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PROTON LIFETIME IN ORTHOGONAL THEORIES OF FAMILY UNIFICATION*

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Abstract

We compute the proton lifetime in recently proposed orthogonal theories of family unification. For $\Lambda_{\overline{MS}} = 100$ MeV, we find a partial lifetime $\tau(p \to e^+ \pi^0)$ of $5.9 \times 10^{31\pm1}$ years, where the error in the exponent comes from uncertainties in the hadronic wave function. Important decay products include electrons, neutrinos and non-strange mesons.

1. Introduction

The most appealing theories of family unification are based on the group O(18). There are many reasons for this [1,2]:

- All the known families fit into just one representation, the 256-dimensional spinor.
- The spinor is complex, so superheavy masses for ordinary families are forbidden.
- The group O(18) is automatically anomaly-free.

Other theories suffer from a variety of afflictions. Some need many different representations to conspire to give three families without anomalies. Others allow superheavy invariant masses. Still others rely on unnatural pseudosymmetries to keep ordinary families light.

Previous attempts to construct theories based on O(18) were plagued by serious difficulties. These stem from the fact that the 256-dimensional spinor contains eight left- and eight right-handed families. Because of the large number of families, asymptotic freedom is lost and coupling constants blow up at a few hundred TeV. These theories are not perturbatively unifiable.

To avoid this problem, it is necessary to split the O(18) spinor, and give some families mass at the grand unified scale M_{GUT}. In Ref. [2] it was shown that this necessarily leads to four left- and four right-handed families in the low-energy world. With eight light families, the theory is pertubatively unifiable, and proton decay is calculable.

Since O(18) predicts both left- and right-handed families in the low-energy world, one must still explain why the right-handed families are heavier than their left-handed counterparts. O(18) group theory provides a natural explanation for this, since it allows the Weinberg-Salam doublet to couple only to the right-handed families [2].* These families receive mass directly at the weak scale, whereas masses for the left-handed families are induced by one-loop radiative corrections.

The O(18) theory of Ref. [2] contains eight light neutrinos, one for each light family. In one version of this theory, all eight neutrinos acquire masses of order ~ 0.1 eV. This is in potential conflict with the simplest version of standard big bang nucleosynthesis [3]. Too many neutrinos can lead to an overabundance of primordial deuterium and/or helium.† Since the number of light neutrinos will soon be measured in Z^0 decays, we do not wish to exclude the possibility that $N_\nu = 8$.

A second version of the O(18) theory contains four light neutrinos, with masses in the electrovolt range, and four heavier neutrinos, with masses between 2 and 35 GeV. The heavier neutrinos receive mass by coupling to an isotriplet of Higgs scalars. The lower limit of 2 GeV is effectively the Lee-Weinberg bound [5], and the upper limit of 35 GeV follows from the experimental fact that the ρ parameter is so close to one. In this second version of the model, the standard big bang picture of nucleosynthesis carries through unchanged.

* For simplicity, we only consider the case of one such doublet.

† It is possible to evade the nucleosynthesis constraints by invoking additional processes such as the photo- or neutrino-dissociation of deuterium [4].

2. Renormalization Group in O(18)

Measurements of the proton lifetime exclude the minimal SU(5) grand unified theory [6,7]. They also exclude a much wider class of models, characterized by the following properties:‡

— The lightest superheavy gauge bosons mediate proton decay in the $e^+ \, \pi^0$ channel;

— The low energy particles form complete SU(5) multiplets;

— SU(3) × SU(2) × U(1) is the effective theory up to the grand unified scale; and

— The SU(3) × SU(2) × U(1) beta functions are well approximated by one-loop results.

All models in this class share a common one-loop unification mass M_{GUT} [9]. Since extra families form complete SU(5) multiplets, they do not change M_{GUT}, unless they lead to a vanishing one-loop beta function. In this case, two-loop contributions dominate the evolution of the gauge coupling constants, and the unification scale can be very different.

This is precisely the case for eight light families. With eight light families, α_3 barely evolves, as we see from the SU(3) beta function [10],

$$\frac{\mathrm{d}g_3}{\mathrm{dt}} = -\frac{1}{3}\frac{{g_3}^3}{16\pi^2} + \frac{302}{3}\frac{{g_3}^5}{(16\pi^2)^2} \quad . \tag{1}$$

The one-loop contribution is small, so the beta function is dominated by the two-loop term. This is illustrated in Figure 1, where we graph the one- and

‡ Other ways of increasing the proton lifetime are considered in Ref. [8].

two-loop evolution of the gauge coupling constants for the standard three-family model, as well as the one- and two-loop results for an eight-family theory. In Fig. 1*d* we see that eight families give a two-loop unification mass significantly larger than the one-loop result. We notice that the running coupling constants at the unification mass are larger than their standard values, which are shown in Fig. 1*b*.

In O(18), eight light families are a direct consequence of family unification [2]. They increase M_{GUT}, as shown in Figure 1, and evade the minimal SU(5) constraints on proton decay. It is important to remember that eight families are a necessary result of splitting the O(18) spinor. They were not added by hand simply to change the one-loop proton lifetime.

3. Proton Decay

The lifetime of the proton depends on the grand unified coupling and on the mass of the lightest gauge boson that mediates proton decay. These are found by solving the coupled system of differential equations describing the two-loop evolution of the SU(3) $\times$ SU(2) $\times$ U(1) coupling constants. For eight families, the SU(3) beta function is dominated by the two-loop contribution, so the usual approximations do not hold. Therefore, we solved the system numerically, integrating repeatedly between M_W and M_{GUT}, determining $\sin^2\theta$ and M_X, the mass of the lightest superheavy gauge boson. We took $\alpha_{em}(M_W)$ and $\alpha_3(M_W)$ as inputs, with $M_W = 80$ GeV and $\alpha_{em}(M_W) = 1/127.7$. We evolved α_3 from the four-flavor regime for $\Lambda_{\overline{MS}}$ between 50 and 200 MeV. At M_X, we took the matching conditions to be

$$\alpha_{GUT}^{-1} - \frac{5}{12\pi} = \alpha_3^{-1} - \frac{1}{4\pi} = \alpha_2^{-1} - \frac{1}{6\pi} = \alpha_1^{-1} \quad , \qquad (2)$$

where α_{GUT} denotes the grand unified coupling at the scale M_X. These are the SU(5) matching conditions [11], which we use because O(18) must be broken to SU(5) at a scale much larger than M_X [2]. We have verified that corrections to the matching conditions induced by the superheavy families and scalars are negligibly small.

In Table 1 we present our results for M_X, α_{GUT} and $\sin^2\theta$. In the first row we list the predictions of the standard SU(5) model. In subsequent rows we display the results for both versions of the O(18) theory, evaluated for different values of $\Lambda_{\overline{MS}}$. As expected, the O(18) values for M_X and α_{GUT} are larger than the minimal SU(5) predictions.

In the last column of Table 1 we present our predictions for the partial lifetimes $\tau(p \to e^+ \pi^0)$. These are scaled from the usual SU(5) result as follows:

$$\frac{\tau(p \to e^+ \pi^0)}{\tau_5(p \to e^+ \pi^0)} = \left(\frac{M_X}{M_5}\right)^4 \left(\frac{\alpha_5}{\alpha_{GUT}}\right)^2 . \tag{3}$$

M_5 and α_5 are the standard SU(5) values for M_X and α_{GUT}, given in Table 1, and τ_5 is the usual SU(5) partial lifetime for $\tau(p \to e^+ \pi^0)$ [7,12]. Note that increased values of α_{GUT} tend to offset increased values of M_X in computing τ/τ_5 .

As we see from Table 1, the values for M_X depend strongly on $\Lambda_{\overline{MS}}$. They are more sensitive to $\Lambda_{\overline{MS}}$ than in ordinary grand unified theories because α_3 is large at all energies. The results in Table 1 are much less sensitive to other effects. We have checked the sensitivity to variations in $\alpha_{em}(M_W)$, M_W and the masses of the superheavy families, as well as to the presence of large Yukawa couplings for the four right-handed families. Even for large variations in these parameters, the changes in the proton lifetime are small compared to the uncertainties inherent in the hadronic wave function.

In Table 1 the masses of the four right-handed families are taken to be 125 GeV. This is the infrared fixed point [13] to which all eight heavy quarks evolve for sufficiently general mixings [14]. We have checked that the results in Table 1 are not sensitive to the precise values of the right-handed masses.

4. Discussion

Since our theory at M_X is an effective SU(5) theory, the decay modes are those of the standard model. We expect $p \rightarrow e^+ \pi^0$ to dominate the decay, and the $e^+\rho^0$ and $e^+\omega$ modes to be substantial [7,12]. The present experimental bounds for $\tau(p \rightarrow e^+ \pi^0)$ are of order 2×10^{32} years [15], a factor of 2000 greater than predicted by standard SU(5),

$$\tau(p \rightarrow e^+ \pi^0) = 1.2 \times 10^{29\pm1} \quad \text{years}, \tag{4}$$

for $\Lambda_{\overline{MS}} = 100$ MeV [7,12]. The error in the exponent stems from uncertainties in the hadronic wave function [7,12]. These uncertainties can account for a factor of 10, so viable models must give $\tau/\tau_5 \gtrsim 200$ in Table 1. We see that this rules out the version of the theory with the weak triplet. In fact, only one model survives. For $\Lambda_{\overline{MS}} = 100$ MeV, this model predicts

$$\begin{aligned} \tau(p \rightarrow e^+ \pi^0) &= 5.9 \times 10^{31\pm1} \quad \text{years,} \quad \text{and} \\ \sin^2\theta &= 0.214 \quad . \end{aligned} \tag{5}$$

Such a lifetime should soon be measured by experiment.

Higher values of $\Lambda_{\overline{MS}}$ increase the proton lifetime, but they also decrease $\sin^2\theta$. For $\Lambda_{\overline{MS}} = 200$ MeV, we find $\tau(p \rightarrow e^+ \pi^0) = 6.2 \times 10^{33\pm1}$ years, and $\sin^2\theta = 0.208$. However, both $\Lambda_{\overline{MS}} = 200$ MeV and $\sin^2\theta = 0.208$ are on the verge of being excluded by experiment. This sets an upper bound on the proton lifetime of order

$$\tau(p \rightarrow e^+\pi^0) \lesssim 6.2 \times 10^{33\pm1} \quad \text{years} \quad . \tag{6}$$

Since $\Lambda_{\overline{MS}} = 100$ MeV is the favored experimental value, and $\sin^2\theta = .214$ is consistent with experiment, the preferred value for $\tau(p \rightarrow e^+\pi^0)$ is $5.9 \times 10^{31\pm1}$ years.

The O(18) theory of Ref. [2] does not specify whether O(18) breaks to SU(5) or O(10). If O(18) breaks to the standard O(10) model [16], the partial lifetime $\tau(p \rightarrow e^+ \pi^0)$ is unchanged. The extra O(10) generators enhance the $p \rightarrow \overline{\nu}_e \pi^+$ and $p \rightarrow \overline{\nu}_e \rho^+$ decay modes relative to the SU(5) case [12,17].

For completeness, we list in Table 2 the predictions of other orthogonal models with six light families [18]. Most of these models are excluded, since they contain six light families, and the renormalization group analysis proceeds exactly as in standard SU(5). One model is marginally acceptable for the extreme value of $\Lambda_{\overline{MS}} = 200$ MeV.

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Table Captions

TABLE 1:

Results for the eight-family O(18) models with and without a weak triplet. In the last column, τ and τ_5 refer to the partial lifetimes of the $e^+ \pi^0$ decay mode. Our preferred predictions are underlined.

TABLE 2:

Results for the six-family models with and without a weak triplet. In the last column, τ and τ_5 refer to the partial lifetimes of the $e^+ \pi^0$ decay mode.

Figure Captions

FIGURE 1:

The SU(3) $\times$ SU(2) $\times$ U(1) running coupling constants:

(a) One-loop results for the standard model,

(b) Two-loop results for the standard model,

(c) One-loop results for the eight-family O(18) theory without weak triplets.

(d) Two-loop results for the eight-family O(18) theory without weak triplets.

Table 1: Eight Family Models

Model	$\Lambda_{\overline{MS}}$ (MeV)	M_X (10^{15} GeV)	$\alpha_{GUT}(M_X)$	$\sin^2\theta$	τ/τ_5
Standard SU(5)	100	0.13	0.024	0.216	1
Triplet	50	0.08	0.11	0.234	7.7×10^{-3}
Triplet	100	0.21	0.14	0.230	2.2×10^{-1}
Triplet	150	0.42	0.16	0.227	2.5
Triplet	200	0.68	0.19	0.225	1.3×10^{1}
No triplet	50	0.52	0.11	0.218	1.2×10^{1}
No triplet	<u>100</u>	<u>1.46</u>	<u>0.14</u>	<u>0.214</u>	$\underline{4.9} \times \underline{10^{2}}$
No triplet	<u>150</u>	<u>3.18</u>	<u>0.17</u>	<u>0.210</u>	$\underline{7.4} \times \underline{10^{3}}$
No triplet	200	5.69	0.21	0.208	5.2×10^{4}

Table 2: Six Family Models

Model	$\Lambda_{\overline{MS}}$ (MeV)	M_X (10^{15} GeV)	$\alpha_{GUT}(M_X)$	$\sin^2\theta$	τ/τ_5
Triplet	50	0.03	0.044	0.234	6.0×10^{-4}
Triplet	100	0.06	0.046	0.231	9.3×10^{-3}
Triplet	150	0.09	0.047	0.228	5.8×10^{-2}
Triplet	200	0.12	0.048	0.227	1.9×10^{-1}
No triplet	50	0.14	0.043	0.219	4.8×10^{-1}
No triplet	100	0.30	0.044	0.215	8.9
No triplet	150	0.49	0.045	0.213	6.2×10^{1}
No triplet	200	0.68	0.046	0.211	2.2×10^{2}

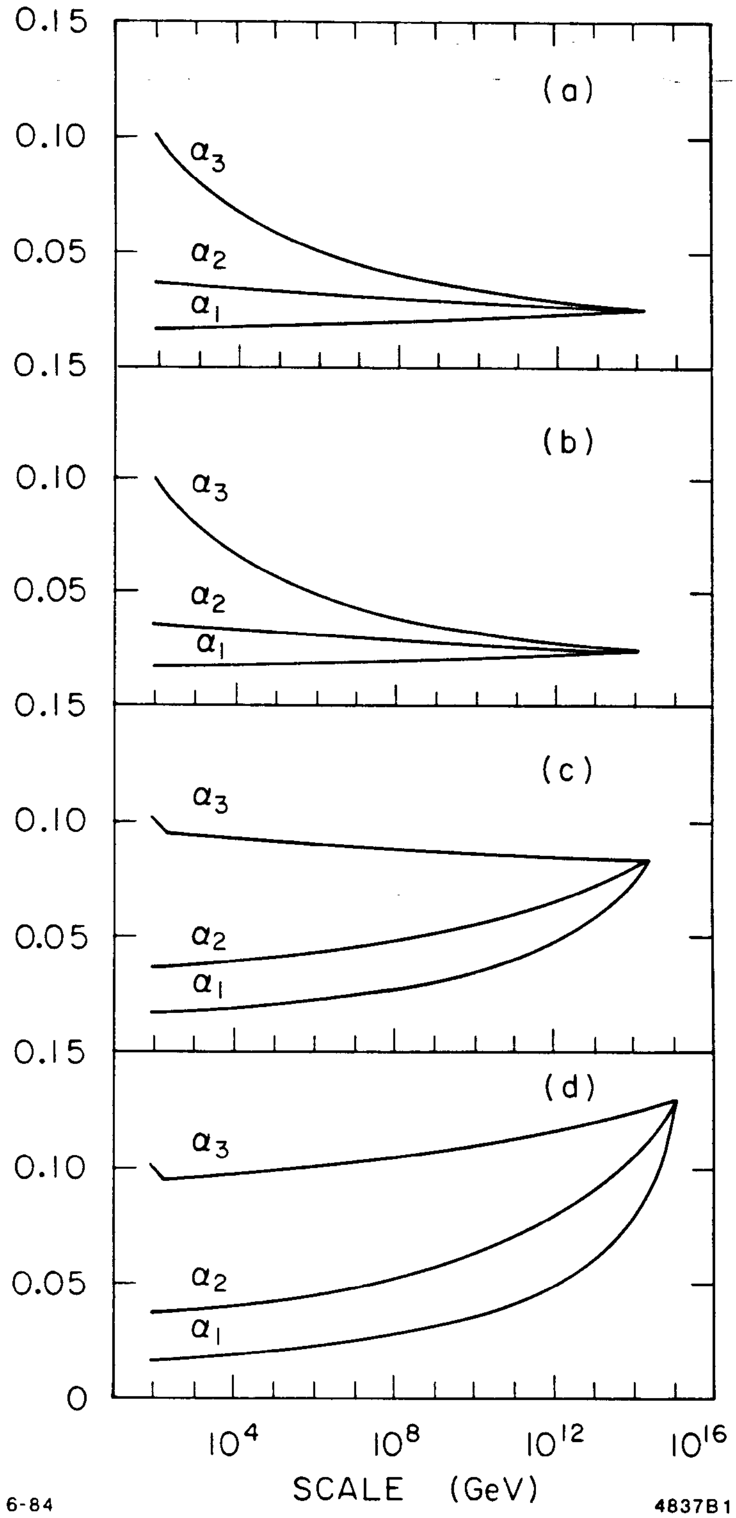
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Fig. 1