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Conformal Symmetry and Exclusive Processes Beyond Leading Order*

by

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ABSTRACT

We present a systematic analysis in perturbative quantum chromodynamics and other renormalization theories of higher-order corrections to quark distribution amplitudes for flavor non-singlet mesons, the wavefunctions which control leading twist exclusive processes. In particular, we investigate the utility of residual conformal symmetry near the light cone. We find that beyond leading order the eigensolutions of the evolution equations are regulator-dependent in renormalizable theories. In a specific calculation for ϕ^3 theory in six dimensions to two loops, we find that the eigensolutions obey conformal symmetry using dimensional regularization for the subset of diagrams which do not contribute to the β function, but conformal symmetry is broken using Pauli-Villars regularization. A comparison with existing calculations of the two-loop kernel for gauge theory with $\beta = 0$ indicates that conformal symmetry does not hold beyond leading order in QCD in dimensional regularization.

I. Introduction

In recent years it has become apparent that many exclusive processes involving large momentum transfer can be analyzed perturbatively in QCD[1,2]. Leading order analyses have been completed for meson (M) and baryon (B) electro-weak form factors,[1,3] meson-photon transition form factors,[1] $\gamma\gamma \to M\bar{M}$ and $B\bar{B}$,[4] $\psi \to B\bar{B}$,[5] and several others. Work has begun on higher order corrections to these processes, with partial analyses of meson-meson[6] and meson-photon form factors.[7]

In this paper we use conformal symmetry[8,9,10] at short distances to give predictions for the quark distribution amplitude $\phi(x,Q)$ for flavor non-singlet mesons ($\phi's,K's,\rho's,etc.$), the wavefunctions which control the behavior of exclusive meson processes at large momentum transfer. These predictions are explicitly confirmed through two-loop order in ϕ^3 theory in six dimensions for a subset of graphs with zero β-function using dimensional regularization, but fail with a Pauli-Villars regulator. In the case of QCD and other gauge theories, conformal symmetry does not appear to hold beyond leading order using dimensional regularization. This unexpected breakdown of conformal symmetry, even for $\beta = 0$, may be due to the sensitivity of gauge theory to infrared cutoffs in both of these regularization schemes. (Of course Pauli-Villars should not be used in QCD due to breaking of non-Abelian gauge invariance.)

In Section II we review the general formalism for analyzing exclusive amplitudes in perturbative QCD. Here and throughout the paper we limit our discussion to flavor non-singlet mesons. We review the leading order analysis, and identify those elements of the second order analysis that are still needed to complete the treatment to that order. The central problem concerns the generalization beyond leading order of the Gegenbauer polynomials $C_n^{3/2}(x_1-x_2)$ that appear in leading order — i.e. an analysis of operator mixing under renormalization.

It has been shown by Parisi[10] that conformal symmetry is satisfied asymptotically at short distance in renormalizable field theories with zero β-function. This result however may only be true for specific ultraviolet regulators. (For a discussion, see Ref. 9). In Sections II and III we postulate the applicability of conformal symmetry to the operator product expansion at short distances and predict the form of the eigensolution of the evolution equation for the distribution amplitude to all orders in perturbation theory. The corrections from $\beta \neq 0$ are then treated in perturbation theory.

In Section IV we show that the predictions of conformal symmetry cannot hold simultaneously beyond leading order in both Pauli-Villars and dimensional regularization. As shown in Appendix C, ϕ^3 theory is six dimensions with dimensional regularization is consistent in two-loop order with the expectations of conformal symmetry. Assuming this also holds in gauge theory we then give detailed predictions for meson distribution amplitudes in QCD, and in Section V apply them to the meson form factors.

We also discuss the problem of generalizing our analysis to flavor-singlet mesons. We briefly summarize the detailed procedure for perturbative calculations of exclusive amplitudes in Appendix A. These are illustrated by a complete one-loop analysis and by parts of the two-loop analysis in the same Appendix.

Recently, three explicit calculations[11] of the two-loop kernel for the meson distribution amplitude in QCD have been performed using dimensional regularization, two in light-cone gauge and the last in Feynman gauge. The results agree with each other, and the diagonal matrix elements are consistent with the second-order non-singlet anomalous dimensions for deep inelastic scattering calcalculated in Ref. 12. The results for the eigensolutions, however, disagree with the predictions of conformal symmetry.

II. Exclusive Amplitudes at Large Momentum Transfer

A. General Formalism

Generally, exclusive amplitudes involving large momentum transfer factor into a convolution of distribution amplitudes $\phi(x_i,Q)$, one for each hadron, with a hard scattering amplitude T_H. The pion's electromagnetic form factor, for example, can be written as [1,2]

$$Q^2 F_\pi(Q) = \int_0^1 [dx] \int_0^1 [dy]\ \phi^*(x_i,Q)\ T_H(x_i,y_i,Q)\ \phi(y_i,Q)\{1 + O\left(\frac{1}{Q}\right)\} \tag{1}$$

where $[dy] = dy_1 dy_2\ \delta(1 - \sum_i y_i)$ and $Q^2 = -q^2$ is large. Here $\phi(y_i,Q)$ is the probability amplitude for finding the valence $q\bar{q}$ Fock state in the initial pion, with the constituents carrying longitudinal momentum $y_1 p_\pi$ and $y_2 p_\pi$, respectively; T_H is the amplitude for scattering the $q\bar{q}$ state from the initial to the final direction; and ϕ^* is the amplitude for the final-state $q\bar{q}$ to fuse back into a pion.

Choosing a frame in which $p_\pi^+ = p_\pi^0 + p_\pi^3 = 1$, the process independent distribution amplitude for a pion is quite naturally defined by[1,2,8]

$$\phi(x_i,Q) = \int \frac{dz^-}{2\pi}\, e^{i(x_1-x_2)z^-/2} \langle 0|\bar{\psi}(-z)\, \frac{\gamma^+\gamma_5}{2\sqrt{2}}\, \psi(z)|\pi\rangle^{(Q)}\Big|_{z^+=z_\perp=0} \tag{2}$$

in $A^+ = 0$ gauge. [In other gauges there is a path ordered factor $\exp(ig\int_{-1}^{1} ds\ A^+(zs)z^-/2)$ between the $\bar{\psi}$ and ψ, making ϕ gauge invariant.] The matrix element in Eq.(2) has an ultraviolet divergence, coming from the light-cone singularity at $z^2 = 0$. This divergence is regulated by introducing a momentum cut-off, or other renormalization scale, equal to Q. Consequently z^2 is in effect smeared over a region of order $z^2 = -z_\perp^2 \sim -1/Q^2$ — the form factor probes distances no shorter than $\sim 1/Q$. Any regulator that is both Lorentz invariant and gauge invariant can be used. For purposes of illustration, we use dimensional regularization and minimal subtraction (with $\mu = Q$) in this section. Other regulators are considered in Appendix A.

Once a regulator is chosen, Eqs. (1) and (2) uniquely specify the gauge invariant hard scattering amplitude T_H. For the pion form factor, as for many other processes, T_H has a perturbative expansion in powers of $\alpha_s(Q)$ with

$$T_H(x_i,y_i,Q) = \frac{1}{Q^n} f(x_i,y_i,\alpha_s(Q)) \tag{3}$$

where $n = 0$, by dimensional analysis. [In general n is the total number of initial and final partons less four.] To leading order in $\alpha_s(Q)$, the distribution amplitude and therefore also T_H are independent of the regulator used in defining ϕ. This is obviously not the case beyond leading order, as will be illustrated in Section III.

The variation of $\phi(x_i,Q)$ with Q is less drastic and somewhat more complicated than T_H. The Q-dependence is determined solely by the ultraviolet structure of the operator $\overline{\psi}(-z)\,\gamma^+\gamma_5\psi(z)$ on the light-cone, and thus can be studied perturbatively. To extract this behavior, we introduce an unrenormalized distribution amplitude $\phi_u(x_i)$ defined in $4-2\varepsilon$ dimensions. Being in $4-2\varepsilon$ dimensions, ϕ_u is ultraviolet finite and therefore Q-independent. It is related to the true distribution amplitude by a 'matrix' of renormalization constants $Z(x_i,y_i,Q)$:

$$\phi_u(x_i) = \int\frac{[dy]}{y_1y_2}\, Z(x_i,y_i,Q)\; \phi(y_i,Q) \tag{4}$$

Differentiating this equation with respect to Q^2, we obtain an evolution equation for ϕ:

$$Q^2\,\frac{\partial}{\partial Q^2}\,\phi(x_i,Q) = \int\frac{[dy]}{y_1y_2}\, V(x_i,y_i,\alpha_s(Q))\;\phi(y_i,Q) \tag{5}$$

where

$$V = -Q^2\,\frac{\partial}{\partial Q^2}\,\ln Z \tag{6a}$$

has a power series expansion in $\alpha_s(Q)$:

$$V(x_i,y_i,\alpha_s(Q)) = \frac{\alpha_s(Q)}{4\pi} V_1(x_i,y_i) + \left[\frac{\alpha_s(Q)}{4\pi}\right]^2 V_2(x_i,y_i) + \ldots \quad (6b)$$

Clearly $\phi(x,Q)$ is only logarithmically dependent on Q; the bulk of the Q-dependence of an exclusive process is due to T_H. A detailed procedure for computing V is illustrated in Appendix A.

In practice, the evolution equation (5) is all that is needed to compute the evolution of ϕ as Q changes. Given some initial distribution $\phi(x_i,Q_0)$, the equation is readily integrated numerically to give $\phi(x_i,Q)$ for any Q. An alternative procedure relates the variation of ϕ to the Q-dependence of moments of the distribution amplitude:

$$\int_0^1 [dx]\ (x_1-x_2)^n \phi(x_i,Q) = \langle 0|\overline{\psi}(0)\, \frac{\gamma^+\gamma_5}{2\sqrt{2}}\, (i\overleftrightarrow{\partial}^+)^n\, \psi(0)|\pi\rangle^{(Q)} \Big|_{p_\pi^+=1}$$

$$= \langle 0|\ (i\partial^+)^m [\overline{\psi}\, \frac{\gamma^+\gamma_5}{2\sqrt{2}}\, (i\overleftrightarrow{\partial}^+)^n\, \psi]|\pi\rangle^{(Q)} \quad (7)$$

independent of m [$i\overleftrightarrow{\partial}^+ \to i\overleftrightarrow{D}^+ = i\overleftrightarrow{\partial}^+ - gA^+$ in gauges other than $A^+=0$ gauge]. Clearly the variation with Q of these moments is identical to the cut-off dependence of the local operators $(i\partial^+)^m\, \overline{\psi}\gamma^+\gamma_5(i\overleftrightarrow{\partial}^+)^n\, \psi$. In general these operators mix under renormalization, but only operators having the same number of derivatives can mix in a Lorentz invariant theory. Consequently, for each integer n, there is a 'tower' of operators $O^{(n)}$, $i\partial^+O^{(n)}$, $(i\partial^+)^2O^{(n)}$, ... where $(a_0^{(n)}\neq 0)$

$$O^{(n)} \equiv \sum_{j=0}^{n} a_j^{(n)}(\alpha_s)\ [(i\partial^+)^{n-j}\ \overline{\psi}\, \frac{\gamma^+\gamma_5}{2\sqrt{2}}\ (i\overleftrightarrow{\partial}^+)^j\ \psi] \quad (8)$$

can be chosen so that each operator is separately multiplicatively renormalizable, all having the same anomalous dimension $\gamma^{(n)}(\alpha_s)$.[12,13] These operators depend implicitly upon the renormalization scale, both through α_s and through the regulator required to define their matrix elements. For our purposes, the renormalization scale is set equal to Q. By introducing the polynomials $\bar{P}_n$,

$$\bar{P}_n(x_1-x_2,\alpha_s) = \sum_{j=0}^{n} a_j^{(n)}(\alpha_s)\,(x_1-x_2)^j\,, \tag{9}$$

we can define moments $\tilde{M}_n(Q)$,

$$\begin{aligned}\tilde{M}_n(Q) &= \int_0^1 [dx]\,\bar{P}_n(x_1-x_2,\,\alpha_s(Q))\,\phi(x_i,Q) \\ &= \langle 0|\,\bar{\psi}\,\frac{\gamma^+\gamma_5}{2\sqrt{2}}\,\bar{P}_n(i\overleftrightarrow{\partial}^+,\,\alpha_s)\psi|\pi\rangle^{(Q)}\end{aligned} \tag{10}$$

that satisfy simple evolution equations:

$$\begin{aligned}Q^2\frac{d}{dQ^2}\tilde{M}_n(Q) &= -\frac{1}{2}\gamma^{(n)}(\alpha_s(Q))\,\tilde{M}_n(Q) \\ \frac{1}{2}\gamma^{(n)}(\alpha_s) &= \frac{\alpha_s}{4\pi}\gamma_1^{(n)} + \left(\frac{\alpha_s}{4\pi}\right)^2\gamma_2^{(n)} + \ldots\end{aligned} \tag{11}$$

Equations (9-11) are equivalent in content to the original evolution equation (Eqs.(5,6)). Given the anomalous dimensions $\gamma^{(n)}$ and the polynomials $\bar{P}_n$, the complete Q-dependence of the distribution amplitude is determined:

$$\phi(x_i,Q) = x_1x_2\sum_{n=0}^{\infty} P_n(x_1-x_2,\,\alpha_s(Q))\,\tilde{M}_n(Q) \tag{12}$$

where, from Eq.(11),

$$\tilde{M}_n(Q) = \tilde{M}_n(Q_0)\,\exp\left[-\int_{Q_0}^{Q}\frac{d\bar{Q}}{\bar{Q}}\;\gamma^{(n)}(\alpha_s(\bar{Q}))\right] \tag{13}$$

and where P_n is defined such that

$$\int_0^1 [dx]\,\bar{P}_n\,x_1x_2\,P_m = \delta_{nm} \tag{14}$$

In general P_n, unlike $\bar{P}_n$, is not a polynomial. The functions P_n and $\bar{P}_n$, and the anomalous dimensions $\gamma^{(n)}$ can all be determined directly from the evolution potential $V(x_i,y_i,\alpha_s)$. One readily obtains defining equations for $P_n,\bar{P}_n$ and $\gamma^{(n)}$ by substituting expansion (12) for ϕ into the evolution equation (5):

$$Q^2 \frac{\partial}{\partial Q^2} \bar{P}_n(y_1-y_2,\alpha_s(Q))\, y_1y_2 = -\frac{1}{2}\gamma^{(n)}(\alpha_s)\, \bar{P}_n(y_1-y_2,\alpha_s)y_1y_2 - \int[dx]\, \bar{P}_n(x_1-x_2,\alpha_s)\, V(x_i,y_i,\alpha_s)$$

$$Q^2 \frac{\partial}{\partial Q^2} x_1x_2P_n(x_1-x_2,\alpha_s(Q)) = \frac{1}{2}\gamma^{(n)}(\alpha_s)\, x_1x_2P_n(x_1-x_2,\alpha_s) + \int[dy]\, V(x_i,y_i,\alpha_s)\, P_n(y_1-y_2,\alpha_s) \tag{15}$$

Being first-order differential equations, these equations must be supplemented by an initial condition or other constraint. The choice of an initial condition is largely a matter of convenience and convention, as will become clear in Section II.C. below.

The formalism outlined in this section is valid to all orders in $\alpha_s(Q)$. Once an ultraviolet regulator has been chosen for defining $\phi(x,Q)$, the evaluation of T_H for some process is straightforward. The process independent distribution amplitudes $\phi(x,Q)$, must be specified at some $Q=Q_0$, either empirically or by some non-perturbative analysis. The variation of $\phi(x,Q)$ with Q can then be computed either directly from the evolution equations (Eqs.(5,6)) or from the moments of ϕ (Eqs.(12-15)). We now specialize our analysis of ϕ to leading and next-to-leading orders.

B. The Distribution Amplitude in Leading Order

The formalism of the previous section simplifies considerable in leading order. The leading-order evolution potential V_1 is readily computed (see Appendix A)

$$V_1(x_i,y_i) = 2C_F\{x_1y_2\,\theta(y_1-x_1)[\delta_{-h,\bar{h}} + \frac{\Delta}{y_1-x_1}] + (1\leftrightarrow 2)\} - C_F\,y_1y_2\,\delta(x_1-y_1) = V_1(y_i,x_i) \qquad (16)$$

where $\Delta\phi(y_i,Q) = \phi(y_i,Q) - \phi(x_i,Q)$.

Functions P_n, $\bar{P}_n$, and the anomalous dimensions $\gamma^{(n)}$ are then determined from Eqs. (15), which in this order simplify to the form

$$\int[dx]\,\bar{P}_n(x_1-x_2,0)\,V_1(x_i,y_i) = -y_1y_2\,\gamma_1^{(n)}\,\bar{P}_n(y_1-y_2,0)$$

$$\int[dy]\,V_1(x_i,y_i)\,P_n(y_1-y_2,0) = -x_1x_2P_n(x_1-x_2,0)\,\gamma_1^{(n)} \qquad (17)$$

Thus in the limit $\alpha_s\to 0$, P_n and $\bar{P}_n$ are eigenfunctions of V_1 corresponding to eigenvalue $-\gamma_1^{(n)}$. Since $V_1(x_i,y_i) = V_1(y_i,x_i)$ is a symmetric operator it is immediately obvious that $P_n = \bar{P}_n$, and that these polynomials form a complete set, orthogonal with respect to weight x_1x_2. The only polynomials orthogonal for this weight are 3/2-Gegenbauer polynomials and therefore

$$\bar{P}_n(x_1-x_2,\alpha_s=0) = P_n(x_1-x_2,0) = \tilde{C}_n^{3/2}(x_1-x_2) \equiv \left[\frac{4(2n+3)}{(2+n)(1+n)}\right]^{\frac{1}{2}} C_n^{3/2}(x_1-x_2) \qquad (18)$$

The anomalous dimensions to one loop then follow easily from Eq. (16):

$$\gamma_1^{(n)} = C_F\left\{1 + 4\sum_{j=2}^{n+1}\frac{1}{j} - \frac{2\,\delta_{-h,\bar{h}}}{(n+1)(n+2)}\right\} \qquad (19)$$

where for pions $\delta_{-h,\bar{h}} = 1$.

C. The Distribution Amplitude to Two Loops

In two-loop order, the polynomials $\bar{P}_n(x_1-x_2,\alpha_s)$ have the general form

$$\bar{P}_n(x_1-x_2,\alpha_s) = \tilde{C}_n^{3/2}(x_1-x_2) + \frac{\alpha_s}{4\pi}\sum_{j=0}^{n-1} d_j^n\, \tilde{C}_j^{3/2}(x_1-x_2) \quad (20)$$

while P_n, no longer a polynomial, must then be given by (see Eq.(14))

$$P_n(x_1-x_2,\alpha_s) = \tilde{C}_n^{3/2}(x_1-x_2) - \frac{\alpha_s}{4\pi}\sum_{j=n+1}^{\infty} d_n^j\, \tilde{C}_j^{3/2}(x_1-x_2) \quad (21)$$

Substituting these expressions into Eq. (15), we obtain

$$\gamma_2^{(n)} = -\int[dx][dy]\,\tilde{C}_n^{3/2}(x_1-x_2)\,V_2(x_i,y_i)\,\tilde{C}_n^{3/2}(y_1-y_2) \quad (22a)$$

$$Q^2\frac{d}{dQ^2}d_j^n = \frac{\alpha_s(Q)}{4\pi}\{(\beta_0-\gamma_1^{(n)}+\gamma_1^{(j)})\,d_j^n - (V_2)_{nj}\} \quad (22b)$$

where $(V_2)_{nj}$ and β_0 are defined by

$$(V_2)_{nj} = \int[dx][dy]\,\tilde{C}_n^{3/2}(x_1-x_2)\,V_2(x_i,y_i)\,\tilde{C}_j^{3/2}(y_1-y_2)$$

$$Q^2\frac{d}{dQ^2}\alpha_s(Q) = \beta(\alpha_s(Q)) = -\frac{\alpha_s^2(Q)}{4\pi}\beta_0 - \cdots$$

To solve for the expansion coefficients d_j^n, we must now deal with the issue of initial conditions for Eq. (22b). At first glance, it seems most natural to choose initial conditions that make the d_j^n constants, independent of Q. However, with this choice, the expansion coefficients equal $(V_2)_{nj}/(\beta_0-\gamma_1^{(n)}+\gamma_1^{(j)})$, which becomes very large when $\beta_0 \approx \gamma_1^{(n)}-\gamma_1^{(j)}$ (e.g., $d_1^{11} \simeq -148$). Such large coefficients are obviously an artifact of the initial conditions, and do not reflect pathologies in the behavior of $\phi(x,Q)$. A far more practical initial condition is

$$d_i^n(Q_0) = 0 \quad (23a)$$

in which case Eq. (22b) implies ($n > j$)

$$d_j^n(Q) = \left\{1 - \left[\frac{\alpha_s(Q_0)}{\alpha_s(Q)}\right]^{(\beta_0 - \gamma_1^{(n)} + \gamma_1^{(j)})/\beta_0}\right\} \frac{(V_2)_{nj}}{\beta_0 - \gamma_1^{(n)} + \gamma_1^{(j)}} . \quad (23b)$$

Then d_j^n is well behaved even in the limit $\beta_0 \to \gamma_1^{(n)} - \gamma_1^{(j)}$:

$$d_j^n(Q) \xrightarrow[\beta_0 \to 0]{} \left[1 - \left(\frac{Q^2}{Q_0^2}\right)^{\frac{\alpha(Q_0^2)}{4\pi}\left(\gamma_1^{(j)} - \gamma_1^{(n)}\right)}\right] \frac{(V_2)_{nk}}{\gamma_1^{(j)} - \gamma_1^{(n)}} .$$

Furthermore $\alpha_s(Q) d_j^n(Q)$ is bounded in magnitude for all $Q \geq Q_0$, since it vanishes both for $Q = Q_0$ and for $Q \to \infty$. Consequently deviations from the leading order result are small throughout this range, provided of course $(V_2)_{nj}$ is not large. An additional convenience of this choice is that the relationship between the moments $\tilde{M}_n(Q)$ and the distribution amplitude $\phi(x, Q)$ is unchanged from the leading order result at $Q = Q_0$, i.e., $P_n = \bar{P}_n = \tilde{C}_n^{3/2}$ is exact both at $Q \to \infty$ and at $Q = Q_0$. This facilitates the determination of the initial moments from the initial distribution amplitude.

From Eq. (22), we learn that Gegenbauer matrix elements $(V_2)_{nj}$ of the two-loop evolution potential determine all $O(\alpha_s)$ corrections to $\phi(x, Q)$. The anomalous dimensions $\gamma^{(n)}(\alpha_s)$ for the operators $O^{(n)}$ (Eq. (8)) have already been determined through two loops for the analysis of moments in deep inelastic scattering.[12,13] Thus the diagonal matrix elements of V_2 (Eq. (22a)) are known. The off-diagonal matrix elements, and therefore also the coefficients d_j^n, are readily determined if conformal symmetry is valid, as we demonstrate in the next section.

III. Consequences of Conformal Symmetry

A. Leading Order

Classical relativistic field theories that are scale invariant and have a renormalizable Lagrangian are also invariant under the conformal group, which consists of the translations, boosts and rotations of the Poincaré group together with dilatations ($x^\mu \to \lambda x^\mu$) and conformal transformations [inversion ($x^\mu \to -x^{\mu}/x^2$) $\otimes$ translations $\otimes$ inversion].[10] Scale invariance and therefore also conformal symmetry are destroyed in QCD by quark masses, and by the renormalization procedure, which inevitably introduces some renormalization scale Λ. However, the evolution potential $V(x_i, y_i, \alpha_s)$ (Eq. (6)) is by definition free of both mass singularities and of all ultraviolet infinities other than those related to charge renormalization. Since there is no renormalization of α_s in leading order, the one-loop potential V_1 must preserve conformal symmetry. As shown in Ref. 8, this constraint implies that the functions P_n that diagonalize V_1 must be Gegenbauer polynomials

$$P_n(x_1 - x_2, 0) \propto \tilde{C}_n^{3/2}(x_1 - x_2) \tag{24}$$

Then the multiplicatively renormalizable operators $0^{(n)}$ defined by P_n transform as irreducible tensors not only under the Lorentz group, but under the full conformal group as well.[8]

B. All Orders Analysis of Conformal Symmetry

Beyond leading order, the functions P_n can be modified by two effects. First the dimension of $0^{(n)}$ in Eq. (25) is increased by the anomalous dimension $\gamma^{(n)}(\alpha_s)$. While this should not affect the conformal symmetry of the evolution potential, it does change the prediction for the P_n.

As shown in Appendix B,[14] the general result for operators $O(n)$ bilinear in spin zero fields in scalar field theory is

$$P_n(x) \propto \frac{1}{(1-x^2)} \frac{d^n}{dx^n} (1-x^2)^{[n+(d_\phi-1)+\frac{1}{2}\gamma_n(\alpha_s)]} \tag{25}$$

where d_ϕ is the canonical dimension of $\phi(d_\phi = 1$ in 4-dimensions, $d_\phi = 2$ in six dimensions). For spin $\frac{1}{2}$ fields, with $O_n(0)$ as defined in Eq. (8), conformal symmetry predicts

$$P_n(x) \propto \frac{1}{(1-x^2)} \frac{d^n}{dx^n} (1-x^2)^{[n+(d_\phi-\frac{1}{2})+\frac{1}{2}\gamma_n]} \tag{26}$$

where d_ϕ is again the canonical dimension of $\phi(d_\phi = \frac{3}{2}$ in 4-dimensions, $d_\phi = \frac{1}{2}$ in 2 dimensions). The results are true in any space-time dimension.

The second effect is due to the breaking of scale invariance by the running coupling constant. This leads to terms in V proportional to the β-function that break the conformal symmetry and therefore modify the P_n's. One expects that all symmetry breaking terms in the potential must be of this second type ($\propto \beta(\alpha_s)$) because mass scales enter V only through charge renormalization.

Each of these effects leads to terms in the two-loop potential V_2 that are not diagonal with respect to the Gegenbauer polynomials $\widetilde{C}_n^{3/2}$. Furthermore, these are the only non-diagonal terms in V_2 and, consequently, the only terms that need be computed to obtain the expansion coefficients d_j^n for $\overline{P}_n$ and P_n (Eq. (23)). Given the expansion coefficients together with the two-loop anomalous dimensions, one can compute the full distribution amplitude. It is useful to study these effects for two different distribution amplitudes, one defined with a Pauli-Villars cut-off and another defined by dimensional regularization ($\overline{MS}$).

In fact we find that conformal symmetry cannot be simultaneously true in both regulators beyond leading order. This is discussed in detail in the next section and Appendix A. This result has been explicitly checked for $[\phi^3]_6$ to two-loop order for the set of (ladder and crossed ladder) graphs that have no contribution to β. The dimensional regularization results agree with conformal symmetry.

IV. Calculations of the Meson Distribution Amplitude in Gauge Theory

A. Pauli-Villars Regulator

By definition, the ultraviolet divergence in the distribution amplitude $\phi(x, Q)$ is removed in Pauli-Villars regularization by subtracting diagrams with the gluon mass set equal to Q. As we shall show, the distribution amplitudes in this scheme and dimensional regularization can be related to each other through a correction to the evolution kernel beyond leading order. In Appendix C we give a complete calculation of the distribution amplitude and the evolution kernel through two loops for $[\phi^3]_6$. By keeping only the crossed ladder and ladder contributions, the model for the distribution amplitude satisfies the Callan-Symansik equation for $\beta = 0$. By explicit calculation through two loops we find, using Pauli-Villars regularization, the polynomials $\overline{P}_n$ a defined in eq. (12) are the Gegenbauer polynomials $\widetilde{C}_n^{\xi_n}(x)$ with index

$$\xi_n = \frac{3}{2} + \frac{\gamma^{(n)}(\alpha_s)}{2} = \frac{3}{2} + \frac{\alpha_s}{4\pi}\gamma_1^{(n)} + \dots \quad .$$

We then find that the functions $P_n(x_1 - x_2, \alpha_s)$, the eigensolutions of the evolution equation for the distribution amplitude, are exactly those predicted by conformal symmetry (Eq. (25), with $d_\phi = 2$), but that this result holds only for dimensional regularization, *not* Pauli-Villars. In this section we show that if one assumes $\bar{P}_n = \tilde{C}_n^{\xi_n}$ in gauge theories in Pauli-Villars regularization, then again the conformal symmetry functions arise for the P_n in the dimensional regularization scheme if $\beta_0 = 0$.

With the above assumption for the $\bar{P}_n$, the polynomials to two loop order for Pauli-Villars regularization are

$$\bar{P}_n(x_1-x_2,\alpha_s) = \tilde{C}_n^{\xi_n}(x_1-x_2)$$

$$= \tilde{C}_n^{3/2}(x_1-x_2) + \frac{\alpha_s}{4\pi}\,\gamma_1^{(n)}\,\frac{d}{d\xi}\,\tilde{C}_n^{\xi}(x_1-x_2)\Big|_{\xi=3/2} + \cdots ,$$

and therefore, from definition (20), d_j^n would be

$$\gamma_1^{(n)} \int [dx]\,\frac{d}{d\xi}\,\tilde{C}_n^{\xi}(x_1-x_2)\Big|_{\xi=3/2}\, x_1x_2\,\tilde{C}_j^{3/2}(x_1-x_2)$$

[Note that we are led to the scheme with constant d_j^n.]

From the discussion in Section II.C, there must therefore be a term V_{2a} in the two-loop potential for which

$$\frac{(V_{2a})_{nj}}{\gamma_1^{(j)}-\gamma_1^{(n)}} = \gamma_1^{(n)} \int [dx]\,\frac{d}{d\xi}\,\tilde{C}_n^{\xi}\Big|_{\xi=3/2}\, x_1x_2\,\tilde{C}_j^{3/2}$$

This expression can be simplified somewhat by using the identity

$$\frac{d}{d\xi}\,\tilde{C}_n^{\xi}\Big|_{\xi=\frac{3}{2}}\, x_1x_2\,\tilde{C}_j^{3/2} = \frac{d}{d\xi}\left[\tilde{C}_n^{\xi}\,(x_1x_2)^{\xi-\frac{1}{2}}\,\tilde{C}_j^{\xi} - \tilde{C}_n^{3/2}\,x_1x_2\,\tilde{C}_j^{\xi}\right]_{\xi=3/2}$$

$$- \tilde{C}_n^{3/2}\,\ln(x_1x_2)\,x_1x_2\,\tilde{C}_j^{3/2}$$

and the orthogonality of $\tilde{C}_n^{\xi}$'s with respect to weight $(x_1x_2)^{\xi-\frac{1}{2}}$. Thus the off-diagonal matrix elements of V_{2a} can be written

$$(V_{2a})_{nj} = \begin{cases} \gamma_1^{(n)}(\gamma_1^{(n)}-\gamma_1^{(j)}) \int [dx]\, \tilde{C}_n^{3/2}\, \ln(x_1x_2)x_1x_2\, \tilde{C}_j^{3/2} & n>j \\ 0 & j>n \end{cases} \quad (27)$$

As we argued above, any symmetry breaking terms in V_2 must be proportional to $\beta(\alpha_s) = -\beta_0\alpha_s^2/4\pi$ where $\beta_0 = 11-2n_f/3$ and n_f is the number of light-quark flavors. The n_f-dependent part of this correction comes entirely from the quark-vacuum-polarization correction to the leading order potential and is easily computed. From it the entire correction is obtained simply by multiplying by $-\frac{3}{2}\beta_0/n_f$. In fact, as we show in Appendix A, there is no symmetry breaking term of this type for the Pauli-Villars regulator. This rather surprising result is easily explained. Any term in V_2 proportional to β_0 should properly be absorbed into the leading order potential by rescaling the argument of α_s. As discussed in Ref. 15, this sets the argument of α_s equal to the mean momentum flowing through the gluons in the leading order diagrams (up to a constant scheme-dependent factor). Generally conformal symmetry will be destroyed if this mean momentum depends upon the longitudinal momenta, as then α_s varies with x_i and y_i. However, the Pauli-Villars regulator automatically sets the mean gluon momentum equal to Q, independent of x_i and y_i, because it regulates divergences by introducing the cut-off Q as a gluon mass. Thus V_{2a} (Eq.(27)) is the only non-diagonal term in the two-loop Pauli-Villars potential.

B. Dimensional Regularization

The two-loop evolution potential obtained using dimensional regularization must again include the conformally symmetric, but non-diagonal, potential V_{2a} (Eq.(27)). In addition there are two symmetry-breaking terms due to the fact that $\beta(\alpha_s) \neq 0$. The first is proportional to β_0, and is readily computed from the vacuum polarization corrections to leading order, as described above (see Appendix A). A second symmetry-breaking term is expected because the coupling constant is not dimensionless in $4-2\varepsilon$ dimensions. Thus the scale invariance of the theory is destroyed, and the β-function is non-zero even in leading order — i.e. $\beta(\alpha_s) = -\varepsilon\, \alpha_s - \ldots$. The extraction of this second term from V_2 is somewhat subtle because it is induced by an $O(\varepsilon)$ effect. It is easier to derive both symmetry-breaking terms together by relating the evolution potentials for Pauli-Villars and dimensional regularization, as we now illustrate.

The distribution amplitudes for the two regulators are related by a finite renormalization constant z:

$$\phi_{PV}(x_i,Q) = \int \frac{[dy]}{y_1 y_2}\; z(x_i,y_i,\alpha_s(Q))\; \phi_{DR}(y_i,Q) \tag{28a}$$

where

$$z = y_1 y_2\; \delta(x_i-y_i) + \frac{\alpha_s(Q)}{4\pi}\, \delta V(x_i,y_i) + \ldots \tag{28b}$$

Substituting this equation into the evolution equation for ϕ_{PV}, we can express one evolution potential in terms of the other:[13]

$$\begin{aligned} V_{DR} &= z^{-1} V_{PV} z - Q^2 \frac{d}{dQ^2} \ln z \\ &= V_{PV} + \left[\frac{\alpha_s(Q)}{4\pi}\right]^2 (V_1\, \delta V - \delta V\, V_1 + \beta_0\, \delta V) + \ldots \end{aligned} \tag{29}$$

where V_1 is the one-loop potential (Eq.(16)) and where, from Appendix A, δV is

$$\delta V = -2C_F \left\{x_1 y_2 \ln \frac{y_1}{x_1} \left(\delta_{-h,\bar{h}} + \frac{1}{y_1-x_1}\right)\theta(y_1-x_1) + (1 \leftrightarrow 2)\right\} \tag{30}$$

$$+ \text{ symmetric terms}$$

Thus the two-loop symmetry-breaking terms in the dimensional regularization potential are contained in

$$V_{2b} = [V_1, \delta V] + \beta_0 \delta V \,, \qquad (31)$$

and all terms that are not diagonal with respect to 3/2-Gegenbauer polynomials are contained in $V_{2a} + V_{2b}$.

The off-diagonal matrix elements of V_{2b} can be greatly simplified. First, using Eqs. (17) we can show that

$$(V_{2b})_{nj} = (\beta_0 - \gamma_1^{(n)} + \gamma_1^{(j)}) \int [dx][dy]\, C_n^{3/2}\, \delta V\, C_j^{3/2}$$

Secondly, and rather remarkably, we show in Appendix A that off-diagonal matrix elements of δV (Eq.(A-11)) are related to the matrix elements of V_{2a} (Eq. (27)):

$$(\delta V)_{nj} = \frac{1}{\gamma_1^{(n)}} (V_{2a})_{nj}$$

Thus the non-diagonal matrix elements of V_2 for this regulator are given by

$$(V_{2a}+V_{2b})_{nj} = (\beta_0 + \gamma_1^{(j)})(\gamma_1^{(n)} - \gamma_1^{(j)}) \int [dx]\, C_n^{3/2}\, \ln(x_1 x_2) x_1 x_2\, C_j^{3/2} \qquad (32)$$

The prediction of conformal symmetry (Eq. (26) with $d_\phi = 3/2$)

$$P_n(x) \propto \frac{1}{1-x^2}\,\frac{d^n}{dx^n}\left\{(1-x^2)^{n+1}\left[1+\frac{\alpha_s}{4\pi}\gamma_n^{(1)}\ell n(1-x^2)\right]\right\}$$

is in complete agreement with Eq. (32) for $\beta_0 = 0$. As shown in Appendix A the essential difference between the Pauli-Villars and dimensional regularization can be traced to the induced contribution to the β function in 4 - 2ϵ dimensions. Despite the consistency of the above approach, we note that explicit calculations[11] of the second order evolution kernel in gauge theories (Abelian QED and SU(N_c) QCD) using dimensional regularization and $\beta_0 = 0$ ($N_F = (11/2)\ N_c$) do not agree with the conformal symmetry prediction. [Although the contributions proportional to β_0 do agree with Eq. (32).] The results have been checked in both light-cone and Feynman gauges. This conflict is unresolved, and hints at an even subtler breakdown of conformal symmetry in gauge theory.

V. Conclusions

In this paper we have shown that the meson distribution amplitude has the form

$$\phi(x_i,Q) = x_1x_2 \sum_{n=0}^{\infty} P_n(x_1-x_2,\alpha_s(Q))\, \tilde{M}_n(Q) \qquad (33a)$$

where

$$P_n = \tilde{C}_n^{3/2}(x_1-x_2) - \frac{\alpha_s(Q)}{4\pi} \sum_{j=n+1}^{\infty} d_n^j(Q)\, \tilde{C}_j^{3/2}(x_1-x_2) + \ldots \qquad (33b)$$

and where $\tilde{M}_n(Q)$ is a moment of ϕ satisfying a standard evolution equation (Eqs.(11) and (13)).

Assuming residual conformal symmetry near the light cone, we found a simple procedure for determining the coefficients d_n^j (See Eqs. (23) and (25)).

However, as we have discussed in the introduction, the predictions of conformal symmetry appear to conflict with explicit two-loop calculations[11] for the distribution amplitude in QCD using dimensional regularization, although they do hold for the analogous calculations for ϕ^3 in six-dimension. Assuming these calculations are correct, this implies that conformal symmetry is broken in a subtle way in gauge theory in dimensional regularization, perhaps due to sensitivity to infrared cutoffs. If the source of this breakdown can be identified, then conformal symmetry could still be useful as a guide to the higher order corrections to the distribution amplitude. More important, this unexpected breakdown points to new effects which control the short distance structure of gauge theory, and give caution to the formal use on conformal symmetry results.

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Appendix A

One-Loop Evolution Potential and Vacuum Polarization Corrections

In this Appendix, we rederive the one-loop evolution potential for mesons and compute the leading corrections due to vacuum polarization. The standard procedure for computing V for some hadron is to compute the distribution amplitude in perturbation theory not for the hadron but rather for a state composed of free quarks (and/or gluons). From this, the renormalization constant Z and then the evolution potential (Eq.(6)) are determined. Since V is insensitive to low momenta it is the same for the hadron as it is for the free quark state.

A related procedure determines the hard scattering amplitude T_H for any process. The amplitude for that process is computed in perturbation theory with all hadrons replaced by free quarks. Using the distribution amplitudes for the free quark states, the hard scattering amplitude is extracted by rewriting the full amplitude in a factorized form, as in Eq.(1); i.e., T_H is obtained by dividing out the distribution amplitudes. In this way collinear mass singularities are systematically removed from T_H, leaving in many cases a well behaved expansion in $\alpha_s(Q)$. This procedure is particularly simple when ϕ is defined using dimensional regularization and minimal subtraction. Then T_H is obtained simply by computing the scattering amplitude for collinear sets of massless valence quarks using dimensional regularization and minimal subtraction to remove the infrared infinities.

Here we examine the distribution amplitude as defined with each of two regulators: dimensional regularization, and Pauli-Villars regularization.[16]

1. Dimensional Regularization

To determine the meson evolution potential $V(x_i,y_i,\alpha_s)$ for a dimensional regulator we first compute the distribution amplitude $\phi_u(x_i)$ in $d=4-2\varepsilon$ dimensions for a free quark and anti-quark carrying momentum y_1p and y_2p respectively ($y_1+y_2=p^+=1$ and $p_\perp=p^-=0$). Schematically, ϕ_u will have the form

$$\sqrt{y_1y_2}\,\phi_u = 1+\frac{\alpha_a}{4\pi\lambda^{2\epsilon}}\left[\frac{b_1}{\tilde{\epsilon}}+a_1\right]+\left[\frac{\alpha_0}{4\pi\lambda^{2\epsilon}}\right]^2\left[\frac{c_2}{\tilde{\epsilon}^2}+\frac{b_2+a_2}{\tilde{\epsilon}}\right]+\cdots\Bigg]+\cdots \quad (A-1)$$

where $1, a_i, \ldots$ should be thought of as operators in x_i-y_i space, $1/\tilde{\varepsilon}=1/\varepsilon-\gamma_E+\ell n\,4\pi$, and λ is some infrared regulator (we use a gluon mass). From this, the renormalized distribution amplitude $\phi(Q)$ is defined by

$$\sqrt{y_1y_2}\,\phi(Q)=1+\frac{\alpha_s(Q)}{4\pi}(a_1-d_1+b_1\ell nQ^2/\lambda^2)+\cdots \quad (A-2)$$

where $\alpha_s(Q)$ is defined by

$$\alpha_s(Q)\equiv\frac{\alpha_0}{Q^{2\varepsilon}}\left(1+\frac{\alpha_0}{Q^{2\varepsilon}}\frac{\beta_0}{4\pi\tilde{\varepsilon}}+\ldots\right) \quad (A-3)$$

so that $\beta(\alpha_s)=-\varepsilon\,\alpha_s-\beta_0\,\alpha_s^2/4\pi-\ldots$. The evolution potential then follows directly from the renormalization constant $Z(Q)=\phi_u\phi(Q)^{-1}$, and is given by [17]

$$V=-Q^2\frac{d}{dQ^2}\ell nZ(Q)=\frac{\alpha_s(Q)}{4\pi}b_1+\left(\frac{\alpha_s(Q)}{4\pi}\right)^2 \left[2(b_2-\beta_0a_1-b_1a_1)+\beta_0d_1+\beta_0b_1(\gamma_E-\ell n4\pi)\right]+\cdots \quad (A-4)$$

This is the basic expression relating ϕ_u to V.

To compute V to leading order, we must compute $\phi_u(x_i)$ for our $q\bar{q}$ state through first order in α_s. The relevant diagrams for $A^+=0$ gauge are shown in Fig. 1. In lowest order (Fig. 1a), ϕ_u for this state is simply

$$\phi_u^a(x_i) = \delta(x_1-y_1)\,\mathrm{Tr}\left|\frac{\gamma^+\gamma_5}{2\sqrt{2}}\ \frac{\gamma_5\not{p}\sqrt{y_1y_2}}{\sqrt{2}}\right| = \delta(x_1-y_1)\sqrt{y_1y_2} \qquad \text{(A-5)}$$

This is all the information needed to define the leading-order hard scattering amplitude T_H for any process involving mesons. One simply computes the amplitude for scattering collinear $q\bar{q}$ pairs (in place of the mesons), and divides by $\sqrt{x_i}$ for each external q or $\bar{q}$, where x_i is the fraction of the meson's momentum carried by that particle.

In the one-loop graph of Fig. 1b, k^+ is set equal to x_1, and the k^- integral can be evaluated using contour integration. The result is

$$\sqrt{y_1y_2}\,\phi_u^b(x_i) = \frac{\alpha_0}{2\pi^2}C_F(2\pi)^{2\varepsilon}\int d^{2-2\varepsilon}k_\perp\left\{x_1y_2\left(1-\varepsilon+\frac{1}{y_1-x_1}\right)\frac{\theta(y_1-x_1)}{k_\perp^2+x_1\lambda^2/y_1} + (1\leftrightarrow 2)\right\} \qquad \text{(A-6a)}$$

$$= \frac{\alpha_0}{4\pi\lambda^{2\varepsilon}}\left\{\frac{2C_F}{\hat{\varepsilon}}\left[x_1y_2\left(1+\frac{1}{y_1-x_1}\right)\theta(y_1-x_1) + (1\leftrightarrow 2)\right] + 2C_F\left[-x_1y_2\theta(y_1-x_1) + x_1y_2\ln(y_1/x_1)\left(1+\frac{1}{y_1-x_1}\right)\theta(y_1-x_1) + (1\leftrightarrow 2)\right]\right\} \qquad \text{(A-6b)}$$

Similarly, the self-energy corrections (Fig. 1c) are

$$\sqrt{y_1y_2}\,\phi_u^c(x_i) = -\frac{\alpha_0}{4\pi^2}C_F(2\pi)^{2\varepsilon}\delta(x_1-y_1)\int[dz]\,d^{2-2\varepsilon}k_\perp \left\{x_1z_2\left(\frac{1-\varepsilon}{x_2}+\frac{2}{z_1-x_1}\right)\frac{\theta(z_1-x_1)}{k_\perp^2+z_2\lambda^2/x_2} + (1\leftrightarrow 2)\right\} \qquad \text{(A-7a)}$$

$$= \frac{\alpha_0}{4\pi\lambda^{2\varepsilon}}y_1y_2\,\delta(x_1-y_1)\left\{-\frac{C_F}{\hat{\varepsilon}} - \frac{2C_F}{\hat{\varepsilon}}\int\frac{[dz]}{y_1y_2}\left[\frac{x_1z_2}{z_1-x_1}\theta(z_1-x_1) + (1\leftrightarrow 2)\right] + C_F\left[\frac{9}{2}-\frac{2\pi^2}{3}\right]\right\} \qquad \text{(A-7b)}$$

Eqs. (A-6) and (A-7) completely determine the one-loop evolution potential and $\phi(x_i,Q)$. By comparing with Eqs. (A-1) and (A-4) we obtain immediately Eq. (16) for $V_1(x_i,y_i)$.

In Section III, we discuss the fermion vacuum polarization corrections to V_1. These are easily obtained from Eqs. (A-6) and (A-7) by including a factor

$$\Pi(\ell^2) = -\frac{2}{3} n_f \frac{\alpha_0}{4\pi(-\ell^2)^{\varepsilon}} \left[\frac{1}{\varepsilon} + \frac{5}{3} \right] \tag{A-8}$$

in the integrands with ℓ equal to the gluon momentum. A typical term has the form

$$\frac{\alpha_0}{4\pi^2} (2\pi)^{2\varepsilon} \int d^{2-2\varepsilon} k_\perp \left\{ \frac{v(x_i,y_i) + \varepsilon\, v'(x_i,y_i)}{k_\perp^2 + \lambda^2 x_1/y_1} \Pi\left(-\frac{y_1 k_\perp^2}{x_1}\right) + (1 \leftrightarrow 2) \right\}$$

$$= -\frac{2}{3} n_f \left(\frac{\alpha_0}{4\pi}\right)^2 \frac{1}{(\lambda^2 x_1/y_1)^{2\varepsilon}} \frac{1}{2\varepsilon} \left\{ \left(\frac{1}{\varepsilon} + \frac{5}{3} - \ln\frac{y_1}{x_1} \right)(v + \varepsilon v') + (1 \leftrightarrow 2) \right\} \tag{A-9}$$

Noting the subtraction $-\beta_0 a_1$ in Eq. (A-4), we see that such a term contributes

$$-\frac{2}{3} n_f \left\{ \left(\frac{5}{3} - \ln\frac{y_1}{x_1} \right) v - v' + (1 \leftrightarrow 2) \right\}$$

to the two-loop evolution potential V_2. Thus, from Eqs. (A-6) and (A-7), the leading correction due to vacuum polarization is

$$V_{VP} = -\frac{2}{3} n_f \left[\frac{5}{3} V_1(x_i,y_i) + \delta V(x_i,y_i) \right]$$

where

$$\delta V = -y_1 y_2\, \delta(x_i - y_i)\, C_F\left(\frac{9}{2} - 2\frac{\pi^2}{3} \right)$$
$$- 2C_F \left\{ x_1 y_2 \ln(y_1/x_1) \left(1 + \frac{1}{y_1 - x_1}\right) \theta(y_1 - x_1) - x_1 y_2 \theta(y_1 - x_1) + (1 \leftrightarrow 2) \right\} \tag{A-10}$$

Only the term in δV containing $\ln y_1/x_1$ is non-diagonal with respect to the polynomials $\tilde{C}_n^{3/2}$ and so it is retained in Section IV. Evaluation of the off-diagonal matrix elements of δV is greatly simplified by noting that $\phi_u^b + \phi_u^c$ (Eqs. (A-6,7)) can be written[18]

$$\phi_u^b + \phi_u^c = \frac{\alpha_0}{4\pi\lambda^{2\varepsilon}} \frac{\tilde{V}_1}{\varepsilon} \frac{\sqrt{y_1 y_2}}{(y_1 y_2)^{1-\varepsilon}} + \text{diagonal terms}$$

where

$$\tilde{V}_1(x_i, y_i) = 2C_F\{(x_1 y_2)^{1-\varepsilon}(1-\varepsilon + \frac{\Delta}{y_1 - x_1})\,\theta(y_1 - x_1) + (1 \leftrightarrow 2)\}$$

$$= V(y_i, x_i)$$

Because $\tilde{V}_1$ is symmetric under interchange of x_i and y_i, it must be diagonal with respect to the Gegenbauer polynomials $\tilde{C}_n^{3/2-\varepsilon}$. The argument here is identical to that given for V_1 in Section II.B, and the result is not surprising since the dimensions of the operators $O^{(n)}$ are reduced by -2ε in $4-2\varepsilon$ dimensions. Thus Gegenbauer polynomials of type $3/2 - \epsilon$ are expected (cf. Eq. (25) with $d_\phi = (3/2 - \epsilon)$ and no γ_*). The eigenvalue equatio for V_1 is then

$$\int[dx]\, \tilde{C}_n^{3/2-\varepsilon}(x_1 - x_2) \frac{\tilde{V}_1(x_i, y_i)}{(y_1 y_2)^{1-\varepsilon}} = (-\gamma_1^{(n)} + O(\varepsilon))\, \tilde{C}_n^{3/2-\varepsilon}(y_1 - y_2)$$

Expanding to first order in ε, we find that (for $n > j$)

$$-\varepsilon\int[dx][dy]\, \tilde{C}_n^{3/2}\, \delta V(x_i, y_i) \tilde{C}_j^{3/2} = (\gamma_1^{(n)} - \gamma_1^{(j)})\varepsilon \int[dx]\, \frac{d}{d\xi} \tilde{C}_n^{\xi} x_1 x_2\, \tilde{C}_j^{3/2}\Big|_{\xi=3/2}$$

Borrowing results from Section IV.A, this can be rewritten

$$\int[dx][dy]\, \tilde{C}_n^{3/2}\, \delta V\, \tilde{C}_j^{3/2} = (\gamma_1^{(n)} - \gamma_1^{(j)}) \int[dx]\, \tilde{C}_n^{3/2} \ln(x_1 x_2) x_1 x_2\, \tilde{C}_j^{3/2} \quad \text{(A-11)}$$

2. Pauli-Villars Regulators

A Pauli-Villars regulator is introduced by subtracting diagrams with the gluon mass set equal to Q. The cut-off distribution amplitude $\phi(x_i,Q)$ for the free $q\bar{q}$ state will have the structure

$$\sqrt{y_1y_2}\;\phi(Q) = \Pi + \frac{\alpha_s(Q)}{4\pi}\left(b_1 \ln\frac{Q^2}{\lambda^2} + a_1\right) + \left[\frac{\alpha_s(Q)}{4\pi}\right]^2\left(c_2 \ln^2\frac{Q^2}{\lambda^2} + b_2 \ln\frac{Q^2}{\lambda^2} + a_2\right) + \ldots) \quad \text{(A-13)}$$

This satisfies an evolution equation with the potential

$$V = \frac{\alpha_s}{4\pi} b_1 + \left(\frac{\alpha_s}{4\pi}\right)^2 (b_2 - \beta_0 a_1 - b_1 a_1) \quad \text{(A-14)}$$

as can be verified by direct substitution. The regulated distribution amplitude is readily computed to first order from Eqs. (A-6) and (A-7):

$$\sqrt{y_1y_2}\;\phi(x_i,Q) = y_1y_2\;\delta(x_i-y_i) + \frac{\alpha_s(Q)}{4\pi} V_1(x_i,y_i) \ln\frac{Q^2}{\lambda^2} + \ldots \quad \text{(A-15)}$$

By comparing this result with that for dimensional regularization, we find that the two distribution amplitudes are related by

$$\phi_{PV}(x_i,Q) = \int\frac{[dy]}{y_1y_2} z(x_i,y_i,\alpha_s)\;\phi_{DR}(y_i,Q)$$

where $z = y_1y_2\;\delta(x_i-y_i) + \frac{\alpha_s}{4\pi}\delta V$ with δV given by Eq. (A.10). Here and in Eq. (29) we are assuming that the same scheme is being used to define α_s for both Pauli-Villars and dimensional regulators. It is, of course, trivial to change from one scheme to another when using either regulator.

To obtain the vacuum polarization corrections for a Pauli-Villars regulator, we insert

$$\Pi(\ell^2) = -\frac{2}{3} n_f \frac{\alpha_s(Q)}{4\pi} \left[\frac{5}{3} - \ell n(-\ell^2/Q^2) \right]$$

into the one-loop integrands with ℓ equal to the gluon's momentum. A typical term has the form

$$\left[\frac{\alpha_s(Q)}{4\pi}\right]^2 \int dk_\perp^2 \left\{ \frac{v(x_i,y_i)}{k_\perp^2+\lambda^2 x_1/y_1} \, \Pi\left(-\frac{y_1 k_\perp^2}{x_1}\right) + (1 \leftrightarrow 2) \right|_{\lambda^2=Q^2}^{\lambda^2}$$

$$= -\frac{2}{3} n_f \left[\frac{\alpha_s(Q)}{4\pi}\right]^2 \left\{ v(x_i,y_i) \left(\frac{\ell n^2 Q^2/\lambda^2}{2} + \frac{5}{3} \ell n\, Q^2/\Lambda^2 \right) + (1 \leftrightarrow 2) \right\}$$

which implies a contribution to V_2 from vacuum polarization of the form

$$-\frac{2}{3} n_f \left[\frac{5}{3} V_1(x_i,y_i)\right]$$

Thus vacuum polarization does not introduce non-diagonal terms into the evolution potential, at least in this order. Because of this result, the β_0 part of V_{DR} as computed from Eq. (29) (using V_{PV}) agrees with the direct calculation leading to Eq. (A-10), as it should.

APPENDIX B

In this appendix[14] we give the general constraints of conformal symmetry for the operator product expansion required to calculate the distribution amplitude for vector and pseudoscalar mesons at large momentum transfer. To leading twist

$$\psi(\tfrac{z}{2})\bar{\psi}(-\tfrac{z}{2}) \sim \sum_{n}^{\infty} \tilde{C}_n(z^2-i\epsilon z_o) \sum_{m=n}^{\infty} \Gamma_\alpha^{(i)} z_{\alpha_1}\ldots z_{\alpha_m} O^{(n)\alpha_1\ldots\alpha_m\alpha}(o) \qquad (B.1)$$

where $i = 1,2$ with

$$\Gamma_\alpha^{(i)} = \begin{cases} \gamma_\alpha & i=1 \\ \gamma_\alpha\gamma_5 & i=2 \end{cases} \qquad (B.2)$$

$$O_{(i)}^{(n)\alpha_1\ldots\alpha_m\alpha}(0) = \sum_{k=o}^{n} d_{mnk}\partial^{\alpha_{k+1}}\ldots\partial^{\alpha_m}\bar{\psi}(0)\Gamma_{(i)}^{\alpha}\overleftrightarrow{D}^{\alpha_1}\ldots\overleftrightarrow{D}^{\alpha_k}\psi(0) \qquad (B.3)$$

D is the covariant derivative and $\tilde{C}_n(z^2 - i\epsilon z_0)$ are singular functions of well defined dimension (powers of logarithms in QCD). In the expansion Eq. (B.1), the operators that appear have also external derivatives like in Eq. (B.3). This does not happen in the discussion of deep inelastic lepton-hadron scattering, since there only forward matrix elements are involved. Thus only $m = n = k$ operators appear there. Also, the expansion of a product of two currents is involved. However, it turns out that the m=n=k operators are identical in the two cases. Thus the $\tilde{\gamma}_k$ that control the Q^2-behavior in form factors are the same as in the k-th moment $M_k(Q^2)$ in deep inelastic lepton hadron scattering (see the first paper of Ref. 8 for more details).

Let us now apply the form of the operator product expansion for two scalar fields in case of exact conformal symmetry[19]

$$A(x)B(o) \sim (x^2-i\epsilon x^o)^{-1/2(\ell_A+\ell_B)} \sum_{n=0}^{\infty}(x^2-i\epsilon x_o)^{1/2(\ell_n-n)}\tilde{C}_n^{AB}x^{\alpha_1}\ldots x^{\alpha_n} \cdot (B.4)$$

$$\cdot \int_0^1 du\, u^{[(1/2)(\ell_A-\ell_B+\ell_n+n)-1]}(1-u)^{[(1/2)(\ell_B-\ell_A+\ell_n+n)-1]} O_{\alpha_1\ldots\alpha_n}(ux)$$

The eigensolutions $P_n(x)$ of the distribution amplitude are proportional to

$$\sum_{m=n}^{\infty} a_{mn} \int dz^- e^{\frac{1}{2} i x z^- p^+} (p^+)(p^+ z^-)^m \qquad (B.5)$$

where

$$a_{mn} = \sum_{k=o}^{n} d_{mnk}\, b_k$$

and

$$\langle o|\bar{\psi}(o)\Gamma_\alpha^{(i)} \overleftrightarrow{D}_{\alpha_1} \cdots \overleftrightarrow{D}_{\alpha_k} \psi(o)|p\rangle = b_k p_{\alpha_1} \cdots p_{\alpha_k} p_\alpha + (g_{\alpha_\ell \alpha_m} \text{ terms}) \qquad (B.6)$$

Comparing (B.1) and (B.4), we then obtain

$$\begin{aligned}
&\sum_{m=n}^{\infty} a_{mn} \int dz^- e^{(i/2) x z^- p^+} (p^+)(p^+ z^-)^m \\
&\quad \propto \int d\xi \xi^n \int_0^1 du\, e^{i\xi(x_2 - u)} [u(1-u)]^{[(1/2)(\ell_n + n) - 1]} \\
&\quad \propto \int d\xi \xi^n \int_{-1}^1 dv\, e^{(i/2)\xi(x - v)} (1 - v^2)^{[(1/2)(\ell_n + n) - 1]} \\
&\quad \propto \frac{d^n}{dx^n} (1 - x^2)^{[(1/2)(\ell_n + n) - 1]}
\end{aligned} \qquad (B.7)$$

For scalars $\ell_n = n + 2d_\phi + \gamma_n$, reproducing Eq. (25). For spinors, where we take the lowest operator to be a vector [Eq. (8)], this is equivalent to $\ell_n = n + 2d_\phi + \gamma_n + 1$, reproducing Eq. (26). [Note that for the ϕ^4 interaction in 4-dimensions and the $(\bar{\psi}\psi)^2$ interaction in 2-dimensions the potential V_1 is a contact potential with measure $(1-x^2)^0$, thus yielding $P_n(x) = \overline{P}_n(x) =$ Legendre polynomials for leading order, in agreement with Eq. (25) and (26). (Actually only $n = 0$ appears in the potential.) In the case of ϕ^3 in 6-dimensions and gauge theory in 4-dimensions, the leading order polynomials are the $C_n^{3/2}$, as expected.]

APPENDIX C

It is straightforward to show that the set of ladder and crossed-ladder graphs in $[\phi^3]_6$ obeys the Callan-Symansik equation with $\beta = 0$. In this appendix we summarize the main results for this model which are applicable to the meson distribution amplitude to two loops. The results are all performed in $d = 6 - 2\epsilon$ dimensions.

As in Section II we define the expansion of the distribution amplitude

$$\phi(x,Q) = (1-x^2)\sum_n P_n(x,\alpha)\widetilde{M}_n(Q)$$

where

$$Q^2 \frac{d}{dQ^2}\widetilde{M}_n(Q) = \frac{\gamma^{(n)}}{2}\widetilde{M}_n(Q)$$

$$\tfrac{1}{2}\gamma_n(\alpha) = -\alpha\gamma_1^{(n)} - \alpha^2\gamma_2^{(n)}$$

The function P_n satisfy

$$Q^2\frac{\partial}{\partial Q^2}P_n(x,\alpha) = -\frac{\gamma^{(n)}(\alpha)}{2}P_n(x,\alpha) + \int_0^1 [dy]\frac{V(x,y,\alpha)}{1-x^2}P_n(y,\alpha)$$

with $V = \alpha V_1 + \alpha^2 V_2 + \ldots$. Since $\beta(\alpha) = 0$, $Q^2\frac{\partial}{\partial Q^2}P_n = 0$. To one loop, we find

$$V_1(x,y) = -\tfrac{1}{2}\left\{(1+x)(1-y)\theta(y>x) + \begin{pmatrix} x \to -x \\ y \to -y \end{pmatrix}\right\} = V_1(y,x)\ .$$

Consequently the P_n to leading order are $\widetilde{C}_n$ normalized Gegenbauer polynomials, and $\gamma_1^{(n)} = 1/(n+1)(n+2)$. To two-loops we expand P_n as in Eq. (21) where $(n > j)$

$$d_j^n = \int_0^1 [dx][dy]\frac{\widetilde{C}_n(x)V_2(x,y)\widetilde{C}_j(y)}{\gamma^{(n)} - \gamma^{(j)}}\ .$$

We have verified that the cross graph kernel is symmetric, so it does not contribute to d_j^n. The double ladder graph in dimensional regularization has the form

$$-g_0^4 \int d^{6-2\epsilon}k \, d^{6-2\epsilon}\ell \, \frac{\delta(x-\eta\cdot k)}{k^2+2p\cdot k} \, \frac{1}{k^2-2p\cdot k} \, \frac{1}{k^2+\ell^2-2k\cdot\ell-\lambda^2} \\ \times \frac{1}{\ell^2+2p\cdot\ell} \, \frac{1}{\ell^2-2p\cdot\ell} \, \frac{1}{\ell^2-2\xi p\cdot\ell} \tag{C.1}$$

Using the usual denominator-combining formulas, and momentum shift, this becomes

$$\alpha^2(\mu)\,\Gamma(2\epsilon) \int \frac{[d\alpha]}{(\alpha_3(1-\alpha_3))^\epsilon} \int [d\beta] \\ \times \int_0^1 d\xi \, \frac{\xi^{\epsilon-1}(1-\xi)^2}{\left(\frac{\lambda^2}{\mu^2}((1-\xi)\beta_3+\frac{\xi}{1-\alpha_3})\right)^{2\epsilon}} \, \delta(x-X(\alpha_i,\beta_i,\xi)) \tag{C.2}$$

where

$$X = -\alpha_1+\alpha_2+\alpha_3\left[(1-\xi)(-\beta_1+\beta_2+\beta_3\xi)+\xi\,\frac{\alpha_2-\alpha_1}{1-\alpha_3}\right] \tag{C.3}$$

and

$$(d\alpha) = d\alpha_1\,d\alpha_2\,d\alpha_3\,\delta\left(1-\Sigma_{i=1}^3\,\alpha_i\right) \ .$$

As in Appendix A, we must now identify parts multiplying $1/\epsilon^2$ and $1/\epsilon$. One power of $1/\epsilon$ comes from $\Gamma(2\epsilon)$, as usual, while the other comes from the ξ integration in the region $\xi = 0$. To draw out the second $1/\epsilon$ we must integrate by parts on ξ. The double ladder can be written as two terms of the form of Eq. (C.2) with $[(1-\xi)^2\,\delta(x-X(\alpha_i,\beta_i,\xi)]$ replaced by

$$[(1-\xi^2)\,\delta(x-X(\alpha_i,\beta_i,\xi))-\delta(x-X(\alpha_i,\beta_i,\xi=0)]$$

and

$$\delta(x-X(\alpha_i,\beta_i,\xi=0) \ ,$$

respectively. For the A term the needed $1/\epsilon$ coefficient is obtained by setting $\epsilon = 0$ in the numerator. For the B term we replace $\xi^{\epsilon-1} = \frac{1}{\epsilon}\,\frac{d}{d\xi}\,\xi^\epsilon$, integrate by parts and obtain

$$B = \alpha^2(\mu) \int [d\alpha]\,[d\beta] \left\{ \frac{1}{2\epsilon^2} - \frac{1}{2\epsilon} \ell n(\alpha_3 \beta_3^2 (1-\alpha_3)) - \frac{1}{\epsilon}((\gamma_E + \ell n \frac{\lambda^2}{\mu^2}) \right\} \times \delta(x - X(0)) \,. \tag{C.4}$$

The $1/\epsilon^2$ terms cancel as required in V. The $1/\epsilon$ contributions to the second order potential is defined from the combination $2(b_2 - b_1 a_1)$ (see Eq. (A.4)). Here we have

$$\begin{aligned} b_2(x,z) = \tfrac{1}{2} \int [d\alpha]\,[d\beta] \Bigg\{ & \int_0^1 \frac{d\xi}{\xi} \, [(1-\overline{\xi})^2 \delta(x - X(\overline{\xi}))]_0^\xi \\ & - \ell n(\alpha_3 \beta_3^2 (1-\alpha_3))\, \delta(x - X(0)) \\ & - 2\left(\gamma_E + \ell n\, \frac{\lambda^2}{\mu^2}\right) \delta(x - X(0)) \Bigg\} (1-\xi^2) \end{aligned} \tag{C.5}$$

and from the one-loop calculation,

$$a_1 b_1 = - \int [d\alpha] \int [d\beta]\, \delta(x - X(0)) \left\{ \gamma_E + \ell n\, \frac{\lambda^2}{\mu^2} + \ell n\, \beta_3 \right\} (1-\xi^2)\,. \tag{C.6}$$

When the $(1-\xi^2)$ factors are due to the choice of the weight. Consequently, the two-loop potential is

$$\begin{aligned} V_2 &= 2\ (b_2 - a_1 b_1) \\ &= \int [d\alpha]\,[d\beta] \Bigg\{ \int_0^1 \frac{d\xi}{\xi} \, [(1-\overline{\xi})^2 \delta(x - X(\overline{\xi}))]_0^\xi \\ &\qquad - \ell n\, [\alpha_3(1-\alpha_3)] \delta(x - X(0) \Bigg\} (1-\xi^2) \end{aligned} \tag{C.7}$$

where, as required, all dependence on μ^2/λ^2 has cancelled.

We have checked numerically that the result (C.7) for the d_n^j agrees with the conformal symmetry prediction, Eq. (32) for $\beta_0 = 0$. As in Section IV, we can show that the Pauli Villars regulator gives a different result due to the term $\beta(\alpha) = -\epsilon\alpha \neq 0$ induced in dimensional regularization. We have also checked numerically that the Pauli-Villars regulator gives results consistent with the extended polynomials for the orthogonal polynomials $\overline{P}_n$. (See Section IV.A).

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16. Here we assume that the $\overline{MS}$ definition for α_s is used for both regulators.

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Table I

Non-zero values (for $n,j \leq 15$) of

$$\int [dx]\, \tilde{C}_n^{3/2}(x_1-x_2)\, \ln(x_1 x_2)\, x_1 x_2\, \tilde{C}_j^{3/2}(x_1-x_2)$$

n/j	0	2	4	6	8	10	12
2	-.374						
4	-.106	-.617					
6	-.047	-.216	-.723				
8	-.026	-.108	-.278	-.783			
10	-.016	-.064	-.149	-.318	-.822		
12	-.010	-.042	-.092	-.177	-.346	-.849	
14	-.007	-.029	-.062	-.113	-.198	-.367	-.869

n/j	1	3	5	7	9	11	13
3	-.525						
5	-.169	-.679					
7	-.081	-.251	-.757				
9	-.046	-.130	-.300	-.804			
11	-.029	-.079	-.164	-.333	-.837		
13	-.020	-.053	-.103	-.188	-.357	-.860	
15	-.014	-.037	-.071	-.122	-.207	-.375	-.877

Table II

Non-vanishing off-diagonal matrix elements of V_2 (Eq. (32)) for pions

n/j	0	2	4	6	8	10	12
2	-18.708						
4	- 7.713	-22.738					
6	- 4.133	-13.261	-20.912				
8	- 2.546	- 8.648	-14.118	-18.791			
10	- 1.711	- 6.061	-10.156	-13.781	-16.942		
12	- 1.221	- 4.468	- 7.642	-10.533	-13.111	-15.398	
14	- 0.911	- 3.419	- 5.947	- 8.304	-10.445	-12.374	-14.110

Table III

$O(\alpha_s)$ corrections to the π-γ form factor (Eqs. (34))

n	T_{on}	T_{1n}	T'_{1n}	$T_{1n}+T'_{1n}$
0	2.4	-1.6	-6.7	-8.2
2	1.5	-3.8	-2.2	-6.1
4	1.2	-5.0	4.3	-0.7
6	1.0	-5.8	10.1	4.4

FIGURE CAPTION

Fig. 1. Diagram contributing to the unrenormalized distribution amplitude ϕ_u through order α_s.

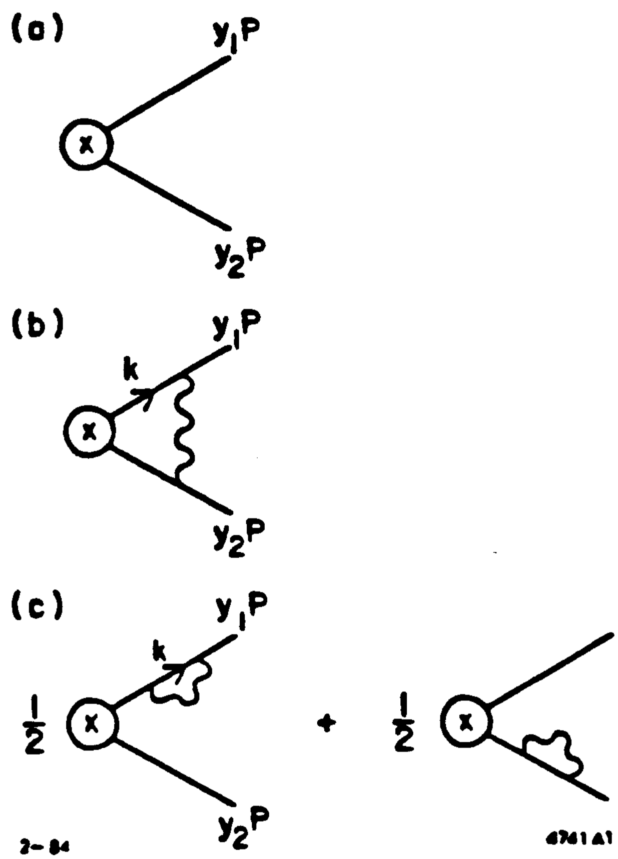

Fig. 1