SLAC-PUB-3351
June 1984
(A)

A VLASOV DESCRIPTION OF THE GRIDDED GAP-ELECTRON FLOW INTERACTION*

S. KHEIFETS, J. JAEGER AND S. YU
Stanford Linear Accelerator Center
Stanford University, Stanford, California 94305

ABSTRACT

Self-consistent solutions of the system of Vlasov equations are found for the case when the electric field in the gap does not depend on the longitudinal coordinate. The solution is valid: *a*) for an arbitrary nonrelativistic particle distribution in velocity and time at the gap entrance, *b*) for any gap length, *c*) for any beam current, and *d*) for a broad class of field dependences on time. In the region of applicability of the small signal approximation (small beam current, small transit angle of the gap), the solution derived reproduces the results of the small signal approximation. Numerical results for the input klystron cavity and for an idler cavity are given and compared with the calculations in small signal approximation. Possible applications of this formulation are discussed. In particular, we argue that the Vlasov description provides a suitable framework for developing one-dimensional models of a multiple cavity klystron.

(Submitted to IEEE Transactions On Microwave Theory and Technique)

---

* Work supported by the Department of Energy, contract DE-AC03-76SF00515.

1. Introduction

The present study is motivated by the modelling of high-power klystrons. The two basic components of a klystron are the resonant cavities and the drift spaces. This paper addresses only the first of these two components. Modelling of the drift spaces is deferred to future work. In one particular case considered here of the drift space between the first (externally powered) and the next (idler) cavities, the ballistic approximation is used for the particle motion. The formulation is not restricted to klystron modelling, but is applicable to any problem involving the interaction of an electron beam with a resonant cavity.

While the theory of klystrons has been worked out in detail in the small signal limit, the problem remains largely unsolved when the signals are large. In particular, the hydrodynamic models of electron beams used to derive the small signal theories fail when particle trajectories cross each other. In this paper, we employ a Vlasov description of the electron beam to study the klystron problem. In the Vlasov formulation we follow the evolution of the electron distribution function in phase space. The general framework can naturally accommodate particle crossing, and the beam dynamics is accurately described even when the signals are large.

While the Vlasov formulation is equivalent in principle to a particle simulation, the mathematical structure of the Vlasov equations makes it relatively easy to build in the steady-state condition. Since in many klystron problems we are interested mostly in the steady-state solution, the Vlasov description is very convenient. This is an advantage that a particle simulation does not share.

The self-consistent solution of the system of Vlasov equations is found under the following assumptions:

a) One dimensional (longitudinally) nonrelativistic particle flow.

b) Electric field uniform in the longitudinal coordinate (gridded gap).

The solution is valid a) for an arbitrary particle distribution of the flow entering the gap, b) for any gap size, c) for all beam intensities, and d) for a broad class of time dependences of the electric field in the gap, although we will be studying in detail the special case of a resonant cavity with a single dominant frequency.

In section 2 the problem is formulated in terms of the Vlasov equations [1,2,3]. In section 3 we present the solution of the Liouville equation for a given gap field. Solutions both for the initial value problem and for the boundary value problem are given. At the end of the section we also write down the gap field excited by a current source. The results of this section are combined in section 4 to produce a self-consistent solution for the Vlasov equations. In the limit of a small beam intensity and/or a small electric field the solution gives the same results as the small signal theory based on the hydrodynamic beam models. This is shown in section 5 for the input klystron cavity and in section 6 for the second (idler) cavity where the expressions for the gap voltage are derived in the small signal approximation. We have also derived a general solution in the limit of small gap size (section 7). The last sections contain a numerical example, comparison with known approximations and some conclusions as well as a discussion of possible applications of the suggested solution to the klystron problem. The results of present paper in greater detail can be found in [4].

2. The Vlasov Equations

The most general and exact description of the electromagnetic interaction of a particle flux with environment is given by a system of equations describing the evolution of the particle phase space distribution function and the electromagnetic field produced by particle charges. This system of equations is referred to usually as the Vlasov equations. For the nonrelativistic one dimensional problem considered here, the Vlasov equations are as follows:

Consider the motion of an electron in the gap in z direction with the velocity $v = dz/dt$:

$$\frac{dv}{dt} = \frac{e}{m} E(t) \quad . \tag{2.1}$$

Here $E(t)$ is the z-component of the electric field assumed to depend on time only. The physical realization of such a field takes place in a gridded gap, for example.

The first two integrals of this equation are

$$v(t) = \frac{e}{m} \int_{t_0}^{t} E(t')dt' + v_0 \tag{2.2}$$

and

$$z(t) = z_0 + v \cdot (t - t_0) \ - \ \frac{e}{m} \int_{t_0}^{t} (t' - t_0) E(t') dt' \tag{2.3}$$

The evolution of a flux of electrons inside the gap can be described by a distribution function ψ of time t, coordinate z and velocity v : $\psi = \psi(z, v, t)$. The continuity equation in the phase space z, v is called the Liouville equation. In our case it looks like ($\mathcal{L}$ is an operator):

$$\mathcal{L}\psi \equiv \frac{\partial \psi}{\partial t} + v \frac{\partial \psi}{\partial z} + \frac{e}{m} E(t) \frac{\partial \psi}{\partial v} = 0 \tag{2.4}$$

Notice that v and z in this equation are considered as independent variables.

The electric field $E(t)$ in general can be produced by the charges and currents of the flux taking into account the environment as well as by external sources. Introducing axial component of the vector-potential $A(\vec{r}, t)$ and the scalar potential $\phi(\vec{r}, t)$ we can describe the electromagnetic field by the following Maxwell equations:

$$\frac{1}{c^2}\frac{\partial^2 A}{\partial t^2}-\frac{\partial^2 A}{\partial z^2}=\frac{4\pi}{c}j(\vec{r},t) \tag{2.5}$$

$$\frac{\partial A}{\partial z}+\frac{1}{c}\frac{\partial \phi}{\partial t}=0 \tag{2.6}$$

$$E=-\frac{\partial \phi}{\partial z}-\frac{1}{c}\frac{\partial A}{\partial t} \tag{2.7}$$

The axial current density $j(\vec{r},t)$ in (2.5) in turn can be expressed as a sum of external current density j_{ext} (produced for example by an external RF generator) and the current density of the flux itself. In our case, we assume for simplicity for the electron current density a uniform dependence on the radial distance from the axis in the interval $0 < r < b$:

$$j(z,r,t)=g(r)I(z,t)+j_{ext} \tag{2.8}$$

$$g(r)=\begin{cases}1/\pi b^2 & r<b\\ 0 & r\geq 0\end{cases} \tag{2.9}$$

$$I(z,t)=e\int_{-\infty}^{\infty} dv\, v\, \psi(z,v,t) \tag{2.10}$$

The system of equations (2.4) through (2.10) are the Vlasov equations. The solution of this system satisfying all the necessary initial and boundary conditions is the self-consistent solution of the problem. The search for such a solution and the study of its properties are the subject of the present paper.

3. Solution Of The Liouville Equation

We solve first the Liouville equation (2.4) assuming for the time being $E(t)$ as a given function of time. It is known, that any function of the integrals of motion is the solution of the Liouville equation. Hence the function

$$\psi(z,v,t) = \bar{\psi}_0\Big(z - v\cdot(t-t_0) + \frac{e}{m}\int_{t_0}^{t}(t'-t_0)E(t')dt' \;,\; v - \frac{e}{m}\int_{t_0}^{t}E(t')\,dt'\Big)$$

is a solution of equation (2.4). $\bar{\psi}_0(z,v)$ corresponds to the initial distribution at $t = t_0$. It is easy to check by direct substitution, that this function indeed satisfies (2.4). We will not do this here since we are interested in the solution of the boundary value problem rather than the initial value problem.

Suppose now that at $z = z_0$ the distribution function is given for all times and velocities:

$$\psi_0 = \psi_0\,(v,t) \tag{3.1}$$

We are interested now in finding a solution $\psi(z - z_0, v, t)$ of (2.4) which goes into (3.1) for $z \to z_0$. This solution will describe the evolution of ψ_0 in z, v and t. In particular, it will give us the distribution function $\psi(\ell, v, t)$ at the exit of the gap $z = z_0 + \ell$.

The aim is achieved in the following way. Introduce first the implicit function $\Theta\;(z - z_0, v, t)$ as a solution of the equation

$$F(z - z_0, v, t, \Theta) \equiv z - z_0 - v\cdot(t-\Theta) + \frac{e}{m}\int_{\Theta}^{t}(t'-\Theta)E(t')dt' = 0 \tag{3.2}$$

which satisfies the condition:

$$\Theta(0, v, t) = t \tag{3.3}$$

Introduce next the function

$$V(z - z_0, v, t) = v - \frac{e}{m}\int_{\Theta\,(z-z_0,v,t)}^{t} E(t')dt' \tag{3.4}$$

From (3.3) it follows immediately

$$V(0, v, t) = v \quad . \tag{3.5}$$

Then

$$\psi(z, v, t) = \psi_0 \Big(V(z - z_0, v, t) \quad , \quad \Theta(z - z_0, v, t) \Big) \tag{3.6}$$

is such a solution of the Liouville equation (2.4) which goes into the boundary value (3.1) when $z \to z_0$. To prove this, note first of all that

$$\mathcal{L}\psi = \frac{\partial \psi}{\partial V} \cdot \mathcal{L}V \; + \; \frac{\partial \psi}{\partial \Theta} \mathcal{L}\Theta$$

Then it is easy to see that

$$\mathcal{L}V \; = \; \frac{e}{m} E(\Theta) \cdot \mathcal{L}\Theta \quad ,$$

therefore the only thing we have to show is that

$$\mathcal{L}\Theta = 0 \tag{3.7}$$

Indeed, if (3.7) is true then $\mathcal{L}\psi = 0$ and (3.3) and (3.5) provide that $\psi|_{z=z_0} = \psi_0(v, t)$. To prove (3.7) find $\mathcal{L}F$ (which is 0 since F = 0)

$$\mathcal{L}F = V \cdot \mathcal{L}\Theta = 0$$

Hence (3.7) is true. In the particular case of a harmonic electric field:

$$E_h(t) = E_0 \, cos(\omega t + \varphi) \tag{3.8}$$

formula (3.4) gives:

$$V_h = v - \frac{eE_0}{m\omega} \Big[sin(\omega t + \varphi) - sin(\omega \Theta_h + \varphi) \Big] \quad , \tag{3.9}$$

Θ_h satisfies the following equation (from formula (3.2)):

$$z - z_0 - v(t-\Theta_h) + \frac{eE_0}{m\omega^2}\left[cos(\omega t+\varphi) - cos(\omega\Theta_h+\varphi)\right] \\ + \frac{eE_0}{m\omega}(t-\Theta_h)\, sin\,(\omega t+\varphi) = 0 \tag{3.10}$$

The solution (3.6) possesses an important feature of periodicity. Namely, if $\psi_0(v,t)$ and $E(t)$ are both periodic in time (T is the period):

$$\psi_0(v, t+T) = \psi_0(v,t) \quad , \tag{3.11}$$

$$E(t+T) = E(t) \tag{3.12}$$

then

$$\psi(z,v,t+T) = \psi_0\Big(V(t+T), \Theta(t+T)+T\Big) \\ = \psi_0\Big(V(t),\Theta(t)\Big) = \psi(z,v,t), \tag{3.13}$$

i.e. it is also periodic.

The correctness of this statement is very easy to see in the simple case of the harmonic electric field with $w = 2\pi/T$. In this case (3.10) is invariant under transformation $t \to t+T, \Theta \to \Theta + T$ and so is (3.9). The proof for more general periodic function $E(t) = \sum_n E_n cos(n\omega t + \varphi_n)$ is more elaborate and we will not give it here.

The constants E, φ in (3.8) are to be found self-consistently from the solution of the Maxwell equation with the current density as a source of the field defined in (2.8) and (2.10).

Define the gap voltage first harmonic for the frequency ω

$$U_1 = -\ell\langle E_1(r)\rangle \quad , \tag{3.14}$$

where $\langle E_1(r)\rangle$ is the average electric field harmonic over the beam cross section $\langle E_1(r)\rangle = \frac{1}{\pi b^2}\int_0^b r dr\, d\varphi\, E_1(r)$ and the gap impedance on the frequency ω:

$$Z_1 = \frac{U_1}{\langle I_1\rangle} \quad , \tag{3.15}$$

where $\langle I_1 \rangle$ is the first harmonic of the full current averaged over the gap (ℓ is the gap length):

$$\langle I_1 \rangle = \frac{1}{\ell} \int_0^\ell dz\, I_1(z) \tag{3.16}$$

Here

$$I_1(z) = \frac{\omega}{2\pi} \int_0^{2\pi/\omega} dt\, e^{-i\omega t}\, I(z,t) \tag{3.17}$$

is the first Fourier harmonic of the current $I(z,t)$.

For the resonant cavity

$$Z_1 = \frac{(R/Q) \cdot Q}{1 + iQ\left(\frac{f}{f_0} - \frac{f_0}{f}\right)} \quad , \tag{3.18}$$

where (R/Q), Q and f_0 are the shunt resistance, the quality and the proper frequency of the cold cavity.

4. Self-Consistent Solution Of The Vlasov Equations

We can rewrite (3.15) in the following way:

$$U_1 \quad = \frac{Z_1}{\ell} \int_0^\ell dz\, I_1(z) \tag{4.1}$$

It is more convenient to consider the current due to electron flow separately from other possible currents, e.g. the current arising from the external generator.

Consider for example the first klystron cavity. Assume that the cold cavity is excited by an external rf generator. Then it is convenient to rewrite (4.1) in the following form:

$$E_1 = E_{1\,ext} - \frac{Z_1}{\ell^2} \int_0^\ell dz\, I_1(z) \quad , \tag{4.2}$$

where now I_1 is the first harmonic of the electron flow current $I(z,t)$ defined in (2.10) and which in turn depends on E_1. $E_{1\,ext}$ is the first harmonic of the field excited in the cold cavity by an external generator. It can be equal zero in particular case of not excited cavity (for example the second klystron cavity).

The complex equation (4.2) constitutes two transcendental equations for the amplitude E_0 and the phase φ (or for the real and imaginary parts) of the first harmonic of the field. Solution of these equations provide the self-consistent field $E_h = E_0 cos(wt + \varphi)$ (3.8). Substitute this field back into the solution (3.6) for the distribution function. One gets now the selfconsistent solution of the Vlasov equation which satisfies the boundary value at $z = z_o$.

As an example, let us assume for the initial electron flow a dc current with no velocity spread:

$$\psi_0(v,t)|_{z=z_0} = \frac{I_0}{ev_0}\delta(v - v_0) \quad , \tag{4.3}$$

where I_0 is the dc electron current, and $v_0 = \sqrt{2eV_0/m}$ is the initial velocity due to the dc gun voltage V_0. At this point it is convenient to introduce the following dimensionless variables

$$x = (z - z_0)/\ell \qquad 0 \leq x \leq 1 \tag{4.4}$$

$$u = v/\omega\ell \qquad -\infty \leq u \leq \infty \tag{4.5}$$

$$\tau = \omega t + \varphi \tag{4.6}$$

$$\tau_0 = \omega\Theta + \varphi \tag{4.7}$$

$$k = eE_0/m\omega^2\ell \tag{4.8}$$

According to (3.6) the distribution function for any later coordinate and time is in the new variables

$$\psi(x,u,\tau) = \frac{I_0}{eu_0}\delta(u - k\,sin\,\tau + k\,sin\,\tau_0 - u_0) \quad , \tag{4.9}$$

where $u_0 = v_0/\omega\ell$ and the function $\tau_0 = \tau_0(x, u, \tau)$ is defined by equation:

$$x - u(\tau - \tau_0) + k[\cos\tau - \cos\tau_0 + (\tau - \tau_0)\sin\tau] = 0. \tag{4.10}$$

From the distribution function (4.9) one can find the beam density current

$$\begin{aligned} I(x,\tau) &= \frac{I_0}{u_0}\int_{-\infty}^{\infty} du\, u\, \delta(u - k\sin\tau + k\sin\tau_0 - u_0) \\ &= I_0\, \frac{\bar{u}(\tau)}{|u_0 + k[\tau - \tau_0(\bar{u})]\cos\tau_0(\bar{u})|} \quad , \end{aligned} \tag{4.11}$$

where $\bar{u} = \bar{u}(\tau)$ is the solution of equation

$$\bar{u} + k\sin\tau_0(\bar{u}) = u_0 + k\sin\tau \tag{4.12}$$

The first harmonic of $I(x,\tau)$ is

$$I_1(x) = \frac{I_0}{2\pi} e^{i\varphi} \int_0^{2\pi} \frac{d\tau\ \bar{u}\ (\tau)\, e^{-i\tau}}{|u_0 + k[\tau - \tau_0(\bar{u})]\cos\tau_0(\bar{u})|} \tag{4.13}$$

Calculate now the average over x of this current and substitute into (4.2). Note that $E_1 = E_0 e^{i\varphi}/2$ and $E_{1ext} = E_{0ext}/2$.

$$k = k_{ext} e^{-i\varphi} - \frac{2Z_1}{\ell^2} \frac{eI_o}{2\pi m\omega^2} \int_0^1 dx \int_0^{2\pi} \frac{d\tau\ \bar{u}(\tau) e^{-i\tau}}{|u_0 + k(\tau - \tau_0)\cos\tau_0|} \quad , \tag{4.14}$$

where $k_{ext} = eE_{0\,ext}/m\omega^2\ell$. The complex equation (4.14) is equivalent to two transcendental equations which define the amplitude E_0 and the phase φ (in respect to the external field) of the field in the gap.

Figures 1-3 are the phase plots of u/k versus x/k for $k = 0.2, 0.25$ and 0.33 respectively, for different values of the time parameter τ. One sees the onset of the crossover of the particle trajectories for the large gap voltage (Fig. 3).

5. Small Signal Approximation. The Input Klystron Cavity.

It is instructive to study the previous results in the limit of a small electric field and to compare them with the known results from the small signal theory.

In the small signal limit the lowest power of the parameter k should be retained in all expansions in power series.

In variables (4.4) - (4.8), equations (3.9), (3.10) for $\bar{u} = V_h/\omega d$ and τ_0 look like:

$$\bar{u} = u - k\,sin\,\tau + k\,sin\,\tau_0 \tag{5.1}$$

$$x - u(\tau - \tau_0) + k(cos\,\tau - cos\,\tau_0 + \tau\,sin\,\tau - \tau_0\,sin\,\tau) = 0 \tag{5.2}$$

The solutions of (5.1) and (5.2) to the first order in k are

$$\bar{u} = u - k\,sin\,\tau + k\,sin\left(\tau - \frac{x}{u}\right) \tag{5.3}$$

$$\tau_0 = \tau - \frac{x}{u} + \frac{k}{u}\left[cos\left(\tau - \frac{x}{u}\right) - cos\,\tau - \frac{x}{u}sin\,\tau\right] \tag{5.4}$$

The terms independent of k here give the ballistic approximation. The last terms in (4.3) and (5.4) represent the influence of the electric field.

Let us assume for simplicity that the distribution function of the electron flow on the entrance of the gap is (4.3).

5.1 Coupling Coefficient

Let us first of all find the expression for the coupling coefficient μ as it follows from our solution. One can define μ as the ratio of the average kinetic energy change to the maximum of the energy gain in the gap [5]. Calculate first the average $\langle u^2 \rangle$ as the function of x:

$$\langle u^2 \rangle = \frac{\int\limits_{-\infty}^{\infty} du\, u^2\, \delta\left[u - k\,sin\,\tau + k\,sin\left(\tau - \frac{x}{u}\right) - u_0\right]}{\int\limits_{-\infty}^{\infty} du\, \delta\left[u - k\,sin\,\tau + k\,sin\left(\tau - \frac{x}{u}\right) - u_0\right]} \tag{5.5}$$

To the first order in k for $x = 1$

$$\langle u^2 \rangle_1 = u_0{}^2 \; + \; 2ku_0\left[sin\,\tau - sin\left(\tau - \frac{1}{u_0}\right)\right] \tag{5.6}$$

From here we get (ϕ denotes $\tau + 1/2u_0$)

$$\mu = \frac{\langle u^2 \rangle_1 - u_0^2}{2k\,cos\,\phi} = \frac{sin\theta}{\theta} \quad , \tag{5.7}$$

where $\theta = \frac{1}{2u_0} = \frac{\omega\ell}{2v_0}$ is the half of the gap transit angle.

5.2 Beam Loading

Now, let us consider the beam loading by the electron flow as it follows from our solution. We need now the expression for the beam current density in the small signal approximation. The charge and the current densities both can found by integration of (4.9)

$$\rho(x,\tau) = \frac{I_0}{u_0\omega\ell}\int_{-\infty}^{\infty} du\;\delta\left[u - k\,sin\,\tau + k\,sin\left(\tau - \frac{x}{u}\right) - u_0\right] \tag{5.8}$$

$$I(x,\tau) = \frac{I_0}{u_0}\int_{-\infty}^{\infty} du\;\;u\delta\left[u - k\,sin\,\tau + k\,sin\left(\tau - \frac{x}{u}\right) - u_0\right] \tag{5.9}$$

Performing the integrations we find to the first order in k:

$$I(x,\tau) = I_0\left\{1 + \frac{k}{u_0}\left[sin\,\tau - sin\left(\tau - \frac{x}{u_0}\right)\right] - \frac{xk}{u_0^2}\,cos\left(\tau - \frac{x}{u_0}\right)\right\} \tag{5.10}$$

$$\rho(x,\tau) = \frac{I_0}{u_0\omega\ell}\left\{1 - \frac{kx}{u_0^2}\,cos\left(\tau - \frac{x}{u_0}\right)\right\} \tag{5.11}$$

It is easy to see that (5.10) and (5.11) satisfy the continuity equation

$$\frac{\partial\rho}{\partial\tau} + \frac{1}{\omega\ell}\frac{\partial I}{\partial x} = 0 \tag{5.12}$$

As one sees from (5.10) in the small signal approximation the current density besides the dc component contains only the first harmonic. Expression (5.10) can be obtained conversely by expanding expression (4.11) in the power series in parameter k.

Using the definition (3.17) the first harmonic of the beam current density is

$$I_1(x) = I_0 e^{i\varphi} \frac{k}{2u_0}\Big(sin\frac{x}{u_0} - \frac{x}{u_0}cos\frac{x}{u_0} - i + icos\frac{x}{u_0} + i\frac{x}{u_0}sin\frac{x}{u_0}\Big) \tag{5.13}$$

This expression is the same as one obtained from formula (2.1) of the paper [5] assuming $E_z = const.$ It also coincides with the corresponding expression for i_v in the book [6].

Let us rewrite equation (4.2) in variables (4.4) - (4.7):

$$E_1 = \frac{Z_1}{\ell}\Big[I_{1ext} - \int_0^1 dx\, I_1(x)\Big] \tag{5.14}$$

Here I_{1ext} is the first harmonic of the external current exciting the cavity. Substitute now (5.13) into (5.14) and take into account that $E_1 = E_0 e^{i\varphi}/2$, $I_{1ext} = I_{0ext}/2$. Then:

$$E_0 = \frac{Z_1}{\ell} I_{0ext} e^{-i\varphi} \; - \; \frac{Z_1}{\ell}\; I_0\, k(B + iA) \quad , \tag{5.15}$$

where

$$B = \int_0^{1/u_0} d\sigma(sin\,\sigma - \sigma\, cos\,\sigma) = 4\theta\, sin\theta\Big(\frac{sin\theta}{\theta} - cos\,\theta\Big) \quad , \tag{5.16}$$

$$A = -\int_0^{1/u_0} d\sigma(1 - cos\,\sigma - \sigma\, sin\,\sigma) = 4\theta\, cos\,\theta\Big(\frac{sin\theta}{\theta} - cos\theta\Big) \quad , \tag{5.17}$$

Here $\theta = 1/2u_0 = \omega\ell/2v_0$ is half of the transit angle for the gap. Solving (5.15) in respect to E_0 (see definition of k in equation (4.8)) one finds:

$$E_0 = \frac{I_{0ext}\, e^{-i\varphi}}{\ell\Big[\frac{1}{Z_1} + \frac{eI_0(B+iA)}{m\omega^2\ell^2}\Big]} \tag{5.18}$$

Substitute now expression (3.18) for Z_1:

$$E_0 = \frac{(R/Q)I_{0ext}\, e^{-i\varphi}}{\ell\left[\frac{1}{Q} + i\left(\frac{f}{f_0} - \frac{f_0}{f}\right) + \frac{I_0(R/Q)(B+iA)}{2V_0 \quad 4\theta^2}\right]} \tag{5.19}$$

From formula (5.19) immediately follow the usual expressions for the loaded quality Q_L and the shifted frequency f_L of the gap:

$$\frac{1}{Q_L} = \frac{1}{Q} + \frac{I_0(R/Q)}{2V_0}\,\frac{sin\theta}{\theta}\left(\frac{sin\theta}{\theta} - cos\theta\right) \tag{5.20}$$

$$f_L = f_0\left[1 - \frac{I_0(R/Q)}{4V_0}\,\frac{cos\theta}{\theta}\left(\frac{sin\theta}{\theta} - cos\,\theta\right)\right] \tag{5.21}$$

6. Small Signal Approximation. Second (Idler) Klystron Cavity.

Let us now proceed to the second (idler) klystron cavity. We want to study how it is excited by an electron flow perturbed by the action of the field in the first cavity and bunched in the drift tube with the length d_1 between the first and the second cavity. We neglect in the present formulation the debunching effects of the space charge in the drift tube and use the ballistic approximation for the particle motion inside the drift tube.

We use the solution for the field in the first cavity k_1 to obtain the distribution function of the electron flow at its exit. for initial distribution function defined in (4.3) we have (see expression (5.3) for $x = 1$):

$$\psi_{1,out}(v,t) = \frac{I_0}{eu_0}\delta[u - k_1 sin\tau_1 + k_1 sin\left(\tau_1 - \frac{1}{u}\right) - u_0] \quad , \tag{6.1}$$

where $\tau_1 = \omega t + \varphi_1$, $u = v/\omega\ell_1, u_0 = v_0/\omega\ell_1$. $k_1 = eE_1/m\omega^2\ell_1$ and φ_1 constitute the solution of equation (4.14). The distribution function at the entrance of the second cavity is obtained from (6.1) in ballistic approximation:

$$\psi_{2,in}(v,t) = \frac{I_0}{eu_0}\delta\left[u - k_1 sin\left(\tau_1 - \frac{d_1}{u\ell_1}\right) + k_1 sin\left(\tau_1 - \frac{\ell_1 + d_1}{u\ell_1}\right) - u_0\right] \tag{6.2}$$

To obtain the distribution function at any place $x = \frac{z}{\ell_1}$ inside the second gap (with the length ℓ_2) according to (3.6) one should substitute in (6.2) $v \to V$ and $t \to \Theta$. Or, in the variables u, τ (see expressions (5.3) and (5.4)):

$$u \to u_2 = u - k_2 sin\tau_2 + k_2 sin\left(\tau_2 - \frac{x}{u}\right) \tag{6.3}$$

$$\tau \to \tau_0 = \tau_2 - \frac{x}{u} + \frac{k_2}{u} cos\left(\tau_2 - \frac{x}{u}\right) - \frac{k_2}{u} cos\tau_2 - \frac{k_2 x}{u^2} sin\tau_2 \tag{6.4}$$

Here $\tau_2 = \omega t + \varphi_2$ and $k_2 = eE_2/m\omega^2\ell_1$. k_2, φ_2 should be found from the solution of (4.14) for the second gap, in which one should put $k_{ext} = 0$. In the linear approximation in k_1 and k_2 we get:

$$\psi(x,u,\tau) = \frac{I_o}{eu_0}\delta\Big[u - k_2 sin\tau_2 + k_2 sin\left(\tau_2 - \frac{x}{u}\right) - k_1 sin\left(\tau_2 + \varphi_1 - \varphi_2 - \frac{x\ell_1 + d_1}{\ell_1 u}\right)$$
$$+ k_1 sin\left(\tau_2 + \varphi_1 - \varphi_2 - \frac{x\ell_1 + \ell_1 + d_1}{\ell_1 u}\right) - u_0\Big] \tag{6.5}$$

The charge (5.8) and current (5.9) densities both can be found from (6.5):

$$\varphi(x,\tau_2) = \frac{I_0}{u_0\omega\ell_1}\Big\{1 - \frac{k_2 x}{u_0^2} cos\left(\tau_2 - \frac{x}{u_0}\right) + \frac{k_1(x\ell_1 + d_1)}{\ell_1 u_0^2} cos\left(\tau_2 + \varphi_1 - \varphi_2 - \frac{x\ell_1 + d_1}{\ell_1 u_0}\right)$$
$$- \frac{k_1(x\ell_1 + \ell_1 + d_1)}{\ell_1 u_0^2} cos\left(\tau_2 + \varphi_1 - \varphi_2 - \frac{x\ell_1 + \ell_1 + d_1}{\ell_1 u_0}\right)\Big\} \tag{6.6}$$

$$I(x,\ \tau_2) = I_0\Big\{1 + \frac{k_2}{u_0} sin\tau_2 - \frac{k_2}{u_0} sin\left(\tau_2 - \frac{x}{u_0}\right) + \frac{k_1}{u_0} sin\left(\tau_2 + \varphi_1 - \varphi_2 - \frac{x\ell_1 + d_1}{\ell_1 u_0}\right)$$
$$- \frac{k_1}{u_0} sin\left(\tau_2 + \varphi_1 - \varphi_2 - \frac{x\ell_1 + \ell_1 + d_1}{\ell_1 u_0}\right) - \frac{k_2 x}{u_0^2} cos\left(\tau_2 - \frac{x}{u_0}\right)$$
$$+ \frac{k_1(x\ell_1 + d_1)}{\ell_1 u_0^2} cos\left(\tau_2 + \varphi_1 - \varphi_2 - \frac{x\ell_1 + d_1}{\ell_1 u_0}\right)$$
$$- \frac{k_1(x\ell_1 + \ell_1 + d_1)}{\ell_1 u_0^2} cos\left(\tau_2 + \varphi_1 - \varphi_2 - \frac{x\ell_1 + \ell_1 + d_1}{\ell_1 u_0}\right)\Big\} \tag{6.7}$$

Expressions (6.6) and (6.7) satisfy the continuity equation (5.12). Substituting (6.7) into (4.14) one obtains for $U_2 = \ell_2 E_2$ and φ_2 the following complex equation:

$$U_2 e^{i\varphi_2} = -\frac{eI_0 U_1 e^{i\varphi_1}(B_{12} + iA_{12})}{m\omega^2 \ell_1 \ell_2 \left[\frac{1}{Z_2} + \frac{eI_0(B_2+iA_2)}{m\omega^2 \ell_2 2}\right]} \quad , \tag{6.8}$$

where $U_1 e^{i\varphi_1}$ is the first gap voltage, Z_2 is the second gap cold unloaded impedance, B_2 and A_2 are the coefficients defined in (5.16) and (5.17), respectively, in which θ should be substituted by $\theta_2 = \ell_2/2\ell_1 u_0$. The coefficients B_{12} and A_{12} are defined as follows:

$$\begin{aligned} B_{12} = {} & 2cos\left(2\theta_2 + \frac{d_1}{\ell_1 u_0}\right) - 2cos\,\frac{d_1}{\ell_1 u_0} \\ & + \left(2\theta_2 + \frac{d_1}{\ell_1 u_0}\right) sin\left(2\theta_2 + \frac{d_1}{\ell_1 u_0}\right) - \frac{d_1}{\ell_1 u_0}\, sin\,\frac{d_1}{\ell_1 u_0} \\ & - 2cos\left(2\theta_2 + \frac{d_1}{\ell_1 u_0} + \frac{1}{u_0}\right) + 2cos\left(\frac{d_1}{\ell_1 u_0} + \frac{1}{u_0}\right) \\ & - \left(2\theta_2 + \frac{d_1}{\ell_1 u_0} + \frac{1}{u_0}\right) sin\left(2\theta_2 + \frac{d_1}{\ell_1 u_0} + \frac{1}{u_0}\right) \\ & + \left(\frac{d_1}{\ell_1 u_0} + \frac{1}{u_0}\right) sin\left(\frac{d_1}{\ell_1 u_0} + \frac{1}{u_0}\right) \end{aligned} \tag{6.9}$$

$$\begin{aligned} A_{12} = {} & 2sin\left(2\theta_2 + \frac{d_1}{\ell_1 u_0} + \frac{1}{u_0}\right) - 2sin\left(\frac{d_1}{\ell_1 u_0} + \frac{1}{u_0}\right) \\ & - \left(2\theta_2 + \frac{d_1}{\ell_1 u_0} + \frac{1}{u_0}\right) cos\left(2\theta_2 + \frac{d_1}{\ell_1 u_0} + \frac{1}{u_0}\right) + \left(\frac{d_1}{\ell_1 u_0} + \frac{1}{u_0}\right) cos\left(\frac{d_1}{\ell_1 u_0} + \frac{1}{u_0}\right) \\ & - 2sin\left(2\theta_2 + \frac{d_1}{\ell_1 u_0}\right) + 2sin\,\frac{d_1}{\ell_1 u_0} \\ & + \left(2\theta_2 + \frac{d_1}{\ell_1 u_0}\right) cos\left(2\theta_2 + \frac{d_1}{\ell_1 u_0}\right) - \frac{d_1}{\ell_1 u_0}\, cos\,\frac{d_1}{\ell_1 u_0} \end{aligned} \tag{6.10}$$

In deriving expressions (6.6) - (6.10) in addition to the small signal approximation conditions $k_1/u_0 < 1$ and $k_2/u_0 < 1$ we assumed also that the length of the drift tube is small enough so that the inequality $k_1 d_1/u_0 \ell_1 < 1$ is still valid.

Expression (6.8) has a simple meaning. Namely, the first harmonic of the current at the entrance to the cavity plays the roll of the external excitation current, while the first harmonic of the current produced by the gap voltage loads the cavity by changing its parameters exactly in the same way as it is described by expressions (5.20) and (5.21).

7. Narrow Gap Approximation

In this section, we derive an analytic solution to the Vlasov equation in the limit of narrow gap. The expansion is not restricted to small signals, but the result is consistent with small signal theory in the proper limit.

The assumption of narrow gap allows us to expand the distribution function in a Taylor series. We have in general

$$\psi(z) = \sum_{n=0}^{\infty} \frac{1}{n!} \left.\frac{\partial^n \psi}{\partial z^n}\right|_{z_0} (z - z_0)^n \tag{7.1}$$

For a narrow gap, the solution is given by the first few terms of the series. We will work out the example of an initial cold distribution, with

$$\psi_0(v, t) = \frac{I_0}{e v_0} \delta(v - v_0) \tag{7.2}$$

for which the general solution is given by

$$\psi(z, v, t) = \frac{I_0}{e v_0} \delta \left(v - \int_{\Theta}^{t} a(t') dt' - v_0 \right) \tag{7.3}$$

where Θ is given by

$$z - z_0 = v(t - \Theta) - \int_{\Theta}^{t} (t' - \Theta) a(t') dt' \tag{7.4}$$

and $a(t) = eE(t)/m$ (see (3.2)). We will perform the expansion of (7.3) up to $n = 3$. The first term in the series ($n = 0$) is given of course by $\psi_0(v, t)$ as defined in Eq. (7.2). To obtain the $n = 1$ term, we need to evaluate

$$\frac{\partial \psi}{\partial z} = \frac{\partial \psi_0}{\partial v}\; a(\Theta)\; \frac{\partial \Theta}{\partial z} \tag{7.5}$$

Differentiating Eq. (7.4), we obtain

$$\frac{\partial z}{\partial \Theta} = -v + \int_{\Theta}^{t} a(t')dt' \tag{7.6}$$

The coefficient of the $n = 1$ term is then given by

$$\left.\frac{\partial \psi}{\partial z}\right|_{z_0} = -\frac{I_0}{ev_0}\delta'(v - v_0)a(t)/v \quad , \quad n = 1 \tag{7.7}$$

The coefficient of the $n = 2$ term is proportional to the second derivative of ψ and is evaluated to be

$$\left(\frac{ev_0}{I_0}\right) \left.\frac{\partial^2 \psi}{\partial z^2}\right|_{z_0} = \delta''(v - v_0)\; \frac{a^2(t)}{v^2} + \delta'(v - v_0)\left[\frac{a'(t)}{v^2} - \frac{a^2(t)}{v^3}\right] \quad , n = 2 \tag{7.8}$$

Note that δ' refers to the derivative of the delta function with respect to velocity while $a'(t)$ is a derivative of the acceleration with respect to time. A superscript with n primes refers to the nth derivative. Finally, the $n = 3$ coefficient is evaluated to be

$$\begin{aligned}\left(\frac{ev_o}{I_0}\right)\left.\frac{\partial^3 \psi}{\partial z^3}\right|_{z_0} &= -\,\delta'''(v - v_0)\; \frac{a^3}{v^3} + 3\delta''(v - v_0)\left[\frac{aa'}{v^3} + \frac{a^3}{v^4}\right] \\ &\quad -\delta'(v - v_0)\left[\frac{a''}{v^3} - \frac{4a'a}{v^4} + \frac{3a^3}{v^5}\right] \quad , n = 3\end{aligned} \tag{7.9}$$

The current is related to the first moment of the distribution function

$$I(z, t) = e\int dv\; v\; \psi(z, v, t) \tag{7.10}$$

In the Taylor series expansion of ψ, the velocity integrals may be evaluated term by term. The $n = 0$ term gives rise to the d.c. component of the current since

$$e\int dv\ v\ \psi_0 = \frac{I_0}{v_0}\int dv\ v\delta(v - v_0) = I_0 \tag{7.11}$$

The $n = 1$ component gives no contribution since

$$e\int dv\ v\ \frac{\partial\psi_0}{\partial z}\bigg|_{z_0} = -\frac{I_0}{v_0}\ a(t)\int dv\ \delta'(v - v_0) = 0 \tag{7.12}$$

The $n = 2$ component of the distribution function has terms which are proportional to $a^2(t)$. However, these two terms cancel exactly when we take the velocity moment of the distribution function. We are then left with a contribution to the rf current

$$e\int dv\ v\ \frac{1}{2}\ \frac{\partial^2\psi_0}{\partial z^2}\bigg|_{z_0}(z - z_0)^2 = \frac{I_0}{2v_0^3}\ a'(t)(z - z_0)^2 \tag{7.13}$$

To evaluate the $n = 3$ component of the current, we take the velocity moment of Eq. (7.9). Again, the terms proportional to $a^3(t)$ vanish and we obtain

$$e\int dv\ v\ \frac{1}{6}\ \frac{\partial^3\psi_0}{\partial z^3}\bigg|_{z_0}(z - z_0)^3 = I_0\left(\frac{5aa'}{v_0^5} - \frac{a''}{3v_0^4}\right)(z - z_0)^3 \tag{7.14}$$

In performing the velocity integrals, we have made use of the delta function identity

$$\int g(v)\ \delta^{(n)}(v - v_0)dv = (-1)^n\ \frac{\partial^n g}{\partial v^n}\bigg|_{v_0} \tag{7.15}$$

Combining these results, we have that to $n = 3$ in the Taylor series expansion,

$$I(z,t) = I_0\left\{1 + \frac{a'(t)}{2v_0^3}(z - z_0)^2 - \left[\frac{a''}{3v_0^4} - \frac{5aa'}{v_0^5}\right](z - z_0)^3\right\} \tag{7.16}$$

The term which is proportional to aa' represents our first explicit nonlinear contribution to the current. However, it is clear that if $a(t)$ is a pure first harmonic, the quadratic term in a can contribute only to the zeroth and second harmonic. Hence, to the order considered, there is no higher order contribution to the rf component of the current.

We now turn to examine more carefully the z-dependence of I_1. The first harmonic of the current is related only to the linear terms in $a(t)$.

For

$$a(t) = \frac{eU_1}{m\ell} cos(\omega t + \varphi) \tag{7.17}$$

we have

$$I_1(z,t) \approx \frac{I_0 \omega}{2v_0^3} \frac{eU_1}{m\ell}(z - z_0)^2 cos\left(\omega t + \varphi + \frac{\pi}{2} - \frac{2\omega(z - z_0)}{3v_0}\right) \tag{7.18}$$

Hence the derived formula predicts a quadratic z-dependence of the amplitude and a linear z-dependence of the phase of I_1. These features are consistent with the numerical results presented in Fig. 4 where the self consistent solution for the amplitude and the phase of the first harmonic of the current are plotted as functions of the dimensionless distance (4.4) inside the gap. The magnitude of the amplitude and rate of phase change are also in agreement.

8. Numerical Results. Comparison With The Small Signal Approximation.

Here we apply derived formulae to the SLAC XK-5 klystron. Table 1 contains its relevant parameters [7]. In general, the small signal approximation gives correct results for the first cavity. This is true due to small values of both the input power and the length of the gap. In addition, the initial distribution of electrons at the cavity entrance in velocities has very small velocity spread and is constant in time. Hence, in this case the results obtained by using the self-consistent solution agree with the small signal approximation [4]. To model the effect of an initial velocity spread, the distribution function at entrance is assumed to be a Gaussian in velocity with the dispersion σ. The Vlasov equations for the two cavities are solved numerically. Figure 5 illustrates the dependence of the first cavity gap voltage k_{1sc} and its phase φ_{1sc} on σ of the initial Gaussian distribution. For σ/v_0 smaller than 10^{-2} the result is the same as for zero spread velocity beam ($\sim \delta(v - v_0)$). At the same time this is the condition of the validity of the small signal approximation.

Similar results are found for the second (idle) cavity. Figure 6 presents the amplitude and phase of the second gap voltage, as functions of σ/v_0 (solid curves). Here again, the small signal approximation (dashed curves) gives correct values for $\sigma/v_0 \lesssim 10^{-2}$.

On Figs. 7 and 8 the amplitudes and phases of the current first harmonic are given at the exit of the first and at the exit of the second cavities of the XK-5 klystron, respectively.

Figure 9 illustrates the self consistent (solid curves) and the small signal (dashed curves) solutions for the voltage and the current first harmonic for the entrance and the exit of second gap in function of the external excitation of the first cavity, respectively. Increasing the external excitation leads to an increasing I_1 at the entrance to the second cavity, as is evident from this plot. Figure 10 presents the same quanitities as functions of the drift length between the cavities.

9. Conclusions

The approach suggested in this work proves to be correct. The results obtained agree to a great accuracy with the small signal approximation. In the limit of a narrow gap the solution gives valid results both for the amplitude and phase of the resonant harmonic of the beam current.

The next questions which should be addressed are how useful and how convenient is the Vlasov approach in general and with respect to the klystron problem in particular. The calculation of the particle distribution along the klystron tube seems to be straight forward although substantial work need to be done.

Nevertheless, the approach looks promising. One can attempt to develop a one-dimensional model of a klystron which will include all important physics of the beam dynamics in multicavity system, including the interaction with the output cavity and crossover of the electron trajectories. The model takes into account the space charge effects in the cavities. The debunching effect of the space charge in drift sections of the klystron can be evaluated in perturbative manner using the ballistic approximation as the unperturbed solution. Such a model might be usefull as a fast and convenient tool for the klystron design. It can also provide information (at least as the first guess) on the amplitudes and the phases of the gap voltages for klystron cavities. That might be useful as the input for more elaborate numerical models of a klystron.

Further work is needed to extend the present formulation into the region of relativistic velocities.

Acknowledgements

The authors are grateful to all members of the Numerical Analysis Group for the interest and stimulating discussions of the problem. Our special gratitude goes to B. Herrmannsfeld and P. Wilson for the encouragement and help in our work.

References

1. Vlasov, A. A., "On the Kinetic Theory of an Assembly of Particles with Collective Interaction." J. Phys. (U.S.S.R.) 9, 25 (1945).
2. See, for example, N. A. Krall and A. W. Trivelpiece, "Principles of Plasma Physics," McGraw-Hill (1973).
3. T. H. Stix, "The Theory of Plasma Waves," McGraw-Hill (1962).
4. S. Kheifets, S. Yu and J. Jaeger, "Analytic solution for the problem of gridded gap - electron flow interaction", SLAC report SLAC/AP-13, January 1984, SLAC, Stanford University.
5. G. M. Branch, Jr., IRE Trans. on Electron Devices, ED-8, #3, May 1961, p. 193.
6. M. Chodorow and C. Susskind, "Fundamentals of Microwave Electronics," McGraw-Hill Company.
7. G. T. Konrad, "High Power RF Klystrons for linear accelerators", SLAC-PUB-3324, April 1984.

Table 1

XK-5 - Klystron

$eV_0 = 0.270000\text{E}+06$ eV

$\sigma = 0.130000\text{E}+06$ m/sec

$I_0 = 293.00$ A

$eV_{ext} = 0.384300\text{E}+04$ eV $\quad (k_{ext} = 0.495415\text{E-}01)$

***** First Cavity *****

$f_0 = 0.28560\text{E}+10$ Hz $\quad f_{1L} = 0.28442\text{E}+10$ Hz

$\ell_1 = 0.006500$ m

$Q_1 = 250.0$ $\quad Q_{1L} = 161.84656$

$(R/Q)_1 = 100.0$ Ohm

$(f - f_0)_1 = 0.40\text{E}+07$ Hz

***** Drift *****

$d_1 = 0.059850$ m

***** Second Cavity *****

$f_0 = 0.28560+10$ Hz $\quad f_{2L} = 0.2871\text{E}+10$ Hz

$\ell_2 = 0.005000$ m

$Q_2 = 2000.0$ $\quad Q_{2L} = 572.54130$

$(R/Q)_2 = 96.0$ Ohm

$(f - f_0)_2 = 0.90\text{E}+07$ Hz

Figure Captions

Fig. 1 Phase trajectories of an electron with the dimensionless velocity $u = 1$ entering the gap with the dimensionless field strength $k = 0.2$. The ratio u/k is plotted versus x/k (the ratio of the dimensionless coordinate x to the field strength k) for the different values of the field phase τ.

Fig. 2 The same as on Fig. 1 but for the field strength parameter $k = 0.25$. The onset of the particle crossover can be seen for large x/k.

Fig. 3 The same as on Fig. 1 but for the field strength parameter $k = 0.33$. The crossover is fully pronounced now.

Fig. 4 Dependence of the first harmonic of the current (relative to the *dc* current of the initial beam) found from a self consistent solution of the Vlasov equations on the coordinate x for small values of x. The left hand side scale is for the amplitude. The right hand side scale is for the phase. Both curves agree with the narrow gap calculation (see text).

Fig. 5 Dependence of the field strength in the first cavity of the klystron XK-5 found from the self consistent solution of the Vlasov equations on the velocity spread in the initial beam. The initial velocity distribution is assumed to be Gaussian with the mean velocity v_0 and the dispersion σ. The left hand side scale is for the dimensionless amplitude k_1 (curve a). The right hand side scale is for the phase φ_1 (curve b). The values of k_1 and φ_1 found from the small signal approximation are 0.191 and 0.451, respectively.

Fig. 6 The same as on Fig. 5 but for the second cavity of the klystron XK-5 (the solid curves). The dashed curves represent the small signal approximation (see section 6).

Fig. 7 Dependence of the first harmonic of the current at the end of the first cavity of the klystron XK-5. The left hand side scale is for the amplitude in A (curve a). The right hand side scale is for the phase (curve b).

Fig. 8 The same as on Fig. 7 but for the second cavity of the klystron XK-5.

Fig. 9 Dependence of the field strength k_2 in the second cavity and of the first harmonic of the current at its exit on the rf excitation in the first cavity. The left hand side scale is for k_2 (solid curve is for the self consistent solution, dashed curve is for the small signal approximation). The right hand side scale is for the amplitude of the current in A. Increase of the rf excitation (bottom scale is for the dimensionless amplitude k_{ext}) brings up to increase of the current harmonic on the entrance to the second cavity. The upper scale represents the corresponding harmonic amplitude in A.

Fig. 10 The same as on Fig. 9 but in function of the drift length (m) between the first and the second cavities. Increase of the drift length (bottom scale) brings up to increase of the current harmonic on the entrance to the second cavity. The upper scale represents the corresponding harmonic amplitude in A.

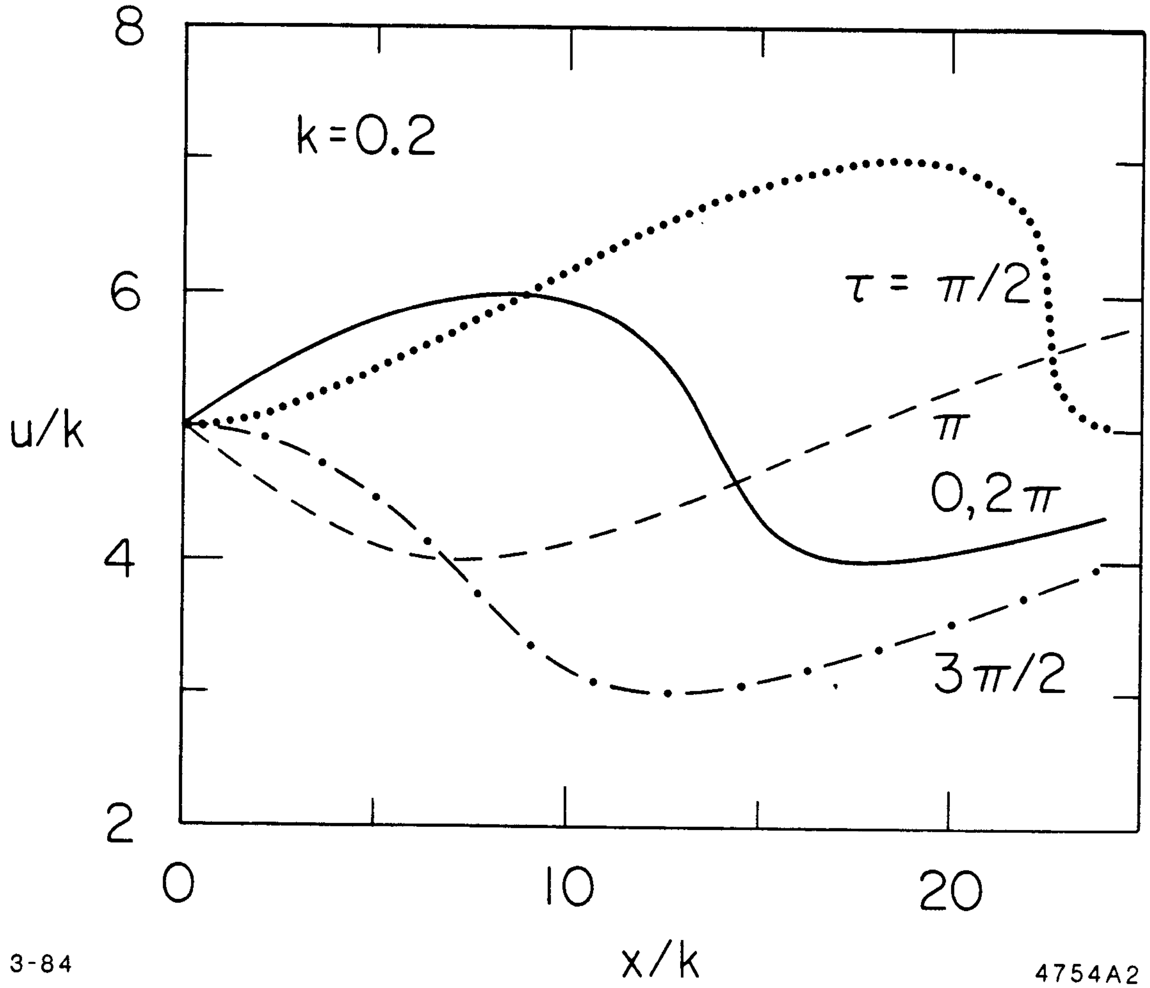


Fig. 1

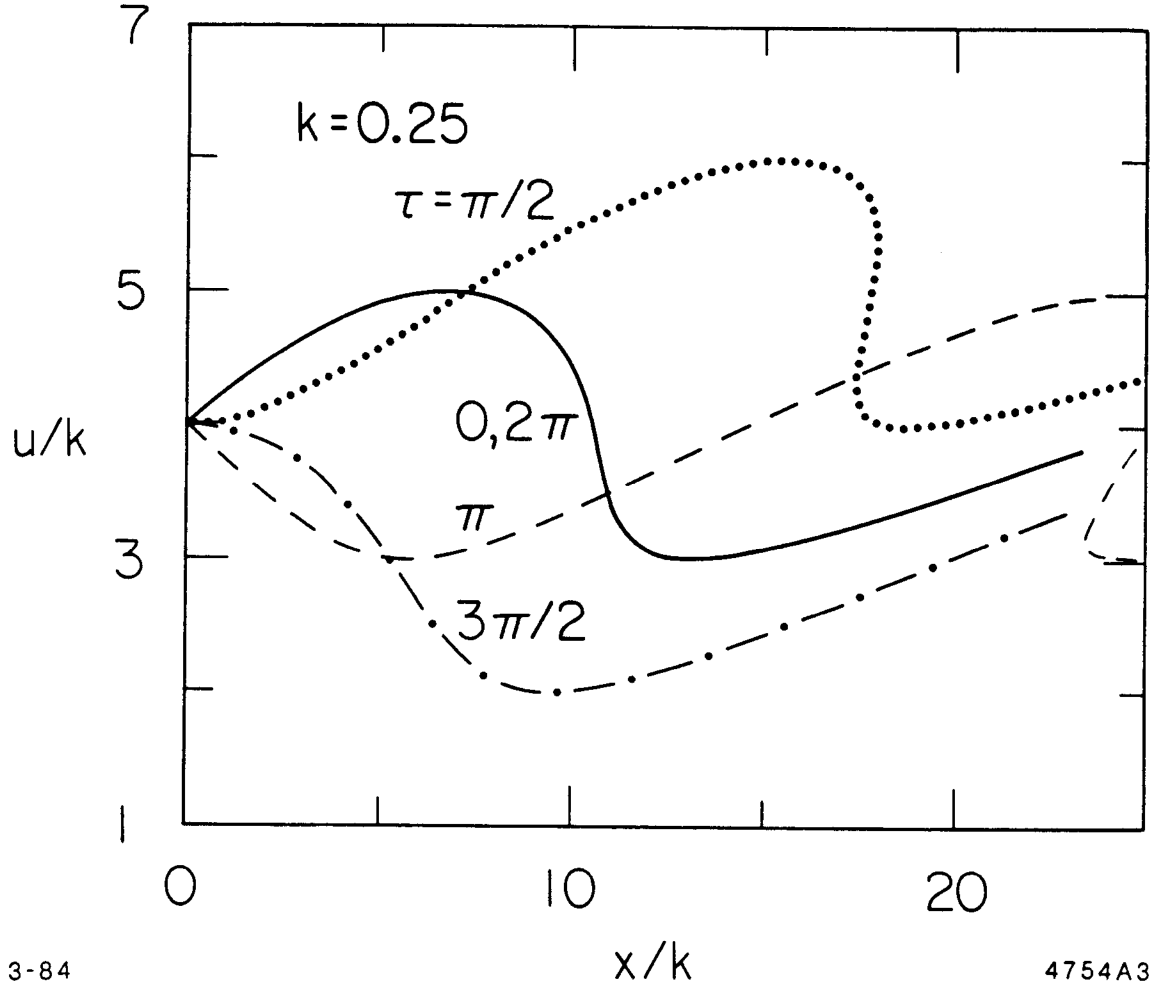


Fig. 2

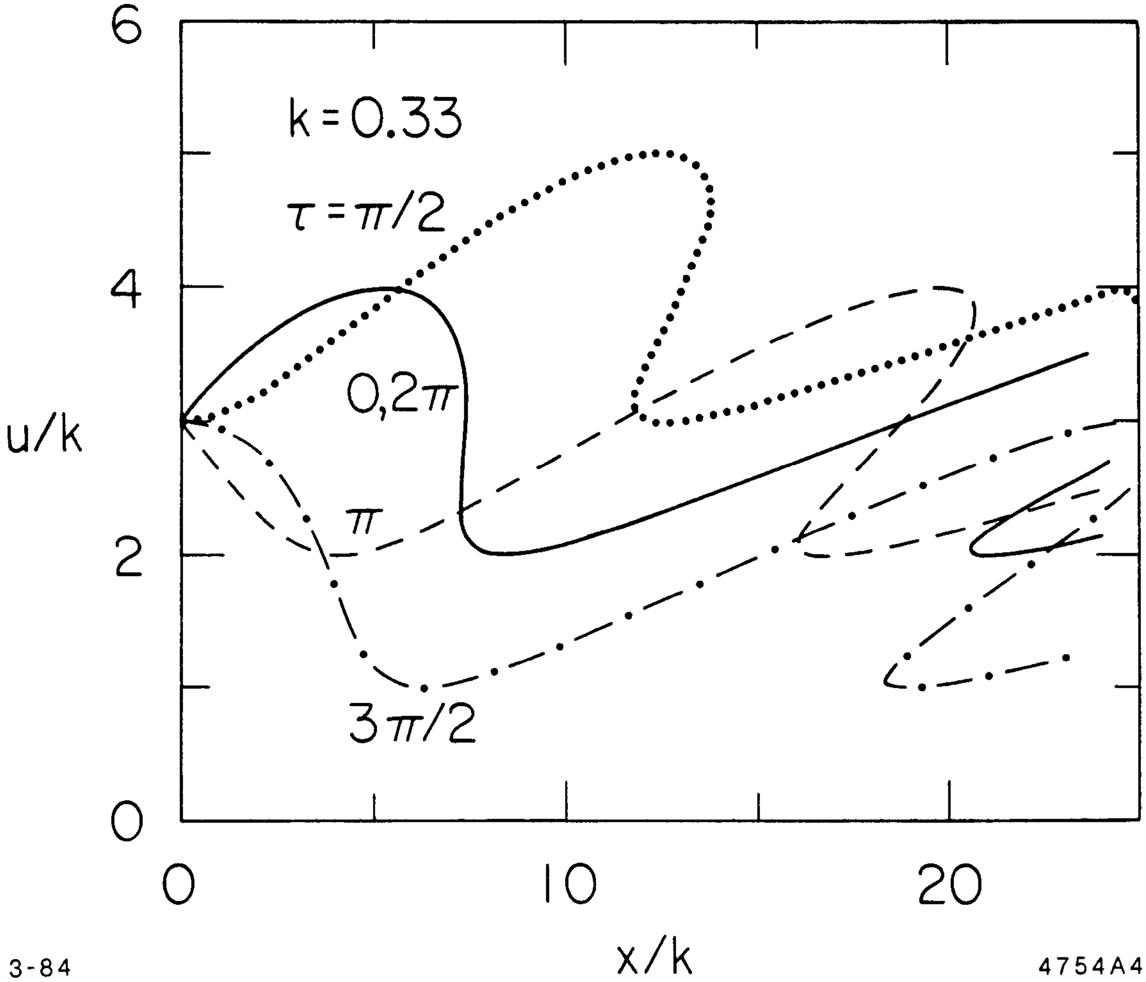


Fig. 3

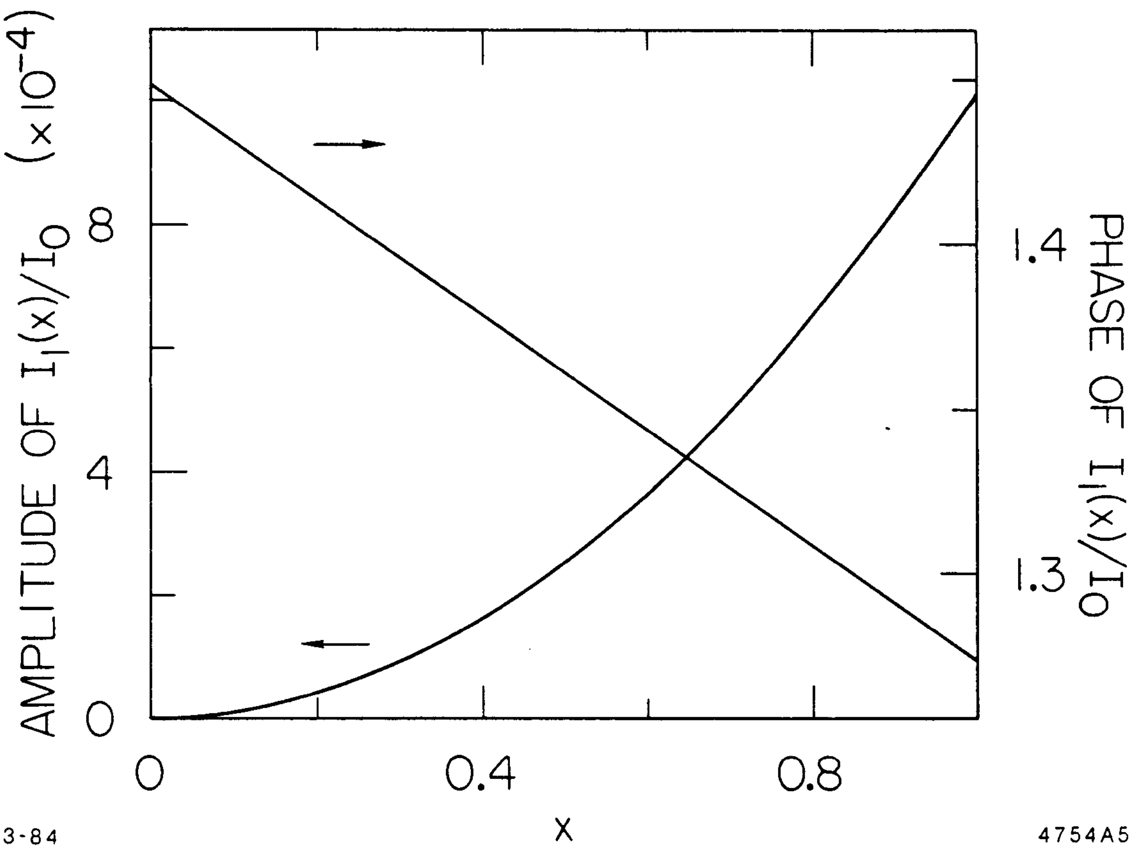

AMPLITUDE OF I1(x)/I0 (×10^-4)
0
4
8
PHASE OF I1(x)/I0
1.3
1.4
0
0.4
0.8
X
3-84
4754A5

Fig. 4

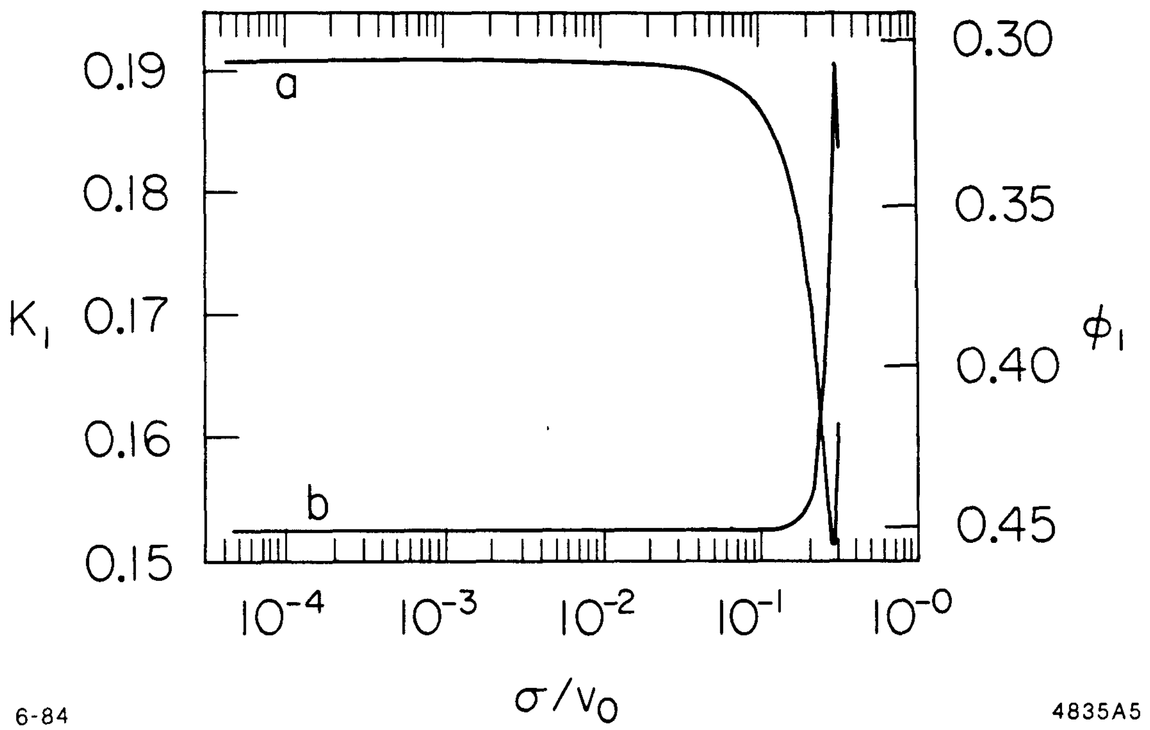

0.19
0.18
0.17
0.16
0.15
K_1
0.30
0.35
0.40
0.45
ϕ_1
a
b
10^{-4}
10^{-3}
10^{-2}
10^{-1}
10^{-0}
σ/v_0
6-84
4835A5

Fig. 5

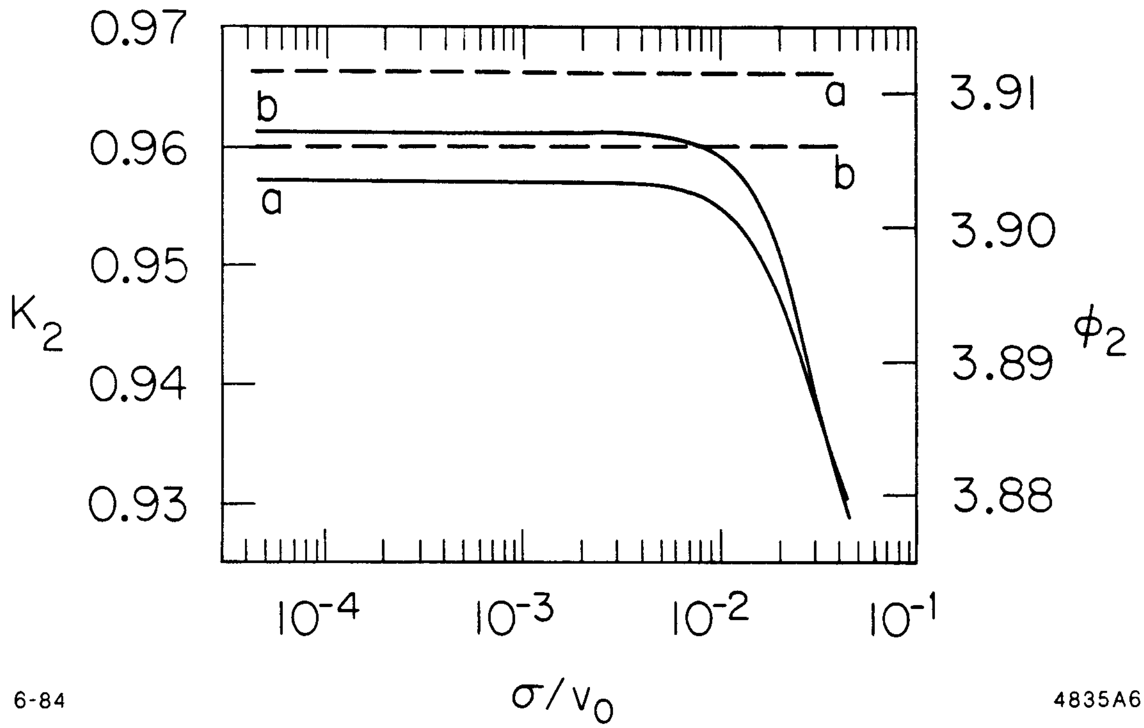
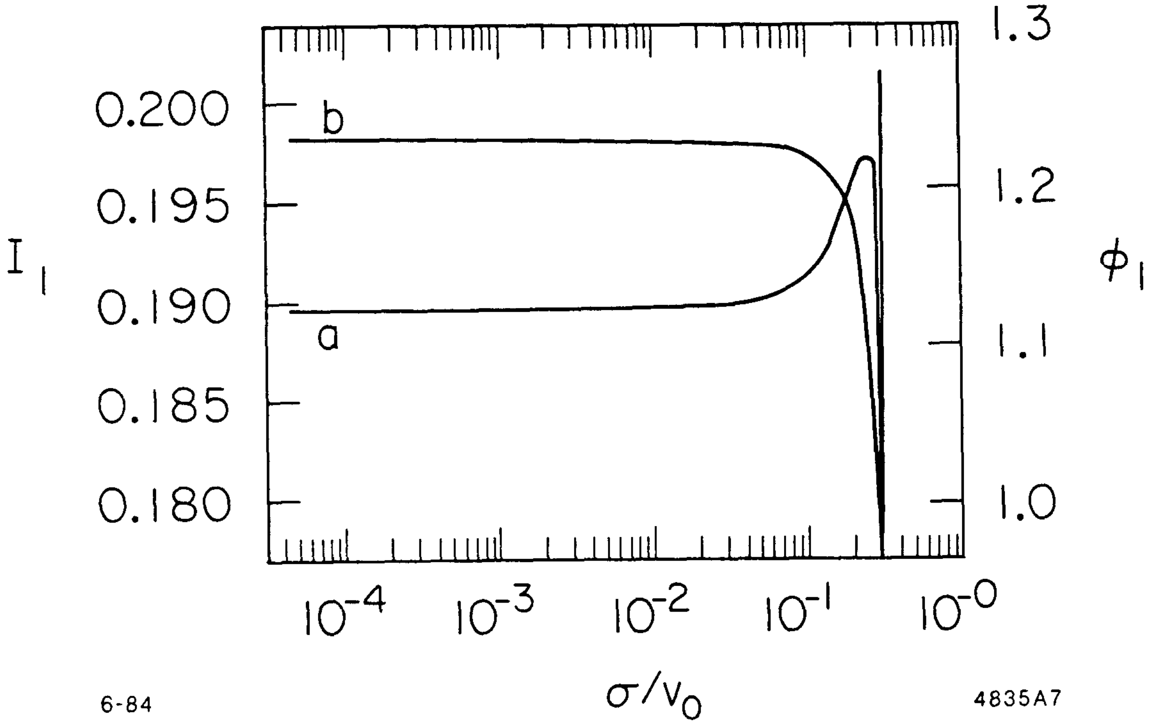
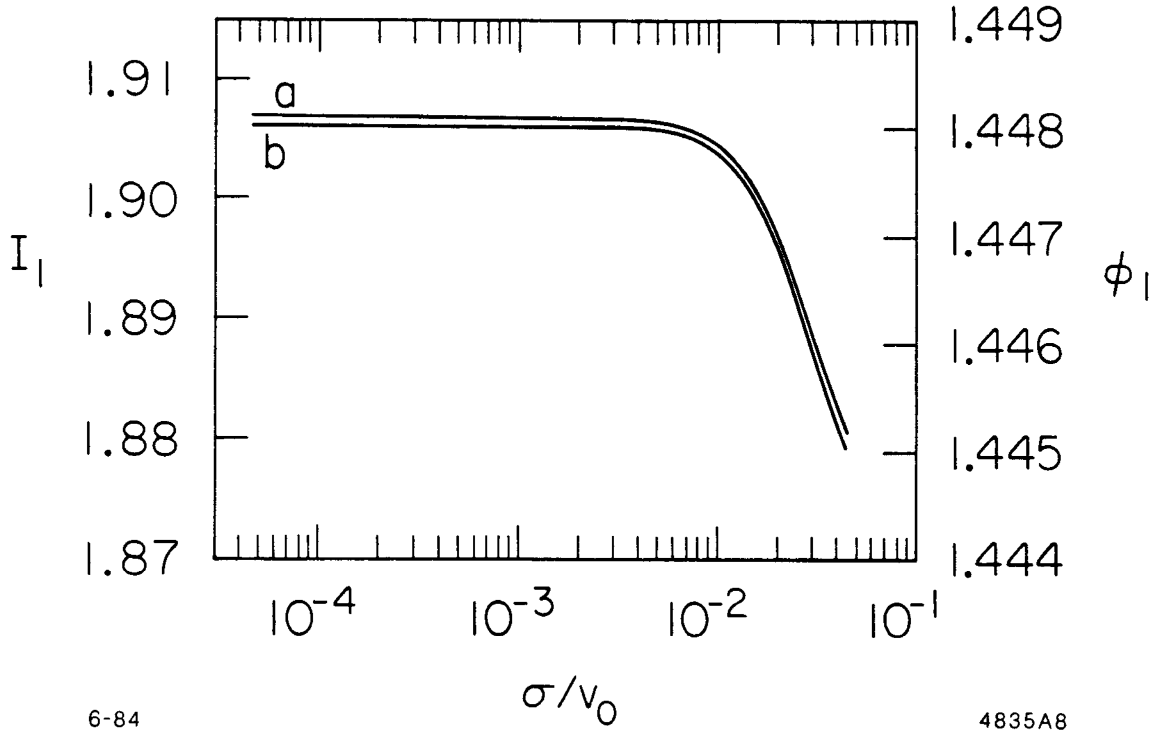

0.97
0.96
0.95
0.94
0.93
K_2
3.91
3.90
3.89
3.88
ϕ_2
a
b
b
a
10^{-4}
10^{-3}
10^{-2}
10^{-1}
6-84
σ/v_0
4835A6

Fig. 6



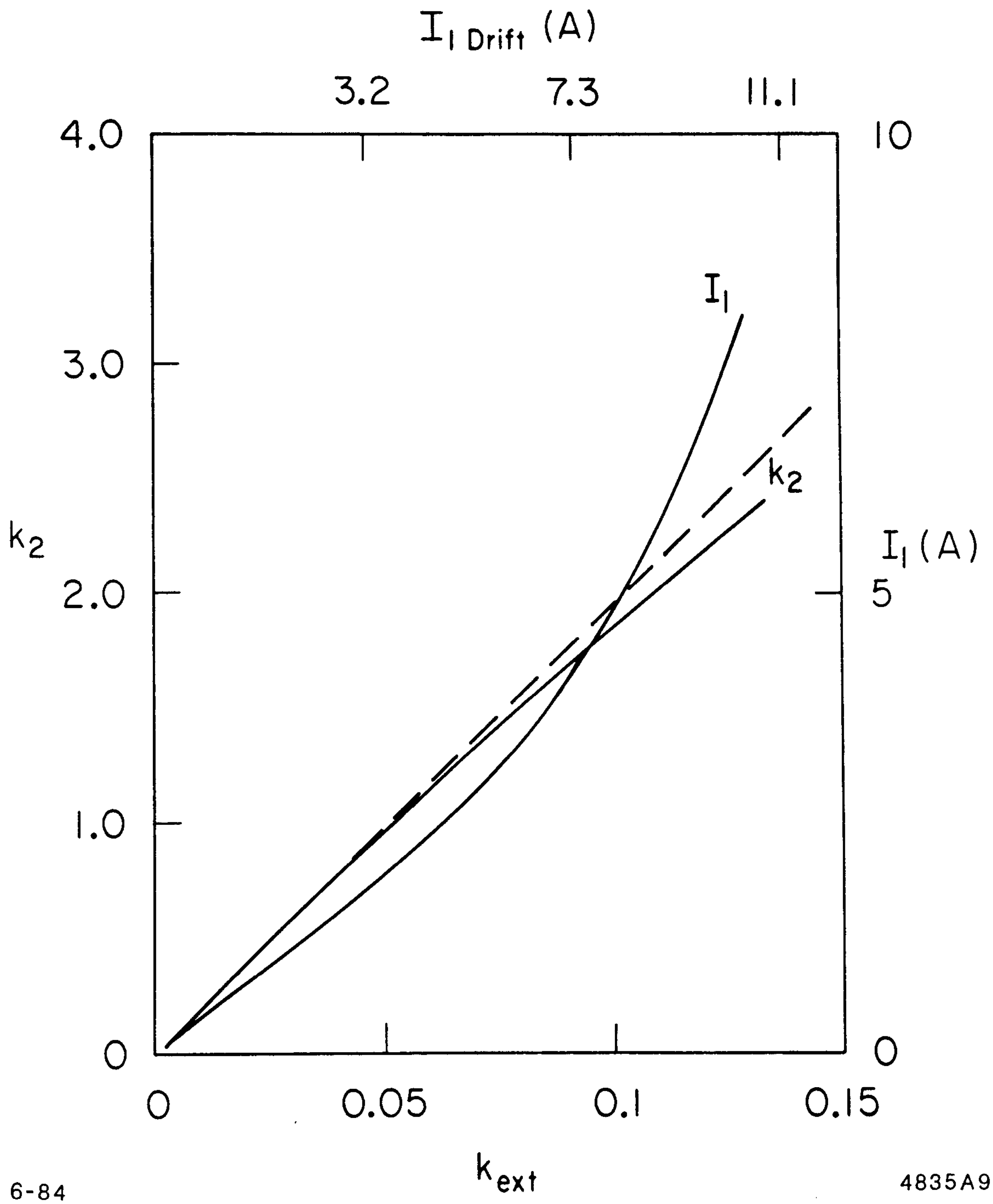
0.200
0.195
0.190
0.185
0.180
1.3
1.2
1.1
1.0
I 1
φ 1
b
a
10-4
10-3
10-2
10-1
10-0
σ/v0
6-84
4835A7

Fig. 7



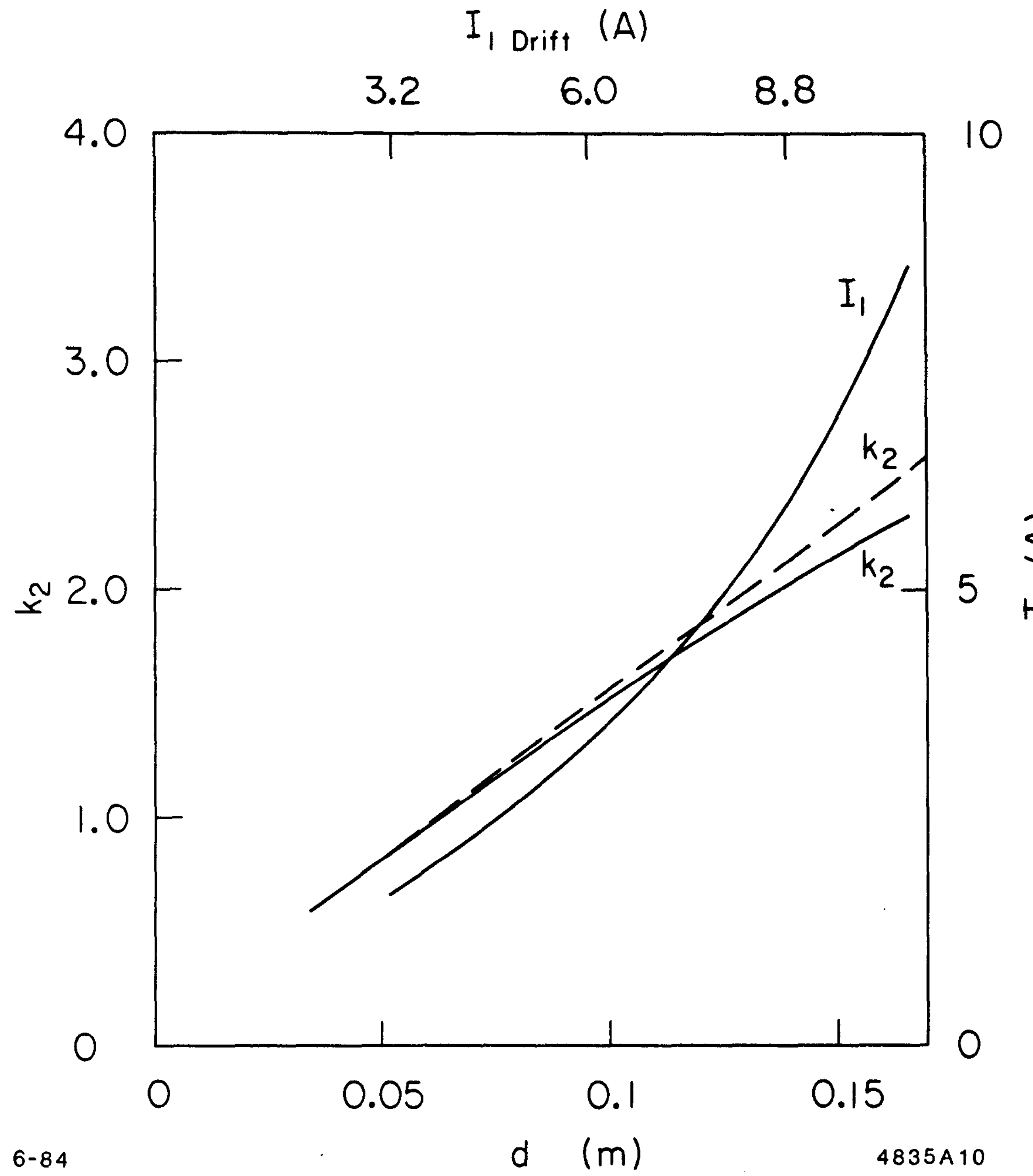
1.91
1.90
1.89
1.88
1.87
I_1
a
b
1.449
1.448
1.447
1.446
1.445
1.444
ϕ_1
10^{-4}
10^{-3}
10^{-2}
10^{-1}
σ/v_0
6-84
4835A8

Fig. 8



$I_{1\,\mathrm{Drift}}$ (A)
3.2
7.3
11.1
4.0
3.0
2.0
1.0
0
10
5
0
k_2
I_1 (A)
I_1
k_2
0
0.05
0.1
0.15
k_{ext}
6-84
4835A9

Fig. 9



$I_{1\ Drift}$ (A)
3.2
6.0
8.8
4.0
3.0
2.0
1.0
0
k_2
10
5
0
I_1 (A)
I_1
k_2
k_2
0
0.05
0.1
0.15
d (m)
6-84
4835A10

Fig. 10