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COMPATIBILITY OF G_A/G_V , r_p AND μ_p IN THE QUARK MODELS*

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ABSTRACT

We examine the question of a simultaneous fit to G_A/G_V , r_p and μ_p in the context of the usual quark models. Neither the nonrelativistic harmonic oscillator quark model nor the MIT bag model (and its variations) is able to reconcile these three quantities. On the other hand, it is found that a model employing a potential of the form $(1 + \gamma_0)[(1/2)Kr^2 + V_0]$ is able to give an excellent fit to these well known low-energy parameters.

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QCD[1] has registered remarkable success in the deep-inelastic region and has provided vital clues to the confinement problem through lattice calculations[2] and computer experiments.[3] However, at present the theory is not in a position to provide a viable description of hadronic spectroscopic data which is usually analyzed in terms of two phenomenological models, *viz.*, the bag model and the nonrelativistic harmonic oscillator model (HO), both essentially incorporate the quark concept in one form or the other. Although both the models have their own merits, the bag in terms of its well defined ingredients and the HO model in terms of its wider applicability, yet it is interesting to note that a simultaneous satisfactory fit to G_A/G_V , μ_p and r_p[4] has eluded these models. Before an attempt is made to reconcile thee three key parameters, it is essential to understand the difficulties faced by the HO model and the bag model in this regard. Let us first have a look at G_A/G_V , r_p and μ_p in the nonrelativistic HO model. In this model, μ_p is usually fitted by adjusting the quark mass[5–8] (m_q) and, in principle, r_p can be fitted by adjusting the shape parameter (α). However, if the most commonly used value[9] of α is considered, we get $r_p \simeq 0.63$ fm compared with the experimental value[10] of 0.88 fm. Irrespective of the value of m_q and α, G_A/G_V is predicted to be 1.66, a value too large compared with the data. The situation does not improve even after taking into account the one gluon exchange effects.[11] However, it is known that the relativistic effects can bring down the value[12] of G_A/G_V , although a satisfactory incorporation of these has not been accomplished.[13]

On the other hand, in the MIT version of the bag model[14] with massless quarks, a typical set obtained for μ_p , G_A/G_V and r_p is

$$\mu_p = 1.9\,(n.m.), \qquad G_A/G_V = 1.09, \qquad \text{and} \qquad r_p = 0.73\,fm \quad . \tag{1}$$

The corresponding experimental values are

$$\mu_p = 2.793, \ ^{15} \qquad G_A/G_V = 1.248, \ ^{16} \quad \text{and} \quad r_p = 0.88\, fm \ ^{17} \qquad . \tag{2}$$

The predictions of the bag model are much lower compared with the experimental numbers. Before one investigates the reasons for this unsatisfactory situation, it is essential to realize that there are two ingredients of the bag model, *viz.,* the radius and the relative 'size' of the 'small' and 'large' components of the quark Dirac spinors, which play a crucial role in the evaluation of these three parameters. It can be easily seen in the bag model that μ_p essentially depends on the interference of 'large' and 'small' components; whereas, in the case of G_A/G_V the contribution of small components with a suitable coefficient, viz., 1/3 in the MIT bag model, is subtracted from the contribution of 'upper' components, and for r_p the contribution of 'small components', is added to the contribution of the upper components. The values mentioned in Eq. (1) correspond to a value of 5 GeV^{-1} bag radius which is well known to be somewhat large, therefore it is not feasible to improve the above numbers by increasing the bag size. Decreasing the size of small components will decreaseμ_p and increase G_A/G_V , therefore on this front also there is very little which can be done in the simplest bag model. Golowich[17] has attempted to fit G_A/G_V by considering massive quarks in the bag model, however in his model the size of the bag becomes uncomfortably large, e.g., 8 GeV^{-1}. Consequently r_p turns out to be too large compared with data, although μ_p registers an improvement compared with the massless case. By incorporating chiral invariance, Theberge *et al.,*[18] have improved the fit although the situation is not entirely satisfactory.

Recently Ravndal[19] has proposed an interesting model which simulates central features of the bag model with harmonic oscillator dynamics (hereafter referred to as HBM). Unfortunately the status of the three parameters in HBM is no better than in other quark models. Interestingly, we have found that a variant of the HBM leads to a satisfactory fit to these three quantities. To facilitate the discussion of results we present certain essential details of the model.

In the HBM, the starting point is a single particle Dirac equation with a potential of the form

$$V(r) = (1+\gamma_0)\left(\frac{1}{2}Kr^2 + V_0\right) \quad . \tag{3}$$

Ravndal, however, does not consider the constant V_0. This kind of potential considerably simplifies the solution and the ground state wave function could be given as

$$\psi(r) = N\left[\begin{pmatrix} \chi \\ \frac{i}{3}(E-m)\vec{\sigma}\cdot\vec{r} \end{pmatrix}\right] exp\left(-\frac{2}{3}K\,\frac{r^2}{E-m}\right) \quad , \tag{4}$$

where E is the quark energy, m is the quark mass and χ is a Pauli spinor. The quantities E, m and K satisfy the eigenvalue relation

$$(E-m)(E+m)^{1/2} = 3K^{1/2}\left(\frac{2}{3}n+1\right) \quad . \tag{5}$$

Ravndal relates energy E to the mass of nucleon, therefore in HBM, through the above relation for $n = 0$, K also gets fixed leaving no free parameters in the model. Within this rigid framework, he obtains the following values for the above mentioned parameters:

$$\begin{gathered} \text{for } m_q = 0, \\ G_A/G_V = 0.93, \quad \mu_p = 2.33 \quad \text{and} \quad r_p = 1.5fm \quad , \end{gathered} \tag{6}$$

and

$$\text{for } m_q = 1.27 \text{ MeV},$$

$$G_A/G_V = 1.28, \quad \mu_p = 2.65 \quad \text{and} \quad r_p = 2.0 fm \quad . \tag{7}$$

For the massless case, except for r_p all the numbers are smaller compared with the data, whereas for the massive quarks, the proton charge radius becomes too large, more than double the experimental value. Without going into a detailed scrutiny of the Ravndal's results, which will be carried out later on, it is essential to realize that his calculations ignore the effect of V_0; moreover, the spring constant turns out to be too small.

At this stage it is interesting to mention that V_0 is negative for confined fermions,[20] in certain potential models[21] it is found to be around -250 MeV. A precise calculation of V_0, however is difficult to make. Moreover, we feel that the success of the HO model depends on a reasonable value of the spring constant which in the HBM is quite small leading to a very large charge radius. Since the purpose of the present note is to make a quick examination of the three above mentioned parameters, therefore it becomes interesting to consider HBM with the above mention value of V_0 as well as a usual value of the spring constant, e.g., $K = 0.01$ GeV^3 corresponding to the shape factor $\alpha = 0.33$ in the HO model. With the above values of K and V_0, in order to reproduce the nucleon mass within HBM, the quark mass has to be 180 MeV. Having fixed the values of V_0, α and M_q, we have evaluated G_A/G_V , r_p and μ_p without any further input.

In Table I, we have presented the results of our calculations and the experimental values. For the sake of comparison we have also presented the results of HO model, the MIT bag model and the HBM. Keeping in mind the simplicity of the model, it is apparent that the fit is excellent. The slight discrepancy in the case of μ_p could perhaps be explained on the basis of pion exchange corrections as has been shown by Theberge *et al.*[18] To understand the role of the particular values we have considered for the various parameters as well as the reasons for the success of present calculations, it becomes interesting to undertake a detailed comparison of the present results with those of HBM.

First of all we examine the situation regarding r_p in the present calculations and the HBM. It is easy to see in the context of present approach as well as in HBM, that r_p is essentially controlled by the exponential factor appearing in the quark wave function. One could check easily that in the HBM the coefficient of R^2 in the exponential [see eq. (4)], which is quite small compared with the corresponding coefficient in the nonrelativistic model becomes still smaller due to the factor $(E - m)$ as one goes from massless quarks to massive quark case, leading to an enormous increase in the size of r_p . In other words, the slower fall of exponential factor with r is the main cause of disagreement in the case of r_p . In the present calculations, the coefficient of r^2 in the exponential not only become larger compared with the massless case of HBM, but also compared with nonrelativistic harmonic oscillator model, leading to the confinement of the quark wave function within a spherical region of radius less than 1 fm. This behavior of the wave function enables us to reproduce a perfect value for r_p by delicately controlling the increase in size due to small component effect. Coming to μ_p and G_A/G_V , first we examine the reasons for the successful reproduction of μ_p and r_p in HBM. While discussing the bag model we have already mentioned that μ_p and G_A/G_V depend upon the relative size of 'large' and 'small' components of quark Dirac spinors. The increase of G_A/G_V for massive quarks compared with the 'massless' quarks is apparent; however, the increase in μ_p seems somewhat intriguing. Apparently one would think that μ_p should decrease as the effect of small components seems to indicate from increase in G_A/G_V . However, a closer look indicates that it is possible that small component effect decreases for G_A/G_V and increases for μ_p as one goes from massless case to massive case. This is possible due to the fact that G_A/G_V depends on $(E-m)^2 r^2$ whereas μ_p depends on $(E-m)r^2$. In HBM, as one goes from 'massless' case to 'massive' case, the coefficient of r^2 in the exponential factor as well as $(E-m)$ decreases. HBM results, therefore, could easily be understood as a consequence of $(E-m)^2$ domination for G_A/G_V and r^2 dominance for μ_p . In the present calculations, $(E-m)$ factor turns out to be somewhat larger compared with HBM; however, the contribution of r^2 for large R is not important due to the sharply decreasing exponential factor. Therefore $(E-m)r^2$ remains sufficiently large to reproduce a reasonable value of μ_p whereas $(E-m)^2 r^2$ in the case of G_A/G_V enables us to reproduce a perfect fit.

To conclude, we would like to mention that we have attempted to achieve a simultaneous fit to three well known parameters, *viz.*, μ_p, r_p and G_A/G_V, within the context of quark model, hitherto eluding the potential as well as confinement models. To this end we have found that a potential model of the form $(1+\gamma_0)[(1/2)Kr^2 + V_0]$ allows us to have a satisfactory fit to the above mentioned quantities.

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Table I

μ_p and r_p are in nm and fm respectively. Sets I and II of HBM correspond to $m_q = 0$ and $m_q = 127$ MeV. Present results correspond to $m_q = 180$ MeV, $K = .01$ GeV^3 and $V_0 = -237$ MeV.

	NRQM	BAG	HBM I	HBM II	Present Results	Experiment
r_p	0.63	0.73	1.5	2.0	0.88	0.88 ± 0.3 [a]
G_A/G_V	1.66	1.09	0.93	1.28	1.24	1.248 [b]
μ_p	2.793 [c]	1.9	2.33	2.65	2.60	2.793 [d]

[a] Ref. 10
[b] Ref. 16
[c] input
[d] Ref. 15