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ON THE COSMOLOGICAL CONSTANT PROBLEM*

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ABSTRACT

We prove that Weyl invariant theories of gravity possess a remarkable property which, under very general assumptions, explains the stability of flat space-time. We show explicitly how conformal invariance is broken spontaneously by the vacuum expectation value of an unphysical scalar field; this process induces general relativity as an effective long distance limit. We show that all ghost degrees of freedom acquire gauge fixing dependent masses, strongly suggesting the unitarity of the theory.

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1. Introduction

Of the four fundamental forces determining low energy physics, three have been adequately described as quantum field theories based on the gauge group $U(1) \times SU(2) \times SU(3)$. The remaining force, gravity, has a similar gauge symmetry, the coordinate invariance, but resists quantization.

Classically, the gravitational force at large distances is very well described by Einstein's general theory of relativity. On the quantum level this theory has serious problems. There is no symmetry forbidding the addition of a cosmological action term, though experimentally no such term has been observed. A single fundamental particle in the Einstein universe induces an infinite contribution to the cosmological constant. Thus, unless severe unnatural fine tuning is employed, general relativity contradicts the observed world. Moreover, the presence of a dimensionful coupling constant is responsible for the perturbative nonrenormalizability of Einstein's gravity [1].

It is reasonable to assume that general relativity is a large distance effective theory of some fundamental renormalizable gravitational Lagrangian [2] in the same way that Fermi's four-fermion interaction is the low energy limit of the standard $U(1) \times SU(2)_L$ electroweak model. As a first step, consider gravitational actions containing linear and quadratic terms in the curvature tensor. These theories are renormalizable [3] and dimensional analysis concludes that the linear Einstein term determines the long wavelength behavior while the quadratic terms dominate at short distances. Nevertheless, these theories are not unitary in the ordinary loop expansion, although nonperturbative techniques seem to suggest that no ghosts are present [4,5]. Independent of their unitarity properties, higher derivative gravitational actions still suffer from the cosmological constant problem.

A new gauge symmetry is needed. At very high energies all elementary particle masses should become negligible. Weyl's generalized Riemannian space [6] naturally incorporates this ideas by possessing an extra symmetry: local conformal invariance. In the resulting Lagrangian, which is quadratic in the Weyl curvature tensor, $C_{\mu\nu\lambda\rho}(x)$, a cosmological constant term is forbidden by the new invariance but Einstein's theory is not reproduced at large distances. However, by analogy with the Higgs mechanism, general relativity can be induced [7] through the vacuum expectation value (VEV) of a scalar field $\phi_0(x)$ which spontaneously breaks the conformal symmetry.

A simple gravitational Lagrangian is:

$$L_G = \sqrt{-g}\Big[-\frac{1}{G^2}\,C^2_{\mu\nu\lambda\rho} - \frac{1}{2}\,(\partial_\mu\phi_0)^2 - \frac{1}{12}\,R\phi_0^2\Big] \tag{1}$$

where G is a dimensionless coupling constant and $R(x)$ the curvature scalar.[1] In the conformal gauge fixing condition $\phi_0(x) = v_0$, assuming a nonzero VEV for $\phi_0(x)$, the unphysical scalar $\phi_0(x)$ is eliminated and the Einstein term emerges. Note that the requirement on (1) to induce the Einstein term with the correct sign forces $\phi_0(x)$ to enter with a negative kinetic energy. The fluctuating quantum fields $h_{\mu\nu}(x)$ and $\sigma(x)$ are defined by:

$$g_{\mu\nu} \equiv \eta_{\mu\nu} + Gh_{\mu\nu} \tag{2a}$$

$$\phi_0 \equiv v_0 + \sigma \tag{2b}$$

where $\eta_{\mu\nu}$ is the Minkowski spacetime metric. Three problems have to be faced immediately: the unitarity of the quantum theory, the presence of conformal anomalies and the reason for the induction of a scalar VEV.

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[1] The metric tensor $g_{\mu\nu}(x)$ has signature $(+---)$, the curvature tensor is defined by $R^{\mu}_{\nu\lambda\rho}(x) \equiv \partial_\lambda\,\Gamma^{\mu}_{\rho\nu}(x) - \ldots$ and $C^2_{\mu\nu\lambda\rho}(x) = R^2_{\mu\nu\lambda\rho}(x) - 2R^2_{\mu\nu}(x) + \frac{1}{3}\,R^2(x)$.

Due to the presence of higher derivative terms [see eq. (1)], the free graviton propagator contains, in addition to the massless spin-two pole, a massive one with negative residue. The unresolved question of unitarity can be powerfully attacked using an expansion in a naturally small scale independent parameter, $1/N$, where N is an effective number of the fundamental matter fields [4]. In general, the Weyl invariant matter part $\mathcal{L}_M$ of the complete Lagrangian will contain N_S scalar fields ϕ, N_F Dirac-fermions ψ and N_V vector bosons A_μ. In this expansion, the quantities G^2N, v_0^2/N and g_i^2N, where g_i denote the matter coupling constants, are fixed. To leading order the β-function of the coupling constant G is [4]:

$$\beta_G = -\frac{1}{240}\,\frac{N}{(4\pi)^2}\,G^3 \tag{3a}$$

where the effective number $N \equiv N_S + 6N_F + 12N_V$. The theory is asymptotically free and so there exists a "QCD like" renormalization group invariant scale Λ at which G becomes strong:

$$\Lambda = \mu\ \exp\left[-\frac{120(4\pi)^2}{G^2N}\right] \tag{3b}$$

with μ the substraction point. The leading order $1/N$ graviton propagator $D_{\mu\nu,\lambda\rho}$ is given by the sum of all graviton diagrams with an arbitrary number of one loop matter corrections (fig. 1). Using the five gauge conditions:

$$\partial_\mu h^{\mu\nu} = 0 \quad ; \quad h^\mu_\mu = 0 \tag{4}$$

we obtain:

$$D_{\mu\nu,\lambda\rho}(p) = \frac{iP_{\mu\nu,\lambda\rho}(p)}{p^2\left[\frac{G^2v_0^2}{24} - \frac{1}{240}\frac{G^2N}{(4\pi)^2}\,p^2\,\ell n\left(-\frac{p^2}{\Lambda^2}\right)\right]} \tag{5}$$

where $P_{\mu\nu,\lambda\rho}$ is the transverse traceless spin-two projector. Notice that $D_{\mu\nu,\lambda\rho}$ has no real massive poles provided $\frac{160\pi^2 v_0^2}{\Lambda^2 N} > \frac{1}{e}$. However, in the complex p^2 plane there is a pair of complex conjugate poles. As a result, the theory is unitary to leading order [4] but requires the Lee-Wick prescription [8] from thereon.

Any renormalizable classically Weyl invariant quantum theory needs a renormalization scale which explicitly breaks the scale invariance [9] and is the source of the conformal, or trace, anomalies [10]. In the context of dimensional regularization this scale μ is naturally introduced once any dimensionless coupling constant e acquires dimensions as the theory is continued to $n < 4$ spacetime dimensions:

$$e = e_0\, \mu^{(4-n)/2} \tag{6}$$

where e_0 is dimensionless. By replacing the scale μ with the unphysical scalar field ϕ_0 raised to an appropriate power, a conformally invariant theory in n dimensions can be written when $e \to e_0 \phi_0^{(4-n)/(n-2)}$ [11]. However, a perturbation expansion exists only around $\phi_0 = v_0 \neq 0$ when the conformal invariance is spontaneously broken [see eq. (2b)]. Therefore, the appropriate substitution is:

$$e \;\to\; e\left(1 + \frac{\sigma}{v_0}\right)^{(4-n)/(n-2)} \quad . \tag{7}$$

The renormalization should be done with a similar spirit [11]. Since the n-dimensional continuation of Weyl invariant theories introduces explicit n dependence in the Lagrangian, the correct algorithm consists of subtracting n-dimensional conformally invariant counterterms. The above method of regularization and renormalization preserves the Ward identities (W-I) of the spontaneously broken theory and trace anomalies do not arise. This has been shown in explicit examples in ref. [11].

It appears that two scales exist, v_0 and the subtraction point μ.[2] Nevertheless, the requirement of a stable perturbation expansion around $\sigma = 0$ will determine the ratio v_0/μ and prove the existence of a single scale parameter for the theory. This direct association between the vacuum expectation value (VEV) v_0 and μ transforms the explicit breaking due to μ to a spontaneous one and substantiates the regularization scheme. The global counterpart of the conformal gauge symmetry is the dilatation invariance [9]. In a Weyl symmetric theory, an infinitesimal dilatation is a particular sum of a coordinate and a conformal transformation. The W-I obtained from dilatations takes the form [15]:

$$\mu \frac{\partial}{\partial \mu} \Gamma^{(m)} = v_0 \, \Gamma_\sigma^{(m+1)} \tag{8}$$

where $\Gamma^{(m)}$ is any one-particle irreducible (1-PI) Green's function and $\Gamma_\sigma^{(m+1)}$ includes an extra zero momentum external σ field. This W-I corresponds to the low energy theorem for the translated scalar field σ and identifies it with the Goldstone boson of the spontaneously broken dilatation symmetry, the dilaton. Furthermore, eq. (8) is compatible with nonzero beta functions in a Weyl invariant theory. However, the $v_0 = 0$ limit of the theory does not exist reflecting the nonanalyticity at $\phi_0 = 0$. This is due to the existence of a nonzero beta function (3) which forces v_0 to be nonzero and the conformal symmetry to be broken.

2. The Vanishing of the Cosmological Constant

Weyl's gravity has a more complicated dynamical structure than general relativity and satisfies a very powerful theorem which, under very general assump-

[2] A systematic treatment of the general properties of Weyl invariant gravitational actions, including detailed proofs of all claims in this letter, can be found in ref. [15].

tions, explains the stability of flat spacetime. The theory contains an unphysical degree of freedom, the dilaton field, that expresses the unavoidable spontaneous breakdown of the conformal component of the full symmetry. The theorem states that the ability to set equal to zero the tadpole of the dilaton field of a Weyl invariant physical system is the necessary and sufficient condition for the vanishing of the cosmological constant.

Consider a Weyl invariant action in n dimensions where the spontaneously broken conformal transformations on our fields take the form:

$$\delta g_{\mu\nu} = 2\Omega g_{\mu\nu} \tag{9a}$$

$$\delta\phi = -\frac{n-2}{2}\,\Omega(v_\phi + \phi) \tag{9b}$$

$$\delta\psi = -\frac{n-1}{2}\,\Omega\psi \tag{9c}$$

$$\delta A_\mu = 0 \tag{9d}$$

The general rule for constructing Lagrangian terms in such a theory is quite simple. Start with any coordinate invariant theory of gravitons and matter fields. Introduce the unphysical scalar field σ, the dilaton, and perform finite conformal transformations with parameter $\Omega = (1 + \frac{\sigma}{v_0})^{2/(n-2)}$ to all the dynamical variables. This parameter has been chosen such that the transformed fields are conformally invariant when the transformation properties of the dilaton itself are taken into account. Then, by making the dilaton a dynamical field we obtain a Weyl invariant theory of the metric, dilaton and matter fields. The "unitarity gauge" $\sigma = 0$ of the conformal symmetry reproduces the original coordinate invariant theory. However, we prove that in a Weyl symmetric theory the spontaneous breakdown of the conformal component provides a relation between the parameters of the theory and enables us to naturally achieve a zero cosmological constant.

The W-I of the theory are derived for the generating functional Γ of the 1-PI Green's functions using BRS methods in arbitrary linear gauge conditions [15]:

$$\sum_{F \neq \bar{c}_a} \frac{\delta\Gamma}{\delta F} \frac{\delta\Gamma}{\delta J_F^s} = 0 \tag{10}$$

where F is a generic field variable, $\bar{c}_a$ is a generic antighost field and J_F^s is the source associated with the BRS transformation of F. The implications of the general W-I (10) for the graviton tadpoles assume a particularly simple form in the Landau type gauges (4) (see fig. 2):

$$\eta_{\mu\nu} \frac{\delta\Gamma}{\delta h_{\mu\nu}} = \frac{n-2}{4} G v_0' \frac{\delta\Gamma}{\delta\sigma'} \quad . \tag{11}$$

The translated field $\sigma' \equiv \phi_0' - v_0'$, the dilaton of the complete physical system, is a linear combination of $\sigma \equiv \phi_0 - v_0$ and all scalars ϕ with nontrivial VEV's. According to the action principle the constant background v_0' should take the value minimizing the effective potential $V(\sigma')$ of the theory. At this value, the dilaton tadpoles $\delta\Gamma/\delta\sigma'$ vanish and (11) automatically guarantees the simultaneous elimination of the graviton tadpoles $\delta\Gamma/\delta h_{\mu\nu}$.[3] Therefore, the cosmological constant is zero to all orders in any perturbation expansion around an extremum of the dilaton potential, if one exists.

The conclusions of our theorem apply to any matter theory. Consequently, the standard model $U(1) \times SU(2) \times SU(3)$ of low energy physics when coupled to Weyl's gravity provides a theory describing all four known interactions with zero

[3]This result can be shown to be valid in arbitrary gauges [15].

cosmological constant. The spontaneous breakdown of the electroweak symmetry, in its simplest form involving elementary scalars, occurs through radiative corrections [12] as dictated by the Weyl invariance. The lack of an appropriate symmetry will eventually drive the electroweak breaking scale up to the Planck energies unless severe fine tuning is performed. There exist alternative Weyl invariant methods to break $U(1) \times SU(2)_L$ and avoid the hierarchy problem: technicolor [13] and possibly conformal supergravity [14] are good candidates. The VEV of any physical scalar field should never equal or exceed the Planck mass since the highly desirable induced Einstein term would vanish or acquire the wrong sign. This is an additional problem scalars could create, in particular those of various grand unified models.

3. Simple Examples

We have carried the renormalization program for a general Weyl symmetric gravitational action along the same lines with ref. [3] and proved the renormalizability of the theory [15]. The most convenient classes of gauges (ς, Φ^μ) and (ξ, Φ), for the coordinate and conformal symmetries respectively, are:

$$\Phi^\mu = \partial_\nu h^{\mu\nu} - \frac{1}{n}\partial^\mu h^\nu_\nu \tag{12a}$$

and

$$\Phi = \frac{1}{2(n-1)}\left(\Box h^\mu_\mu - \partial_\mu \partial_\nu h^{\mu\nu}\right) \quad . \tag{12b}$$

These choices force the ghost propagator matrix of the theory to be diagonal to lowest order. By taking $\varsigma = 0$ and introducing extra derivatives in the conformal gauge fixing term all ξ-dependent divergences are eliminated. Furthermore, in the $1/N$ expansion, due to the better ultraviolet convergence properties of the graviton propagator the renormalization becomes very simple.

Consider the simplest framework for studying the spontaneous breaking of the conformal symmetry; this is achieved when only the scalar field ϕ_0 receives a VEV v_0 and the renormalized Lagrangian, in the $1/N$ expansion, is[4] :

$$\mathcal{L} = \sqrt{-g}\left[-\frac{Z_G}{G^2}\left(1+\frac{\sigma}{v_0}\right)^{2\frac{n-4}{n-2}} \frac{n-2}{2(n-3)}\, C^2_{\mu\nu\lambda\rho} - \frac{1}{2}\,(\partial_\mu\sigma)^2 - \frac{n-2}{8(n-1)}\, R(v_0+\sigma)^2\right] + \mathcal{L}_M + \mathcal{L}_{GF} \tag{13}$$

where Landau type gauges (4) are assumed in $\mathcal{L}_{GF}$ which contains the gauge fixing and ghost interaction terms. The effective potential $V_{eff}[\sigma]$ of the dilaton to leading order in $1/N$ (see fig. 3) is given by [15]:

$$V_{eff}[\sigma] = -\frac{5i}{2}\int \frac{d^nk}{(2\pi)^n}\ \ell n\left[1+2\,\frac{k^2\ell n\,(1+\frac{\sigma}{v_0})+\frac{80\pi^2}{N}\,(\sigma^2+2v_0\sigma)}{\frac{160\pi^2}{N}\,v_0^2-k^2\ell n\left(-\frac{k^2}{\Lambda^2}\right)}\right] \quad . \tag{14}$$

The necessary and sufficient condition for $V_{eff}[\sigma]$ to have no imaginary part is $\frac{160\pi^2 v_0^2}{\Lambda^2 N} > \frac{1}{e}$ and this inequality is identical to the unitarity condition (5). If the integral is defined by dimensional continuation, it can be computed using methods of complex analysis and a finite answer is obtained:

$$V_{eff}[\sigma] = \frac{5}{32\pi}\,\Lambda^4\left[\left(1+\frac{\sigma}{v_0}\right)^4-1\right](\sin\,2\theta)\,e^{-2\theta/\tan\theta} \quad ; \quad 0\le\theta\le 2\pi \tag{15a}$$

where the angle θ is such that:

$$\frac{160\pi^2 v_0^2}{\Lambda^2 N} = \frac{\theta}{\sin\theta}\ e^{-\theta/\tan\theta} \quad . \tag{15b}$$

[4]All remaining Weyl invariant terms can be consistently set equal to zero to leading order in $1/N$ without altering the physical conclusions [15].

At the minimum of $V_{eff}[\sigma]$ for $\sigma = 0$ the first derivative vanishes and we infer:

$$\theta = \frac{\pi}{2} \quad ; \quad v_0^2 = \frac{N}{320\pi}\,\Lambda^2 \quad . \tag{16}$$

In terms of the Planck mass, $v_0 = \sqrt{3/4\pi}\,M_{P\ell} \sim \frac{M_{P\ell}}{2}$ and $\Lambda^2 = \frac{240 M_{P\ell}^2}{N}$. At extremely high energies we have an asymptotically free Weyl invariant theory. The physical spectrum of the standard model predicts $N = 292$ and, therefore, the gravitational coupling constant G gets strong very close to the Planck mass which emerges as the natural scale of the theory. Using the $1/N$ expansion we could penetrate the strong coupling regime of (13) and demonstrate the spontaneous breaking of the conformal invariance in terms of a single scale parameter v_0 whose existence also induces Einstein's term.

Notice that for $\theta = \pi/2$ the complete effective potential (15) is zero. This realizes the field σ as the "Goldstone mode" of the spontaneously broken dilatation invariance. The most striking property of the dilaton field is its dual role as a Higgs scalar, acquiring a VEV that breaks a local symmetry, and a massless Goldstone boson, depositing its degree of freedom to the metric field and disappearing from the physical sector.

Having found v_0/Λ, we can check to one loop the vanishing of the cosmological constant as predicted by the W-I (11). The graviton tadpole $t_{\mu\nu}$ (see fig. 4) can be calculated:

$$t_{\mu\nu} = -\frac{5i}{64\pi}\,G\Lambda^2(\sin 2\theta)\,e^{-2\theta/\tan\theta}\eta_{\mu\nu} \quad . \tag{17}$$

and verifies (11) (see fig. 2):

$$\eta^{\mu\nu}\,t_{\mu\nu} = \frac{1}{2}\,v_0 G\left(-i\,\frac{dV_{eff}[\sigma]}{d\sigma}\bigg|_{\sigma=0}\right) \quad . \tag{18}$$

For $\theta = \pi/2$, $t_{\mu\nu}$ vanishes and so does the cosmological constant to this order. When the dilaton and, therefore, the graviton tadpole is zero Weyl invariance, through its W-I, implies the vanishing of an arbitrary 1-PI function involving only dilaton and graviton external legs at zero momentum [15].

The generalization of the above analysis to an arbitrary Weyl invariant Lagrangian is straightforward. In particular, $\mathcal{L}_M$ may contain some scalars ϕ_i which acquire VEV's v_i and provide masses to vector bosons and fermions. Furthermore, the unphysical field σ can self-interact and mix with these scalars. A Lorentz rotation on the fields σ and ϕ_i, dictated by the relative sign difference of their kinetic energies, gives the dilaton σ' and Higgs scalars ϕ_i' of the theory; the VEV of σ' is $v_0' = \sqrt{v_0^2 - v_i^2}$. The scalar effective potential should be minimized in the Higgs and dilaton directions. Alternatively, all the VEV's are determined by requiring the Higgs and dilaton tadpoles to vanish. Then, the W-I (11) will automatically imply a zero cosmological constant. An explicit example of this mechanism, whose essential points are presented here, is given in ref. [15]. Let $t_{\mu\nu}$, t_0 and t_i denote the gravitational, dilaton and Higgs tadpoles respectively. The contributions to these quantities up to one-loop come from the scalar quartic interactions $t^{(\phi)}$, as well as the vector boson, fermion and graviton loops $t^{(A)}$, $t^{(\psi)}$ and $t^{(h)}$ respectively (see fig. 5). We have:

$$t_{\mu\nu} = t_{\mu\nu}^{(\phi)} + t_{\mu\nu}^{(A)} + t_{\mu\nu}^{(\psi)} + t_{\mu\nu}^{(h)} \tag{19a}$$

$$t_0 = t_0^{(\phi)} + t_0^{(A)} + t_0^{(\psi)} + t_0^{(h)} \tag{19b}$$

$$t_i = t_i^{(\phi)} + t_i^{(A)} + t_i^{(\psi)} + t_i^{(h)} \quad . \tag{19c}$$

Since the Lagrangian possesses classical scale invariance, the gauge symmetry breaking occurs by radiative corrections and requires $\lambda_i \sim g_j^4$, where λ_i and g_j

represent various quartic and gauge coupling constants [12]. The W-I (11) is true diagram by diagram:

$$\eta^{\mu\nu}\, t^{(\mathcal{F})}_{\mu\nu} = \frac{n-2}{4}\; G v'_0\, t^{(\mathcal{F})}_0 \quad ; \quad \mathcal{F} = \phi, A, \psi, h \quad . \tag{20}$$

To achieve a stable perturbation theory we determine v'_0 and v'_i such that the dilaton and Higgs tadpoles are zero. Then, $t_{\mu\nu} = 0$ and the cosmological constant vanishes to this order. When the ratio v_i/v_0 is very small, as it would be in the standard model, the value of v'_0 is essentially equal to v_0 and, therefore, gravitation supplies the dominant contribution.

4. Gauge Invariance and Unitarity

The presence of a massive spin-two ghost state is a characteristic of all gravitational actions quadratic in the curvature and is the source of their nonunitarity in ordinary perturbation theory. We will argue that Weyl's gravity should be unitary in the $1/N$ expansion without use of the Lee-Wick prescription: all physical quantities, like S-matrix elements, involving only helicity-two massless gravitons and physical matter particles as external lines never need unphysical degrees of freedom as intermediate states. It is straightforward to extend the standard demonstration of the gauge invariance of physical quantities to this theory and guarantee the gauge fixing independence of the S-matrix.

The decoupling of the dilaton has already been shown; we now prove the gauge independence of its VEV v_0 and the gauge dependence of the spin-two complex mass M. Notice that in gauge theories exactly the opposite happens: the VEV of the Higgs field is gauge variant while the mass of the vector boson gauge invariant. But in gravity the physical quantity is the Planck mass, which is associated with the VEV of the dilaton, while the spin-two ghost is an unphysical particle.

Consider the gauge fixing conditions (12) used to prove the renormalizability of the theory. The derivation of the W-I for the ξ-dependence of 1-PI Green's functions follows that of ref. [3]. A new source term $-\eta\,\bar{c}\,\Phi$ is introduced in the effective Lagrangian, where η is an anticommuting constant and $\bar{c}$ the conformal antighost. The W-I replacing (10) is [15]:

$$2\eta\xi\,\frac{d\Gamma}{d\xi}=\sum_{F\neq\bar{c}_a}\frac{\delta\Gamma}{\delta F}\,\frac{\delta\Gamma}{\delta J_F^s}\quad. \tag{21}$$

Unlike gauge theories, since all the counterterms in the theory are ξ-independent [3,15], there is only explicit ξ-dependence in the Green's functions and the $d/d\xi$ appearing in (21) gives the total gauge fixing dependence of the 1-PI functions. By using v_0 such that the dilaton (and, therefore, the graviton) tadpole is zero, we find from (21):

$$\frac{d}{d\xi}\,\frac{\delta\Gamma}{\delta\sigma}=0\;\Rightarrow\;\frac{d}{d\xi}\,v_0=0 \tag{22}$$

establishing the gauge independence of v_0 and, consequently, the Planck mass. Moreover, employing (21) for the graviton self-energy we infer [15] the gauge parameter dependence of the complex pole M^2:

$$\frac{d}{d\xi}\,M^2=\frac{1}{N}\left(\frac{3}{32\pi^2}\,G^2N\,M^2_{(0)}\right)+O\left(\frac{1}{N^2}\right) \tag{23}$$

where

$$M^2=M^2_{(0)}+\frac{1}{N}\,M^2_{(1)}+\dots \tag{24}$$

As $\xi\to\infty$, the pole M^2 goes to infinity along a ray in the complex plane; both its real and imaginary parts become gauge parameter dependent. Physical quantities, being gauge invariant, should not depend on the gauge variant pole M. Thus, the spin-two ghost associated with the pole should be unphysical.

Acknowledgements

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Figure Captions

1. Leading $1/N$ graviton propagator.
2. Ward identity for the cosmological constant in Landau-type gauges.
3. Dilaton effective potential to leading order in $1/N$.
4. Leading $1/N$ graviton tadpole.
5. Graviton, dilaton, Higgs tadpoles up to one loop in scalar QED.

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Fig. 1

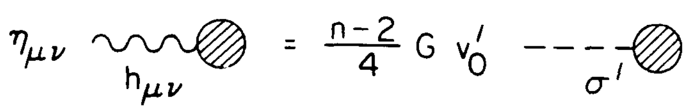

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Fig. 2

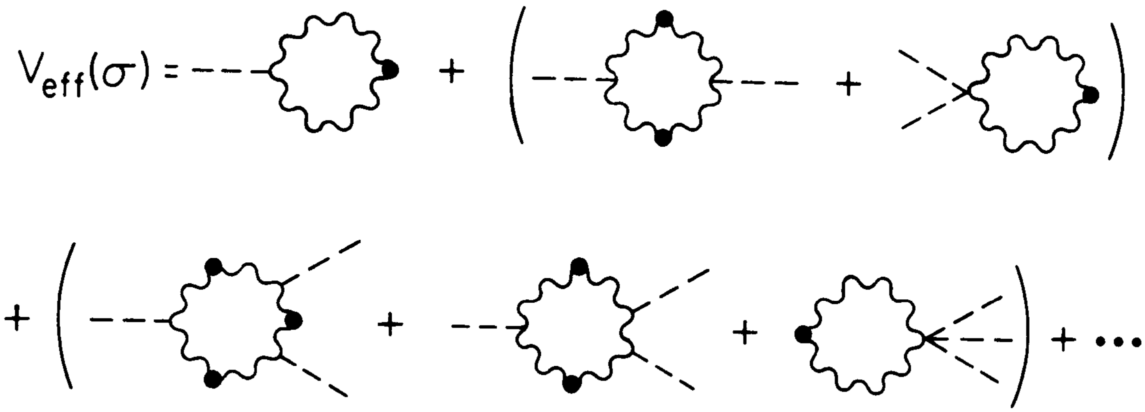

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Fig. 3

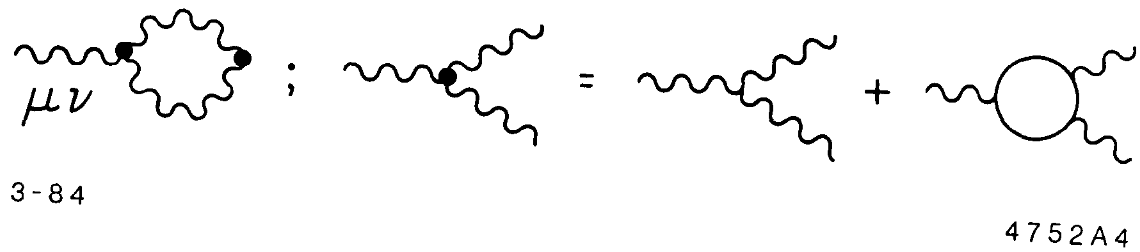

Fig. 4

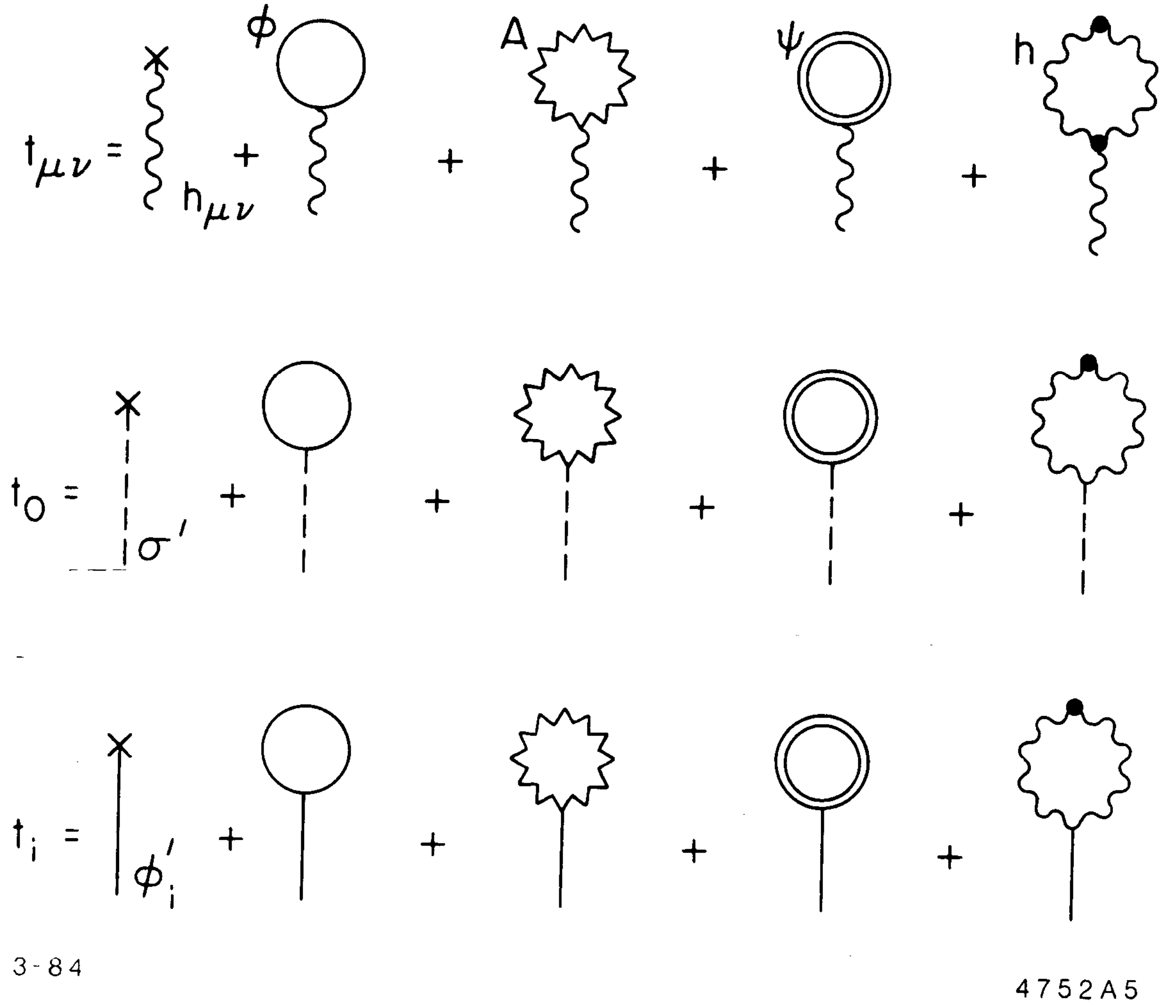

Fig. 5