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QUANTUM CHROMODYNAMIC EVOLUTION OF MULTIQUARK SYSTEMS*

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Abstract

We present a new technique which extends the quantum chromodynamic evolution formalism in order to predict the short distance behavior of multiquark wavefunctions. In particular, predictions are given for the deuteron reduced form factor in the high momentum transfer region, and rigorous constraints on the short distance effective force between two baryons are predicted. These new techniques can be generalized in order to analyze the short distance behavior of multibaryon systems.

1. Introduction

A systematic analysis in quantum chromodynamics (QCD) of exclusive processes involving the transfer of large momenta has been presented in Ref. 1. A large number of experimentally accessible phenomena including the elastic and inelastic electromagnetic and weak form factors and large-angle elastic scattering processes can be analyzed in terms of a simple picture for exclusive processes based on light-cone perturbation theory. For example, the baryon form factor at large Q^2 is represented by the factorized form (see Fig. 1)[2]

$$
\begin{aligned}
F_B(Q^2) &= \int_0^1 [dx] \int_0^1 [dy]\; \phi^*(y_i,\tilde{Q}_y)\, T_H(x_i,y_i,Q)\, \phi(x_i,\tilde{Q}_x) \left[1 + O\left(\frac{m_i^2}{Q^2}\right)\right] \\
&= \frac{32\pi^2}{9}\, \frac{\alpha_s^2(Q^2)}{Q^4} \sum_{n,m} b_{nm} \left(\ell n\, \frac{Q^2}{\Lambda^2}\right)^{-\gamma_n-\gamma_m} \left[1 + O\left(\alpha_s(Q^2), \frac{m_i^2}{Q^2}\right)\right] \\
&\to C \left(\frac{\alpha_s(Q^2)}{Q^2}\right)^2 \left(\ell n\, \frac{Q^2}{\Lambda^2}\right)^{-2\gamma_0} \quad (\text{as } Q^2 \to \text{large}) \quad , \qquad (1.1)
\end{aligned}
$$

where x_i is the light-cone longitudinal momentum fraction of i^{th} quark $x_i = (k_i^0 + k_i^3)/(p^0 + p^3)$, $[dx] \equiv dx_1\, dx_2\, dx_3\, \delta\,(1 - \sum_i x_i)$ and $\tilde{Q}_x \equiv \min_i(x_i Q)$. The

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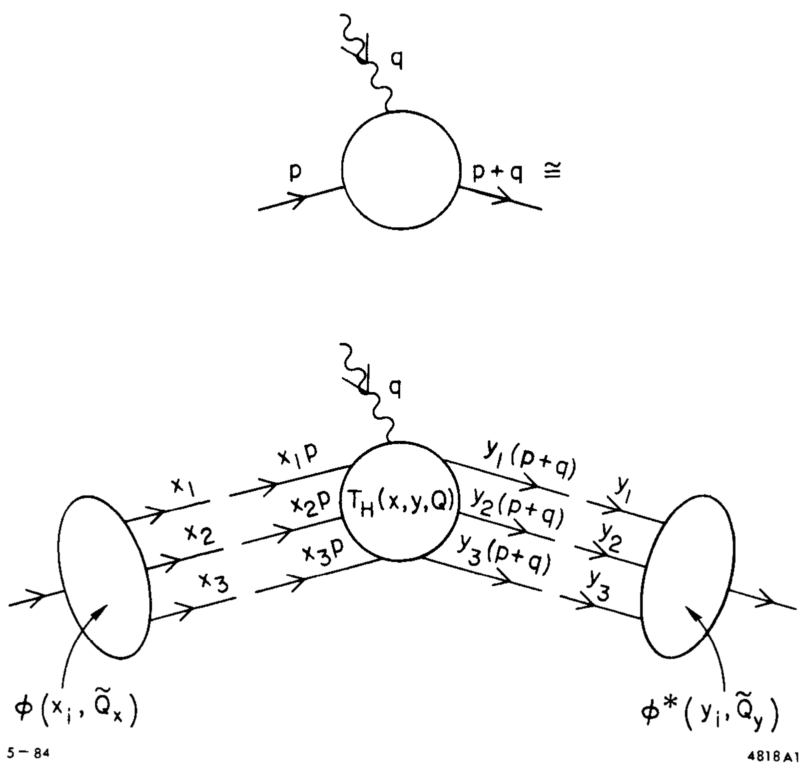

Fig. 1. The general structure of the baryon magnetic form factor at large Q^2.

dominant Q^2 dependence $(\alpha_s(Q^2)/Q^2)^2$ is derived from the hard scattering amplitude $T_H(x_i, y_i, Q)$. For $\gamma^* + 3q \rightarrow 3q$ with the only weak (logarithmic) Q^2 dependence coming from quark distribution amplitude $\phi(x_i, Q)$ (γ_0 is the leading anomalous dimension). The essential feature of Eq. (1.1) is that a very complicated process can be simply represented by the factorization into product of three amplitudes, and, thus, the main calculation for this process turns out to be the calculation of T_H (sum of tree diagrams for $\gamma^* + 3q \rightarrow 3q$). The distribution amplitude $\phi(x_i, Q)$ is in principle determined by nonperturbative bound state physics, and it is independent of the process.

The quark distribution amplitude $\phi(x_i, Q)$ is the amplitude for converting the baryon into three valence quarks at impact separation $b_\perp \sim O(1/Q)$. It is related to the equal $\tau = t + z$ hadronic wave function $\psi(x_i, \vec{k}_{\perp i})$[3]:

$$\phi(x_i, Q) \propto \int^{Q} \prod_{i=1}^{3} d^2\vec{k}_{\perp i}\, \delta^2\left(\sum_i \vec{k}_{\perp i}\right) \psi(x_i, \vec{k}_{\perp i}) \quad , \tag{1.2}$$

and contains the essential physics of that part of the hadronic wave function which affects exclusive processes with large momentum transfer. In this talk, we present a new technique[4] for constructing $\phi(x_i, Q)$ in order to predict the short distance behavior of multiquark systems.

The distribution amplitude for a baryon is determined by an evolution equation which can be derived from the Bethe-Salpeter equation at large transverse momentum projected on the light-cone:

$$\left(Q^2 \frac{\partial}{\partial Q^2} + \frac{3C_F}{2\beta}\right) \phi(x_i, Q) = \frac{C_B}{\beta} \int [dy]\; V(x_i, y_i)\, \phi(y_i, Q) \quad , \tag{1.3}$$

where $C_F = (n_c^2 - 1)/2n_c = 4/3$, $C_B = (n_c + 1)/2n_c = 2/3$, $\beta = 11 - (2/3)n_f$, and $V(x_i, y_i)$ is computed to leading order in α_s from the single-gluon-exchange kernel.[5] The general solution of this equation is

$$\phi(x_i, Q) = x_1 x_2 x_3 \sum_{n=0}^{\infty} a_n \left(\ell n \frac{Q^2}{\Lambda^2}\right)^{-\gamma_n} \tilde{\phi}_n(x_i) \quad , \tag{1.4}$$

where the anomalous dimensions γ_n and the eigenfunctions $\tilde{\phi}_n(x_i)$ satisfy the characteristic equation:

$$x_1 x_2 x_3 \left(-\gamma_n + \frac{3C_F}{2\beta}\right) \tilde{\phi}_n(x_i) = \frac{C_B}{\beta} \int_0^1 [dy]\; V(x_i, y_i)\, \tilde{\phi}_n(y_i) \quad . \tag{1.5}$$

In the large Q^2 limit, only the leading anomalous dimension γ_0 contributes, as indicated in Eq. (1.1).

In the simple three quark case, the color singlet property of the baryon system guarantees all three quarks have different quantum numbers. Thus, we do not necessarily have to antisymmetrize the system according to Pauli's principle and $\tilde{\phi}_n(x_i)$ may be derived by expanding $V(x_i, y_i)$ on a polynomial basis $\{x_1^m x_3^n\}_{m,n=0}^{\infty}$. However, if we consider multibaryon systems of $3n$ quarks, then the color singlet requirement does not guarantee that all the quarks of the system have different quantum numbers and antisymmetric representations in the total quantum space are needed. This is the essential point of the new technique.

The antisymmetrization technique will be presented in detail the next section. If we apply this method to three quark system, then we have consistency with preceding results[6] but additionally we obtain a distinctive classification of nucleon (N) and delta (Δ) wave functions and the corresponding Q^2 dependence which discriminates N and Δ form factors. In Section 3, we derive QCD predictions for the reduced form factor [7] of the deuteron and compare them with the available experimental results. Furthermore, we can combine these results with the fractional parantage technique of Harvey,[8] and we can derive constraints on the effective force between two baryons at short distances. This will be explained in Section 4. Conclusions follow in Section 5.

2. The Antisymmetrization Technique

In order to solve the evolution equation (1.3) we use the following procedure:

1. Construct the representations in each of the quantum spaces color (C), isospin (T), spin (S), and orbital (O) using Young diagramatic techniques.[9] Each quantum state is constructed by filling up the Young tableaus with corresponding specific quantum numbers. "Orbital" states are classified by polynomials $\prod_i x_i^{n_i}$, with the minimal powers dominant in the high Q^2 region. We use the symmetry of the x_i dependence of $\tilde{\phi}_n(x_i)$ in analogy with the orbital dependence of nonrelativistic wave functions as far as permutation symmetry is concerned. After an orthonormalization procedure, the orbital functions satisfy the condition:

$$\int [dx]\ \omega(x)\ \tilde{\phi}^*_m(x_i)\ \tilde{\phi}_n(x_i) = \delta_{mn} \quad , \tag{2.1}$$

   where $\omega(x) = \prod_i x_i$.

2. Construct the inner-product of Young diagrams in order to produce completely antisymmetric representations in the CTSO total space. The Clebsch-Gordan coefficients of the permutation group are used. A convenient algebraic method will be discussed in detail in Ref. 4.

3. Calculate the QCD kernel matrix in the basis of completely antisymmetric representations. For example, the one gluon exchange kernel for the three

quark system is given by $(i, j, k = 1, 2, 3)$

$$
\begin{aligned}
V(x_i, y_i) = &\left(\sum_{a=1}^{8} \frac{\lambda_a}{2} \cdot \frac{\lambda_a}{2}\right) \int_0^1 [dy] \sum_{i \neq j} \theta(y_i - x_i)\, \delta(x_k - y_k) \\
&\times \frac{y_j}{x_j} \left(\frac{\delta_{h_i \bar{h}_j}}{x_i + x_j} + \frac{\Delta}{y_i - x_i} \right) \quad , \quad k \neq i, j
\end{aligned}
\tag{2.2}
$$

where the λ_a are $SU(3)_c$ Gell-mann matrices, $\Delta\phi(y_i) = \phi(y_i) - \phi(x_i)$, and $\delta_{h_i \bar{h}_j} = 0(1)$ when the helicities of constituents are antiparallel (parallel). From this kernel, we find the following QCD evolution properties:

(a) Color singlet states are preserved by the action of V.

(b) Isospin cannot be changed, i.e. N and Δ cannot mix with each other.

(c) Spin states can mix by the spin annihilation term $(\delta_{h_i \bar{h}_j})$.

(d) Orbital states can also mix, with total $n = \sum_i n_i$ preserved.

As an example, let's consider the leading $n = 1$ amplitude of the s^2p excited nucleon state[10] of $p^*_{(\frac{1}{2},\frac{1}{2})}$ and $p^*_{(\frac{3}{2},\frac{1}{2})}$ (the x dependence of ϕ is given by the orbital Young diagram).

$$
\begin{aligned}
\phi_{\frac{1}{2}} &= \begin{array}{|c|}\hline r \\ \hline y \\ \hline b \\ \hline \end{array} \times \begin{array}{|c|c|}\hline u & u \\ \hline d & \multicolumn{1}{c}{} \\ \cline{1-1} \end{array} \times \begin{array}{|c|c|}\hline \uparrow & \uparrow \\ \hline \downarrow & \multicolumn{1}{c}{} \\ \cline{1-1} \end{array} \times \begin{array}{|c|c|}\hline 0 & 0 \\ \hline 1 & \multicolumn{1}{c}{} \\ \cline{1-1} \end{array} \; , \\
\phi_{\frac{3}{2}} &= \begin{array}{|c|}\hline r \\ \hline y \\ \hline b \\ \hline \end{array} \times \begin{array}{|c|c|}\hline u & u \\ \hline d & \multicolumn{1}{c}{} \\ \cline{1-1} \end{array} \times \begin{array}{|c|c|c|}\hline \uparrow & \uparrow & \downarrow \\ \hline \end{array} \times \begin{array}{|c|c|}\hline 0 & 0 \\ \hline 1 & \multicolumn{1}{c}{} \\ \cline{1-1} \end{array} \; .
\end{aligned}
\tag{2.3}
$$

If we split the kernel V in terms of a spin annihilation term V_δ and the remainder V_Δ, we find

$$
\begin{aligned}
\int [dy]\; V_\delta(x, y) \begin{bmatrix} \phi_{\frac{1}{2}}(y) \\ \phi_{\frac{3}{2}}(y) \end{bmatrix} &= \begin{bmatrix} \frac{2}{3} & \frac{1}{6} \\ \frac{1}{6} & \frac{2}{3} \end{bmatrix} \begin{bmatrix} \phi_{\frac{1}{2}}(x) \\ \phi_{\frac{3}{2}}(x) \end{bmatrix} \\
\int [dy]\; V_\Delta(x, y) \begin{bmatrix} \phi_{\frac{1}{2}}(y) \\ \phi_{\frac{3}{2}}(y) \end{bmatrix} &= \begin{bmatrix} -\frac{3}{2} & 0 \\ 0 & -\frac{3}{2} \end{bmatrix} \begin{bmatrix} \phi_{\frac{1}{2}}(x) \\ \phi_{\frac{3}{2}}(x) \end{bmatrix} \; .
\end{aligned}
\tag{2.4}
$$

4. Diagonalize the kernel matrix ($V = V_\delta + V_\Delta$) to determine the eigenvalues and eigensolutions. For example, from Eq. (2.4) we find the following results:

$$b = \begin{cases} \frac{2}{3} \\ 1 \end{cases} \text{ for } \tilde{\phi} = \begin{cases} \frac{1}{\sqrt{2}}\phi_{\frac{1}{2}} + \frac{1}{\sqrt{2}}\phi_{\frac{3}{2}} \\ -\frac{1}{\sqrt{2}}\phi_{\frac{1}{2}} + \frac{1}{\sqrt{2}}\phi_{\frac{3}{2}} \end{cases} , \tag{2.5}$$

where $\gamma = (2bC_B + (3/2)C_F)/\beta$.

Following the above procedures (1 through 4), we can find the anomalous dimensions and construct the corresponding eigenfunctions for arbitrary multi-quark systems. In the three quark case, we find that the results coincide with preceding calculations,[6] but here we can unambiguously resolve the N and Δ states wave functions and discriminate their form factors.

3. Deuteron Reduced Form Factors

As outlined above, we can derive the six-quark leading anomalous dimensions and predict the Q^2 dependence of the reduced deuteron form factor. It is conventional to represent the deuteron form factor by the standard "impulse approximation" form:

$$F_d(Q^2) = F_d^{\text{body}}(Q^2)\, F_N(Q^2) \quad , \tag{3.1}$$

where F_N is the on-shell nucleon form factor. However, as easily seen, for example using ϕ^3 theory, the impulse approximation form can only be valid in the nonrelativistic regime $Q^2 \lesssim 2M_d\epsilon_d$ (M_d and ϵ_d are the mass and the binding energy of deuteron) because the struck nucleon is necessarily off-shell at large momentum transfer. However, from much more general considerations, in particular, from constituent interchange, we can readily show[11] that the correct form factor factorization in terms of on-shell nucleon form factors for $\epsilon_d \to 0$ is

$$F_d(Q^2) = f_d(Q^2)\, F_N^2\left(\frac{Q^2}{4}\right) \quad , \tag{3.2}$$

which is valid at all range of Q^2. Thus, in order to make correct and experimentally accessible predictions at high Q^2, we define the reduced form factor[7] from Eq. (3.2):

$$f_d(Q^2) \equiv \frac{F_d(Q^2)}{F_N^2\left(\frac{Q^2}{4}\right)} \quad . \tag{3.3}$$

Since the leading anomalous dimension for the helicity zero deuteron is given by $\gamma_0 = 6C_F/5\beta$,[12] the QCD prediction for the asymptotic Q^2-behavior of $f_d(Q^2)$ is[13]

$$f_d(Q^2) \sim \frac{\alpha_s(Q^2)}{Q^2}\left(\ell n\,\frac{Q^2}{\Lambda^2}\right)^{-\frac{2}{5}\frac{C_F}{\beta}} \quad , \tag{3.4}$$

where $-(2/5)(C_F/\beta) = -8/145$ for $n_f = 2$. Although the QCD prediction is for asymptotic momentum transfer, it is interesting to compare Eq. (3.4) directly with available high Q^2 data[14] and this is shown in Fig. 2. The results appear consistent with experiment even for Q^2 as low as 1 GeV^2. The effect of nonleading anomalous dimension should make only small modifications.[2]

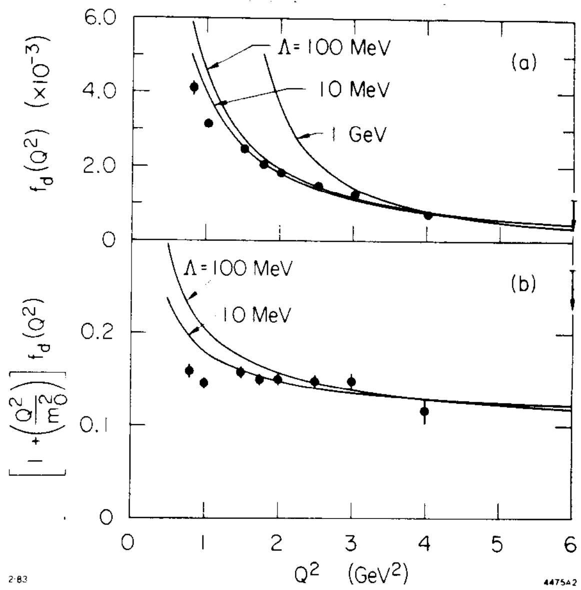

Fig. 2. (a) Comparison of the asymptotic QCD prediction $f_d(Q^2) \propto \frac{1}{Q^2}[\ell n(Q^2/\Lambda^2)]^{-1-(2/5)(C_F/\beta)}$ with data of Ref. 14 for the reduced deuteron form factor, where $F_N(Q^2) = (1 + Q^2/0.71\ \text{GeV}^2)^{-2}$. the normalization is fixed at the $Q^2 = 4$ GeV^2 data point. (b) Comparison of the prediction $[1 + (Q^2/m_0^2)]\, f_d(Q^2) \propto [\ell n(Q^2/\Lambda^2)]^{-1-(2/5)(C_F/\beta)}$ with the above data. The value $m_0^2 = 0.28$ GeV^2 is used (see Ref. 7).

4. The Effective Force Between Baryons

In Section 2, we have shown how we can solve QCD evolution equations in order to predict the short distance behavior of multiquark systems using Young diagramatic methods. Since the eigensolutions obtained in this way have definite permutation symmetry, we can apply the fractional parantage technique[8] for the multibaryon system in order to relate the eigensolutions to cluster representations which have physical baryon, or alternatively, "hidden-color" degrees of freedom.

For example, if we apply this technique to the simple case of the 4 quark system under $SU(2)_c$,[15] then we find the transition matrix given by Table I (T = S = 0 case) which relates the symmetry basis represented by four-quark eigensolutions and the physical basis represented by "toy"-dibaryon and hidden-color degrees of freedom. From this table we can expand the distribution amplitudes of the physical basis in terms of eigensolutions:

$$
\begin{aligned}
\phi_{NN}(x_i,Q) &= 0.07\phi_1(x_i)\left(\ell n\frac{Q^2}{\Lambda^2}\right)^{0.13C_F/\beta} - 0.64\phi_2(x_i)\left(\ell n\frac{Q^2}{\Lambda^2}\right)^{-0.02C_F/\beta} + \ldots \\
\phi_{\Delta\Delta}(x_i,Q) &= -0.07\phi_1(x_i)\left(\ell n\frac{Q^2}{\Lambda^2}\right)^{0.13C_F/\beta} - 0.59\phi_2(x_i)\left(\ell n\frac{Q^2}{\Lambda^2}\right)^{-0.02C_F/\beta} + \ldots \\
\phi_{CC}(x_i,Q) &= -0.70\phi_1(x_i)\left(\ell n\frac{Q^2}{\Lambda^2}\right)^{0.13C_F/\beta} - 0.35\phi_2(x_i)\left(\ell n\frac{Q^2}{\Lambda^2}\right)^{-0.02C_F/\beta} + \ldots
\end{aligned}
\tag{4.1}
$$

where $C_F = 3/4$ in this case.

Table I. The relationship between four-quark anitsymmetric SU(2) color representations and effective two-cluster representations (T = S = 0 case). Isospin singlet and triplet states both with color singlet are denoted N and Δ, while color triplet state is represented by C. The square and curly brackets represent orbital (O) and spin-isospin (TS) symmetries separately.

	[4] {22}	[22] {22}	[22] {4}
NN	$-\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{\sqrt{2}}$
$\Delta\Delta$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$
CC	$\frac{1}{\sqrt{2}}$	$-\frac{1}{\sqrt{2}}$	0

Thus, we find that the NN, $\Delta\Delta$ and CC states have completely different Q^2 evolution. As Q^2 goes to infinity, the NN and $\Delta\Delta$ components are negligible but the CC components are large. In other word, the dominant degrees of freedom at the origin of the dibaryon system at zero impact separation are hidden-color states rather than physical baryon states. This indicates that the physical dibaryons have a repulsive core at the origin[16] while the colorful hidden-color clusters behave as in an attractive well. In this way, we derive constraints on the effective force between two baryons.[17] Analogous results hold for the six-quark states in $SU(3)_c$.

5. Conclusion

By using a new technique based on completely antisymmetric representations, we can analyzed the quark distribution amplitudes $\phi(x_i, Q)$ in QCD in order to predict the short distance behavior of multiquark systems. Through this analysis we find that multiquark wave functions have the following QCD evolution properties:

1. Color singlet states are preserved.
2. Isospin states cannot be changed.
3. Different spin states can mix with each other.
4. Different orbital polynomials with same n are mixed. (These eigensolutions are the $L_Z = 0$ projection of true solution to the valence Fock states of multiparticle systems).

Since the new technique is based on permutation symmetry, we can readily classify the multiquark systems. In the 3-quark case, we can resolve the N and Δ form factors. In the multibaryon system, this technique is essential since it cannot be guaranteed that all quarks have different quantum numbers.

The QCD predictions for the Q^2 dependence of the deuteron reduced form factor in the high Q^2 regime above 1 GeV^2 agree well with the available experimental data. We have also decomposed the multiquark systems into multibaryon physical components and hidden color components, and expanded each component in terms of the QCD eigensolutions. Through the evolution of each components we can derive constraints on the effective force between the clusters. Using the toy-$SU(2)_c$-dibaryon analysis, we find that colorless clusters tend to be repulsive but colorful clusters are attractive at short distances.

In conclusion, we have developed a new technique which is essential and useful for analyzing the short distance behavior of multiquark systems.

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