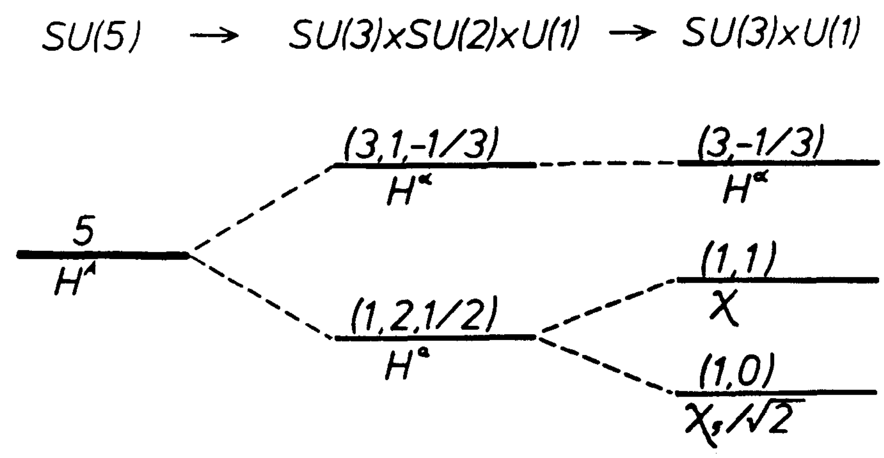

Figure 1

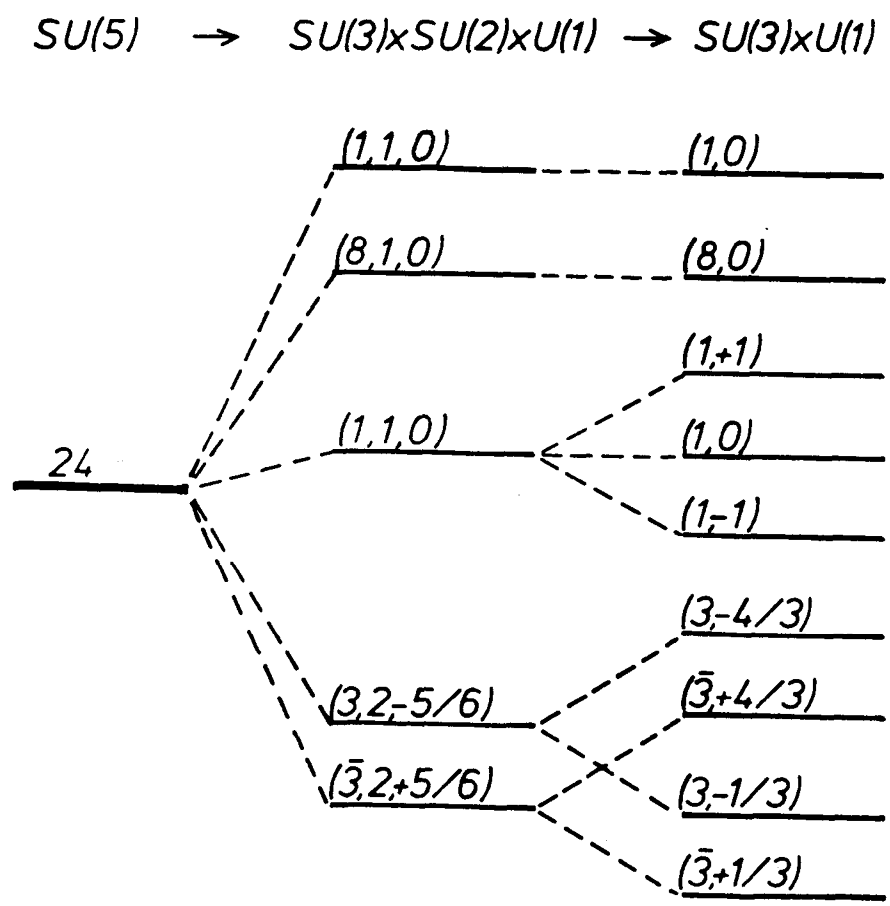

Figure 2

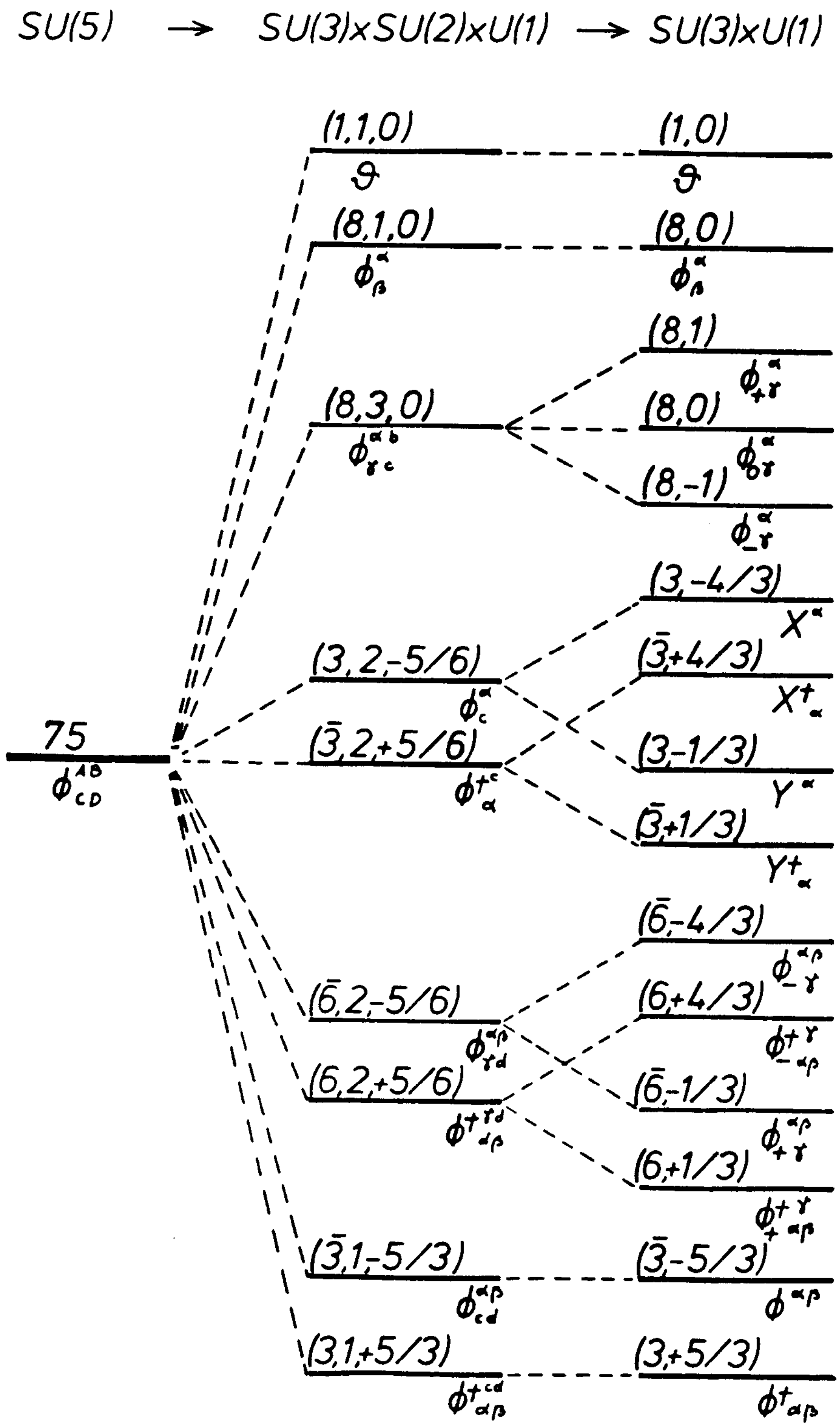
SU(5) → SU(3)xSU(2)xU(1) → SU(3)xU(1)
75
(1,1,0)
(1,0)
(8,1,0)
(8,0)
(8,1)
(8,3,0)
(8,0)
(8,-1)
(3,-4/3)
(3,2,-5/6)
(3̄,+4/3)
(3̄,2,+5/6)
(3,-1/3)
(3̄,+1/3)
(6̄,-4/3)
(6̄,2,-5/6)
(6,+4/3)
(6,2,+5/6)
(6̄,-1/3)
(6,+1/3)
(3̄,1,-5/3)
(3̄,-5/3)
(3,1,+5/3)
(3,+5/3)

Figure 3