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HEAVY PARTICLE PRODUCTION AT THE SSC*

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ABSTRACT

Predictions for the production of heavy quarks, supersymmetric particles, and other colored systems at high energy due to intrinsic twist-six components in the proton wavefunction are given. We also suggest the possibility of using asymmetric collision energies (*e.g.*,via intersecting rings at the SSC) in order to facilitate the study of forward and diffractive particle production processes.

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I. INTRODUCTION

One of the most important areas of investigation at the SSC will be the production of heavy particles carrying color quantum numbers, *e.g.*,new quark flavors, supersymmetric partners, technicolor constituents, *etc*. The simplest and most obvious QCD production mechanism, gluon fusion: $gg \to Q\bar{Q}$ only accounts for a fraction of the charm cross section observed at the ISR,[1] neither reproducing the shape of the cross section in longitudinal momentum nor the magnitude of the cross section [see Fig. 1]. Similarly, the usual perturbative QCD process $\gamma^* g \to Q\bar{Q}$ (which is equivalent to evolution of the structure function in leading order) predicts that charm as well as other heavy quarks are produced dominantly at low $x(x < 0.1)$, whereas data from the EMC collaboration[2] indicates that the charm structure function of the nucleon (at large W^2) has substantial contributions at $x \sim 0.3$ and beyond [see Fig. 2].

It is thus natural to investigate other production mechanisms for heavy particles in QCD. Here we will focus on the virtual heavy particle constituent $Q\bar{Q}$ pairs <u>intrinsic</u> to the nucleon bound state wavefunction,[3] in analogy to the virtual heavy lepton pair contributions to atomic systems in QED. To leading order in $1/m_Q^2$ such intrinsic contributions correspond to twist six terms in the effective QCD Lagrangian:[4]

$$\mathcal{L}_{QCD}^{eff} = -\frac{1}{4} F_{\mu\nu a} F^{\mu\nu a} - \frac{g^2 \frac{N_C}{2}}{60\pi^2 m_Q^2} D_\alpha\, F_{\mu\nu a}\, F_{\mu\nu a}\, D^\alpha\, F^{\mu\nu a}$$

$$+ C\, \frac{g^3}{\pi^2\, m_Q^2}\, F_\mu^{a\nu}\, F_\nu^{b\tau}\, F_\tau^{c\mu}\, f_{abc} + O\left(\frac{1}{m_Q^4}\right) \quad .$$

For QED the $e^2(\partial_\alpha F_{\mu\nu})^2/60\,\pi^2\, m_\ell^2$ term gives the standard Serber-Uehling vacuum polarization contribution to the mass shift of an atom due to heavy lepton pairs.[5] In QCD the corresponding $\alpha_s(D_\alpha F_{\mu\nu})^2/m_Q^2$ term yields a heavy quark contribution to the proton state with two to six gluon attachments to the nucleon constituents [see Fig. 3]. Note that as in the atomic case, the running coupling constant $\alpha_s(k^2)$ is evaluated at the soft momentum scale of the bound state, not at the heavy particle mass scale. Since the coupling constant is large and the number of contributing graphs is large, it is not unreasonable that the probability for the existence of a charm quark pair in the nucleon wavefunction is of the order of 1% or so.

In addition to causing a shift in the proton mass, the twist-six contributions imply the existence of new virtual Fock state components of the proton wavefunction containing an extra $Q\bar{Q}$ pair. At a given time $\tau = t + z$ on the light-cone the n-particle state is off the $p^- = p^\circ - p^z$ energy shell by

$$\epsilon_n = M_p^2 - \sum_{i=1}^{n} \left(\frac{m^2 + k_\perp^2}{x}\right)_i \quad .$$

Eventually lattice gauge theory or the light-cone equation of state will provide a full QCD solution for the proton wavefunction. At this point we can deduce[3] the following semiquantitative properties for intrinsic states such as $|u\,u\,d\,Q\,\bar{Q}\rangle$:

1) The probability of such states in the nucleon is nonzero and scales as m_Q^{-2}.

2) The maximal wavefunction configurations tend to have minimum off-shell energy, corresponding to constituents of equal velocity or rapidity, *i.e.*,

$$x_i \equiv \frac{(k^\circ + k^z)_i}{p^\circ + p^z} \quad \alpha \quad \sqrt{(k_\perp^2 + m^2)_i}$$

Thus the heavy quarks tend to have the largest momentum fraction in the proton wavefunction, just opposite to the usual configuration assumed for sea quarks. A simple model[3] for the $|q\,q\,q\,Q\,\bar{Q}\rangle$ state which incorporates counting rule behavior at $x_i \sim 1$ and inverse power behavior in ϵ_n predicts a valence-like momentum distribution for the heavy quarks in the nucleon. As seen in Fig. 2, the EMC data for the charm structure function is consistent with this prediction.

3) The transverse momenta of the heavy quarks are roughly equal and opposite and of order m_Q, whereas the light quarks tend to have soft momenta as set by the hadron wavefunction.

II. PARTICLE PRODUCTION FROM INTRINSIC STATES

Intrinsic Fock states can be materialized in high energy hadronic collisions by the (multigluon) exchange of longitudinal momentum. As shown in Fig. 4, the simplest gluon exchange diagrams give both inclusive and diffractive excitations of heavy colored particle systems. If the time of collision ($\sim 1/\sqrt{s}$) is much shorter than the time of internal excitation($1/m_Q$) than one is in the region of the sudden approximation, and the state will be produced with kinematics similar to the virtual configuration. Since the wavefunction for $|q\,q\,q\,Q\,\bar{Q}\rangle$ has the usual hadronic size and the $Q\,\bar{Q}$ subsystem has nonzero color, we can estimate the hadron cross section as the normal geometric cross section multiplied by the probability that the virtual state exists. Thus we estimate the production cross section for $pp \to Q\,\bar{Q}\,X$ well above the $Q\,\bar{Q}$ threshold as

$$\sigma_{Q\bar{Q}}^{diff} = 2P_{Q\bar{Q}/p}\,\sigma_{elastic}$$

$$\sigma_{Q\bar{Q}}^{TOTAL} = 2P_{Q\bar{Q}/p}\,\sigma_{TOTAL}$$

The ISR and EMC data are consistent with $P_{c\bar{c}/p} \sim 0.5\%$ corresponding to $\sigma_{c\bar{c}} \sim 0.5$ mb. At low energies one expects[3] a threshold factor $\sim (1 - s_{TH}^{Q\bar{Q}}/s)^4$. Since $P_{Q\bar{Q}} \propto 1/m_Q^2$ this implies[3] $\sigma_{Q\bar{Q}} \sim (m_c^2/m_Q^2)\,\sigma_{c\bar{c}}$ asymptotically, *i.e.*,$\sigma_{b\bar{b}} \sim$ 100 μb, $\sigma_{t\bar{t}} \sim 1$ μb, for $m_t = 30$ GeV, *etc.* This prediction is three orders of magnitude larger than the corresponding cross sections expected from gluon fusion.[6] It is, however, possible that the intrinsic contribution to $\sigma_{Q\bar{Q}}$ could decrease somewhat faster than $1/m_Q^2$, *e.g.*,because of coherent cancellations or contraction of the $|q\,q\,q\,Q\,\bar{Q}\rangle$ wavefunction size.

Intrinsic contributions to heavy particle production cross sections are additive with the usual contribution expected from gluon fusion. The intrinsic state production mechanism obviously can be extended to supersymmetric particles such as color octet gluinos (which have an enhanced color factor) (see Fig. 5), and to technicolor constituents in those models in which the technicolor particles also carry ordinary $SU(3)_C$ quantum numbers. We discuss these possibilities in detail in Section III. Diffractive production $pp \to p\mathcal{M}$ in events in which the scattered proton is identified may be very useful for finding new states $\mathcal{M}$ since one can then kinematically identify the diffractive system as a baryonic state of specific mass and 3–momentum. The complete kinematic specification of the massive baryonic system will allow much tighter constraints on searches for missing momentum and energy, the usual signals for supersymmetric particle decay. The essential feature that heavy constituents have relatively large longitudinal momentum fractions in the proton wavefunction implies that heavy particles and their decay products are produced at large x_L as well as large p_T. Since the forward rapidity regions are not normally accessible in usual detectors, it could be interesting to investigate asymmetric beam energy collisions, *e.g.*, by colliding the SSC with a booster ring or other intersecting stored beams. We discuss the kinematics of such possibilities in Section IV.

The existence of intrinsic heavy particle Fock states also can be extended to photon and weak vector boson states: the existence of twist-six operators in the effective Lagrangian imply that a real photon state contains via vector dominance intrinsic charm Fock state components $|u\,\bar{u}\,c\,\bar{c}\rangle$, *etc.*, with finite probabilities $P_{c\bar{c}}/\gamma$, *etc.*, This implies diffractive excitation photoproduction cross section at high energies with cross sections of the order $P_{c\bar{c}/\gamma}\ \sigma_\gamma^{TOT} \sim \mathcal{O}(1\ \mu\mathrm{b})$, which again must be added to the usual fusion $\gamma g \to c\,\bar{c}$ contribution. In fact, as reviewed in Ref. [7], inelastic photoproduction of charm states is observed to be much larger than predicted by the fusion model. A similar intrinsic production mechanism for the W leads to a significant $c\,\bar{c}$ cross section in charged current neutrino-production and could be at least in part responsible for the same-sign dilepton anomaly.[7]

III. INTRINSIC SUPERSYMMETRIC AND TECHNICOLOR COMPONENTS OF THE PROTON

It is amusing to consider some of the consequences of a non-negligible intrinsic component of gluinos, scalar quarks (the supersymmetric partners of the quarks), or techniquarks inside the proton. First, we consider the case of the gluinos. The supersymmetric content of the proton is usually computed to leading order in perturbative QCD using the Altarelli-Parisi evolution equations. Results of such calculations have been given in Ref. [8]. Here, we simply wish to emphasize that such calculations could be a significant underestimate, especially at large x, in the same way that the charm content of the proton is underestimated by perturbative QCD calculations (see Fig. 2). In principle, precise measurements of structure functions at very large energy *ep* machines could uncover evidence for new constituents, although such experiments are clearly very difficult.

The most likely way to uncover the existence of gluinos ($\tilde{g}$) and scalar-quarks ($\tilde{q}$) is by direct observation of their production and decay. The expected decay signatures depend on their masses. If $M_{\tilde{g}} > M_{\tilde{q}}$ then $\tilde{g} \to g\tilde{q}$ and $\tilde{q} \to q\tilde{\gamma}$; if $M_{\tilde{g}} < M_{\tilde{q}}$, then $\tilde{q} \to q\tilde{g}$ and $\tilde{g} \to q\bar{q}\tilde{\gamma}$, where the photino ($\tilde{\gamma}$) is assumed to escape the detector (similar to the case of a neutrino). Thus, the basic property which distinguishes such decays from ordinary heavy quarks is the loss of transverse energy (due to the $\tilde{\gamma}$) <u>without</u> an accompanying charged lepton. (A serious exception is $Q \to q\tau\nu$ where $\tau \to \nu +$ hadrons; such a background will have to be calculated and should be carefully studied.)

The case of technicolor is somewhat different. Here, one imagines finding $\bar{Q}_T\, Q_T$ pairs inside the proton where Q_T is a techniquark which carries both color and technicolor quantum numbers. The characteristic technicolor scale Λ_T is nearly 10^3 that of QCD. Due to the necessity of neutralizing the technicolor quantum numbers in the final state, associated production analogous to $\Lambda_c \bar{D}$ does not occur. The most natural final states thus contain a spectrum of massive technicolor-singlet bound states at the scale of the mass of the technicolor quarks. These states decay dominantly into the lighter mass technihadrons which in turn decay into pairs of W's and Z's.

There are, in addition, very light-states pseudo-Goldstone boson (technipions) predicted by many technicolor theories which behave like the charged and neutral Higgs bosons of ordinary electroweak models.[9] Since they are color singlets, they can only be produced intrinsically by probes with momenta of order of the technicolor scale, as opposed to their light mass. Thus intrinsic technipions in the proton are again characterized by the Λ_T scale of the technicolor theory, and will comprise only a fraction of the technicolor particles produced in diffractive events. If such states are produced diffractively, they will carry large longitudinal momentum fractions in contrast to the small momentum fractions usually predicted for elementary Higgs particles. In addition, in the technicolor scenario, the longitudinal components of the $W^{\pm}$ and Z° are bound states of techniquarks, and one thus also predicts intrinsic

$W^{\pm}$ and Z° components of the proton at the Λ_T scale.

A general conclusion from the above discussion is that cross sections for new physics, in certain kinematic regions, may be seriously underestimated by leading twist perturbative QCD calculations. In principle, this indicates that the ability to discover new phenomena may be much greater than previously supposed. On the other hand, one would like to place more restrictive limits on various candidate models for new physics if no new phenomena are detected. All the consequences discussed in this paper depend on the correctness of the intrinsic Fock state picture. If this picture is experimentally verified through further study of charm and observation of the predicted enhancement of b and t quark production (as discussed in Sections I and II), then one can begin to use the intrinsic state approach to obtain stronger restrictions on the parameters of the various models of new physics beyond the Standard Model.

IV. DETECTION OF HEAVY PARTICLE DIFFRACTIVE REACTIONS

We turn now to the kinematics of diffractive heavy particle production. Imagine a diffractively produced state of mass M where M lies at the threshold for production of new heavy quarks /SUSY particles, *etc.* Some percentage of the time this system will in fact contain heavy particles and will decay roughly isotropically in its rest frame. Assuming adequate cross section, the problem is to have detectors which see enough of this "isotropic" decay, that it is distinguishable from a background event in which M contains only light quarks, fragmenting to light hadrons, produced primarily along the collision center-of-mass axis. The difficulty lies in the fact that, in a colliding beam configuration, the system M arising from the diffractive excitation of one beam will be moving rapidly in the laboratory and some of its decay products will be lost at small angles. We discuss the kinematics of this situation and limits placed upon the new heavy particle M values observable for a given beam configuration and detector acceptance.

Consider a head-on collision of proton beams of energies and momenta $p_1 = (E_1, 0, 0, \sqrt{E_1^2 - M^2})$, and $p_2 = (E_2, 0, 0, -\sqrt{E_2^2 - M^2})$. We take $E_2 \leq E_1$ and diffractively excite proton #2 to a state of mass M. The system will move in the negative z direction. The momentum of p_1, after the collision, is denoted by p_1'.

We consider an event with minimum momentum transfer $q^2 = (p_1 - p_1')^2 \approx -[(M^2/s)M]^2$. We define p_M to be the 4-vector of the state M. Then

(a) $q^+ = q^o + q^z = \frac{M^2}{2E_2}\ ,\ \vec{q}_T = 0\ ;$

(b) $p_M^+ = \frac{M^2}{2E_2}\ ,\ p_M^- \approx 2E_2\ ,\ \vec{p}_M^{\,T} = 0\ ;$

(c) The boost variable $y_{\mathcal{M}}$ from the lab frame to the rest frame of $\mathcal{M}$ is $y_{\mathcal{M}} = -\ell n\ \frac{2E_2}{\mathcal{M}}$. (Recall $a^{+\prime} = e^{-y}\ a^{+}\ ;\ a^{-\prime} = e^{y}\ a^{-}$.)

(d) A particle with mass $\ll \mathcal{M}$ produced at a small angle θ_R with respect to the $-\hat{z}$ direction of $\vec{p}_2$ in $\mathcal{M}$'s rest frame appears at

$$\theta_L = \theta_R\ \frac{\mathcal{M}}{2E_2}$$

with respect to $-\hat{z}$ in the lab frame.

(e) The phase space available to a particle of mass m_π, transverse energy E_T, and rapidity y_R in the $\mathcal{M}$ rest system is defined by the $E_T - y_R$ plot of Fig. 6. ($E_T = \sqrt{m_\pi^2 + p_T^2}$; laboratory 4–momenta for this particle are $p_L^z = E_T\ \sinh\ y_L, E_L = E_T\ \cosh\ y_L$ where $y_L = y_R + y_{\mathcal{M}}$). In the plot we assume that $\mathcal{M}$ is large enough that $\frac{2E_2}{\mathcal{M}} < \frac{\mathcal{M}}{m_\pi}$. Phase space corresponding to the excluded forward and backward laboratory angles smaller than θ_0 ($\ll 1$) is indicated by the hatched area. The outer boundary of the phase space region is defined by

$$|y_{max}(E_T)| = \ell n \left[\frac{\left(\frac{\mathcal{M}}{2} + \sqrt{\frac{\mathcal{M}^2}{4} - E_T^2} \right)}{E_T} \right] \quad .$$

The critical E_T^c below which a particle is lost to small angles $< \theta_0$ is given by

$$\frac{p_T^{2c}}{E_T^{2c}} = 1 - \frac{m_\pi^2}{E_T^{2c}} = \theta_o^2\ \sinh^2 \left(y_R - \ell n\ \frac{2E_2}{\mathcal{M}} \right) \quad .$$

For $(\theta_0 E_2)/\mathcal{M} \ll 1$ we give several key points in Fig. 6. For instance the largest E_T that can be lost, $\bar{E}_T = \sqrt{m_\pi^2 + \theta_0^2 E_2^2}$, requires $y_R = -\ell n\ \mathcal{M}/\bar{E}_T$.

The limits on heavy particle $\mathcal{M}$ system detection will now be discussed.

1. We require $\mathcal{M}/\sqrt{s} \lesssim 1/5$ to avoid phase space suppression of the cross section as discussed earlier. For $s = 4E_1E_2$ and $E_1 = 2 \times 10^4$ GeV, this implies

$$\mathcal{M}\,(\mathrm{GeV}) \lesssim 60\ \sqrt{E_2\ (\mathrm{GeV})} \quad .$$

2. We demand, for an isotropically decaying heavy system $\mathcal{M}$, that we cover at least 95% of the solid angle. This means, if $\bar{\theta}_R$ is the smallest rest frame

angle (with respect to $-\hat{z}$) observable, that $\int_{\pi-\bar{\theta}_R}^{\pi} \sin\theta_R d\theta_R < .05 \times 2$, or $\bar{\theta}_R^2 < 0.2$. Requiring the corresponding laboratory angle to be θ_0 implies

$$\left(\frac{2E_2}{\mathcal{M}}\theta_0\right)^2 \leq 0.2 \quad .$$

If we want to be able to contain systems with $\mathcal{M}$ as low as 30 GeV, we require $E_2 \lesssim 7 \times 10^3$ GeV $\times(10^{-3}/\theta_0)$. With such an E_2 we can avoid phase space suppression up to $\mathcal{M} \leq 5$ TeV. For $E_2 = 2 \times 10^4$ GeV, $\mathcal{M} = 90$ GeV $\times(\theta_0/10^{-3})$ is the lower limit and $\mathcal{M} < 8$ TeV the upper limit.

3. Minimum bias events do not appear to be a significant problem. Even if one loses $\langle n\rangle_{\mathcal{M}}\ 2m_\pi$ in transverse energy for such an event (we have taken $\langle E_T\rangle = 2m_\pi$), this will still be very small compared to $\mathcal{M}$ since $\langle n\rangle_{\mathcal{M}}$ grows only like $\ell n\ \mathcal{M}$. In addition this lost energy is generally symmetrically distributed and will not confuse a missing E_T trigger.
4. The most significant problem is to distinguish an isotropic heavy particle $\mathcal{M}$ system from rare background events in which significant transverse energy resides in QCD jets. This problem is endemic to heavy particle searches at a hadron machine whether produced diffractively or otherwise. As usual, lepton signals, vertex detectors, and event topology measures will be required to bring the signal above background level.
5. The physics can be greatly constrained by measuring p_1' in order to completely determine the $p_{\mathcal{M}}$ for diffractive events. This requires being able to measure the momentum of the forward scattered proton, p_1', with

$$\frac{p_1^z - p_1'^z}{p_1^z} = \frac{q_1^z}{p_1^z} = \frac{\mathcal{M}^2}{4E_2E_1} = \frac{\mathcal{M}^2}{s} \quad .$$

In summary, there appears to be considerable merit for considering an asymmetric colliding beam configuration with $E_2 \approx 1/5$ of E_1 and a specialized detector with good coverage down to $\theta_0 \sim 10^{-3}$. Since the intrinsic state model predicts substantial diffractive cross sections for new particle production, this region should not be ignored in designing the super collider.

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REFERENCES

1. K. L. Giboni *et al.*, Phys. Lett. 85B, 437 (1979); D. Drijard *et al.*, Phys. Lett. 81B, 250 (1979; W. Lockman *et al.*, Phys. Lett. 85B, 443 (1979). For a complete summary see F. Halzen, *Proc. of High Energy Physics, Paris,* p. 381 (1982).
2. J. J. Aubert *et al.*, Nucl. Phys. 213B, 31 (1983). Figure 2 is a rebinning of the data due to C. Gossling, CERN-EP Seminar and private communication (1983)
3. S. J. Brodsky, P. Hoyer, C. Peterson and N. Sakai, Phys. Lett. 93B, 451 (1980). S. J. Brodsky, C. Peterson and N. Sakai, Phys. Rev. D23, 2745 (1981). S. J. Brodsky and C. Peterson, unpublished. For alternative explanations based on charm quarks at low x, see P. D. B. Collins and T. P. Spiller, Durhan preprint DTP/84/4 (1984) and references therein.
4. For other examples of twist-six contributions in QCD see S. C. Generalis and D. J. Broadhurst, OUT-4102-10 (1983) and references therein. The F^3 term has a negligible effect on the examples cited here.
5. See, *e.g.*, J. D. Bjorken and S. D. Drell, *Relativity Quantum Mechanics,* McGraw Hill.
6. See, *e.g.*, C. Quigg, these Proceedings.
7. F. Halzen, ref. 1.
8. B. A. Campbell, J. Ellis and S. Rudaz, Nucl. Phys. B198, 1 (1982); I. Antoniadis, C. Kounnas and R. Lacaze, Nucl. Phys. B211, 216 (1983); C. Kounnas and D. A. Ross, Nucl. Phys. B214, 317 (1983); S. K. Jones and C. H. Llewellyn Smith, Nucl. Phys. B217, 145 (1983).
9. K. Lane, in *"Elementary Particle Physics and Future Facilities,"* Snowmass (1982).

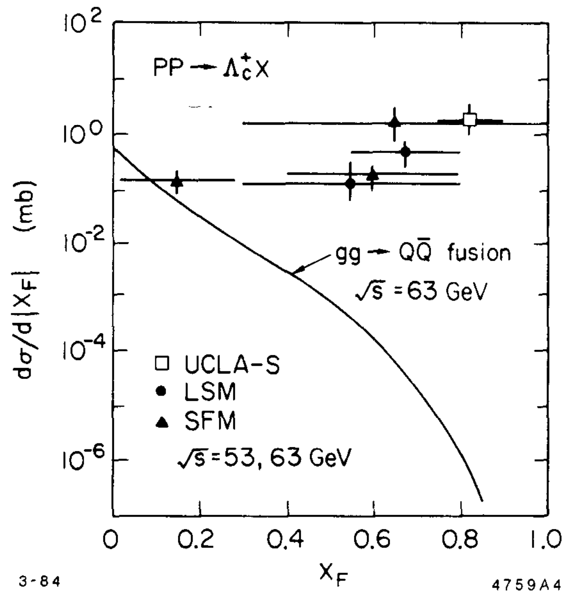

Figure 1. ISR data[1] for Λ_c inclusive production compared with the charm quark longitudinal momentum distribution predicted by the fusion model. The total charm cross section includes additional contributions from D production.

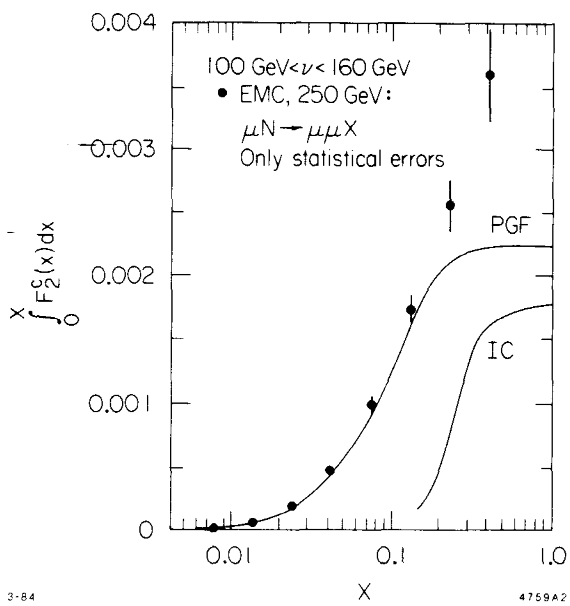

Figure 2. EMC data[2] for the integrated charm quark x_{Bj} distribution in the nucleon deduced from $\mu N \to \mu\mu X$. The photon-gluon fusion (PGF) model predicts that the integrated momentum distribution saturates at $x_{Bj} = 0.1$ whereas there is a substantial contribution beyond $x_{Bj} = 0.4$. The intrinsic charm (IC) prediction is from Ref. 3.

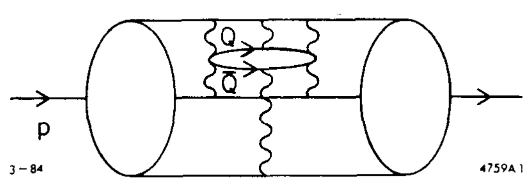

Figure 3. Example of a heavy quark contribution to the nucleon state in QCD at order $1/M_Q^2$. Two to six gluons can attach to up to six constituent legs in the nucleon wavefunction.

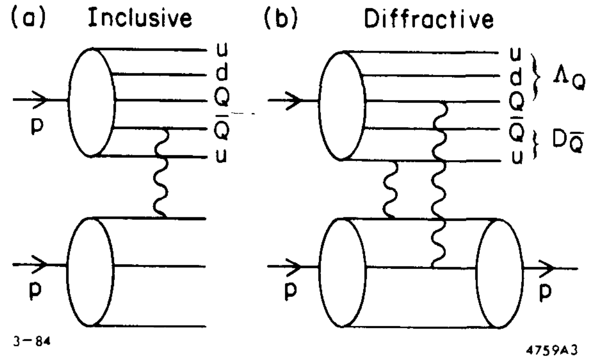

Figure 4. Simplest QCD contributions to inclusive and diffractive production of heavy quark states in high energy pp scattering. The diffractive contribution is characterized by the absence of particles produced in the central rigidity region.

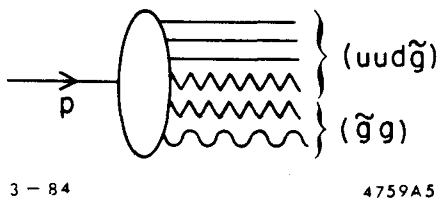

Figure 5. Example of associated production of supersymmetric hadrons in pp scattering due to intrinsic gluino pair states.

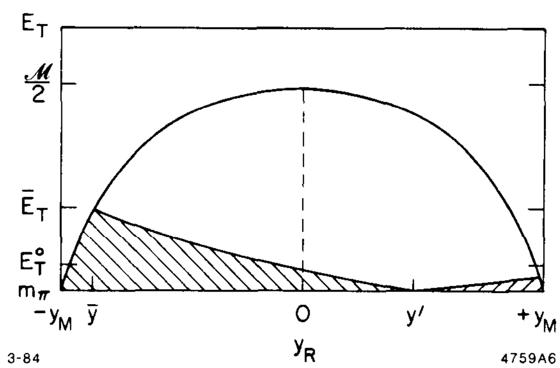

Figure 6. The phase space for $\mathcal{M}$ decay in terms of the transverse energy E_T and $\mathcal{M}$-restframe rapidity y_R of a particle of mass m_π. The maximum E_T is $\mathcal{M}/2$ and the maximum and minimum rapidities are $y_R = \pm y_M$, with $y_M = \ell n\ (\mathcal{M}/m_\pi)$. The rapidity $y' = \ell n\ (2E_2/\mathcal{M})$ represents the zero laboratory rapidity point. For $y_R < y'$, the hatched region is lost inside a backward detector of minimum acceptance angle θ_0. The maximum transverse energy lost in the backward region is $\bar{E}_T = \sqrt{m_\pi^2 + \theta_0^2 E_2^2}$ and occurs at $y_R = \bar{y} = -\ell n\ (\mathcal{M}/\bar{E}_T)$. The $y_R = 0$ crossing point is $E_T^0 = m_\pi / \sqrt{1 - (\theta_0^2 E_2^2 \beta^2)/\mathcal{M}^2}$ where $\beta = 1 - (\mathcal{M}^2/4E_2)$.