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CONSTRAINTS ON HEAVY UNSTABLE NEUTRINOS FROM GALAXY FORMATION*

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ABSTRACT

A new bound is derived on the lifetime τ_H of nonradiatively decaying heavy neutrinos ν_H from the requirement that the universe be dominated by nonrelativistic matter after the epoch of recombination, in order that galaxies and clusters have time to grow from small fluctuations in the early universe. The resulting constraint on τ_H is much stronger than that obtained by requiring only that the ν_H decay products not contribute more than a critical density today. The implications for a number of modes of ν_H decay, in particular $\nu_H \to \nu_L \nu_L \bar{\nu}_L$ via Z^o or Higgs bosons and $\nu_H \to \nu_L +$ familon or Majoron, are discussed in detail.

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1. INTRODUCTION

It is by now generally appreciated that cosmology sets constraints on stable neutrino masses, lifetimes, and number of species much more stringent than those provided by laboratory experiments [1]. The aim of the present paper is to derive a new bound on the lifetime of decaying neutrinos from the requirement that galaxies and clusters have time to form despite the observational upper bounds on small scale fluctuations in the microwave background radiation. This requirement leads us to assume that the universe is matter dominated during the entire period after hydrogen recombination. We explore the implications for a number of particle physics models in which a heavy neutrino decays nonradiatively.

Our constraint turns out to be by about five orders of magnitude stronger than that of Dicus, Kolb and Teplitz (DKT) [2]. Like the latter, it applies only to nonradiative neutrino decay modes with light or massless stable [$F0$] particles in the final state. The constraints on neutrinos with radiative decay modes are quite severe [1], and we will therefore not consider that case. That is, throughout this paper we will never consider heavy neutrinos with significant branching ratios for decays producing either photons or electrons, like $\nu_H \to \nu_L e^+ e^-$, $\nu_H \to \nu_L \gamma$, $\nu_H \to \nu_L \gamma\gamma$.

The paper organized as follows: sections 2 and 3 review the DKT constraint; section 4 discusses the implications of galaxy formation; sections 5 and 6 derive and discuss our new, stronger constraint on massive unstable neutrinos; and section 7 discusses the implications for particle physics models. Section 8 summarizes our conclusions.

2. STANDARD CONSTRAINTS

The present total cosmological density is $\rho_T(t_0) = 1.1 \times 10^4 h^2 \Omega$ eV cm^{-3}, where h is Hubble's constant $H(t_0)$ in units of 100 km sec^{-1} Mpc^{-1} (observationally, $1/2 \lesssim h \lesssim 1$) and the cosmic density parameter Ω is the ratio of $\rho_T(t_0)$ to the present critical density $\rho_c(t_0) \equiv 3H(t_0)^2/8\pi G$.

The determination of the present neutrino number density $n_\nu(t_0)$ runs as follows. In the early universe all neutrinos are kept in thermal equilibrium by collisions down to a temperature $T_D \approx 1\,\text{MeV}$. However, neutrinos with $m_\nu >$ $1\,\text{MeV}$ start to annihilate as the temperature drops to their mass and continue until they go out of chemical equilibrium at a "freezing" temperature T_F which typically exceeds T_D and depends on m_ν. Obviously, for $m_\nu \lesssim 1\,\text{MeV}$, $T_F = T_D$ is fixed. Now, the standard cosmology allows one to compute $n_\nu(t_0)$ in terms of T_F. The dependence of T_F on $m_\nu \geq 1\,\text{MeV}$ is displayed in the Table (from ref. [2]) together with the corresponding value of $n_\nu(t_0)$. Clearly, the direct or indirect [F1] neutrino contribution to the present cosmic energy density must not exceed $\rho_T(t_0)$. We shall discuss this apparently innocuous bound in its various — old and new — ramifications.

The present energy density for stable massive neutrinos is $\rho_\nu(t_0) = m_\nu n_\nu(t_0)$. By making use of the results in the Table, we see that the constraint $\rho_\nu(t_0) \lesssim \rho_T(t_0)$ can be met by having a small m_ν or, alternatively, a small $n_\nu(t_0)$. Correspondingly, two bounds arise:

$$\sum m_\nu \lesssim 100 h^2 \Omega \ \text{eV} \qquad [3] \tag{1}$$

$$m_\nu \gtrsim 5 h^2 \Omega \ \text{GeV} \qquad [4] \tag{2}$$

As a consequence, a forbidden mass gap emerges for ***stable*** neutrinos [F2].

Massive neutrinos are however generally ***unstable*** on the age of the universe in practically all realistic gauge models, and this circumstance can weaken the above constraints. Indeed, it has been claimed [2] that an unstable neutrino is allowed to have a mass in the forbidden gap $100h^2\Omega\ \text{eV} \lesssim m_\nu \lesssim 5h^2\Omega\ \text{GeV}$, provided:

i) its stable decay products satisfy eq. (1),

ii) its lifetime is sufficiently short to allow the energy density of the decay products to be red-shifted enough by the Hubble expansion so as not to exceed $\rho_T(t_0)$ today.

Denoting the "heavy" decaying neutrino [F3], its mass, and its lifetime as ν_H, m_H, τ_H, the quantitative form of such a constraint reads [F4]:

$$m_H(\tau_H/t_0)^{1/2}\, n_H(t_0) \lesssim 10^4 h^2 \Omega \text{ eV cm}^{-3} \tag{3}$$

where $t_0 \equiv$ age of the universe and $n_H(t_0)$ is the ν_H number density extrapolated to the present as given in the Table. The tacit, obvious, but fundamental assumption behind these considerations is that the decay products do not thermalize. We stress that the DKT bound (3) is relevant only for nonradiative decays, and it is well known [1] that stronger constraints arise when a photon is present in the final state.

3. COSMOLOGY WITH HEAVY DECAYING NEUTRINOS

The evolution of the universe according to the standard cosmology [5] is remarkably simple. The universe first lives in a radiation-dominated phase (i.e., gravitationally dominated by photons, neutrinos, and other relativistic species) until the scale factor $a(t)$ (normalized to unity at the present: $a(t_0) = 1$) reaches the value $a(t_{eq}) \approx 4.0 \times 10^{-5} h^{-2}\Omega^{-1}(1+\gamma)/1.681$. (We assume here that there are N_ν species of very light or massless neutrinos, and $\gamma = \rho_\nu/\rho_\gamma = (7/8)(4/11)^{4/3}N_\nu$ ($= 0.681$ for $N_\nu = 3$).) This occurs at a time $t_{eq} \approx 2 \times 10^{10} h^{-4}\Omega^{-2}$ sec. At this point the universe becomes dominated by (nonrelativistic) matter, which continues until the present.

The existence of a heavy neutrino ν_H decaying into light stable particles complicates such a picture in a manner that depends on m_H.

Consider first the situation $m_H \lesssim T_D \approx 1\,\text{MeV}$. Then no change occurs until the temperture drops (at a time t_D) to T_D. Two scenarios are now possible for the ν_H decay: ν_H either (1) nonrelativistic or (2) relativistic at decay. In either case, two additional possibilities must be considered: (A) all the decay products are still relativistic when matter domination occurs, or (B) one or more of the decay products become themselves the nonrelativistic gravitationally dominant dark matter.

In more detail: (1) For lifetimes sufficiently short but anyway longer than $(1\,\mathrm{MeV}/m_H)^2 t_D$, ν_H becomes nonrelativistic when the universe cools down to $T \approx m_H$, and so ν_H starts to matter-dominate. This phase continues up to $a(t_H)$ when ν_H finally decays at $t_H \approx \tau_H$ [F5]. (2) Alternatively, for lifetimes shorter than $(1\,\mathrm{MeV}/m_H)^2 t_D$, ν_H remains relativistic until decaying, and no matter domination by ν_H occurs. Since we consider only decay products which satisfy eq. (1), the universe (1) again enters, or (2) remains, in a radiation-dominated phase for $t > \tau_H$ until $a(t)$ grows to $a(t_{eq})$. Then either (case A) the contribution of the still relativistic decay products to the cosmic energy density equals the matter contribution [F6], or (case B) *before* that happens at least one decay product becomes nonrelativistic.

Things are different for $m_H > 1\,\mathrm{MeV}$. Obviously ν_H is now always nonrelativistic after thermal decoupling, but this does not mean that it matter-dominates the universe. Actually now $n_\nu(T_D)$ becomes increasingly smaller than $n_\gamma(T_D)$ as m_H increases, as a consequence of the ν_H annihilation. Correspondingly, the above picture of evolution of the universe still holds but develops smoothly into the standard scenario, which is indeed fully recovered for $m_H \gtrsim 5h^2\Omega\,\mathrm{GeV}$.

The point to be stressed here is that thus far no assumption has been made about the value of $a(t_{eq})$ in discussing the bounds given in the preceding section. In fact, $a(t_{eq}) = 1$ when the DKT bound (3) is attained, but $a(t_{eq}) \ll 1$ for eq. (3) satisfied but for from being saturated [F7]. This means that the universe could have been equally well either matter or radiation-dominated over its recent part.

It is precisely this arbitrariness that we would like to call into question. Specifically, we argue that the universe should have been matter-dominated more or less as in the standard scenario [5]. Correspondingly, we get that $a(t_{eq}) \ll 1$ should always hold true. Clearly then we expect eq. (3) to get replaced by a much stronger constraint. We explain the reason for this in the next section.

4. FORMATION OF GALAXIES AND CLUSTERS

In the standard hot big bang cosmology [5], it is generally assumed that all large structures in the universe — galaxies, clusters, superclusters and voids — grew gravitationally from initially small fluctuations in the matter-energy density. The origin of the fluctuations is uncertain but not necessarily problematic, since inflationary models [6] or cosmic strings [7] can give rise to fluctuations of the appropriate magnitude.

The most important observational constraint on the magnitude of the primordial fluctuations is provided by measurements of fluctuations in the cosmic background radiation, on which the most stringent present upper limit [8] is of order $\Delta T/T \lesssim 3 \times 10^{-5}$ on an angular scale $\theta = 4.5$ minutes of arc. This corresponds to a comoving length given by $\lambda_o = 2c\theta/\Omega H_o$, which equals $8h^{-1}\Omega^{-1}$ Mpc for $\theta = 4.5$ min. There are no measured anisotropies in the cosmic background radiation apart from the dipole anisotropy due primarily to our motion relative to the mean Hubble flow, and upper limits are of order 10^{-4} on all angular scales from a few arc minutes to 90° (quadrupole).

On angles greater than $\Omega^{1/2}\,2°$ (corresponding to comoving lengths larger than the last scattering shell at recombination, $\lambda > \lambda_r = 200h^{-1}\Omega^{-1/2}$ Mpc), the magnitude of the root mean square curvature fluctuations, $\epsilon(\lambda) = (\delta\rho/\rho)_{rms}$ when scale λ first enters the horizon, is constrained by these observations to be $\epsilon(\lambda) \lesssim 10^{-4}$. On smaller angular scales, the constraint on ϵ is more complicated and depends on details of the fluctuation spectrum and the assumed properties of the dark matter.

The most natural assumption is that the fluctuation spectrum is of the constant curvature or "Zeldovich" form [9], i.e. $\epsilon(\lambda) =$ constant. This is the spectrum predicted by both inflation and cosmic string models for the origin of fluctuations, and we will assume it here for definiteness and simplicity. We will also assume that the fluctuations are adiabatic rather than isothermal, *i.e.* that the matter and radiation fluctuate together, as suggested by particle physics models for baryosynthesis.

Consider fluctuations which enter the horizon during the radiation-dominated era. Ordinary matter, consisting of baryons and leptons, is ionized until recombination ($z_r = 1300$), and locked to the radiation by Compton drag. After the fluctuation enters the horizon, relativistic neutrinos freely stream away and the radiation-matter fluid undergoes acoustic oscillations which for adiabatic fluctuations of galaxy size and below are eventually strongly damped by photon diffusion (Silk damping). Thus the amplitude of ordinary matter fluctuations does not begin to grow until recombination, after which it grows proportional to the scale factor $a = (1+z)^{-1}$ if the universe is matter dominated. In the simplest scheme, in which the universe consists of light neutrinos, radiation, and ordinary matter, the magnitude of the fluctuations would thus be $\epsilon z_r \ll 1$ today, too small for any structure to have formed.

This is a powerful argument for some form of nonbaryonic dark matter as the gravitationally dominant component of the universe. The salient feature of this dark matter is that it does not interact with radiation; thus dark matter fluctuations are uninhibited by Compton drag and unaffected by Silk damping, and can grow as the scale factor a as soon as the universe becomes (dark) matter dominated. There is also the possibility of some growth during the radiation-dominated era if the dark matter is "cold" (e.g., axions or massive photinos) or "warm", although fluctuations in "hot" dark matter (light neutrinos or majorons) damp by free streaming. While isothermal (entropy) fluctuations in the dark matter can grow only by a factor of 2.5 between horizon crossing and matter domination ("Meszaros effect"), adiabatic fluctuations of galaxy size grow by an additional order of magnitude ("stagspansion") [10]. After recombination, the baryons "fall in" to the dark matter fluctuations. The result is that it is (barely) possible in these cases to grow the observed large scale structures in the universe without violating the $\Delta T/T$ constraints (see for example the review [11] and references therein).

In the theories just discussed, enough growth of fluctuations occurs because (dark) matter dominates before recombination and for all times thereafter. In theories in which the universe is gravitationally dominated after recombination by the relativistic decay products of an unstable massive neutrino, there will be less than a factor of 10^3 growth in the amplitude of the fluctuations after recombination and it will be impossible for the fluctuations to grow to nonlinearity by the present. Thus we are led to assume that the universe remains matter dominated after recombination; i.e.,

$$a(t_{eq}) \lesssim 10^{-3}. \tag{4}$$

It is possible that by relaxing some of our assumptions, for example the Zeldovich primordial fluctuation spectrum, consistency with the $\Delta T/T$ constraints could be obtained without satisfying eq. (4). But such theories would probably have to invoke reionization of the universe by some hypothetical early strongly radiating objects in order to smear out the small angle $\Delta T/T$ fluctuations, and involve enough other complications to require detailed discussion. For example, dissipationless structures such as the dark matter halos of galaxies and clusters retain a density roughly equal to that of the background universe at the time they first became nonlinear, and thus cannot form too early. Our point in this paper is that a theory which does not satisfy eq. (4) cannot rely on the only theoretical ideas — inflation or cosmic strings — which have so far been shown to produce fluctuations on scales larger than the horizon, and must bear the burden of proof that galaxies and large scale structure can form without violating the $\Delta T/T$ constraints.

5. A NEW STRONGER CONSTRAINT

The question now arises as to what constraint on τ_H is implied by eq. (4). According to our assumptions, we consider the general decay mode:

$$\nu_H \rightarrow L_1 + \cdots + L_N \tag{5}$$

where L_i with masses m_i are light or massless stable neutral particles, which should therefore satisfy eq. (1). Two possibilities now arise, depending on whether ν_H is relativistic or nonrelativistic at decay. Obviously, for $m_H >$

1 MeV only the latter option occurs. Moreover, the reader will immediately realize that the former option yields a more stringent bound on τ_H. Since what is really relevant is the *least* stringent constraint implied by eq. (4), we consider first the case of ν_H nonrelativistic at decay.

5.1 HEAVY NEUTRINOS NONRELATIVISTIC AT DECAY

The ν_H is nonrelativistic at decay whenever the lifetime exceeds the age of the universe when its temperature drops to m_H. This means

$$\tau_H > (1\,\text{MeV}\,/m_H)^2\, t_D \tag{6}$$

Unfortunately, we don't know t_D exactly. The best we can say is that it certainly cannot be much greater than some seconds. The two cases A and B discussed in section 3 will now be considered separately.

CASE A It is not difficult to realize that eq. (4) above is in the present instance not sufficient by itself to provide a well-defined answer. Again, the above-discussed constraint that the present total energy density of L_i should not exceed $\rho_T(t_o)$ plays a decisive role. An equivalent but now more suitable way to phrase the same thing is that the total energy density of L_i at t_{eq}, $\rho_L(t_{eq}) \approx \sum E_i(t_{eq}) n_H(t_{eq})$, should not exceed $\rho_T(t_{eq})$, i.e. $\rho_T(t_o)$ extrapoloted back to t_{eq} in a matter-dominated universe. Since $n_H(t)$ and $\rho_T(t)$ scale the same way from t_{eq} to t_0, we get

$$\sum_i E_i(t_{eq}) n_H(t_0) \lesssim 10^4 h^2 \Omega\, \text{eV cm}^{-3} \tag{7}$$

where the sum is over the N relativistic decay products of ν_H, eq. (5). We are considering ν_H nonrelativistic at decay, therefore

$$\begin{aligned} E_i(t_{eq}) &\approx (m_H/N) \times (a(\tau_H)/a(t_{eq})) \approx \\ &\approx (m_H/N) \times (\tau_H/t_{eq})^{1/2} \approx (m_H/N) \times (\tau_H/t_0)^{1/2} a(t_{eq})^{-3/4} \end{aligned} \tag{8}$$

Correspondingly, eq. (7) becomes

$$m_H\, (\tau_H/t_0)^{1/2} a(t_{eq})^{-3/4} n_H(t_0) \lesssim 10^4 h^2 \Omega\, \text{eV cm}^{-3} \tag{9}$$

By now making use of eq. (4), we finally get our bound:

$$m_H\,(\tau_H/t_0)^{1/2} n_H(t_o) \lesssim 56 h^2 \Omega\,\mathrm{eV\,cm^{-3}} \qquad (10)$$

Comparsion between eqs. (3) and (10) shows that our constraint on τ_H is a factor 3×10^{-5} stronger than that of DKT [F8].

CASE B Denote by L_1 the heaviest decay product, i.e. $m_1 \geq m_i$ for $i > 1$. Then for $m_1 \lesssim 100 h^2 \Omega\,\mathrm{eV}$ (but not too small) it can well happen that the universe becomes matter-dominated by L_1. Specifically from the very definition of t_{eq} this means $E_1(t_{eq}) \approx m_1$. Eq. (8) then yields

$$(m_H/N) \times (\tau_H/t_o)^{1/2}\, a(t_{eq})^{-3/4} \approx m_1 \qquad (11)$$

and consequently eq. (4) implies

$$m_H(\tau_H/t_0)^{1/2} \lesssim 5.6 \times 10^{-3}\, N\, m_1 \qquad (12)$$

which is our bound in the present situation. Since eq. (4) has already been taken into account, eq. (12) has only to agree with the DKT bound. It can be immediately seen that this is indeed the case by multiplying eq. (12) by $n_H(t_0)$ and noting that $5.6 \times 10^{-3}\, N\, m_1\, n_H(t_0) \ll m_1\, n_H(t_0) \lesssim 10^4\, h^2 \Omega$ eV cm^{-3} from $m_1 \lesssim 100 h^2 \Omega\,\mathrm{eV}$ and $n_H(t_0) \lesssim 100\,\mathrm{cm^{-3}}$ (see Table). Moreover, the bound (12) turns out to be ***weaker*** than the bound (10) for

$$10^4 (h^2 \Omega/N)(1\,\mathrm{cm^{-3}}/n_H(t_0))\,\mathrm{eV} < m_1 \lesssim 10^2 h^2 \Omega\,\mathrm{eV} \qquad (13)$$

Clearly, for $m_H \lesssim 1\,\mathrm{MeV}$ this happens for $10^2 h^2 \Omega/N < m_1/eV \lesssim 10^2 h^2 \Omega$ and it can be easily seen that m_H cannot be greater than 6-8 MeV.

5.2 HEAVY NEUTRINOS RELATIVISTIC AT DECAY

We turn now to the case of ν_H relativistic at decay. This situation occurs when the energy of ν_H at decay exceeds its mass. Obviously, this can only happen for $m_H \lesssim 1\,\mathrm{MeV}$. We have

$$\begin{aligned} E_H(\tau_H) &\approx E_H(t_D)\,(a(t_D)/a(\tau_H)) \approx \\ &\approx T_D\,(t_D/\tau_H)^{1/2} \end{aligned} \qquad (14)$$

Remarkably enough, in the present scenario we do know what t_D is. Indeed, due to the absence of matter domination by ν_H we are practically in the same situation as in the standard cosmology, and so we can reliably take $t_D \approx 1$ sec [F9]. Consequently, the condition $E_H(\tau_H) > m_H$ takes the form:

$$\tau_H < (1\,\mathrm{MeV}\,/m_H)^2\,\mathrm{sec} \tag{15}$$

Obviously, the same conclusion follows by requiring that ν_H decays before the universe cools down to $T = m_H$. Eq. (15) is our bound in the present situation. Of course, we have still to consider whether eq. (4) is satisfied or not. Before doing that, however, let us compare the bound (15) with the bounds (10) and (12). To this end, we rewrite (15) as

$$m_H(\tau_H/t_0)^{1/2} < 1.8 \times 10^{-3}\,\mathrm{eV} \tag{16}$$

since $t_0 > 10^{10}y$. Then the bound (15) turns out to be ***stronger*** than (10) for $h^2\Omega > 3.2 \times 10^{-3}$ [F10].(Observational evidence [10] implies $\Omega h^2 > 5 \times 10^{-2}$.) The bound (15) is even stronger than (12) as long as $m_1 > 0.16\,\mathrm{eV}$.

Before discussing the value of $a(t_{eq})$ in the cases A and B, it is convenient to compute $E_i(t_{eq})$. By making use of eq. (14), we obtain

$$\begin{aligned} E_i(t_{eq}) &\approx (E_H(\tau_H)/N) \times (a(\tau_H)/a(t_{eq})) \approx \\ &\approx (T_D/N) \times (t_D/t_{eq})^{1/2} \approx (T_D/N) \times (t_D/t_0)^{1/2} a(t_{eq})^{-3/4} \quad . \end{aligned} \tag{17}$$

CASE A By proceeding in exactly the same way as in the analogous case of ν_H nonrelativistic at decay, we get eq. (7) which has to be satisfied. Inserting now eq. (17) into eq. (7) we obtain the desired condition [F10]:

$$T_D \times (t_D/t_0)^{1/2} \times a(t_{eq})^{-3/4} \lesssim 10^2 h^2\Omega\ \mathrm{eV} \tag{18}$$

which implies

$$a(t_{eq}) \gtrsim 4.8 \times 10^{-7} h^{-8/3}\Omega^{-4/3} \tag{19}$$

where we have taken $t_0 = 10^{10}$ y just to get the most pessimistic lower bound on $a(t_{eq})$ [F11]. Therefore eq. (4) is satisfied for

$$h^2\Omega > 3.2 \times 10^{-3} , \tag{20}$$

which certainly holds.

CASE B Assuming as before that L_1 is the heaviest decay product, the condition $E_1(t_{eq}) \approx m_1$ becomes now

$$(T_D/N)(t_D/t_0)^{1/2} a(t_{eq})^{-3/4} \approx m_1 \tag{21}$$

thanks to eq. (17). Consequently we get

$$a(t_{eq}) \approx 2.3 \times 10^{-4}\ N^{-4/3}\ (1\,\mathrm{eV}\,/m_1)^{4/3} \tag{22}$$

where we have taken again $t_0 = 10^{10}$ y for the same reason as before. Hence eq. (9) is satisfied provided:

$$m_1 \gtrsim (0.33/N)\ \mathrm{eV}\ . \tag{23}$$

6. DISCUSSION

We would like to summarize our conclusions in a physical manner. Very short lifetimes — smaller than $(1\,\mathrm{MeV}\,/m_H)^2$ sec — imply that ν_H is relativistic when it decays, and this is obviously possible only for $m_H \lesssim 1\,\mathrm{MeV}$. Unfortunately, since the distinction between cases A and B does *not* depend on either m_H or τ_H, we cannot know in general whether one or the other situation occurs. All we can tell is that for all decay products lighter than $(0.33/N)\,\mathrm{eV}$ option B is excluded, and so the present universe cannot be dominated by the decay products of ν_H. Longer lifetimes — greater than $(1\,\mathrm{MeV}\,/m_H)^2$ sec — imply a ν_H nonrelativistic at decay. Now the distinction between cases A and B is under control. In fact, for very light decay products — eq. (13) *not* satisfied — the bound (12) is *stronger* than the bound (10). Physically, this means that the decay products cannot become nonrelativistic before their energy density equals the baryonic contributon. Again, the present universe cannot be dominated by the decay products of ν_H. The case of not too light decay products – eq. (13) satisfied — allows for an interesting possibility. Actually, for lifetimes satisfying eq. (10), the above scenario still obtains, but for longer lifetimes τ_H satisfying the condition

$$10^4(1\,\mathrm{eV}/m_H)^2(n_H(t_0)\,\mathrm{cm}^3)^{-2} \lesssim \tau_H/t_0 \lesssim 10^{-4}N^2(m_1/m_H)^2 \qquad (24)$$

the present universe would be dominated by the decay products of ν_H. This would provide interesting possibilities for the cosmological dark matter, in which the heaviest ν_H decay product L_1 which would behave as either hot or cold dark matter [10]. The optimum values for the hot scenario to occur are $m_H \approx 1\,\mathrm{MeV}$, $m_1 \approx 10^2\,\mathrm{eV}$, and τ_H obeying the constraint $10^{-12} \lesssim \tau_H/t_0 \lesssim N^2 \times 10^{-12}$. Alternatively, m_H and m_1 could be much heavier, with $n_H(t_0)$ suppressed by annihilation.

7. IMPLICATIONS FOR NEUTRINO DECAY MODELS

A variety of possibilities have been considered over the last few years to allow unstable neutrinos with masses in the forbidden gap (1), (2) to satisfy the DKT bound.

We now review these models in their essential features and carefully discuss whether they also meet the stronger constraints we have derived. Whenever the quantities h, Ω, t_0 have to be specified, we shall take illustratively $h \approx 1/2$, $\Omega \approx 1$, $t_0 \approx 10^{18}$ sec to get the most optimistic results. According to this strategy, the intergenerational mixing parameter ϵ in the neutrino neutral current sector will be taken illustratively as $\epsilon \approx 10^{-1}$ [F 12].

7.1 GAUGE BOSON MEDIATED PURE LEPTONIC DECAY

Even in the standard $SU(2) \otimes U(1)$ model with massive neutrinos, the Z°-mediated decay mode $\nu_H \to \nu_L \nu_L \bar{\nu}_L$ is ***not*** GIM suppressed when both Dirac and Majorana mass terms are present [12]. As a consequence, the hope arises that such a decay can proceed sufficiently fast so as to satisfy eq. (3). Specifically, the lifetime is

$$\tau_H \approx 4\epsilon^{-2}(m_\mu/m_H)^5\,\tau_\mu \tag{25}$$

Now, when Majorana masses for the right-handed neutrinos are present, the "survival hypothesis" [13] strongly suggests that they should be much greater than the $SU(2)\otimes U(1)$ breaking scale, which implies that the masses of observed neutrinos should be much lighter than the charged lepton masses. This is just the "see-saw" mechanism [14], which is in the present context [F13] the only known way to ***naturally*** explain the neutrino-charged lepton mass splitting. Denoting by M_D, M_L, M_R the Dirac and Majorana masses for left and right-handed neutrinos, we have $\epsilon \approx (M_D/M_R)^2$. Since the light neutrino masses are typically $m_\nu \approx M_D^2/M_R$, from the electron family we argue $M_D/M_R \lesssim 10^{-5}$ and so $\epsilon \lesssim 10^{-10}$. Correspondingly, the DKT bound is satisfied for $m_H \gtrsim 300\,\mathrm{MeV}$ [F14]. On the other hand, our relevant bound is clearly eq. (10), which is met for $m_H \gtrsim 1\,\mathrm{GeV}$ [F14]. Unfortunately, eq. (2) becomes $m_\nu \gtrsim 1.2\,\mathrm{GeV}$ for $h \approx 1/2$, $\Omega \approx 1$. Therefore, ***no*** neutrino can be removed from the forbidden gap (1), (2) by the decay mode considered here, as long as we stick to the natural "see-saw" scenario. Nevertheless, if fine-tuning is allowed to keep neutrino masses ***artificially*** small, then the DKT bound is satisfied for $m_H \gtrsim 2\times 10^5\,\mathrm{eV}$ [F14]. Instead, our bound (10) is met for $m_H \gtrsim 5\,\mathrm{MeV}$ [F14]. This practically excludes the possibility that the ν_L coming from ν_H decay can matter-dominate the present universe.

7.2 HIGGS BOSON MEDIATED PURE LEPTONIC DECAY

The same decay mode $\nu_H \to \nu_L\nu_L\bar{\nu}_L$ can also proceed via the exchange of a neutral Higgs boson Δ° [16], as it occurs automatically in the context of left-right symmetric models based on gauge group $SU(2)_L\otimes SU(2)_R\otimes U(1)_{B-L}$ [17]. Again, this process is not GIM suppressed, and the lifetime is [F15]

$$\tau_H \approx \epsilon^{-2}(m_{\Delta^\circ}/m_W)^4/(m_\mu/m_H)^5\,\tau_\mu \tag{26}$$

Notice that here the "see-saw" mechanism is at work, and so neutrinos are naturally light. Since Δ° carrys isospin ***one***, singly and doubly charged Higgs bosons Δ^+, Δ^{++} are present, which mediate the decays $\nu_H \to \nu_L\gamma$ [18] and $\mu \to ee\bar{e}$ [19], respectively. Therefore, the parameters of the model [16] have to be adjusted so as to keep these processes below the experimental bounds [18,19]. Moreover, to have enough suppression of Higgs-induced $\Delta S = 2$ neutral currents, the requirement $m_{\Delta^\circ} \gtrsim 10\,\mathrm{GeV}$ [20] has to be met. Then, it has been shown that at best [F14] for $m_H \gtrsim 10^4\,\mathrm{eV}$ all the experimental constraints are satisfied and the DKT bound is met. Just as before, our relevant bound is eq. (10), which is met at best [F14] for $m_H \gtrsim 2.5 \times 10^5\,\mathrm{eV}$.

Amusingly, for a suitable range of parameters the present universe would be matter dominated by the ν_L produced in the ν_H decay. According to our preceeding discussion, such a scenario would occur for $33h^2\Omega\,\mathrm{eV} < m_1 \lesssim 10^2 h^2\Omega\,\mathrm{eV}$ and e.g. $\epsilon \approx 10^{-2}$, $m_{\Delta^\circ} \approx 30\,\mathrm{MeV}$, $m_H \approx 1\,\mathrm{GeV}$ and $t_0 \approx 10^{18}$ sec. There is however a potential dramatic danger. The ν_L and $\bar{\nu}_L$ could annihilate into a pair of very energetic photons would both distort the cosmic background radiation too much [21] and destroy a significant amount of light elements by photofission [22], since $\tau_H \approx 4 \times 10^6$ sec in the present case. In fact, these dangers do not actually occur. The physical reason is that the cross-section for $\nu_L \bar{\nu}_L \to \gamma\gamma$ [23] is so small that the annihilation mean free path λ exceeds by many orders of magnitude the universe horizon at the time τ_H. Assuming the above-considered values of the parameters, a straightforward calculation yields

$$\lambda_{\nu\bar{\nu}\to\gamma\gamma}(\tau_H) \approx \frac{1}{n_H(\tau_H)\sigma(\nu\,\bar{\nu} \to \gamma\gamma)} \approx 10^{38} h^{-4} \Omega^{-2}\,\mathrm{cm} \qquad (27)$$

whereas the horizon is simply $c\tau_H \approx 10^{17}$ cm.

7.3 FAMILONS

The attempt to predict fermion masses as well as to find a justification for the emergence of the Peccei-Quinn symmetry $U(1)_{PQ}$ [24] led to the suggestion of a spontaneously broken global horizontal symmetry, G_{HOR}, with the ensuing Goldstone bosons called *familons*, and denoted generically by f [25,26,27]. The astrophysical constraint that not too much energy is carried away by familons from red giants stars as well as the laboratory bounds from K and μ decays imply that the scale F at which G_{HOR} gets broken should satisfy the constraint $F \gtrsim 10^9$ GeV [25,26][F16].

As far as neutrinos are concerned, their masses arise via the "see-saw" mechanism provided Higgs singlets under $SU(2) \otimes U(1)$ are introduced, and so they are naturally light. Then heavier neutrinos can decay into lighter ones by familon emission, and this can lead to the evasion of the cosmological bounds for ν_H masses in the forbidden gap (1), (2). This issue has been discussed recently [28], with the result that only for $m_H \gtrsim 10^5\,\mathrm{eV}$ does this possibility actually work. We investigate here the same question in the light of our stronger constraints. The lifetime for $\nu_H \to \nu_L f$ is [25,26]:

$$\tau_H(\nu_H \to \nu_L f) \approx 3 \times 10^9 (F/10^{10}\,\mathrm{GeV})^2 \times (10^5\,\mathrm{eV}/m_H)^3\ \mathrm{sec} \tag{28}$$

The relevant bound is again eq. (10).

Our results can be summarized as follows [F14]. Values $m_H \lesssim 1.5\,\mathrm{MeV}$ do *not* satisfy our bound for $F \gtrsim 10^9$ GeV. As F increases, the lower bound on m_H which satisfies our constraint (10) increases as well. Such a lower bound is 1.5 MeV for $F = 10^9$ GeV, 12 MeV for $F = 10^{10}$ GeV, 24 MeV for $F = 10^{11}$ Gev, 60 MeV for $F = 10^{12}$ Gev, 150 MeV for $F = 10^{13}$ GeV, 200 MeV for $F = 10^{14}$ GeV, and so on.

A possibility is that $U(1)_{PQ} \subset G_{HOR}$ [26], in which case F coincides with the Pecci-Quinn symmetry breaking scale and the picture of the "invisible axion" emerges [29]. Then it has been shown [30] that in order that the axion energy density does not overdominate the present universe, the upper bound $F \lesssim 10^{12}$ GeV has to be met. This has no direct relevance for the present paper, but for $F \approx 10^{12}$ GeV axions would be the gravitationally dominant "cold" dark matter and perhaps help to explain galaxy formation [31,10]. Then only neutrinos with $m_H > 60$ MeV would be allowed.

As a final remark, we observe that since the possibility $m_H \approx 1\,\mathrm{Mev}$ is excluded, the range of parameters allowing for a present ν_L-dominated universe from ν_H familon decay is so narrow that this case looks purely academic to us.

7.4 SINGLET MAJORON MODEL

In the first model we considered, B-L was an explicitly broken symmetry in the Lagrangian, whereas the second model had $U(1)_{B-L}$ as a spontaneously broken gauge symmetry. Within the framework of the familon model, either option is allowed. Now we take a different attitude, which is the common element of this and the following models. The B-L symmetry might well be global and spontaneously broken, with the ensuing Goldstone boson called the ***Majoron*** and denoted by χ.

The present version [32] consists in the standard $SU(2)\otimes U(1)$ model augmented with a Higgs singlet Φ which carrys $B-L=2$ and couples only to right-handed neutrinos. The "see-saw" scenario ensures naturally light neutrinos for $\langle\Phi\rangle \gg 250$ GeV. Via Higgs mixings, χ couples extremely weakly to charged particles and only the coupling with neutrinos are not negligibly small. As expected, the Majoron is "invisible" in all present laboratory experiments [32] but can be relevant to remove the cosmological constraints on neutrinos in the forbidden gap (1), (2). Specifically, the decay $\nu_H \to \nu_L \chi$ arises at the one-loop level and the lifetime is expected to be [33]

$$\tau_H(\nu_H \to \nu_L\chi) \approx 32\pi h^{-2}\epsilon^{-2}(\langle\Phi\rangle/m_H)^2 m_H^{-1} \tag{29}$$

where h is here a Yukawa constant. It was claimed [33] that [F14] the DKT bound is obeyed for any value of m_H in the forbidden gap (1), (2), provided $\langle\Phi\rangle \lesssim 10^6$ GeV, which is in turn a nice value to get sufficiently light neutrino masses. Unfortunately, a more detailed analysis [34] has shown that the couplings in eq. (29) must be so tiny that τ_H exceeds the age of the universe. Our feeling is that this difficulty probably can be overcome by sufficiently complicating the model, which however then loses much of its attractivness. For this reason we only note that by assuring that eq. (29) can work for say $h \approx 10^{-2}$, $\epsilon \approx 10^{-1}$, our constraint (10) would be satisfied for any value of m_H provided $\langle\Phi\rangle \lesssim 5.3\times 10^4$ GeV.

7.5 TRIPLET MAJORON MODEL

Alternatively, the standard $SU(2)\otimes U(1)$ model can be extended by adding a complex Higgs triplet Φ carrying $B-L=2$ but no "right-handed" neutrinos. Then, as $\langle\Phi\rangle \neq 0$ the Majoron χ arises in the physical spectrum and neutrinos get masses proportional to $\langle\Phi\rangle$[35,36]. The constraint that red giant stars do not lose too much energy via Majoron emission implies $\langle\Phi\rangle \lesssim 5 \times 10^4$ eV [36, 37, 38], which in turn leads to the conclusion that all neutrinos should be lighter than 5.10^4 eV in this model. We see that — even by a very different strategy – neutrinos are naturally light again. Among the various peculiar features of the model, all neutrinos annihilate into Majorons during the evolution of the universe, as soon as the temperature drops to their masses [36]. Therefore, no cosmological constraint applies to neutrinos in this model, and stable Majorana neutrinos as heavy as 5.10^4 eV are perfectly allowed! The same holds true in various generalized versions [39].

7.6 HORIZONTAL LEPTON NUMBERS MODEL

This model [40] is a variation of the Majoron models in which different lepton numbers are given to different generations. Clearly, more than one Higgs Φ - singlet or triplet - is needed to make all neutrinos massive upon spontaneous breakdown of global B-L symmetry. Since more than one Φ is present and they have different "horizontal lepton numbers", tree-level decays $\nu_H \rightarrow \nu_L \chi$ are allowed, and proceed with fast enough lifetimes. The modification of the triplet Majoron model is irrelevant to our considerations since all neutrino species annihilate into Majorons. On the contrary, the modification of the singlet Majoron model leads exactly to eq. (29), which is now truly exact.

8. CONCLUSION

A clear moral emerges from the present paper. At the present status of the model building art, neutrinos with masses in the range $100h^2\Omega$ eV to 5 Gev $h^2\Omega eV$ are ***naturally*** consistent with cosmology only in extensions of the standard $SU(2) \otimes U(1)$ models which contain Majorons. Models not of this kind look rather contrived and are necessarily unnatural (in the technical sense).

TABLE. Freeze-out temperature T_F, number and mass densities today as a function of the mass m_ν of a heavy neutrino (adapted from Ref. 2).

m_ν MeV	T_F $°K$	$n_\nu(t_0)$ cm^{-3}	$\rho_\nu(t_0)$ keV cm^{-3}
5×10^3	2.62×10^{12}	2×10^{-6}	1×10^1
2.5×10^3	1.45×10^{12}	1.45×10^{-5}	3.6×10^1
1×10^3	6.74×10^{11}	1.9×10^{-4}	1.9×10^2
7.5×10^2	5.33×10^{11}	4.3×10^{-4}	3.2×10^2
5×10^2	3.85×10^{11}	1.3×10^{-3}	6.5×10^2
2.5×10^2	2.24×10^{11}	9×10^{-3}	2.3×10^3
1×10^2	1.14×10^{11}	1.1×10^{-1}	1.1×10^4
7.5×10^1	9.39×10^{10}	2.3×10^{-1}	1.7×10^4
5×10^1	7.23×10^{10}	6.6×10^{-1}	3.3×10^4
2.5×10^1	4.89×10^{10}	3.7×10^0	9.3×10^4
1×10^1	3.50×10^{10}	2.6×10^1	2.6×10^5
7.5	3.35×10^{10}	4.1×10^1	3.1×10^5
5	3.30×10^{10}	6.4×10^1	3.2×10^5
2.5	3.36×10^{10}	9.1×10^1	2.3×10^5
1	3.40×10^{10}	1.0×10^2	1.0×10^5

FOOTNOTES

[F0] By *stable* we mean throughout stable on the age of the universe, and not necessarily absolutely stable.

[F1] By *indirect* we mean the contribution due to the decay products of decaying neutrinos.

[F2] Globular cluster age estimates imply $t_o \gtrsim 15 \times 10^9$ y, which for a Friedmann universe requires $h^2\Omega \lesssim 0.3$. (This constraint is relaxed if there is a positive cosmological constant.)

[F3] Our discussion will be confined to left-handed neutrinos *only*. A naive extension to "right-handed" neutrinos cannot be trusted since their weak interactions, if any, are much smaller and they decouple much before $T_D \approx$ 1 MeV in the thermal history of the universe.

[F4] Eq. (3) is the form of the DKT bound [2] obtained by assumming that all neutrinos decay at the same time. The derivation of their bound assumes that the decay-products radiation-dominate the universe up until today, and this is true only if the bound is attained. This point will be discussed in detail later on.

[F5] Throughout this paper, we will always make the realistic approximation $\tau_H \gg t_D$. Recall that $t_D \approx 1$ sec in the standard cosmology [5].

[F6] Which evolves to $\rho_T(t_0)$ today.

[F7] One can check this graphically by plotting $\log \rho_T(t)/\rho_T(t_0)$ versus $\log a(t)$.

[F8] Clearly, the same holds true by more correctly treating ν_H decaying according to the exponential law instead of all at τ_H, as we have assumed for simplicity.

[F9] This attitude is corroborated by the fact that an increase by a factor α in the neutrino number density would change t_D by $\alpha^{2/3}$.

[F10] Remember that $n_H(t_0) \approx 100\,\mathrm{cm}^{-3}$ since in the present case $m_H \lesssim 1\,$Mev.

[F11] An age of the universe $t_0 = 10^{18}$ sec would change the factor 4.8 into 2.5

in eq. (19).

[F12] Since the purpose of the present analysis is essentially illustrative, we neglect possible constraints on ϵ from neutrino oscillation experiments.

[F13] In all models considered in this paper neutrinos are Majorana particles. Both experimentally and theoretically, it is by now totally unclear whether massive neutrinos should be either Majorana or Dirac particles. A strategy for producing *naturally* small Dirac masses for neutrinos has been recently proposed [15] which differs from the "see-saw" mechanism.

[F14] In the most optimistic situation defined above.

[F15] The Yukawa couplings are taken equal to the gauge couplings, according to our optimistic attitude.

[F16] All the various Yukawa couplings have been taken of the same magnitude for simplicity, as in refs. [25,26].

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