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GOLDSTONE FERMIONS IN SUPERSYMMETRIC THEORIES AT FINITE TEMPERATURE*

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ABSTRACT

The behavior of supersymmetric theories at finite temperature is examined. It is shown that SUSY is broken for any $T \geq 0$ because of the different statistics obeyed by bosons and fermions. This breaking is always associated with a Goldstone mode(s). This phenomenon is shown to take place even in a free massive theory, where the Goldstone modes are created by composite fermion-boson bilinear operators. In the interacting theory with chiral symmetry, the same bilinear operators create the chiral doublet of Goldstone fermions, which is shown to saturate the Ward-Takahashi identities up to one loop. Because of this spontaneous SUSY breaking, the fermion and the bosons acquire different effective masses. In theories without chiral symmetry, at tree level the fermion-boson bilinear operators create Goldstone modes but at higher orders these modes become massive and the elementary fermion becomes the Goldstone field because of the mixing with these bilinear operators.

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1. INTRODUCTION

The supersymmetric theories at finite temperature have been investigated by several authors. Das and Kaku[1] first investigated the one-loop effective potential (free energy) at finite temperature and suggested that supersymmetry (SUSY) behaves differently from regular symmetries in that it is always broken at finite temperature, regardless of whether it is broken or unbroken at zero temperature. Furthermore, by observing that the fermion field develops an inhomogeneous term in its SUSY transformation law, they suggested that this breaking is spontaneous and the Goldstone fermion appears. Then Girardello et.al.[2] clearly established that SUSY is broken at finite temperature by showing that the (effective) mass-splitting between the bosons and fermions occurs at one-loop level. They also noted that this symmetry breaking is because of the fact that fermions and bosons obey different statistics. However, they concluded that this breaking was *explicit* in the sense that there was no Goldstone fermion associated with it.

Later several authors[3–6] looked at the SUSY transformations, which involve an anti-commuting (Grassman) parameter and claimed that proper account of this parameter leads to the conclusion that unbroken SUSY at $T = 0$ stays unbroken at any $T > 0$. Their analysis, however, involves unphysical correlation functions that have *periodic* boundary conditions in imaginary time for *both* bosons and fermions.

In more recent articles,[7–8] SUSY theories at finite temperature were investigated with the attention to the *physical* correlation functions. By calculating them at the one-loop order or at the leading order of the $1/N$-expansion, it was found that SUSY was broken at $T > 0$ because of a nonzero thermal average of the auxiliary field $\langle F\rangle_\beta$, in agreement with the work of Girardello et. al.. However they also found that this breaking is associated with a Goldstone fermion, namely a massless fermion that couples to the supercurrent. Therefore they concluded that SUSY can be thought to be broken *spontaneously*.

The understanding of this phenomenon involves a subtlety associated with the quantization of field theories at finite temperature. In ref. 2, the imaginary-time (Matsubara) formalism was used. Whereas this formalism is suitable for studying the perturbative aspects of a theory, there are two main problems with it. The first is that the study of the dynamical response functions (for example the correlation functions) involves an analytic continuation to Minkowski (real) time[9-10] which is not straightforward in the Fourier transform of these quantities. The second problem is associated with the fact that the imaginary-time formalism *explicitly* breaks Lorentz covariance (frequencies are discrete while momenta are continuous). Since SUSY is deeply related to the Lorentz covariance (the anti-commutator of the supercharges are proportional to $\gamma^\mu P_\mu$), it comes as no surprise that in Matsubara formalism SUSY appears to be broken explicitly at $T > 0$. Indeed the computations done in refs. 7,8 used a covariant formalism to reveal the interesting physics underlying SUSY breaking at finite temperature.

However several puzzling questions were left unanswered. Among them are the following: Why is SUSY breaking found at higher orders (of either the loop-expansion or the $1/N$-expansion) of the theory? While Das and Kaku suggested that this phenomena may be a kind of dynamical symmetry breaking, the different statistics of fermions and bosons are evident in the tree-level propagators (see eqs. (2.1) and (2.2)). This suggests that SUSY may be broken at tree level. This also leads to the following question about free field theory: Does the difference of the statistics has any consequences in SUSY breaking? If so, what kind of breaking is it; are the broken-symmetry Ward-Takahashi (WT) identities satisfied with a contribution from the Goldstone fermion mode?

Yet another problem exists in SUSY models with R-invariance. In these models, the quantum numbers prevent the auxiliary fields from acquiring a nonzero thermal average and the fermion from coupling to the supercurrent (even if it is massless). This kind of model has *not* been studied before, although its physics is interesting in relation to the above questions; according to the general argument, the difference of statistics of bosons and fermions should result in the SUSY breaking even in these models. Then what is a good order parameter? Is there a Goldstone fermion?

In this paper, we answer these questions and expose the physics of SUSY breaking at finite temperature. In particular, we are interested in understanding whether there are Goldstone fermion(s); namely the massless excitations that couple to the supercurrent. We restrict ourselves to the models in which SUSY is unbroken at zero temperature.

This paper is organized as follows. In section 2, we briefly review the formalisms for quantizing field theories at finite temperature. Our emphasis is on their relevance to the analysis of the SUSY theories. We also mention the general WT identities that are essential in the following sections. In section 3, a free SUSY field theory at finite temperature is studied. This simple example is interesting in the sense that its physics is quite non-trivial. The analysis of this section forms a basis for the investigations of the interacting models in the following sections. In section 4, we analyze the chiral (R-invariant) Wess-Zumino model which is mentioned above. Section 5 is devoted to reviewing the finite-temperature aspects of the Wess-Zumino model without chiral symmetry. We study this theory in the context of the results obtained in the sections 3 and 4. Our conclusions are summarized in section 6.

A comment on the notation: For brevity, we omit the signs for the T-products throughout this paper. Namely, any product of operators $\hat{O}_1\hat{O}_2...\hat{O}_n$ means the T-product of the respective operators, $\mathrm{T}\{\hat{O}_1\hat{O}_2...\hat{O}_n\}$.

2. FINITE TEMPERATURE FIELD THEORIES AND SUPERSYMMETRY

As was mentioned in the introduction, there are two formalisms that allow the quantization of field theories at finite temperature. They are the imaginary time formalism and the real time formalism. In this section, we briefly review the features of both formalisms that are relevant for our later analysis of the SUSY theories.

In the imaginary time formalism,[11–12] the Green's functions are generated from the Euclidean action that is an integration over a finite imaginary time range $0 \leq \tau \leq \beta$ $(\equiv 1/kT)$. The boson fields obey the periodic boundary condition while the fermion fields obey the anti-periodic boundary condition. Thus the frequencies are discrete whereas the (three-)momenta are continuous. This renders this formalism non-covariant. As has been pointed out, the perturbation theory using this quantization method is quite straightforward. However, in order to extract the dynamical information of the system, one has to continue analytically to the real time.[9–10] (This is evident when one wants to study the linear response functions, since they are equivalent to the Green's functions in the real time.) This continuation is easily carried out in the *configuration* space, but in Fourier space this procedure is more subtle and involves the spectral representation of the correlation functions.[9,11]

In the real-time formalism, the Lagrangian is not affected by the temperature. It only affects the (real-time) Green's functions through the asymptotic boundary condition. The Green's function for (pseudo-)scalar bosons is given as follows:[11,13]

$$D_\beta(k) = \frac{i}{k^2 - m^2 + i\epsilon} + 2\pi n_B(k)\,\delta(k^2 - m^2) \quad , \tag{2.1a}$$

where in the rest frame of the thermal equilibrium state,

$$n_B(k) \equiv \frac{1}{e^{\beta|p_0|} - 1} \quad . \tag{2.1b}$$

Similarly for spin $\frac{1}{2}$ fermions,

$$S_\beta(k) = (\not{k} + m)\left(\frac{i}{k^2 - m^2 + i\epsilon} - 2\pi n_F(k)\,\delta(k^2 - m^2)\right) \quad , \tag{2.2a}$$

where

$$n_F(k) \equiv \frac{1}{e^{\beta|p_0|} + 1} \quad . \tag{2.2b}$$

Before going any further, we interpret the unfamiliar terms in the above, which are crucial for later calculations and interpretations: At finite temperature, the thermal equilibrium state is a "plasma" of excitations. Namely, *all* the energy levels are populated with real (on-shell) particles. The probability of occupation is given by the Bose-Einstein or Fermi-Dirac statistical factors. When the Heisenberg fields are expanded in terms of the creation and annihilation operators, $a^\dagger$ and a, the propagators contain $aa^\dagger$ and $a^\dagger a$ terms. The second terms in the propagators arise from either creating or annihilating a particle in the populated energy levels. The on-shell δ-functions are because of the particles in the thermal equilibrium state being on mass-shell. The relative sign between D_β and S_β is of course because of the commutation relations.

This formalism can be made explicitly covariant by inclusion of the four-velocity vector u_μ of the thermal equilibrium state.[13] (The necessary changes are $p_0 \rightarrow pu$ in (2.1b) and (2.2b).) Dolan and Jackiw[11] noted that the perturbation method that uses the above propagators encounters ambiguities at higher orders because of the product of δ-function terms. It was found that in order to avoid this ambiguity and complete the real-time formalism one can double the degrees of freedom (Thermo Field Dynamics - TFD).[14] Namely, for each of the dynamical degrees of freedom φ_i in the zero-temperature theory, an extra degree of freedom (ghost) $\tilde{\varphi}_i$ has to be introduced. The vertices of the perturbation theory consist of the usual ones plus the ones among ghosts themselves. The ghost sector contributes through the "off-diagonal" elements of the tree propagators $\langle\tilde{\varphi}_i\varphi_i\rangle$. In this way, the ghosts lift the ambiguities in the naive perturbation theory and the real-time Green's function of the usual fields can be calculated directly.

For analyzing the SUSY theories at finite temperature, we use this real-time formalism. The technical reason is that we want to investigate the real-time Green's function, which can be directly calculated in that formalism. It is also because, in order to avoid an explicit breaking of SUSY, it is necessary to work with a covariant formalism as mentioned in the introduction.

Although a full analysis of the theory would require the machinery of the TFD formalism, we will content ourselves with understanding the relevant physics in low orders in perturbation theory where the usual real-time approach is unambiguous. In particular, because of the above properties of the ghost sector, the one-loop *inverse* propagators receive no contributions from ghosts. Consequently, there will be no need to consider the ghosts in the computations that we will do in the following sections (except only one insignificant case). This will be further mentioned in the appropriate places.

In ref. 7, the WT identities of the real-time operators were derived by an analytical continuation from the imaginary-time expressions. It was shown that the WT identities at finite temperature in real time are the same as the zero temperature case provided that we replace vacuum expectation values of operators by thermal averages. The thermal average of an operator is written as:

$$\langle \hat{O} \rangle_\beta \equiv \frac{Tr\,(\hat{O} e^{-\beta H})}{Tr\, e^{-\beta H}} \quad , \tag{2.3}$$

where $\hat{O}$ is a *Heisenberg* (as opposed to Matsubara) operator and H is the Hamiltonian of the theory. The expression (2.3) is correct in the rest frame of the thermal-equilibrium state and can be written in a fully covariant fashion.[13] For completeness we give the necessary expressions below: The principal WT identity resulting from the symmetry of the transformation $\varphi_i \rightarrow \varphi_i + \epsilon\delta\varphi_i$, ($\varphi_i$ represents either a bosonic or fermionic fields) is,

$$\langle \partial^\mu S_\mu(z) + \sum_i \delta\varphi_i(z) \rangle_\beta = 0 \quad , \tag{2.4}$$

where S_μ is the Noether current of the symmetry. This WT identity is true in the presence of the external sources J_i for the fields φ_i. All the necessary identities are obtained by taking functional derivative of the eq.(2.4) in terms of the sources. The above identity can also be obtained directly in the TFD formalism. In the path-integral expression of the generating functional, the WT identities can be obtained as the results of the invariance of the functional integral under the (local) change of integration variables, $\varphi_i(z) \to \varphi_i(z) + \epsilon(z)\delta\varphi_i(z)$. The identity (2.4) is derived by doing this change only in the sector of the usual degrees of freedom.

3. FREE MASSIVE MODEL

We have noted in the introduction that there are still some unanswered questions on the SUSY breaking at finite temperature. One of the questions was how the symmetry is realized in "free" models, where particles interact only with the heat bath but not with each other. In this section, we study this question in the context of the massive free Wess-Zumino model.

The most general Wess-Zumino model[15] is defined by a supermultiplet $\Phi = (Z, \psi, \mathcal{F})$, where $\mathcal{F}$ is a auxiliary field and ψ a Majorana spinor. In terms of the component fields, $Z = \frac{1}{\sqrt{2}}(A + iB)$, and $\mathcal{F} = \frac{1}{\sqrt{2}}(F + iG)$. The fields A and F are scalars and B and G are pseudoscalars. The Lagrangian reads,

$$\begin{aligned} \mathcal{L} &= \partial_\mu Z^\dagger \partial^\mu Z + \frac{1}{2}\bar{\psi} i \partial\!\!\!/ \psi + \mathcal{F}^\dagger \mathcal{F} \\ &+ \mathcal{F} P'(Z) + \mathcal{F}^\dagger P'^*(Z^\dagger) - \frac{1}{2}\bar{\psi}(\gamma_+ P''(Z) + \gamma_- P''^*(Z^\dagger))\psi \quad , \end{aligned} \tag{3.1}$$

where $\gamma_\pm = \frac{1}{2}(1 \pm \gamma^5)$ and $P(Z)$ a polynomial of at most third order in Z. The SUSY transformations can be found in the literature and will not be repeated here. Under these transformations, the change in the action is,

$$\delta \int d^4x \mathcal{L} = \int d^4x \,\bar{\epsilon}\, \partial^\mu S_\mu \quad , \tag{3.2}$$

where ϵ is the Grassman transformation parameter. The supercurrent is given as follows,

$$S_\mu = -\sqrt{2}\{\partial\!\!\!/(\gamma_+ Z + \gamma_- Z^\dagger) + i(\gamma_- P'(Z) + \gamma_+ P'(Z^\dagger))\}\gamma_\mu \psi \quad . \tag{3.3}$$

In this section, we chose $P(Z) = \frac{1}{2}mZ^2$ (m: real), which defines a free theory with A, B and ψ all of the same mass m.

Since we want to know the consequences of the different statistics in the propagators of bosons and fermions, we need the WT identity that relates them. It is,

$$-i\int d^4z\,\partial_z^\mu \langle S_\mu(z)A(x)\bar{\psi}(y)\rangle_\beta = \langle \psi(x)\bar{\psi}(y)\rangle_\beta + i\,\not{\partial}_y\langle A(x)A(y)\rangle_\beta + \langle A(x)F(y)\rangle_\beta \quad . \tag{3.4}$$

The steps leading to (3.4) can be found in refs. 2, 7. The finite temperature propagators for ψ, A and B are given in (2.2) and the AF propagator is,

$$\langle A(x)F(y)\rangle_\beta = -m\langle A(x)A(y)\rangle_\beta \quad . \tag{3.5}$$

The right hand side of (3.4) is easily found to be,

$$r.h.s. = -2\pi\int \frac{d^4p}{(2\pi)^4}(\not{p}+m)\delta(p^2-m^2)[n_B(p)+n_F(p)]e^{-ip(x-y)} \quad , \tag{3.6}$$

where n_B (n_F) is the Bose (Fermi) statistical factor. The corresponding terms in the left hand side of (3.4) can be written as,

$$l.h.s. = i\int d^4z\,\partial_z^\mu\{\,[(\not{\partial}_z + im)\langle A(z)A(x)\rangle_\beta]\,\gamma_\mu\langle\psi(z)\bar{\psi}(y)\rangle_\beta\,\} \quad , \tag{3.7}$$

or,

$$l.h.s. = i\int \frac{d^4p}{(2\pi)^4}d^4q\,(\not{p}+m)\,(\not{p}-\not{q})S_\beta(q)D_\beta(p)\,e^{iqy-ipx}\,\delta^4(p-q) \quad . \tag{3.8}$$

From the form of the propagators (2.2), we see that in the product $S_\beta(q)D_\beta(p)$ there is a term:

$$I = 2\pi i(\not{q}+m)\left[\frac{\delta(p^2-m^2)n_B(p)}{q^2-m^2+i\epsilon} - \frac{\delta(q^2-m^2)n_F(q)}{p^2-m^2+i\epsilon}\right] \quad , \tag{3.9}$$

that as $p \rightarrow q$ brings a singularity that can cancel $(\not{p} - \not{q})\,\delta(p-q)$ in the numerator of (3.8). It is easy to see that only (3.9) has this singularity. The contribution of (3.9) to (3.7) gives,

$$l.h.s. = i \int \frac{d^4p}{(2\pi)^4} d^4q \,[(p^2 - m^2)(\not{q} + m) - (\not{p} + m)(q^2 - m^2)]$$
$$\times \left[\frac{\delta(p^2 - m^2) n_B(p)}{q^2 - m^2 + i\epsilon} - \frac{\delta(q^2 - m^2) n_F(q)}{p^2 - m^2 + i\epsilon} \right] e^{iqy - ipx}\, \delta^4(p - q) \quad . \tag{3.10}$$

After some algebra, we find this expression to be equal to (3.6) (in the limit $\epsilon \rightarrow 0$) therefore $l.h.s = r.h.s.$ and the WT identity is fulfilled. This is rather surprising because the left hand side of (3.4) is the integral of a total derivative and it would vanish unless there is massless state. Since the contribution to the right hand side arises from (3.7), we will look for this state in the corresponding contribution to the ψA propagator.

In this non-interacting theory, the composite ψA propagator is the product of the "free" propagators;

$$\langle \psi A \bar{\psi} A \rangle_\beta = S_\beta\left(\frac{P}{2} + r\right) D_\beta\left(\frac{P}{2} - r\right) \quad , \tag{3.11}$$

where we introduced the center of mass momentum P and the relative momentum r. The temperature dependent part of the above product contains a singularity for $P \cdot r = 0$. For small P, (3.11) is given as follows,

$$\langle \psi A \bar{\psi} A \rangle_\beta \approx \langle \psi A \bar{\psi} A \rangle_{T=0}$$
$$+ 2\pi i (\not{r} + m) \left(\frac{n_B(r) + n_F(r)}{2P \cdot r} + 2\pi i \, n_B(r) n_F(r) \delta(2P \cdot r) \right) \delta(r^2 - m^2) \quad . \tag{3.12}$$

Using the identity,

$$\not{r}+m = 2P\cdot r\frac{1}{\not{P}} - \frac{1}{\not{P}}(\not{r}-m)\not{P} \quad , \tag{3.13}$$

we project out the massless pole,

$$\langle\psi A\bar{\psi}A\rangle_\beta = \frac{i}{\not{P}}2\pi(n_B(r)+n_F(r))\delta(r^2-m^2)+\ldots\ldots \quad , \tag{3.14}$$

where we wrote only the contribution of the first term in (3.14). The dots stand for terms that do not contribute to the WT identity (3.4). It is because the singular terms that come from the second term in (3.13) are multiplied by $(\not{r}+m)\not{P}$ from the left, as in eq. (3.8). Then, this term yields zero because of $\delta(r^2-m^2)$. Therefore the massless pole given in (3.14) is the one which plays the role of the Goldstone pole. Note that for $T\rightarrow 0$ the residue of this pole vanishes as $e^{-\sqrt{m^2+|\vec{r}|^2}/T}$. Although we have only mentioned the composite operator ψA, the same can be said for $\gamma^5\psi B$.

The physical interpretation for (3.14) is the following: As we mentioned before, the thermal-equilibrium state consists of all the excited states of the system being populated by *real particles* with probabilities given by the statistical factors. The excitation spectrum of the theory is supersymmetric; bosons and fermions have the same masses. Therefore if we create say a fermion in one of these energy levels and destroy a boson (from the thermal-equilibrium state) in the same energy level, this process costs no energy. The same is true for creating a boson and destroying a fermion. The eq. (3.14) is the sum of these two amplitudes.

The similar reasoning indicates that if the masses of the fermions and bosons are m_B and m_F respectively it would cost $|m_B-m_F|$ to create this excitation with zero center of mass momentum (see Sec.5). It is also easy to see that the amplitude for a process involving two bosons or two fermions vanishes.

With this analysis of the very simple non-interacting model we have learned several things. As we suspected SUSY is broken at finite temperature because fermions and bosons have different statistics. However what is surprising is that the symmetry is realized in the Goldstone fashion. Namely, the WT identity relating the Green's functions of bosons and fermion is non-trivially satisfied indicating the presence of a massless excitation that couples to the supercurrent. While in this model the auxiliary field cannot be an order parameter (always $\langle F \rangle_\beta = 0$) because of the lack of the interaction, the order parameter of SUSY breaking is given by the right hand side of the WT identity (3.4), a linear combination of the fermion and boson propagators. Therefore we interpret this breaking as the spontaneous breaking of SUSY and this massless excitation as the Goldstone fermion.

4. INTERACTING MODEL WITH CHIRAL SYMMETRY

In the last section, we found a new and interesting feature of SUSY at finite temperature. The analysis was done in a noninteracting theory. Therefore the next question is what do we expect in an interacting theory. Are the excitations created by the composite operators ψA and $\gamma^5\psi B$ still massless? How does it affect the other channels like a single ψ? In order to answer these questions, we study the interacting Wess-Zumino models in the following sections. As we mentioned before, we are also interested in what happens to the R-invariant models, where the auxiliary field is not allowed to be the order parameter. Therefore, we first investigate an R-invariant (chiral) model in this section.

By chosing $P(Z) = \frac{1}{6}gZ^3$ in the Lagrangian (3.1), we obtain an interacting theory that is symmetric under the following chiral transformation,

$$Z \rightarrow e^{i\alpha} Z \ , \quad \psi \rightarrow e^{-i\frac{\alpha}{2}\gamma^5}\psi \ , \quad \mathcal{F} \rightarrow e^{2i\alpha}\mathcal{F} \ . \tag{4.1}$$

This is a special case of R-invariance possible for SUSY theories. At zero temperature to the tree level, both SUSY and the chiral symmetry are explicit (unbroken). Because of the chiral symmetry, the fermion is massless. Therefore its SUSY partners A and B are massless. It has been shown that the Coleman-Weinberg mechanism does not take place in this theory![16]

Let us now investigate the model at finite temperature. At tree level, we find that SUSY is broken but the chiral symmetry is not. The WT identity (3-4) is satisfied in the same way as in the previous section (except that now $m = 0$). Namely, the fact that the fermion and boson propagators have different statistics indicates that the operators ψA and $\gamma^5\psi B$ create massless excitations that couple to the supercurrent. These are interpreted as Goldstone fermions that form a doublet under the unbroken chiral symmetry. Although a single ψ is massless at this order, it does not couple to the current and therefore is not a Goldstone fermion.

In higher orders, we expect the same breaking pattern to persist: the chiral symmetry is unbroken while SUSY is broken. For the chiral symmetry, it is known that if it is unbroken at zero temperature, the finite temperature effects leave it unbroken. In fact, from the one-loop effective potential, we find

$$\langle \mathcal{Z} \rangle_\beta = \langle \mathcal{F} \rangle_\beta = 0 \quad . \tag{4.2}$$

We expect SUSY to be broken because even in the interacting theory the fermion and the boson propagators are essentially different because of the statistics. This effect will be reflected in the corresponding spectral representations of the propagators.

For this reason, the WT identity for the propagators, eq. (3-4), should be satisfied with a *nonzero* right hand side. Thus *there must be a massless Goldstone mode* contributing to the left hand side. In order to see explicitly how this massless mode contributes, we examine the WT identity for the inverse propagators. This may be derived from (3.4) by multiplying both sides by the inverse propagators Γ_ψ and Γ_{AA} (in the functional sense),

$$\begin{aligned}\int d^4x_1 d^4y_1 d^4z \;\; \partial_z^\mu \langle S_\mu(z)\bar{\psi}(x_1)A(y_1)\rangle \, \Gamma_\psi(x_1,x)\, \Gamma_{AA}(y_1,y) \; = \\ -i\,\not{\partial}\Gamma_\psi(x,y)+\Gamma_{AA}(x,y) \quad .\end{aligned} \tag{4.3}$$

We have dropped the terms which contain Γ_{AF}, because these are zero because of the chiral symmetry (F and A do not mix because of their different chiral charge). We have also neglected the "ghost" fields since the result is not affected by them up to the one-loop order. The reason for working with the inverse Green's functions and not with the propagators themselves is because the latter require knowledge of the spectral density whereas the former do not involve this quantity and are easier to compute. Furthermore, the one-loop calculation of the inverse propagators does not involve contributions from the "ghost" sectors and therefore are straightforward. That is, the right hand side of (4.3) is given by the usual Feynman diagrams for the self-energy correction, but with the thermal Green's functions (2.1) and (2.2) instead of the usual ones. The one-loop Feynman diagrams for the left hand side of (4.3) are illustrated in Fig.1. (Actually the external A and ψ lines are truncated by the corresponding Γ's.) Among the four of them, only (a) and (b) can have the necessary singularity to give a nonvanishing contribution. This can be seen from the fact that only in these diagrams ψA and $\gamma^5\psi B$ are intermediate states. The same mechanism that gave rise to the singularity in the ψA channel at tree level also works in the loops. Diagram (b), however, vanishes because the bare masses vanish. (Actually, there is a corresponding diagram with "ghost" lines. But it identically vanishes too.) Careful calculation shows that the identity (4.3) is indeed satisfied *nontrivially*, with the l.h.s. being,

$$l.h.s. = -2\pi g^2 \int \frac{d^4k}{(2\pi)^4} \delta(k^2)(n_F(k) + n_B(k)) \frac{\not{k}\not{p}}{(k+p)^2} \quad , \tag{4.4}$$

where p is the external momentum. The fact that *only* diagram (a) gives a nonzero contribution to the left hand side of (4.3) indicates that *the composite operator ψA (and $\gamma^5\psi B$) creates a massless excitation that couples to the supercurrent*. That is, even in this interacting theory, there is the chiral doublet of Goldstone fermions in the same composite channel as at the tree level.

It is interesting to note that not all the ψA and $\gamma^5\psi B$ couple to the current. So far we have been dealing with bilinear operators with two arbitrary coordinates, for example $\psi(x)A(y)$. However the local operator $\psi(x)A(x)\equiv\chi_A(x)$ (or, in momentum space, the linear superposition of all the states with relative momentum r with equal weight) decouples from the supercurrent,

$$\int d^4z\,\partial_z^\mu\langle S_\mu(z)\psi(x)A(x)\rangle = 0 \quad . \tag{4.5}$$

and the same holds for $\gamma^5\psi B$. Although the states created by these local operators are massless, the coupling to the supercurrent vanishes up to one-loop order because of the zero bare mass.

Next let us look at the ψ channel. At finite temperature, chiral symmetry does not prevent the fermion from acquiring a mass. Indeed chiral symmetry only requires $\{\gamma^5, \Gamma_\psi(p)\} = 0$. This relation can be satisfied with $\Gamma_\psi(p) = F_1(p^2, pu)\not{p}+F_2(p^2, pu)\not{u}$, where u_μ is the four velocity of the thermal equilibrium state and $F_{1,2}$ are scalar functions.[13] In this way, the zeroes of $\Gamma_\psi(p)$ may not be at $p_\mu = 0$. The chiral symmetry has further consequences for the SUSY breaking. In fact, the following WT identity relevant for the single fermion,

$$\int d^4z\,\partial_z^\mu\,\langle S_\mu(z)\bar{\psi}(x)\rangle_\beta = \langle F(x)\rangle_\beta \quad , \tag{4.6}$$

cannot be satisfied with a nonvanishing $\langle F\rangle_\beta$. The reason is that there is no singlet under the chiral symmetry in either side, so they both vanish. This shows that ψ *cannot be a Goldstone fermion* (this is in contrast to what happens in the case without chiral invariance).

In fact, the one-loop calculation[17] shows that the fermion acquires an effective mass $\frac{1}{4}gT$. It is because of the mixing between the ψ and $\chi_{A,B}$: The one-loop fermion self-energy is essentially given by the propagators $\langle\chi(-p)\bar{\chi}(p)\rangle$, which is singular at $p_\mu = 0$. This singularity leads to the following one-loop inverse fermion propagator for small momentum p_0 (at $\vec{p} = \vec{0}$),

$$\Gamma_\psi(p_0, \vec{p} = \vec{0}) \simeq -i\gamma_0\left(p_0 - \frac{g^2T^2}{16}\frac{1}{p_0}\right) \quad . \tag{4.7}$$

This shows that the fermion mass arises from the masslessness of the state created by $\chi(x)$. This also indicates that the amplitude for the process $\psi(p) \rightarrow \psi(p-q) + A(q)$ has a singularity as $p \rightarrow 0$. The one loop contribution is depicted in Fig.2.

For the bosons A and B, the one loop calculation shows that they acquire the effective mass $\frac{1}{2\sqrt{2}}gT$. This difference between the effective masses of the fermion and the bosons is because of the spontaneous breaking of the SUSY: the presence of the Goldstone fermion mode in the ψA and $\gamma^5\psi B$ renders the left hand side of (4.3) nonzero, so that the poles in the fermion propagator and the boson propagator do not coincide.

5. INTERACTING MODEL WITHOUT CHIRAL SYMMETRY

In the previous section, the chiral symmetry (4.1) prevented the fermion from becoming a Goldstone fermion. In this section, we investigate the physical aspects of a model that does not have this symmetry. One of those models is given by $P(Z) = -lZ + \frac{1}{6}gZ^3$ in the Lagrangian (3.1).

Although this model has been studied in ref. 7, here we would like to understand the role of the excitation created by the bilinear operators ψA and $\gamma^5\psi B$: In this non-chiral case we know that ψ becomes a Goldstone fermion at one-loop level. The auxiliary field develops a nonvanishing thermal average. Therefore there is a question as to whether the excitations created by these composite operators remain Goldstone fermions at higher orders. And if not, it still has to be clarified how it decouples from the supercurrent.

In order to investigate this theory, we start by shifting the fields in the Lagrangian by their thermal averages. In component notation, $A = A_c + A_f$, and $F = F_c + F_f$, where $A_c \equiv \langle A \rangle_\beta$ and $F_c \equiv \langle F \rangle_\beta$ are determined by the minimization conditions of the effective potential. The induced masses of the physical particles (obtained after eliminating the fluctuations of the auxiliary fields) are:

$$
\begin{aligned}
m_\psi^2 &= \frac{g^2}{2}A_c^2 \quad , \\
m_A^2 &= \frac{g^2}{2}A_c^2 - \frac{g}{\sqrt{2}}F_c \quad , \\
m_B^2 &= \frac{g^2}{2}A_c^2 + \frac{g}{\sqrt{2}}F_c \quad .
\end{aligned}
\tag{5.1}
$$

At tree level, the minimization condition yields $A_c = 2/\sqrt{gl}$ and $F_c = 0$. At this level, this model describes bosons and fermions, all of the same mass $m = \sqrt{g/l}$. In this sense, this model is a direct extension of the free model discussed in the section 3. As before, the excitations created by the operators ψA and $\gamma^5\psi B$ are massless, and saturate the (tree level) WT identity (3.4) as in the free theory. Hence at tree level these composite operators create the Goldstone modes arising from the breaking of SUSY.

At one loop level, F developes a nonzero thermal average that is proportional to $e^{-m/T}$ for $T \ll m$. Thus according to (5.1) the induced masses are different. This results in a shift of the massless pole in the free ψA (and ψB) propagators: Instead of (3.12), the ψA-propagator now contains a singularity when $2P\cdot r + m_A^2 - m_\psi^2 = 0$. Applying the following identity instead of (3.13),

$$\frac{\not P}{2} + \not r + m_\psi = (2P\cdot r + m_A^2 - m_\psi^2)\frac{1}{\not P + m_A - m_\psi} + \frac{1}{\not P + m_A - m_\psi}\left(\frac{\not P}{2} - \not r + m_A\right)(\not P + m_\psi - m_A) \quad , \tag{5.2}$$

we obtain the shifted pole,

$$\langle \psi A \bar\psi A\rangle_\beta \propto \frac{i}{\not P + m_A - m_\psi} \quad . \tag{5.3}$$

As mentioned in sec. 3, this shift can be interpreted physically. Destroying a boson of mass m_A from the statistical ensemble, and creating a fermion of mass m_ψ (or vice-versa) in the same energy level (with zero center-of-mass momentum) costs energy $\simeq |m_A - m_\psi|$.

The explicit calculation of the one-loop fermion self-energy indicates a pole at zero momentum.

$$\Gamma_\psi(p_\mu = 0) = 0 \quad . \tag{5.4}$$

The WT identity (4.6) is saturated with this massless pole, which is then identified as a Goldstone fermion.

In this model, the WT identities for the inverse propagators are given by (4.3) with an extra term $-i\not{\partial}_x \Gamma_{FA}(x,y)$ on the right hand side. This identity is now satisfied in a different fashion. Namely, we find that diagram (a) of fig. 1 no longer contributes to the left hand side. This can be traced to the fact that because of the shifting of the pole the propagator (5.3) is *finite* as $p \to 0$. For the same reason, the diagram (b) does not contribute either. Some of the nontrivial contributions to the WT identity are shown in fig. 3. These are nonvanishing because the one-loop propagator of the field ψ has a pole at zero momentum. Therefore we see that the mechanism by which the WT identity (4.3) was nontrivially fulfilled (at one-loop in the model with chiral symmetry) no longer applies in this case since $F \neq 0$. Instead, the fermion ψ becomes a Goldstone excitation and saturates the WT identities.

In the last section, we mentioned that the masslessness of the excitations created by $\chi_{A,B}(x)$ was responsible for generating a chiral-nonbreaking mass for the fermion at the one-loop level. This was because there is a mixing term $\bar{\psi}(x)\chi_{A,B}(x)$ in the Lagrangian. In this non-chiral model, however, the contribution of the $\langle\chi(-p)\bar{\chi}(p)\rangle_\beta$ to the fermion self-energy cancels the mass term for $p_\mu = 0$. This is the mechanism that makes the fermion massless at one-loop.[18]

This is a very interesting phenomenon: At tree level ψA and $\gamma^5\psi B$ create a pair of Goldstone fermions at $T \neq 0$ because of the lack of the mass splitting. But at one-loop level, the auxiliary field acquires a nonzero thermal average. As a result, the singularities are transferred to the ψ field through mixing with $\bar{\psi}A$ and $\bar{\psi}\gamma^5 B$. In this way, the ψ becomes massless and ψA and $\gamma^5\psi B$ become massive.

6. CONCLUSION

In this paper we have investigated the physics of symmetry breaking of SUSY theories induced by finite temperature effects.

The breaking of SUSY arises because fermions and bosons obey different statistics. We have quantized these theories at finite temperature in a Lorentz covariant way (real-time formalism) and have found that this breaking is always associated with massless excitations that couple to the supercurrent. These are identified as Goldstone fermions.

Even in the simple case of a free field theory, there are Goldstone excitations created by the fermion-boson composite operators, ψA and $\gamma^5\psi B$. These states nontrivially saturate the WT identities that relate boson and fermion propagators. Their different statistics yield a temperature-dependent piece in the difference of the propagators, which then is canceled by the contribution of the zero-momentum coupling of the massless composite states to the supercurrent.

In an interacting theory where there is a chiral symmetry (R-invariance), the auxiliary field is prohibited from having a non-zero expectation value. A WT identity then forbids the elementary fermion from coupling to the supercurrent at zero momentum. We have shown that this type of model posesses essentially the same features as the free theory. That is, the same composite operators ψA and $\gamma^5\psi B$ create Goldstone excitations, which now form a doublet under the chiral symmetry. These excitations are explicitly shown to saturate the WT identity up to the one-loop order. These excitations appear as an intermediate state in the process $\psi \rightarrow \psi + A(B)$, rendering the amplitude for this process singular as the incoming momentum goes to zero.

In the model without chiral symmetry, there is an interesting phenomenon.

At tree level, the thermal average of the auxiliary field vanishes. The elementary fermion is massive and composite operators create Goldstone fermions. At the one-loop level, the auxiliary field acquires a nonzero thermal average. The elementary fermion becomes massless and is the Goldstone fermion. The excitations created by ψA and $\gamma^5\psi B$ are no longer massless. It is seen that the local couplings of ψ to $\bar{\psi}A$ and $\bar{\psi}\gamma^5 B$ are responsible for ψ becoming massless. This clarifies the structures of the Goldstone fermions found previously. If one looks only at the single fermion channel, one finds the Goldstone fermion at one-loop level, and one might think that this phenomenon is similar to the dynamical breaking. However, in the composite sector, the symmetry breaking is manifest at the tree level. It becomes visible in the single fermion channel through mixing, only at higher orders in the perturbation theory.

In the free theory example, the Goldstone modes do not contribute to the thermo-dynamic properties because of the lack of interactions. In interacting theories, however, we expect these modes to contribute to the thermo-dynamic properties. In the non-chiral case, for example, the Goldstone pole of the fermion would give a power law behavior to the specific heat were it not for the fact that the probability (residue) of this pole is $\simeq e^{-m/T}$ at low temperatures.

We believe that aspects of SUSY at finite temperature deserve to be studied further since they posesss new interesting physical properties that are not known in the zero temperature theories. It would also be very interesting to study these aspects in gauge theories.

After completion of this work, we received a paper by Matsumoto et. al.[19] where it is also realized that there are massless excitations created by bilinear operators.

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FIGURE CAPTIONS

1. The one loop graphs relevant for the left hand side of the WT identity (4.3). The solid lines stand for the fermion and the wavey lines for bosons. The circle with the cross stands for the supercurrent. The external lines are actually truncated, but are shown here for clarity.

2. The vertex correction graph which generates a singularity for $p \to 0$ in the process $\psi(p) \to \psi(p-q) + A(q)$.

3. Some of the one-loop contributions to the WT identity of the inverse propagators ((4.3) plus the FA-term). The double solid lines represent the one-loop dressed fermion propagators. The external ψ and A lines are actually truncated.

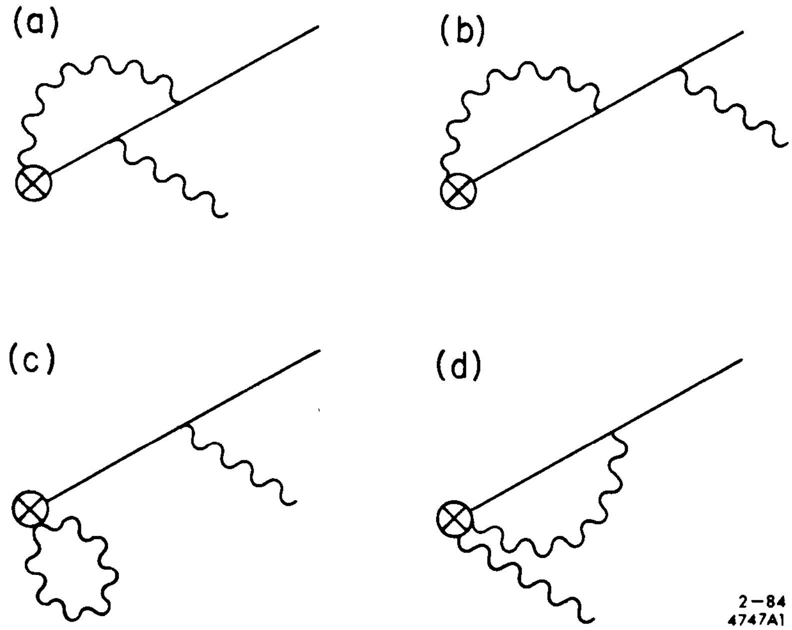

Fig. 1

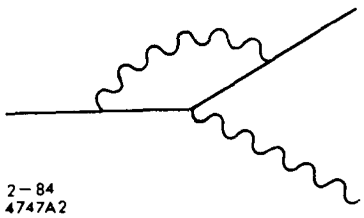

Fig. 2

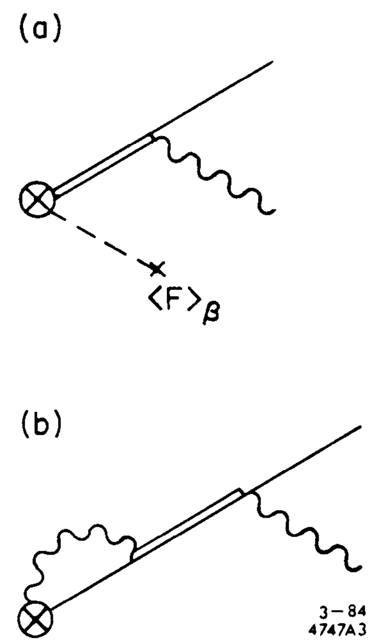

Fig. 3