THE PROPOSITION–AS–RULES IDEA*

Christoffer Gefwert

Academy of Finland, Helsinki

and

Stanford Linear Accelerator Center

Stanford University, Stanford, California 94305

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...a single insight at the start is worth
more than ever so many in the middle.

Ludwig Wittgenstein

0. Introduction: Brouwer and Wittgenstein.

On 10th of March 1928, L.E.J.Brouwer, the main proponent of the intuitionist philosophy of mathematics, came to Vienna to deliver a lecture entitled *Mathematics, Science and Language*.[1] Wittgenstein was present, together with Herbert Feigl and Friedrich Waismann, among the public attending Brouwer's lecture.[2] According to Feigl, perhaps this was the turning point, because afterwards, in a café, Wittgenstein suddenly began talking philosophy.[3] It seems, in retrospect, that Brouwer's lecture made a lasting impression on Wittgenstein. That he seems to have been influenced by Brouwer, is documented in a letter from Bertrand Russell to G.E.Moore in 1930, where we read: "He (Wittgenstein) uses the words 'space' and 'grammar' in peculiar senses, which are more or less connected with each other. He holds that if it is significant to say 'This is red' it cannot be significant to say 'This is loud'. There is one 'space' of colours and another 'space' of sounds. These 'spaces' are apparently given *a priori* in the Kantian sense, or at least not perhaps exactly that, but something not so very different. Mistakes in grammar result from confusing 'spaces'. Then he has a lot of stuff about infinity, which is always in danger of becoming what Brouwer has said, and has to be pulled up short whenever this danger becomes apparent".[4]

Although Wittgenstein is in sympathy with much of the intuitionist program, he completely rejects its *philosophical* foundations: "Intuitionism is all bosh — entirely. Unless it means an inspiration".[5] In particular he denies the intuitionist claim that what is essential in mathematics is the internal mental state, and that the external manifestations of that state , the linguistic accompaniments, are irrelevant. In fact, he claims that it is precisely the external, overt use of language which is critical in a mathematical activity, and that the reference to

internal mental states is not only not necessary, but cannot possibly provide us with a basis for the objectivity of mathematical inference: "An 'inner process' stands in need of outward criteria".[6]

Wittgenstein's remarks on the relevance of mental images to mathematics are clearly in conflict with the classical intuitionists emphasis on internal mental constructions.[7] His attack is primarily directed against the idea that *understanding* in mathematics is the possession of some mental state or image, and this can be seen as a refutation of the intuitionist claim that to know a mathematical proposition is to contemplate an image or be engaged in the process of mental constructions: "An investigation is possible in connexion with mathematics which is entirely analogous to our investigation of psychology. It is just as little a *mathematical* investigation as the other is a psychological one".[8]

Despite the rejection of the metaphysics and epistemology of the classical intuitionists, Wittgenstein has learned a great deal from them, especially Hermann Weyl. He read and discussed Weyl's *Die heutige Erkenntnislage in der Mathematik* and *Philosophie der Mathematik und Naturwissenschaft,*[9] and showed special interest in Weyl's remark that the formalists conceive the axioms of mathematics as analogous to rules of chess.[10] The general convergence of ideas between intuitionism and Wittgenstein's later work, is crucially important when reading Wittgenstein's later texts on the nature of philosophy. Nevertheless, one is not to be misled by this general convergence of ideas. Wittgenstein wanted to show that as far as meaning is concerned, the difference between mathematics and natural science, notably physics, is much smaller than what was usually believed. In fact, what he wanted to show, was that the alledged difference is an illusion.

Already, when Waismann expounded Wittgenstein's philosophical view on

the nature of mathematics at the Königsberg symposium (where, incidentally, Kurt Gödel first announced his celebrated incompleteness theorem), he stressed the generality of Wittgenstein's approach.[11] This lecture should have been a very important event in the "...development of the ideas of the Vienna Circle, since some of the members looked to Wittgenstein for a synthesis of logicism and intuitionism that would give a constructivist account of mathematics developed on an acceptable empiricist basis".[12] In this lecture Wittgenstein emphasized the point that the principles holding for empirical language hold also for mathematical language. His idea of the principles holding both for empirical and mathematical language is an application on a more general philosophical idea regarding the connection between meaning and understanding.[13] Our understanding of a mathematical statement can be identified with our *knowing what* would verify it, since "(t)o establish the sense of a mathematical proposition, one must make clear how it is to be verified...the sense of a mathematical proposition is the method of its verification".[14] Wittgenstein emphasized that e.g. physics and mathematics have common principles of verification. He did *not* stress that they have *empiricist* principles of verification. It it this method of verification advocated by Wittgenstein that we want to understand. We want to understand his statement that: "Asking whether and how a proposition can be verified is only a particular way of asking 'How d'you mean?' The answer is a contribution to the grammar of the proposition".[15]

1. THE LIMIT OF LANGUAGE

According to Wittgenstein the tool of thought is a *proposition*.[16] The importance he attached to this idea is clearly expressed when he says that "(m)y *whole* task consists in explaining the nature of the proposition, i.e. in giving the nature

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of all facts, whose picture the proposition *is*".[17] A proposition applies to reality because its *form* is mirrored in the form of Language, and "(w)hat finds its reflection in language, language cannot represent. What expresses *itself* in language, *we* cannot express by means of language. Propositions *show* the logical form of reality".[18] That is, a proposition shows reality because there are internal relations between propositions and facts *(Tatsache)* and we deal with a logically perfect Language. Here we encounter the idea of a logically perfect (well-formed) Language. A logically perfect Language is a language which contains nothing but well-formed propositions. Wittgenstein's view of a logically perfect Language originates from Frege who wrote that "(a) logically perfect language *(Begriff-schrift)*...should satisfy the conditions that every expression grammatically well constructed as a proper name out of signs already introduced shall in fact designate an object, and that no new sign shall be introduced as a proper name without being secured a meaning".[19] In contrast to Russell who only included formal languages in his definition of a logically perfect language, Wittgernstein stressed that also natural languages are capable of expressing every sense, since "(i)n fact, all the propositions of our everyday language, just as they stand, are in perfect logical order".[20] The structure of Language is in good logical order although a philosophical investigation is required to make it explicit.

An elementary proposition is constituted by *names*, and to every name in the proposition corresponds *one* object: "In a proposition a name is the representative of an object".[21] An object is simple. It is the object which must exist for there to be meaning. This object is, literally, the sense of a name. There is a chosen set of predicates, i.e. the categories are stable. The names must be suitably chosen: there are facts *(Gegenstände)*. Wittgenstein advocates here,

as André Maury points out, a form of Kantianism.[22] If now the senses of the Language, being facts, are the *limit of the world*, then it is relatively simple to state that a proposition is either true or false, which is exactly the property that the proposition has sense *(Sinn)*, i.e. it is such that it *can* apply to reality.[23] The situation that one name corresponds to one object can be said to be a logically perfect case. This implies that Frege's requirement that "a proper name...shall in fact designate an object" is always fulfilled.[24] As far as meaning goes it is a redundant requirement. Language, by itself, contains its own application. It embraces the world, i.e. the limit of Language coincides with the limit of the world.[25] Here we meet the important notion of a "limit of language", which, according to von Wright, is a *Leitmotiv* which runs through all of Wittgenstein's work.[26] We can say that this idea implies that what *can* happen, or be the case, is exactly what is describable (in principle) by Language. Logic sets limits to Language, not to human thought.[27] In other words, what we can think of or conceive may well be more limited than what can be expressed or what makes sense, so that the limits of thought might lie somewhere within the limits of Language. It cannot, of course, lie outside these limits, for what cannot be said cannot be thought either: "The correct explanation of the form of the proposition, 'A makes the judgement p', must show that it is impossible for a judgement to be a piece of nonsense. (Russell's theory does not satisfy this requirement.)".[28] Therefore, the traditional empiricist criterion for what is logically possible, viz. what I can imagine, may be too narrow, and is in any case defective by being psychological. In 1914 Wittgenstein wrote: "In order that you should have a language which can express or *say* everything that *can* be said, this language must have certain properties; and when this is the case, *that* it has them can no

longer be said in that language or *any* language".[29] Language shows the form of reality because Language *determines* that form. This insight, that there is a sense in which Language cannot be investigated without the investigation being circular vis-a-vis *the form of the Language* is an extremely important insight. In this sense Language excludes description of itself.

2. FORM AND JUDGMENT

In his later philosophical texts Wittgenstein attacks the Tractarian conception of Language. According to his new ideas the view formulated in the *Tractatus* tells us next to nothing concerning Language as it is. Especially he is critical of his former conception of Language as having a uniform structure: "We see that what we call 'proposition' and 'language' has not the formal unity that I imagined, but a family of structures more or less related to each other".[30] When understanding the notion of Language, we realize that it is a mistake to presuppose a uniform structure of Language, the idea of which have forced philosophers to be "craving for generality".[31]

As has been stated elsewhere,[32] to engage in a philosophical investigation, comes to realizing that it is what we actually *do*, i.e. our concrete activity (will), that is real or concrete, "...since it is our *acting* which lies at the bottom of the language-game". [33] What Wittgenstein wants to convey here is the deep insight that one *engages* in philosophy. A philosopher, as long as he is doing philosophy, is not putting emphasis primarily on observation, that is, any kind of seeing. He is not observing at all. He is doing something. He is asserting. Now one is to realize that to *assert* is to act. This is to say: we take "assertion" to stand for the german *Urteil (judgment)*. Wittgenstein never uses the word *Urteil;* his word is "thought" *(Gedanke)*. Nevertheless, he holds that a *Gedanke* is an *Urteil.*[34]

Asserting is not something one can *see* happening, in the same sense as one can e.g. see a car on the street. To assert is to perform an act of codifying canonical steps.[35] If asserting is to engage in constructing by canonical codification, then, "(e)very instruction can be construed as a description, every description as an instruction".[36] It is to peform an act of the will. It is to engage in a task. The insight of a philosophical assertion as an activity of canonical codification by a *participator* will be of crucial importance when we are to understand Wittgenstein's comments on "logical grammar".[37] The very idea behind the promotion of an observer to participator rests on the insight that relations between Language and reality are not open to meaningful description given that the criterion of propositional identity is provided by a canonical step (it makes the criterion inaccessible). This is to say that such a criterion is embodied in our propositions and is not to be found anywhere else and, furthermore, that no proposition is a description of itself. One could say that this (inaccessible) criterion of identity expresses a "principle of novelty" providing knowledge of meaning *sui generis*.[38] In order to engage as a participator in a task we must never ask what a certain body of knowledge is about, but ask *what* someone can do who understands the Language, or who possesses that knowledge. This puts the core of the problem as a problem of *form*, since to ask what someone can do who understands a particular Language requires that the person has grasped the form of the Language. Otherwise the person could not do anything with the Language since he could not use it. If a person has grasped e.g. the form of the language of arithmetic he can engage in a practical arithmetical task. And it is the same with other Languages.

Furthermore, one must realize that the form of a Language may be almost

anything. If the Language is written, we are concerned with two dimensional arrangements of alphabetical symbols. The forms of expressions are characteristic patterns of of such things, possibly involving a combination of symbols peculiar to the form in question. An expression is something which we read as built up by successively arranging already obtained expressions in such patterns. Then the expressions in the variable places of the pattern are the constituents of the expression that is formed. If the Language is spoken, we are concerned with temporal sequences of the phonemes that are used by its speakers. The form of expressions are patterns of such things, again, involving phonemes peculiar to the form. An expression is the sound which we hear as formed by successively arranging already obtained expressions in such patterns, these being the constituents of the expression that is formed. A more exotic variant is to imagine _Languages based on temporal sequences of electrical impulses on a wire or of configurations of holes punched on a tape, as used in computers. In each case the *form* of the Language will be something different. There may be a countless number of different forms. What the forms are does not belong to the *essence* of a Language, but is something accidental. The main point is that a Language has *a* form: this is what makes it into what it is - a Language.

As we said earlier, the form of a proposition excludes the possibility that the sense of the proposition can be justified by reference to fact. The sense, or the (logical) form, of a proposition cannot be justified except by *repeating* the proposition. Consequently, it is *sense*, rather than sentence used, which is the criterion of propositional identity. This has by Maury been coined as "Wittgenstein's principle".[39] The principle itself is expressed by Wittgenstein in the following way: "Die Grentze der Sprache zeigt sich in der Unmöglichkeit, die

Tatsache zu beschreiben, die einem Satz entspricht (seine Übersetzung ist), ohne eben den Satz zu wiederholen. (Wir haben es hier mit der Kantischen Lösung des Problems der Philosophie zu tun)".[40] The principle tells us that a concrete activity, being the sense (canonical step), is the criterion of propositional identity and thereby the criterion of "repetition". Indeed, the criterion is inaccessible: "(f)or of course I don't make use of the agreement of human beings to affirm identity. What criterion do you use, then? None at all".[41] It is impossible to *assert (judge)* the identity of the sense of two propositions. For in order to be able to assert (judge) anything concerning their sense, a precondition is, that one knows their sense, and this, in turn, requires that one knows whether what they mean is the same or different. It requires that one *already* know their sense (and thereby the criterion of "already"). It presupposes that one already knows how the ordinary forms of expressions are to be understood. One understands their *logical form.* But this is exactly what a philosophiel investigation is supposed to do. It determines by canonical steps *how* the ordinary forms of expressions are to be understood. It determines their logical form. Consequently, one can only assert (judge) the sense of a proposition *once.*[42] It is to assert (judge) according to the

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proposition-as-rules idea

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by a canonical step, i.e. a "...proposition has the dignity of a rule".[43] It is to engage in the codification of a canonical assertion (judgment) when applied, that is, a "...proposition is to shew us what it makes SENSE to say".[44] We can say that a Language is a collection of instantiations of propositions as rules. These

(canonical) instantiations are, in Wittgenstein's terminology, "depth grammatical" expressions and sentences.[45] One can therefore realize that to engage in a philosophical investigation is *itself* the criterion of propositional identity. A philosophical investigation *shows* the "principle of novelty", which, in effect, says that the criterion of identity is never available. And, according to Maury, "there is no other possibility. The defining feature of a proposition is surely its sense".[46] Translations and logical analyses of propositions, which are to be sensepreserving, are, in the end, nothing but repetitions of those propositions. This is the core of the proposition-as-rules idea. Sense and concrete activity are intrinsically connected. Thus to engage in a philosophical investigation is always to engage in a *task*, the completing of which is to terminate the investigation. This insight becomes important when engaging as a participator in asserting (judging), and can be applied "to give an account of assertions about meaning themselves, regarded as assertions *within* our language".[47]

3 On Assertions and Judgments

In Frege's *Begriffschrift* the assertion-sign "⊢" *(Urteilstrich)* is introduced to mark the expression (act) of judgment, or assertion.[48] It is composed of two constituents: the judgment-stroke "|" the content-stroke "−". These represent Frege's crucial distinction between an act of judging and its object, i.e. a judgeable content. One way of understanding the assertion-sign is to maintain, like Frege, that it belongs to psychology, i.e. "always to separate sharply the psychological from the logical, the subjective from the objective".[49] One of Frege's main aims was to free logic from psychologism, to draw a sharp boundary between the provinces of logic and psychology. Wittgenstein accepts this program in the *Tractatus* and gives a fairly explicit account of where to draw the boundary. For

Frege the distinction follows from the distinction between knowledge and truth and from the independence of sense from the facts. Whether I know that p or whether I am justified in asserting p involve investigation into my state of mind. It involves investigation into what evidence *I have* for the truth of p. The truth of p, however, is independent of my state of mind, indeed of anybody's state of mind. Logic concerns inferences only, i.e. the assignment of truth to statements relative to the truth of others. Since sense *(Sinn)* is independent of the facts, it is independent of any knowledge of what is the case. It is not a construct out of anything subjective,[50] and therefore logic is independent of psychology, the science of the subjective.

Such an attitude seems also to have been embraced by Russell and Whitehead in the *Principia Mathematica* when they adopted both the idea and the symbol from Frege.[51] That Wittgenstein regarded them to embrace such an attitude is confirmed by his comment in the *Notebooks 1914-16* that "(t)he assertion-sign is logically quite without significance. It shows, in Frege and Russell, that these authors hold the propositions so indicated to be true. A proposition cannot possibly assert of itself that it is true. Assertion is merely psychological".[52] Notions such as assertion, evidence, proof, knowledge, understanding and meaning are psychological. The difference between assertion and questions, orders, etc. is psychological.[53] So too is any consideration of what justifies an assertion as opposed to a supposition. Proofs in logic and mathematics are merely "mechanical expedients" to bring about an appropriate "intuition", the recognition of a statement as expressing a tautology or a (true) mathematical equation.[54] Similarly, any evidence that a person might have for p is merely a symptom of his knowledge that p. Perfect knowledge, such as God might have, would not depend on

proof and evidence. It is a truth of psychology that mere mortals need proofs and evidence in order to recognize the truth of certain statements. The limit of human knowledge are obvious from reflection on the fact that we do not immediately recognize every tautology as a tautology.

In his later thought Wittgenstein considerably changed his opinion concerning the assertion-sign. According to Kripke, the result was "that Wittgenstein proposes a picture of language based, not on *truth-conditions*, but on *assertability-conditions* or *justification-conditions*: under what circumstances are we allowed to make a given assertion?".[55] Is this position a correct reading of Wittgenstein? Is Wittgenstein really proposing a picture of language *based* on assertability-conditions and *not* on truth-conditions? I think not. But then the problem is: how are we to understand as *participators* the notion of "assertion" in connection with Wittgenstein's remarks concerning "logical grammar"? When discussing the notion of assertion (judgment) Wittgenstein wrote: "...assertion is not something that gets added to the proposition, but an essential feature of the game we play with it...Imagine it were said: A command consists of a proposal ("assumption") and the commanding of the thing proposed...(W)hat does a proposition's 'being true' mean? 'p' *is true* = p. (That is the answer.) So we want to ask something like: under what circumstances do we assert a propositon? Or: how is the assertion of the proposition used in the language-game?".[56]

Here it seems that Wittgenstein is making a distinction between (1) the concrete activity of asserting (judging) and (2) of that which is asserted, i.e. the assertion (judgment) as an object of knowledge. He is trying to make us appreciate the difference between judgments and propositions. In its first sense an assertion is taken to stand for engagement in a philosophical investigation in or-

der to provide understanding. It stands for the concrete activity of engaging in a construction by canonical steps of a depth grammar. It is when we are engaged in a task of constructing a depth grammar that we engage in the investigation by formulating and explaining rules of the form "p is a proposition". A word of warning: It is important to note that the expression "depth grammar", as Wittgenstein used it, is not to be confused with, what linguists following Noam Chomsky in the tradition of generative grammar, understand by the very same expression.[57] The *concrete activity* of codifying propositions is an essential part of Wittgenstein's depth grammar. It is to engage in creating *a* language-game.[58] In a philosophical investigation we *create* by canonical steps a depth grammar, and by doing this we create *a* Language. In this sense we can say that e.g. mathematics and physics are creative activities. By the expression "Language" is meant the whole codified *result* of a concrete philosophical investigation: *the* language-game.[59]

To explain the meaning of an assertion as an object of knowledge one has to explain an assertion of the form "*p* is a proposition". This form has to be explained before all others. To give an explanation of this form is to explain what someone must know in order to set out on the task of a philosophical investigation. One has to give an account of, so to speak, the essential insight of the investigation. To explain the essential point is to engage in asserting (judging) in a *non-redundant* manner, since one is not to presuppose understanding of what the investigation is "about". On the contrary, the task *is* to determine what the investigation is "about". To explain what the investigation is "about" constitutes the philosophical task. It is to engage in an *essential* explanation. As an analogy of an essential and an inessential explanation of meaning (of "about")

one could give the following example. A person who has never seen a combustion engine wants to know how it works. Especially he wants to know the function of the spark plugs. He wants to have information "about" the function of spark plugs. Accordingly, we engage in giving an explanation of meaning concerning the *function* of spark plugs in a combustion engine. There is no *point*, in such an explanation, to simply say that the engine works in virtue of the spark plugs. Such an explanation is a redundant explanation vis-a-vis the meaning, or the function, of the spark plugs. E.g. if the person to whom we are explaining suggests that the function of the spark plugs in an internal combustion engine is: to be connected by cables via the distributor to the battery, we must explain to the person *how* the engine works. This is to explain the function in a non-redundant manner. By this explanation we are to convince the person that the function of the spark plugs is to ignite the mixture of gasoline and air which is sucked into the cylinders from the carburettor. Not that it is wrong to say that the spark plugs are connected by cables via the distributor to the battery: they certainly are. But that is not what we should essentially pay our attention to in order to understand the *essential function* of the spark plugs in the running of the engine.

Now, let us investigate this a little bit further. First, to codify the rules of the essential use of a functional expression (e.g. "spark plug") when engaging in a philosophical investigation, is, consequently, like codifying the internal mechanism of the combustion engine if we use the analogy above. Similarly with the case of understanding a proposition. One cannot presuppose understanding of what a proposition is "about", since to engage in explaining the meaning of a proposition of the form "*p* is a proposition", is precisely to give the rule from

which the meaning of the proposition is determined. As Wittgenstein expressed this insight: "The causes of our belief in a proposition are indeed irrelevant to the question what we believe. Not so the grounds, which are grammatically related to the proposition, and tell us what proposition it is".[60] It determines *what* we are doing with the proposition. It determines a proposition of the form "*p* is a proposition" as a functional expression. And this is exactly to read "*p* is a proposition" according to the proposition-as-types idea put forward by Per Martin-Löf.[61] One must realize that it is essential to a functional expression that one knows the rule for its use (what to do) and that the rule is unambiguous. Thus if a functional expression produces a value in a philosophical investigation, it will always produce the same unambigous value. This is *not* something we prove of a functional expression. For having understood something as a func--tional expression, we have *no doubt* concerning what to do with it, that is, we have already understood the instructions as unambiguous at every turn. In such a case: we obey the rule blindly.[62] In the *Tractatus* Wittgenstein thought of a functional expression as a formal (internal) property, i.e. as a property of objects. This explains why he said that "(a) property is internal if it is unthinkable that its object should not possess it".[63] The *task* in investigating a proposition of the form "*p* is a proposition" is to determine *what* we are doing with *p* as a functional expression. Should we, in a philosophical investigation, omit the rules for functional expressions, there would be no functional expressions, no propositions, and consequently no investigation.

Secondly, one has to explain affirmation, that is an assertion (judgment) of

the form "p is true", or in Frege's notation:

$$\vdash p$$

To be entitled to assert a proposition of the form "p is true" one must be able to give direct evidence for the truth of p. That is to say: "'The proposition is either true or false' only means that it must be possible to decide for or against it. But this does not say what the ground for such a decision is like".[64] Thus there can be no question of asserting p to be true unless one has previously asserted p to be a proposition. To be entitled to assert a proposition of the form "p is true" requires in concrete practice ability to give a proof of p. If one reads "assertion" as an object of knowledge then one can say that to assert is to prove, i.e.:

$$A \leftrightarrow \exists a \; (a \text{ is a proof of } A)$$

In a concrete practical situation to be able to provide a proof of p requires that one already have understood a proposition of the form "p is a proposition". Wittgenstein expressed it thus in the *Tractatus*: "To understand a proposition means to know what is the case if it is true. (One can understand it, therefore, without knowing wheteher it is true.)".[65] Later, in the *Investigations*, he wrote "What *engages* with the concept of truth (as with a cogwheel), is a proposition".[66] This insight we can see clearly expressed by Wittgenstein when he writes the formula:[67]

$$\text{`` `}p\text{' is true} = p\text{''}$$

One is entitled to assert a proposition of the form "p is true" if one can give the circumstances, or the direct grounds, for affirming p: "...under what circumstances do we assert a proposition? Or: how is the assertion of the proposition

used in the language-game?". To answer these questions in a non-redundant manner vis-a-vis meaning is exactly to require that we already are in a position to assert a proposition of the form "p is a proposition", i.e. we have already understood the circumstances when it is a functional expression. All we can say *in general* is the formula above. Truth, consequently, is a form of judgment. It is not knowledge independent. Indeed, to explain the notion of truth is precisely to explain *what one must know* in order to have the right to judge a proposition to be true, since, as Wittgenstein says, "(i)t would be nonsense to say that we regard an assertion as justified because it is certainly true".[68] Wittgenstein's point is to correct an unjustified optimism of those who think that the notions of truth and falsity will by themselves provide a sufficient basis for coping with questions concerning *meaning.* If truth is the common form of affirmative judgments, *then truth-conditions cannot be but assertion-conditions.* This is, again, precisely the position adopted by Martin-Löf.[69] Now we can also realize that Wittgenstein did not (*pace* Kripke) propose a picture of Language based on assertion-conditions *instead* of truth-conditions. As Peter Winch expresses it: "To say that he *replaced* 'truth- conditions' by 'assertion-conditions' (another term of art) in the 'theory of meaning' is misleading in its suggestion that he was offering an alternative theory of meaning".[70]

4 WITTGENSTEIN ON ASSERTION

What is being argued here is that Wittgenstein can be seen as making a distinction between assertions (judgments) as concrete philosophical investigations and assertions (judgments) as objects of knowledge. He is making a distinction between assertions (judgments) and propositions. When engaging in explaining the latter in virtue of a philosophical investigation the explanation splits into

two parts of the form "p is a proposition" and "p is true". If we allow for this distinction we are also in a position to understand his point that the words "proposition" and "true" are interwoven.[71] One has furthermore to realize that there are not two kinds of assertions (judgments). As Wittgenstein put the point: "One might think: if philosophy speaks of the use of the word 'philosophy' there must be a second-order philosophy. But it is not so...".[72] The distinction concerning assertions made above is only to be made when engaging in a philosophical investigation, that is, when one is engaged in a philosophical task of explaining problems of meaning.

In his later stages Wittgenstein continued to stress that the problems concerning philosophy of mathematics cannot, essentially, be separated from his philosophical method in general.[73] The fundamental thesis of this new generalized approach is, as noted, that sense is to be explained in terms of assertion (judgment).[74] We identify the sense of a proposition p with the act of asserting p. The proposition p is then a regulative assertion-condition (A-condition). It is also a regulative truth-condition. We identify the sense of p with the possible grounds for claiming *what to do with p*, i.e. what we must know in order to assert (judge) p to to be a proposition. The A-conditions regulating a *person program* establishes *certainty*, in Wittgenstein's sense, on the regulated program. That is to say: the codified A-conditions are *criteria* of understanding, i.e. defeasible regulative conditions conferring certainty *ceteris paribus*.[75] That Wittgenstein understood by "philosophy" *person programs* of this kind, is supported by a vast amount of evidence. It is clearly stated in a quite general way when he states that, "There is not *a* philosophical method, though there are indeed methods, like different therapies".[76] He also states that "(w)hat counts as an adequate test

of a statement – belongs to logic. It belongs to the description of the language-game".[77] For Wittgenstein *logic* is the study of the essential features of Language. Here the notion of *logic* is used by Wittgenstein as standing for the code of the person program (logical grammar), reminiscent of Husserl's and Kant's use of the term "transcendental logic". In his writings from 1929 onwards, "logic" is very often called "grammar".[78] Logic, in Wittgenstein's sense, shows the "logical form" of a Language. Although there is a significant parallel between Kant's transcendental logic as the "...rules of understanding in general"[79] and Husserl's formulation of "ausgangsfragen der Transcendental– Logischen problematik: Die Grundbegriffsprobleme",[80] on the one hand, and Wittgenstein's grammar as the rules for meaning and sense, on the other, there is also an important difference. E.g. Kant models his transcendental logic on Aristotelian logic: the categories are stable. Wittgenstein, on the other hand, states that *logic* stands for many different language-games, and wishes to free grammar from any particular system of rules. Accordingly, we can say that logic must accomodate itself with understanding and not the other way around. Wittgenstein's grammar cannot be like Kant's transcendental logic, which "treats of understanding without any regard to difference in the objects to which the understanding may be directed".[81] Logical grammar, in Wittgenstein's sense, is a revolt against self-sufficient logic, in the sense that grammar is more fundamental than "mathematical logic", and that before questions of truth and falsity there are more basic questions of sense *(Sinn)* and nonsense *(Unsinn)*: " 'Mathematical logic' has completely deformed the thinking of mathematicians and of philosophers, by setting up a superficial interpretation of the forms of our everyday language as an analysis of the structures of facts. Of course in this it has only continued to build on the Aristotelian

logic".[82]

5 The Grammatical Necessity

Wittgenstein wanted to suggest, not merely that there is error in the way in which we ordinarily interpret our recognition of necessity, but more: the whole notion of recognition, of discovery, is misappropriated in the case of necessary statements. That a given statement is necessary always consists in our having expressly adopted that very statement as unassailable by a canonical step. We treat it as *certain*. We never question such an expression or statement since "nothing in our *Weltbild* speaks in favour of the opposite".[83] With this step Wittgenstein considerably expands the arena of necessities. His expanded arena of necessity crops up also in his central contention that every proposition-as-rule (necessity) is part of a form of representation *(Darstellungsform)*, or a picture *(Bild* or *Weltsbild)*.[84] Every proposition-as-rule is context-bound vis-a-vis logical form. Here it is important to note the distinction between "form" and "logical form". Any expression is *a* form, but only an expression which is *correct* is a logical form. Read in this way the notion "logical form" contains a normative element. It contains the *point* (the essence) of the activity one is engaged in. In other words: "*Essence* is expressed by grammar".[85] And when we express necessities, we do not *discover* anything: "In grammar you cannot discover anything (since) grammar is not accountable to any reality. It is grammatical rules that determine meaning (constitute it) and so they themselves are not answerable to any meaning and to that extent are arbitrary".[86] One does not *discover* grammatical rules: codification by canonical steps (abstraction) is a creative activity. Now one can realize that it is impossible, by engaging in a *person program,* to instantiate a string of eternally (in a temporal sense) valid regulative rules: "Die

Rolle der 'logischen Analyse'. Wie kann ich den Satz *jetzt* verstehen, wenn die Analyse soll zeigen können, *was* ich eigentlich verstehe. //was es ist was ich verstehe. //Hier spielt die Idee des Verstehens als seltsamen geistigen Vorgangs hinein".[87] Indeed, "analysis" is to show us *what* we know. It provides knowledge concerning what to do. But activities are subject to grammatical indeterminacy. Grammatical rules may change as a result of our activity. That is to say: the (logical) form of Language may change as a result of spontaneous action. But we cannot *formulate* and explain the change, and we cannot whistle it either, to use Frank Ramseys famous expression.[88] To attempt this is to step outside the form of Language; it is to run against the limit of Language. We could also characterize this idea as follows: "'It's impossible for us to discover rules of a new type that hold for a form with which we are familiar. If they are rules which are new to us, then it isn't the old form. The edifice of rules must be *complete*, if we are to work with a concept at all - *we cannot make any discoveries in syntax.* -For, only the group of rules *defines* the sense of our signs, and any alteration (e.g. supplementation) of the rules means an alteration of the sense. Just as we can't alter the marks of a concept without altering the concept itself.(Frege)".[89]

6 SYNTAX AND SEMANTICS

To engage in a philosophical investigation is to stress the concrete engagement of constructing a logical depth grammar (person program) in order to explain problems of meaning. It is to show the logical form of a Language. In a philosophical investigation this coincides with the task of dissolving problems of sense. This is what justifies a philosophical investigation to be called a *constructivist* investigation. In this sense Languages are constructive. Here one can find a trace of the influence on Wittgenstein by Brouwer's intuitionism. The

codification of a practice is a secondary activity. This secondary activity we call *syntax*. By syntax is to be understood assertions (judgments) of the form "p is a proposition". The (verbalized) explanation of the practice (syntax), that is, the explanation of the principles (rules) which regulate it and which we *formulate* when we engage in syntax, is again a third, derivative, or "higher order" activity. To engage in an explanation of this third kind is what we call *semantics*. By semantics is consequently to be understood the act of formulating and verbally explaining assertions (judgments) of the form "p is a proposition". One can say that semantic judgments are *founded* on syntactic judgments.[90] Consequently, to engage in syntax and semantics, respectively, is to formulate and explain the logical form of a Language. This is the aim of a philosophical investigation (person program).

When we engage in a philosophical investigation we assert propositions constituting the assertion-, or judgment-conditions required by Kripke.[91] In the *Philosophical Grammar* Wittgenstein wrote that "...grammatical explanations (are) explanations which *create* language".[92] We create a Language by engaging in the construction (act of asserting) a logical depth grammar, i.e. "Language *means* the totality of propositions".[93] By "Language" is to be understood *semantics* as the notion is used here. Codified, and verbally explained, propositions-as-rules, constitute a logical depth grammar. To construct, and explain, the depth grammar in question is to construct the Language in question. A depth grammar (Language) is made up of two parts: a *formal* and a *non-formal* part. In order to start the investigation one has to explain *formal* assertions (judgments) of the form "p is a proposition", since "(t)he sense of a proposition (or thought) isn't anything spiritual; its what is given as an answer to a request for an explanation

of sense".[94] When we set out on the task of doing this, we should be able to start the investigation according to the general formal rule:

$$\frac{(a)}{p}$$

which is the *general* expression of the "proposition-as-rules" idea. Whenever one engages in a task (a) of this kind, one notices that from (a) follows p. Recall that, to understand a rule, we must understand the conclusion under the assumption that the premises have been understood. A rule of this form gives knowledge of *what* to do. An investigation of this kind can accordingly be seen as a program (task) that gives information of its own evaluation. A proposition taken as a *canonical (normal) rule* consequently always has itself as value. Propositions of the form "p is a proposition" are statements which are analytic *a priori*, in Kant's terminology, since in statements of this kind the predicate is contained in the subject, provided, like Wittgenstein, we take the subject to be the object.[95] A statement of this kind can only be understood. There is no question of it being true or false. It has to be understood in order to successfully engage in a philosophical investigation. This is stated by Wittgenstein in a general way when he says that "(a) general propositional form determines a proposition as part of a calculus".[96] By calculus Wittgenstein here means a Language. Alternatively, we can say that an investigation of this kind provides the "data" or "facts" read as canonical rules. Wittgenstein expressed it thus: "The only correlate in language to an intrinsic necessity is an arbitrary rule. It is the only thing which one can milk out of this intrinsic necessity into a proposition".[97]

On the other hand, a non-formal rule explains what is essential to an activity in order for it to be successfully terminated. A rule of this form shows the *purpose*

of the activity being investigated. It shows the *point* of the activity.[98] It shows
what one must know in order to *successfully* engage in a certain activity. It is a
non-formal proposition-as-rule. It shows what one must know in order to be able
to complete a practical task in a successful way. The rules of this kind are the
rules one must formulate in order to terminate the philosophical investigation.
A philosophical investigation must terminate. (As is well known this was an
absolut requirement by Wittgenstein concerning philosophical investigations). In
doing this we achieve closure of practice and philosophy vis-a-vis meaning. Non-
formal rules provide *objective* understanding of when a subjective task (e.g.
proof, experiment) is completed.[99] Consequently, a philosophical investigation
must have a purpose, *a point,* in the construction of a depth grammar, in order
to be successfully terminated, "(s)o I am inclined to distinguish between the
essential and the inessential in a game too. The game, one would like to say, has
not only rules but also a *point*".[100]

Codified propositions-as-rules of these two kinds constitute a logical depth
grammar, which, in turn, constitutes a Language, since "'(a) proposition be-
longs to a language'. But that just means: it is units of language that I call
'propositions'".[101] A Language, consequently, does not only have a *descriptive*
grammar. It also contains a normative component. It is required in order for the
investigation to be able to show the logical form of the Language. The logical
depth grammar shows the logical form of the Language. A Language can also
be seen as a constructivist *Begriffschrift* to use a term by Frege. This is also
indicated by Wittgenstein's use of the "account book" analogy.[102] To engage in
formulating a depth grammar can be seen as a task, or program, of codifying,
or writing, an "account book" containing expressions and sentences asserted by

canonical steps. When we engage in syntax and semantics in order to formulate and explain the meaning of expressions and sentences by a constructivist *Begriffschrift* we must realize that a *sentence* is the meaning carrying unit of a Language. This can be seen as a reaffirmation of Frege's *Context Principle* which says that we are "never to look for the meaning of a word in isolation, but only in the context of a sentence".[103]

The meaning of a particular sentence is explained by explaining how it is formed and explaining the rules (syntax) by means of which it is formed. Thus, when we have explained the rules (syntax) of the Language, we have also explained the meanings of the sentences that can be formed by means of these rules, that is, the well-formed (logically adequate) sentences of the Language. A rule is explained by explaining how the meaning of the conclusion of an instance of the rule depends on the meanings of the premises of that instance. This is the same as explaining the meaning of the conclusion under the assumption that the meanings of the premises are already known. However, before a rule can be explained, it must be explained, for each one of the sentence forms occurring in the rule, what is the meaning of a sentence of that form. In a person program these *general* meaning explanations are associated with the sentence forms of the Language and come *before* the explanations of the rules, whereas the explanation of the meaning of a *particular* sentence comes *after* the explanations of the rules by means of which it has been formed. This is the *general* way person programs are to be executed. This is the *general* way person programs do bring about the realization of Frege's contextual principle. To engage in a person program *is* to bring about the realization of this methodological principle.

To explain is to engage in semantics. A philosophical investigation, con-

sequently, is to engage in semantics. When we have completed the investigation (person program) by engaging in semantics we have complete clarity, since "grammar are all the conditions (the method) necessary for ...the understanding (of sense)".[104] We have formulated and explained the logical form of a Language: the person program has terminated.

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3. Feigl, Herbert., *The Wiener Kreis in America*, in, *Perspectives in American History*, Vol.II., Harvard (1968), *639.*

4. Russell, Bertrand., *The Autobiography of Bertrand Russell 1914-44*, Little, Brown and Company, Boston, Toronto (1968), *297.*

5. Diamond, Cora ed., *Wittgenstein's Lectures on the Foundations of Mathematics, Cambridge 1939*, New York (1976), *237.*

6. Wittgenstein, Ludwig., *Philosophical Investigations*, Oxford (1968), §580.

7. Wittgenstein, Ludwig., *Philosophical Grammar*, Oxford (1974), *106:*"In the consideration of our problems one of the most dangerous ideas is the idea that we think *with*, or *in, our head.* The idea of a process in the head in a completely enclosed space, makes thinking something occult. 'Thinking takes place in the head' really means only 'the head is connected with thinking'. -Of course one says also 'I think with my pen' and this localisation is at least as good. It is a travesty of the truth to say 'Thinking is an activity of our mind, as writing is an activity of the hand"'.

8. Wittgenstein, L., *Investigations* (cf.n.6), *232.*

9. McGuinness, B. ed., *Wittgenstein and the Vienna Circle*, Oxford (1979), *37,81-2,84*; Weyl, H., *Die heutige Erkenntnislage in der Mathematik*, Symposion *1, 1-32*; *Philosophie der Mathematik und Naturwissenschaft*, München und Berlin(1926).

10. Ibid. *103-5;* cf.Witgenstein, Ludwig., *Philosophical Remarks*, Oxford (1975), *170,176.*

11. At Königsberg, in September 1930, there was a discussion concerning the foundations of mathematics. The logicist school was presented by Carnap, the intuitionist by Heyting and the formalist by von Neumann. These lectures are all published in Benacerraf-Putnam eds., *Philosophy of Mathematics* (cf.n.67), *41-65.* Due to unfortunate circumstances Waismann's contribution: *Über das Wesen der Mathematik: Der Standpunkt Wittgensteins*, which outlined Wittgenstein's position, was never published. Cf. Baker, G.P., *Verehrung und Verkehrung: Waismann and Wittgenstein*, in Luckhardt, C.G. ed., *Wittgenstein: Sources and Perspectives*, Harvester (1979), *271.*

12. Ibid. *271.*

13. Wittgenstein, L., *Investigations* (cf.n.6), §81.

14. Baker, G.P., *Verehrung and...* (cf.n.11), *271;* In his notes on Wittgenstein's lectures (Moore, G.E., *Philosophical Papers*, London (1959), *266*) G.E.Moore wrote: "Near the beginning of (Lent Term 1930) he (Wittgenstein) made the famous statement, 'The sense of a proposition is the way in which it is verified'; but in (May Term 1932) he said this only meant 'You can determine the meaning of a proposition by asking how it is verified', and went on to say 'This is necessarily a mere rule of thumb, because 'verification' means different things'...He went on to say 'Verification determines the meaning of a proposition only where it gives the grammar of the proposition in question"'.

15. Wittgenstein, L., *Investigations* (cf.n.6), §353.

16. Ibid. §§11,22-3.

17. Wittgenstein, Ludwig., *Notebooks 1914-16*, Oxford (1969), *39*.

18. Wittgenstein, L., *Tractatus* (cf.n.16), §4.121.

19. Frege, Gottlob., *Philosophical Writings of Gottlob Frege*, (Black, Max andGeach, Peter eds. and transl.), Oxford (1970), *70*.

20. Wittgenstein, L., *Tractatus* (cf.n.16), §5.5563.

21. Ibid. §3.22.

22. Maury, André., *Wittgenstein and the Limits of Language*, Acta Philosophica Fennica, Vol.*32*, Helsinki (1981), *157*.

23. Wittgenstein, L., *Tractatus* (cf.n.16), §2.223.

24. Frege, G., *Philosophical Writings...* (cf.n.19), *70*.

25. Wittgenstein, L., *Tractatus* (cf.n.16), §5.6.

26. von Wright, Georg Henrik., *Wittgenstein on Certainty*, in von Wright ed., *Problems in the Theory of Knowledge*, The Hague (1972), *55*.

27. Cf. Lee, Desmond., *Wittgenstein's Lectures: Cambridge 1930-1932*, Oxford (1980), *45*:"Thought is therefore autonomous, complete in itself; and anything not given in my thought cannot be essential to it. Thought does not point outside itself. We think it does because of the way in which we use symbols. We compare the symbol with something else or translate it into a description in other terms or an action within our control. A description must be entirely determined by the fact described plus the grammar and vocabulary used. I can *choose* the language which I use, but

my description is then determined by the grammar and vocabulary of the language chosen".

28. Wittgenstein, L., *Tractatus* (cf.n.16), §5.5422.

29. Wittgenstein, L., *Notebooks 1914-16* (cf.n.17), *107*.

30. Wittgenstein, L., *Investigations* (cf.n.6), §108; By the term *Satz* is here to be understood a proposition, not a sentence.

31. Wittgenstein, Ludwig., *Blue and Brown Books*, Oxford (1969), *17*.

32. Gefwert, Christoffer., *A Participator: A Metaphysical Subject*, SLAC-PUB-3277, Stanford University, December (1983), *19*.

33. Wittgenstein, Ludwig., *On Certainty*, Oxford (1974), §204; cf. also *Investigations* (cf.n.6), §23.

34. Griffin, James., *Wittgenstein's Logical Atomism*, Oxford (1964), *114-5*.

35. Gefwert, C., *A Participator:...* (cf.n.32), *22*.

36. Wittgenstein, L., *Phil. Remarks* (cf.n.10), *59*.

37. Gefwert, C., *A Participator:...* (cf.n.32), *19-21*.

38. Cf. Wittgenstein, Ludwig., *Ursache und Wirkung: Intuitives Erfassen*, edited with supplementary notes by Rush Rhees, and with a translation by Peter Winch: *Cause and Effect: Intuitive Awareness*, Philosophia, vol.6, nos 3-4 (1976), *420:* "Language - I want to say - is a refinement, *im Anfang war die Tat* ('in the beginning was the deed'). First there must be firm, hard stone for building and the blocks are laid rough- hewn on one another. *Afterwards* it's certainly important that the stone can be trimmed, that it's not *too* hard".

39. Maury, André., *Wittgenstein and...* (cf.n.22), *153;* Cf. Wittgenstein, L., *Phil. Remarks* (cf.n.10), *208:* "A form cannot be described: it can only be presented".

40. Wittgenstein, Ludwig., *Vermischte Bemerkungen*, edited by G.H. von Wright in collaboration with Heikki Nyman, Frankfurt am Main (1977), *27;* The English translation of the above work is *Culture and Value*, P.Winch (transl.), Oxford (1980). By the expression *Satz* is here to be read a *proposition*, not a sentence, contrary to Winch translation.

41. Wittgenstein, Ludwig., *Remarks on the Foundations of Mathematics*, Revised Edition, Oxford (1978), VII §40.

42. cf. Wittgenstein, L., *Phil. Remarks* (cf.n.10) *111:*"A co-ordinate of reality may only be determined *once*"; cf. Lee, D.,*Wittgenstein's Lectures...* (cf.n.27), *24:* "Proposition and judgment are the same thing, except that the proposition is the "type" of which the judgment (made in a particular place by a particular person at a particular time) is the token".

43. Wittgenstein, L., Remarks on the... (cf.n.41), I §165.

44. Ibid., III §28: "The proposition proved by means of the proof serves as a rule - and so as a paradigm. For we *go by* the rule. But does the proof only bring us to the point of going by this rule (accepting it), or does it also shew us *how* we are to go by it. For the mathematical proposition is to shew us what it makes SENSE to say".

45. Wittgenstein, L., *Investigations* (cf.n.6), §664: "In the use of words one might distinguish 'surface grammar' from 'depth grammar'; cf. also *Tractatus* (cf.n.16) §4.0031: "All philosophy is a 'critique of language' (though

not in Mauthner's sense). It was Russell who performed the service of showing that the apparent logical form of a proposition need not be its real one".

46. Maury, André., *Wittgenstein and...* (cf.n.22), *151*.

47. Kripke, Saul A., *Wittgenstein on Rules and Private Language: An Elementary Exposition*, in Block, Irving ed., *Perspectives on the Philosophy of Wittgenstein*, Oxford (1981), *276*.

48. Frege, Gottlob., *Begriffschrift, eine des aritmetischen nachgebildete Formelsprache des reinen Denken*, Hildesheim (1964), §2.

49. Frege, Gottlob., *The Foundations of Arithmetic*, translated by J.L.Austin, Oxford (1950), *x*.

50. cf. Wittgenstein, L., *Tractatus* (cf.n.16), §5.552: "The 'experience' that we need in order to understand logic is not that something or other is the state of things, but rather that something *is:* that, however, is *not* an experience. Logic is *prior* to every experience - that something *is so*. It is prior to the question 'How?', not prior to the question 'What?"'.

51. Russell, Bertrand and Whitehead, Alfred North., *Principia Mathematica*, Cambridge (1925), secon ed., Vol.I, *96n*.

52. Wittgenstein, L., *Notebooks 1914-16* (cf.n.17), *96*.

53. Ibid. *96*.

54. Wittgenstein, L., *Tractatus* (cf.n.16) §6.1262: "Proof in logic is merely a mechanical expedient to facilitate the recognition of tautologies in complicated cases"; cf. also §§6.2331, 6.234: "The process of *calculating* serves to bring about that intuition. Calculation is not an experiment; Mathematics

is a method of logic".

55. Kripke, Saul., *Wittgenstein on Rules...* (cf.n.47), *274*.

56. Wittgenstein, L., *Remarks on the Foundations...* (cf.n.41), Appx.III, §§2,3,6; cf. *Tractatus* (cf.n.16), §4.063: "...in order to be able to say ' *"p"* is true (or false)', I must have determined in what circumstances '*p*' is true, and in so doing I determine the sense of the proposition".

57. Chomsky, Noam., *Aspects of the Theory of Syntax*, Cambridge, Massachusetts (1965), *24:* "A grammar can be regarded as a theory of language...", and especially *199:* "The term 'depth grammar' and 'surface grammar' are familiar in modern philosophy in roughly like the sense here intended (cf. Wittgenstein's distinction of *'Tiefengrammatik'* and *'Oberflächgrammatik'*,(1953, p.168)...". Chomsky's claim is mistaken. As Wittgenstein used the term "depthgrammar", it is *not* a hypothesis. In fact Wittgenstein repudiated *any* attempt to make philosophy having the status of hypothesis.

58. Wittgenstein, L., *Investigations* (cf.n.6), §7: "We can also think of the whole process of using words in (2) as one of those games by means of which children learn their native language. I will call these games 'language-games' and will sometimes speak of a primitive language as a language-game".

59. Ibid., §7: "I shall also call the whole, consisting of language and the actions into which it is woven, the 'language- game"'.

60. Wittgenstein, L., *Zettel*, (G.E.M.Anscombe and G.H.von Wright, eds.), Oxford (1967), §437.

61. Martin-Löf, Per., *On the Distinction between Propositions and Judgments*, in abstracts of section I of the 7th International Congress of Logic, Methodology and Philosophy of Science, Salzburg, Austria, July 11th-16th, 1983, *24*.

62. Wittgenstein, L., *Investigations* (cf.n.6), §219: "'All the steps are really already taken' means: I no longer have any choice. The rule, once stamped with a particular meaning, traces the lines along which it is to be followed through the whole of space. –But if something of this sort really were the case, how would it help? No; my description only made sense if it was to be understood symbolically. –I should have said: *This is how it strikes me*. When I obey a rule, I do not choose. I obey the rule *blindly*"; cf. *Tractatus* (cf.n.16) §4.064: "Every proposition must *already* have a sense: it cannot be given a sense by affirmation. Indeed its sense is just what is affirmed".

63. Wittgenstein, L., *Tractatus* (cf.n.16) §4.123.

64. Wittgenstein, L., *On Certainty* (cf.n.33), §200.

65. Wittgenstein, L., *Tractatus* (cf.n.16) §4.024.

66. Wittgenstein, L., *Investigations* (cf.n.6), §136.

67. Wittgenstein, L., *Remarks on the...* (cf.n.41), Appx.III §6; cf. also Wittgenstein, Ludwig., *Philosophical Grammar*, Oxford (1974), *124:* "It can also be put thus: The proposition ' *"p"* is true' can onlybe understood if one understands the grammar of the sign *"p"* as a propositional sign; not if *"p"* is simply the name of the shape of a particular ink mark. In the end one can say that the quotation marks in the sentence ' *"p"* is true' are simply superfluous". Wittgenstein is here adhering to what we might call

the *equivalence thesis* of truth. However, the *locus classicus* of the equivalencethesis is provided by Frege (*Logical Investigations*, Oxford (1977), *8)*: "We express acknowledgment of truth in the form of an assertoric sentence. We do not need the word 'true' for this. And even when we do use it, the properly assertoric force does not lie in it, but in the assertoric sentence-form...". Frank Ramsey expressed the same point by saying that the statement "'He is always right' could be expressed by 'for all *a, R, b,* if he asserts *aRb*, then *aRb*, to which 'is true' would be an obvoiusly superfluous addition...and it is clear that the problem is not as to the nature of truth and falsehood, but as to the nature of judgment or assertion, for what is difficult to analyze in the above formulation is 'He asserts *aRb*"'(Ramsey, F., *The Foundations of Mathematics and other Logical Essays*, R.B.Braithwaite, ed. London (1931), *143*.

68. Wittgenstein, L., *On Certainty* (cf.n.33), §197.

69. Martin-Löf, P., *On the Distinction...* (cf.n.61), *24;* According to Paul Benacerraf and Hilary Putnam this is also the position adopted by Michael Dummett which "...is a theory of truth for propositions, mathematical and non-mathematical alike, in which truth-conditions are the assertability-conditions"(Benacerraf, P. and Putnam, H., *Philosophy of Mathematics*, second edition, Cambridge (1938), *24*. Dummett, however, never attributes this position to Wittgenstein. Instead Wittgenstein is attributed with the equivalence thesis understood as a *redundancy thesis* (Dummett offers no argument for this identification), which he in fact never embraced.(cf.Dummett, Michael., *Truth and other enigmas*, London (1978), *xx*.

70. Winch, Peter., *Im Anfang war die Tat*, in Block, Irvin ed., *Perspectives on the Philosophy of Wittgenstein*, Oxford (1981), *171.*

71. Wittgenstein, L., *Investigations* (cf.n.6), §225.

72. Ibid., §121.

73. Wittgenstein, L., *On Certainty* (cf.n.33), §651: "I cannot be making a mistake about 12 × 12 being 144. And now one cannot contrast *mathematical* certainty with the relative uncertainty of empirical propositions. For the mathematical proposition has been obtained by a series of actions that are in no way different from the actions of the rest of our lives, and are in the same degree liable to forgetfulness, oversight and illusion".

74. Wittgenstein, L., *Remarks on the...* (cf.n.41), Appx.III, §6; cf. also note 56.

75. Baker, G.P. and Hacker, P.M.S., *Wittgenstein: Understanding and Meaning*, Oxford (1980), *607.*

76. Wittgenstein, L., *Investigations* (cf.n.6), §133.

77. Wittgenstein, L., *On Certainty* (cf.n.33), §82; cf. *Remarks on the...* (cf.n.41), I §134: "Logic, it may be said, shews us what we understand by 'proposition' and by 'language"'.

78. Maury, André., *Reality and Logical Form, Synthèse, 56* (1983), *171.*

79. Kant, Immanuel., *Critique of Pure Reason*, (translated by N.Kemp-Smith), London (1961), *93.* Kant makes the point very clear when he states on page *93* that: "Logic...can be treated in a twofold manner, either as logic of the general or as logic of the special employment of the understanding. The former contains the absolutely necessary rules of thought without which

there can be no employment whatsoever of the understanding".

80. Husserl, Edmund., *Formale und Transcendentale Logik: Versuch einer Kritik der logischen Vernunft*, reprinted in: *Husserliana*, Band XVII, Den Haag (1974), *184*.

81. Kant, I., *Critique of Pure Reason* (cf.n.79), *93*.

82. Wittgenstein, L., *Remarks on the...* (cf.n.41), V §48.

83. Wittgenstein, L., *On Certainty* (cf.n.33), §93.

84. Wittgenstein, L., *Investigations* (cf.n.6), *184,198,223; On Certainty* (cf.n.33), §§92,94,162,262,292; *Remarks on the...* (cf.n.41), IV §30.

85. Wittgenstein, L., *Investigations* (cf.n.6), §371.

86. McGuinness, B. ed., *Wittgenstein and the Vienna Circle* (cf.n.9), *77*, and
    Wittgenstein, L., *Philosophical Grammar* (cf.n.67), *184*.

87. Wittgenstein, L., Manuscript 157(a), *55;*For an English translation cf., G.P.Baker and P.M.S.Hacker., *Wittgenstein: Understanding and Meaning*, Oxford (1980), *495:* "The role of 'logical analysis'. How can I understand a proposition *now* if it is for analysis to show me *what* I really understand. //what it is that I understand. //Here there sneaks in the idea of understanding as a peculiar mental process"; cf. also *Zettel* (cf.n.58) §445.

88. Ramsey, Frank P., *The Foundations of Mathematics*, London (1931), *233:*"But what we can't say we can't say, and we can't whistle it either...".

89. Wittgenstein, L., *Phil. Remarks* (cf.n.10), *182*.

90. The expression *"founded"* is here analogously used with Husserl's expression "grounded" *"fundiert"* (cf. Husserl, Edmund., *Ideas: general introduction to pure phenomenology*, New York (1962), *51*. The *"fundierende"*

act providing essential insight is the primary act that "underlies" the secondary, the latter being "consolidated" through and with it. Husserl formulates it by saying that "essential insight *is* intuition, and if it is insight in the pregnant sense of the term, and not a mere, and possibly vague, representation, it is a *primordial* dator Intuition, grasping the essence in its "bodily" selfhood. But, on the other hand, it is an intuition of a fundamentally *unique* and *novel* kind, namely in contrast to the types of intuition which belong as correlatives to the object-matter of other categories, and more specifically to intuition in the ordinary narrow sense, that is, individual intuition"(*49-50*).

91. Kripke, Saul., *Wittgenstein on Rules...* (cf.n.47), *274*.

92. Wittgenstein, L., *Philosophical Grammar* (cf.n.67), *143*.

93. Wittgenstein, L.,*Phil. Remarks* (cf.n.10), *113;*cf. *Tractatus* (cf.n.16), §4.001: "The totality of propositions is language".

94. Wittgenstein, L., *Philosophical Grammar* (cf.n.67), *131*.

95. Wittgenstein, L., *Tractatus* (cf.n.16), §5.632: "The subject does not belong to the world: rather, it is a limit of the world".

96. Wittgenstein, L., *Philosophical Grammar* (cf.n.67), *125*.

97. Ibid., *184*.

98. Wittgenstein, L., *Investigations* (cf.n.6), §496: "Grammar does not tell us how language must be constructed in order to fulfil its purpose, in order to have such-and-such an effect on human beings. It only describes and in no way explains the use of signs".

99. cf. Gefwert, C., *A Participator...* (cf.n.32), *21-2*.

100. Wittgenstein, L., *Investigations* (cf.n.6), §564.

101. Wittgenstein, L., *Philosophical Grammar* (cf.n.67), *170*.

102. Ibid., *87:*"Grammar is the account books of language. They must show the actual transactions of language, everything that is not a matter of accompanying sensations".

103. Frege, G., *The Foundations of Arithmetic*, sec.ed., Oxford (1980), *x*,Cf. *71.*, "...we ought always to keep before our eyes a complete proposition. Only in a proposition have the words really a meaning...It is enough if the proposition as a whole has a sense; it is this that confers on its parts also their content"; Cf. Kripke, Saul A., *Wittgenstein on Rules...* (cf.n.47), *306 n.47:*"Perhaps the best conception of Wittgenstein's relation to Frege here is to say that Wittgenstein would regard the spirit of Frege's contextual principle as sound but would critizise Frege for using 'name and object' as a catch-all for uses of language that are 'absolutely unlike' (PI,§10)".

104. Ibid., *88.*