SLAC – PUB – 3287
February 1984
(T/E)

O(18) REVIVED: SPLITTING THE SPINOR*

JONATHAN BAGGER

Stanford Linear Accelerator Center
Stanford University, Stanford, California 94305

and

SAVAS DIMOPOULOS†

Department of Physics, Stanford University
Stanford, California 94305

Submitted for Publication

*Work supported by the Department of Energy, contract DE-AC03-76SF00515, and by the National Science Foundation, contract NSF-PHY-83-10654.

†Alfred P. Sloan Foundation Fellow.

Abstract

We propose an O(18) theory which is perturbatively unifiable and which accounts for the absence of right-handed families in the low-energy world. The theory predicts a fourth left-handed family as well as four right-handed families at energies near the weak scale. It also implies the existence of eight light neutrinos, all of which contribute to the width of the Z^0. Cosmological arguments suggest that four of these neutrinos should have masses between 2 and 35 GeV, and that the other four should be much lighter. They also suggest the existence of a doubly-charged scalar ϕ^{++} and a singly-charged scalar ϕ^+. Dramatic signatures include the production of four right-handed charged leptons and eight right-handed quarks, $Z^0 \rightarrow \nu'_R + \bar{\nu}'_R \rightarrow \nu_R \gamma + \bar{\nu}_R \gamma$, and $e^+e^- \rightarrow \phi^{++} + \phi^{--}$. The lightest right-handed charged quark should be surprisingly long-lived ($\tau \gtrsim 10^{-2}$ sec) for a particle of mass $\simeq$ 100 GeV.

1. Introduction

Some of the most pressing problems in particle physics concern the question of families. We still do not know why quarks and leptons come in families, nor why the families repeat. Neither do we understand why the observed families have only left-handed weak interactions. Conventional grand unified theories [1] shed some light on the first question. However, they do not begin to address the multiplicity of families. Extra families are simply added at will.

There have been many attempts to unify families with flavors. The most promising of these are based on the group O(18) [2]. There are several reasons for this:

- All the known families are contained in just one representation of O(18), the 256-dimensional spinor.
- The O(18) spinor is *complex*. This means that there are no O(18)-invariant mass terms that make everything heavy and leave the low-energy world free of matter.
- The group O(18) is anomaly-free.

It is important to note that O(18) is the only sensible theory that satisfies all three points. Other theories suffer from a variety of afflictions. Some need many different representations to conspire to give three families without anomalies. Others, such as E(8), allow superheavy invariant masses. Still others, such as O(22), contain O(18) as a subgroup, and are ridiculously large. (For example, O(22) predicts 128 quark flavors.)

Previous attempts to construct realistic theories based on O(18) have been plagued by serious difficulties [2]. These problems stem from the fact that the 256-dimensional spinor contains eight left-handed and eight right-handed families. This may be seen by decomposing the 256 under O(10) $\times$ O(8),

$$256 \rightarrow (16, 8') + (\overline{16}, 8'') \quad . \tag{1}$$

Here O(8) is a horizontal family group, and the 16 and the $\overline{16}$ are the usual left- and right-handed families of O(10). Note that the 16 and the $\overline{16}$ belong to *different* representations of O(8). This simply reflects the fact that the spinor of O(18) is complex with respect to its O(10) $\times$ O(8) subgroup. It has the important consequence that there are no O(10) $\times$ O(8) -invariant mass terms which make all fermions superheavy.

The fact that the spinor of O(18) is complex with respect to O(10) $\times$ O(8) allows one to construct models in which all the elements in the 256-dimensional spinor are protected from acquiring mass down to $M_W \simeq 250$ GeV. In spite of their aesthetic appeal, these models have major problems. In particular, because of the large number of families involved, asymptotic freedom is lost and coupling constants blow up at a few hundred TeV. These theories are not perturbatively unifiable.

Another possibility is that the family group O(8) is completely broken at the grand unified scale M_{GUT}. One might imagine that some of the left- and right-handed families pair up and become massive, leaving a few unpaired left- and right-handed families. This approach is unreasonable because the unpaired families are not protected by any symmetry from becoming superheavy [3].

The only sensible alternative is the intermediate possibility in which O(8) breaks at M_{GUT} to a subgroup H which protects some of the families from becoming superheavy. All unprotected families are assumed to be superheavy and absent from the low-energy spectrum. We refer to this mechanism as splitting the O(18) spinor. In the Appendix and in Section 2 we classify and discuss all ways of splitting the spinor via continuous subgroups H of O(8). In Section 3 we show how to make the light right-handed fermions heavier than their left-handed counterparts. In Section 4 we address the question of the light doublet neutrinos. We examine the cosmological constraints and discuss how to split the light neutrino masses by introducing a weak isotriplet of Higgs scalars. The neutrino spectrum gives rise to dramatic experimental signatures which should be evident in Z^0 decays. In Section 5 we discuss the fate of the right-handed families. We show that they decay into left-handed particles and should not be present in the visible universe. We conclude in Section 6 with a summary of some of the expected experimental signatures.

2. Splitting Heavy from Light

In the previous section we have seen how the 256-dimensional spinor of O(18) splits into eight left- and eight right-handed families under $G = \mathrm{O}(10) \times \mathrm{O}(8)$. In this section we describe a mechanism which allows some fermions to become heavy, and the rest to remain light. Our basic idea is simple: We find all continuous symmetries $H \subseteq \mathrm{O}(8)$ under which the 256 contains a complex representation of G. We assume that fermions in real representations of G pair off and become massive at the grand unified scale M_{GUT}. We then use the fact that the remaining fermions are in a complex representation to guarantee that they remain massless down to low energies.

All possible continuous family symmetries H are classified in the Appendix, and the results are summarized in Table 1. We see that there are only a few family symmetries which protect fermions from acquiring large masses. As discussed in Section 1, the full family group O(8) leaves eight light left-right generations. In addition, the subgroups SU(3) $\times$ U(1) and SU(2) $\times$ U(1) each give six light generations, and four different U(1)'s protect either four, six or eight light generations. An odd number of light left-right families is not allowed by any continuous symmetry H.

In this paper we only consider the case of four light generations. With six or eight light families, asymptotic freedom and perturbative unification are lost. As mentioned in the introduction, if there are eight light families, α_S becomes strong at about 10^6 GeV, and if there are six light families, α_S blows up at about 10^{11} GeV. With four light families, the low energy theory is still asymptotically free. The color beta function is almost flat, so α_S barely evolves. At the one-loop level, the values of M_{GUT} and $\sin^2(\theta_W)$ are not affected by the extra low-energy families.

As we see from Table 1, there are only two continuous symmetries H which leave four light left- and right-handed families. Both of these symmetries are abelian. They act on the various O(8) representations as shown in Table 2. Focussing on the $8'$ and the $8''$, we see that each U(1) allows four families to pair off at the grand unified scale M_{GUT}. The other four families form complex representations of H.* They are protected from acquiring masses down to low energies.

The situation we have in mind is illustrated in Figure 1. The group O(18) breaks to O(10) $\times$ O(8) at a superheavy energy scale characterized by the vacuum expectation value $\langle\omega\rangle$. The family group O(8) then breaks to U(1) at a scale $\langle\psi\rangle$

---

* We forbid the degenerate cases $a = 0, c = 0$ and $a = \pm c$ for the first U(1), and $a = 0, a = \pm c$ and $a = -\frac{1}{3}c$ for the second.

of order M_{GUT}. The flavor symmetry is broken to SU(3) $\times$ SU(2) $\times$ U(1) at a similar scale. One might imagine that $\langle\omega\rangle \simeq \langle\psi\rangle$, although in principle $\langle\omega\rangle$ and $\langle\psi\rangle$ are distinct, and $\langle\omega\rangle$ could be as large as the Planck mass.

For the preceeding scenario to work, there must be directions in the symmetric product $(256 \times 256)_S$ which preserve O(10) $\times$ U(1). To verify this, we must find the O(10) $\times$ U(1) decomposition of

$$(256 \times 256)_S = [1] + [5] + [9] \quad . \tag{2}$$

In Eq. (2), the symbol $[m]$ denotes the (self-dual) antisymmetric tensor with m indices. It turns out that [1] contains a $(1,8_V)$ under O(10) $\times$ O(8), and [5] contains a $(1,56_V)$. From Table 2 we learn that the 8_V does not contain a singlet under either U(1). The 56_V, however, contains directions which preserve either U(1) for any choice of a and c. These directions develop vacuum expectation values when O(8) is broken to U(1). We see that it is always possible for four families to marry off at M_{GUT}, as shown in Figure 2, leaving four light generations protected by either U(1) for any choice of a and c.

Note that we cannot preserve the U(1) family symmetry down to the weak scale M_W. Otherwise, there would be gauge bosons mediating flavor-changing neutral currents. From experimental limits, we know that the family U(1) must be broken at a scale $\langle\chi\rangle \gtrsim 10^5$ GeV. This leads immediately to the question of what protects the left- and right-handed fermions from joining together once the U(1) family symmetry is broken. This problem may be avoided by arranging the Higgs χ to have charge n, incommensurate with the charges of any possible left-right mass terms. In this case the discrete symmetry $Z_n \subseteq$ U(1) continues to forbid left-right masses even though the family U(1) is broken. The scale $\langle\chi\rangle$ can lie anywhere between 10^5 GeV and $M_{GUT} \simeq 10^{15}$ GeV. The discrete symmetry Z_n can be broken at the weak scale M_W.†

A simple example would be to choose $a = 1$ and $c = 2$ in the first column of Table 2. The right-handed family charges are $\pm\frac{1}{2}$, $\pm\frac{7}{2}$, and the left-handed charges are given by $\pm\frac{5}{2}$, $\pm\frac{5}{2}$. Left-right masses are forbidden if the U(1)-breaking Higgs χ has a charge incommensurate with the integers 1, 2, 3, and 6. In this example, a typical Higgs χ might have charge 4, leaving a discrete Z_4 family symmetry. Note that in general the Z_n family symmetry does not necessarily forbid left-left or right-right masses below the weak scale M_W.

3. Splitting Left From Right

Having arranged our model in such a way that some families are heavy and some are light, we must still explain why the light right-handed families are heavier than their left-handed counterparts. We do this by ensuring that the Weinberg-Salam Higgs doublet ϕ_W couples only to right-handed families. This is easy to enforce if one considers the O(8) multiplication laws

$$\begin{aligned} 8' \times 8' &= 1 + 28 + 35' \\ 8'' \times 8'' &= 1 + 28 + 35'' \quad . \end{aligned} \tag{3}$$

If ϕ_W is contained in the $35''$, it couples only to right-handed fermions. Invoking the extended survival hypothesis [5], we assert ϕ_W is the *only* light Higgs doublet.‡ This guarantees that only the right-handed families get direct masses at the weak

† There should be no domain wall problems since Z_n is contained in O(18) [4].

‡ Each light Higgs requires a separate fine-tuning, so the extended survival hypothesis minimizes the number of unnatural fine-tunings [5].

scale. Masses for the left-handed families are suppressed since they are generated by radiative corrections.

As in Section 2, we must be sure that the appropriate Weinberg-Salam Higgs doublet lies in the symmetric product $(256 \times 256)_S$. It is straightforward to verify that appropriate Higgs representations lie in the [5] and [9] of O(18). In fact, the [5] contains a $(10,35'')$ under $O(10) \times O(8)$, and the [9] contains a $(\overline{126}, 35'')$. Higgs doublets pointing in either of these directions couple only to right-handed fermions.§

A schematic picture of the right-handed masses is shown in Figure 3. The Weinberg-Salam Higgs doublet ϕ_W develops a vacuum expectation value at the weak scale M_W. The right-handed families join with themselves and develop masses of order $\langle\phi_W\rangle$, so we expect the right-handed quark and lepton masses to lie between 100 and 1000 GeV.

The left-handed fermions cannot acquire masses directly from the weak doublet ϕ_W. They get their masses through radiative corrections. Contributions to the left-handed fermion masses are shown in Figure 4. It is easy to estimate the magnitude of these contributions, because if O(8) were unbroken, these diagrams must vanish. This implies that the left-left masses must be of order $(\alpha_{GUT}/2\pi)\, M_W \langle\psi\rangle^2/\langle\omega\rangle^2$, where $\alpha_{GUT} \simeq \alpha_S \simeq .1$. If $\langle\psi\rangle \simeq \langle\omega\rangle$, we have left-left masses of order $(\alpha_{GUT}/2\pi)\, M_W$, in reasonable accord with experiment.

§ Note that the $(120,35'')$ cannot give mass to the $(\overline{16}, 8'')$ because of Fermi statistics.

4. Splitting the Neutrinos

There are 32 neutrinos in the spinor of O(18). The 16 neutrinos which belong to superheavy generations get O(10) $\times$ U(1)-invariant masses and disappear from the low-energy spectrum. Of the remaining neutrinos, eight are SU(2) $\times$ U(1) singlets, and the rest are doublets. The eight singlets obtain masses at the O(10) $\supset$ SU(3) $\times$ SU(2) $\times$ U(1) $\times$ U(1)-breaking scale, of order $\langle\psi\rangle \simeq M_{GUT}$. This happens because the eight singlet neutrinos come in pairs with opposite family charges. At low energies, we are left with eight isodoublet neutrinos. The natural upper bound for the mass of these neutrinos is on the order of $M_W^2/\langle\psi\rangle \simeq 0.1$ eV.

Such a large number of light neutrinos is in conflict with the simplest version of standard big bang nucleosynthesis [6,7]. Large numbers of light neutrinos lead to excessively large primordial helium and deuterium abundances. This situation is illustrated in Figure 5, where we have graphed the primordial helium and deuterium abundances, $X(^4\mathrm{He})$ and $X(^2\mathrm{H})$, for two, four and eight light neutrinos.

The bulk of observational and theoretical analyses strongly suggest that $N_\nu \lesssim 4$. It is, of course, possible to evade this constraint. For example, the primordial mass fraction $X(^4\mathrm{He})$ might be as large as 0.30. Alternatively, post-nucleosynthesis decays might have caused significant photo- or neutrino- disintegration of deuterium [8]. Since the number of light neutrinos will soon be measured in Z^0 decays, we do not wish to exclude the possibility that $N_\nu = 8$.

If we take the standard nucleosynthesis constraints seriously, however, we are forced to consider at most four light ($\lesssim 1$ MeV) neutrinos. The remaining four neutrinos (if long-lived) must have masses higher than 2 GeV [9]. The simplest way to implement this constraint is to introduce an SU(2) isotriplet $\vec{\phi}$ which gives direct masses to the four SU(2)-doublet neutrinos in the four right-handed families. Such a field lives in the $35''$ of O(8) and the [9] of O(18). Then, as in the previous section, $\vec{\phi}$ couples directly to the right-handed doublet neutrinos, but only in higher orders to their left-handed counterparts. The right-handed doublet neutrinos gain a mass $m_{\nu_R} \simeq g\langle\vec{\phi}\rangle \gtrsim 2$ GeV, where g is a Yukawa coupling of order one.

Although $\vec{\phi}$ cannot couple directly to left-handed neutrinos, it can couple to them through loops, as shown in Figure 6. These graphs are further suppressed if the U(1) or Z_n family charge of $\vec{\phi}$ is incommensurate with various left-left charge combinations. The fact that the electron neutrino Majorana mass is less than 46 eV restricts the strength of the effective $\vec{\phi}\nu_e\nu_e$-coupling to be less than (46 eV/ 2 GeV) $\simeq 2 \times 10^{-8}$. The restrictions on the other diagonal and off-diagonal couplings of the $\vec{\phi}$ are not quite so severe. Note that the $\vec{\phi}$ couples trilinearly to the Weinberg-Salam doublet, $M_W \vec{\phi} \cdot \phi_W \vec{\tau} \phi_W$. This interaction explicitly breaks any effective low-energy lepton number symmetries, and avoids the problems associated with pseudo-Goldstone Majorons [10].

It is important to observe that there is also an *upper* bound on the mass of the right-handed doublet neutrinos. This stems from the fact that the ρ parameter is very close to 1, $\rho \equiv (m_W/m_Z\cos\theta_W)^2 = 1.02 \pm 0.02$, where m_W and m_Z are the W- and Z-boson masses, and θ_W is the weak mixing angle [11]. The fact that the ρ parameter is so close to 1 implies that $v \equiv \langle\vec{\phi}\rangle \lesssim 35$ GeV, or equivalently, $m_{\nu_R} \lesssim 35$ GeV. Therefore, if we take the cosmological constraints seriously, we are led to expect a spectrum of four right-handed neutrinos in the range 2 GeV $\lesssim m_{\nu_R} \lesssim 35$ GeV. The upper bound on m_{ν_R} can be violated if some Yukawa coupling is greater than one.

The heavy right-handed neutrinos should decay radiatively into the lighter right-handed neutrinos via the graphs of Figure 7. These graphs are not necessarily GIM-suppressed since the masses and splittings of the right-handed families are presumably comparable to the weak scale. If we make this assumption, we find the lifetimes of the heavy right-handed neutrinos to be of order $\tau \simeq (\pi^4/\alpha^5) M_W^4/m_{\nu_R}^5$. For m_{ν_R} between 2 and 35 GeV, τ ranges from 10^{-4} to 10^{-10} seconds. Of course, it is possible that the Z_n symmetry might suppress these decays, either directly because of its quantum numbers, or indirectly through mass degeneracies. In any case, it is important to emphasize that the upper bound of 35 GeV on these neutrinos is less than half the Z^0 mass, so they should be seen in Z^0 decays. If they decay as in Figure 7, they yield dramatic signatures of the form $Z^0 \to \nu'_R \bar{\nu}'_R \to \nu_R \bar{\nu}_R \gamma \gamma$, with acoplanar hard photons and lots of missing energy.

The SU(2) isotriplet Higgs that we introduced to give mass to the right-handed doublet neutrinos contains a doubly charged scalar ϕ^{++}, a singly charged scalar ϕ^{+}, and a neutral component ϕ^0. These fields should all have masses between tens and hundreds of GeV. The scalar masses are more uncertain than the neutrino masses because they are more dependent on the parameters of the Higgs potential. The scalars $\vec{\phi}$ have important experimental consequences, and give rise to dramatic signatures in high energy e^+e^- and hadronic machines (see, for example, Figure 8). The ϕ^{++} should be especially spectacular because of its double charge. If any of the scalars are lighter than half the Z^0 mass, they should also contribute to the Z^0 width.

The decays of the scalars $\vec{\phi}$ depend crucially on their unknown masses and couplings. If they are sufficiently heavy, they should decay into right-handed leptons, $\phi^{++} \rightarrow L^+ L^+$, $\phi^+ \rightarrow L^+ \nu_R$ and $\phi^0 \rightarrow \nu_R \nu_R$, or into each other, $\phi^{++} \rightarrow \phi^+ W^+$, $\phi^+ \rightarrow \phi^0 W^+$, etc., with partial widths Γ of order $(g^2/4\pi) m_{\vec{\phi}}$. If phase space forbids these decays, then the $\vec{\phi}$ widths should be much narrower, since the couplings of the $\vec{\phi}$ to the left-handed leptons are much smaller. The partial width of $\phi^{++} \rightarrow e^+ e^+$ is of order $\Gamma \simeq 10^{-16} m_{\phi^{++}}$. Even for $m_{\phi^{++}} \simeq 100$ GeV, this gives a long partial lifetime, of order 10^{-10} seconds, with a track length of about 3 cm. The partial widths of $\phi^{++} \rightarrow \tau^+ \tau^+$, $\mu^+ \mu^+$, $\tau^+ \mu^+$, etc. could be substantially larger.

It is amusing to reflect that the existence of these new relatively light doubly- and singly-charged scalars was suggested by big bang nucleosynthesis. However, whether or not we believe in cosmology, we predict that eight neutrinos contribute to the width of the Z^0. If all eight neutrinos are ultralight ($\lesssim 1$ MeV), nucleosynthesis constraints need to be reexamined. If only four are ultralight, there should also be Higgs scalars ϕ^{++}, ϕ^+ and ϕ^0, as well as four right-handed neutrinos, with masses between 2 and 35 GeV. The upper bound of 35 GeV comes from the fact that the ρ parameter is close to one.

5. Decays of the Right-Handed Families

According to the standard scenario [12], the observed excess of matter over antimatter is generated at energies on the order of the grand unified scale M_{GUT}. At such high energies, our theory includes both left- and right-handed families, which can be transformed into each other by O(18) gauge transformations. This leads us to expect comparable amounts of left- and right-handed matter in the early universe. Since no right-handed matter has ever been seen, we must ensure that the right-handed particles decay rapidly into their left-handed counterparts.

We shall first discuss the fate of the right-handed charged leptons. Depending on their mass, the right-handed charged leptons should decay either via the graphs of Figure 9*a* (if they are heavy) or via the graphs of Figure 9*b* (if they are light). The lifetimes for these decays are less than 10^{-21} seconds.

The right-handed quarks should decay into left-handed quarks and pairs of Higgs scalars via the graphs of Figure 10*a*. Note that the Higgs scalars must carry net Z_n charge. If the intermediate scalar has a mass of order 10^{14} GeV, the induced dimension-five operator gives a lifetime of order $\tau \gtrsim (10^{14}\ \text{GeV})^2/(10^2\ \text{GeV})^3 \simeq 10^{22}\ \text{GeV}^{-1} \simeq 10^{-2}$ seconds. These decays occur when the temperature of the universe is about 10 MeV, so they do not affect nucleosynthesis. (The Higgs scalars decay immediately into fermions, and a slight reheating occurs.) The right-handed quarks can also decay via the dimension-six operators of Figure 10*b*. The rates for these processes compete with the dimension-five operators of Figure 10*a* if the intermediate particle has a mass of about 10^8 GeV.

The right-handed neutrinos should first decay radiatively into their lightest right-handed partner through the graphs of Figure 7. The lightest partner can be stable or almost stable since its mass exceeds the Lee-Weinberg bound. One way in which it might decay is through its Cabibbo mixings with the left-handed neutrinos. Such mixings can be caused by the Z_n-breaking graph of Figure 11. The Cabibbo mixing induced by this graph is very small; for $m_{\nu_R} = 10$ GeV,

$$\theta_C \simeq \frac{M_W^2}{m_{\nu_R} M_{GUT}} \simeq \frac{10^5}{10 \cdot 10^{14}} \simeq 10^{-10} \quad . \tag{4}$$

This implies a lifetime for $\nu_R \to \ell^+ \, \ell^- \, \nu_\ell$ or $\nu_\ell \, \bar{\nu}_{\ell'} \, \nu_{\ell'}$ of order

$$\begin{aligned} \tau &\simeq 2 \times 10^{-6} \text{ sec} \left(\frac{m_\mu}{10 \text{ GeV}}\right)^5 \left(\frac{1}{10^{-10}}\right)^2 \\ &\simeq 2 \times 10^4 \text{ sec} \quad . \end{aligned} \tag{5}$$

These decays occur when the temperature of the universe is about 10 keV and are harmless since the ν_R's are dilute.*

* It is interesting to note that these late decays will disintegrate some of the deuterium that that had been synthesized in an earlier epoch. This should be a small effect.

6. Experimental Signatures

In this section we shall summarize some of the main experimental signatures of our model.

(1) Eight light neutrinos should be discovered in Z^0 decays. Four should be left-handed, with masses less than 1 MeV. The other four should be right-handed, with masses between 2 and 35 GeV. The right-handed neutrinos should decay radiatively into each other. For example, the cascade

$$Z^0 \to \nu'_R + \bar{\nu}'_R \to \nu_R \gamma + \bar{\nu}_R \gamma \tag{6}$$

should yield two hard photons and lots of missing energy. The lifetime of the ν'_R should be of order 10^{-4} to 10^{-10} seconds. Thus it is possible that the ν'_R might travel a macroscopic distance before it decays.

(2) In very high energy e^+e^- machines, one would expect to see

$$e^+e^- \to \phi^{++} + \phi^{--} \quad . \tag{7}$$

If the ϕ^{++} is sufficiently heavy, it should decay into $\phi^+ W^+$, $\phi^+ \ell^+ \bar{\nu}_\ell$, etc., or into L^+L^+ pairs. If it is light, it could be very narrow and travel a macroscopic distance before decaying into an ordinary charged lepton pair. If $m_{\phi^{++}} \lesssim 45$ GeV, the ϕ^{++} should be seen in Z^0 decays.

(3) The four right-handed charged leptons should be produced directly in e^+e^- machines, or possibly as decay products of the ϕ^{++}. The right-handed charged leptons decay immediately via

$$L^+ \to \nu_R + W^+ \tag{8a}$$

or

$$L^+ \to \nu_R + \ell^+ + \bar{\nu}_\ell \quad , \tag{8b}$$

with very short lifetimes of less than 10^{-21} seconds.

(4) The eight right-handed quarks should be produced in high energy hadronic machines. They should decay weakly into each other,

$$Q'_R \to Q_R + W \tag{9a}$$

or

$$Q'_R \to Q_R + q_L \bar{q}_L, \quad Q'_R \to Q_R + \ell^+ \bar{\nu}_\ell, \quad \text{etc.,} \tag{9b}$$

until they cascade down to their lightest right-handed partner. The lightest right-handed quark should be relatively long-lived ($\tau \gtrsim 10^{-2}$ sec) and should decay via the graph of Figure 10*a*,

$$Q_R \to q_L + \phi\phi' \quad , \tag{10}$$

where ϕ and ϕ' are generic light Higgs scalars.

Acknowledgments

It is a pleasure to thank Howard Georgi for several illuminating conversations, and especially for pointing out the existence of the U(1) family symmetries. We would also like to thank Richard Slansky for teaching us about maximal little groups. We have also benefited from discussions with R. Bond, M. Claudson, L. Krauss, S. Raby, G. Senjanovic, M. Turner and R. Wagoner.

Appendix: Splitting the Spinor

In this Appendix we classify all continuous family symmetries $H \subseteq O(8)$ that protect some left- and right-handed fermions from gaining masses at the grand unified scale M_{GUT}. We find all subgroups H of O(8) subject to the following conditions:

- The subgroups H must ***stabilize*** the product $8' \times 8'' = 8_V + 56_V$. In other words, the 8_V or the 56_V must contain a singlet under H.
- The O(8) representation $8' + 8''$ must decompose into a ***complex*** representation of H.

This task is best approached by examining the maximal little groups of the 8_V and the 56_V. A maximal little group G of a representation R is defined to be a little group of R which is not contained in any larger little group of R [13]. Clearly, if the O(8) representation $8' + 8''$ decomposes into a real representation of G, it decomposes into a real representation of any subgroup of G as well. Therefore we are primarily interested in finding all maximal little groups of the 8_V and the 56_V under which the O(8) representation $8' + 8''$ is complex.

The only maximal little group of the 8_V is O(7). Under O(7), the $8'$ and the $8''$ both decompose into a real spinor 8. Since $8 + 8$ is a real representation of O(7), we cannot use this group or any of its subgroups to protect light fermions.

The maximal little groups of the 56_V are collected in Table 3, along with the decompositions of the $8'$ and the $8''$. From Table 3 we see that the only maximal little groups with complex fermions are SU(3) $\times$ U(1) and U(1). The group SU(3) $\times$ U(1) leaves six light families, and the U(1) leaves eight. Considering all possible subgroups of these groups leads directly to the family symmetries listed in Table 1.

References

1. H. Georgi and S. Glashow, Phys. Rev. Lett 32 (1974) 438; J. Pati and A. Salam, Phys. Rev. D10 (1974) 275.

2. M. Gell-Mann, P. Ramond and R. Slansky, in P. van Nieuwenhuizen and D. Freedman, eds., *Supergravity* (North-Holland, Amsterdam, 1979); F. Wilczek and A. Zee, Phys. Rev. D25 (1982) 553.

3. H. Georgi, Nucl. Phys. B156 (1979) 126.

4. G. Lazarides and Q. Shafi, Phys. Lett. 115B (1982) 21.

5. S. Dimopoulos and H. Georgi, Stanford preprint ITP-759 (1984).

6. R. Wagoner, W. Fowler and F. Hoyle, Ap. J. 148 (1967) 3; V. Shvartsman, JETP Lett. 9 (1969) 184; R. Wagoner, Ap. J. 179 (1973) 343; D. Schramm and R. Wagoner, Ann. Rev. Nucl. Sci. 27 (1977) 37.

7. J. Yang, D. Schramm, G. Steigman and R. Rood, Ap. J. 227 (1979) 697; K. Olive, D. Schramm, G. Steigman, M. Turner and J. Yang, Ap. J. 246 (1981) 557; J. Yang, M. Turner, G. Steigman, D. Schramm and K. Olive, Enrico Fermi Institute preprint 83-43 (1983).

8. M. Turner, private communication.

9. B. Lee and S. Weinberg, Phys. Rev. Lett. 39 (1977) 165; L. Krauss, Phys. Lett. 133B (1973) 169.

10. G. Gelmini and M. Roncadelli, Phys. Lett. 99B (1981) 411; H. Georgi, S. Glashow and S. Nussinov, Nucl. Phys. B193 (1981) 297.

11. W. Marciano and A. Sirlin, Brookhaven preprint BNL-33819 (1983).

12. A. Sakharov, JETP Lett. 5 (1967) 24; M. Yoshimura, Phys. Rev. Lett. 41 (1978) 281; (E) 42 (1979) 740; S. Dimopoulos and L. Susskind, Phys. Rev.

D18 (1978) 4500; J. Ellis, M. Gaillard and D. Nanopoulos, Phys. Lett. 80B (1979) 360; D. Toussaint, S. Treiman, F. Wilczek and A. Zee, Phys. Rev. D19 (1979) 1036; S. Weinberg, Phys. Rev. Lett. 42 (1979) 850.

13. R. Slansky, Phys. Reports 79 (1981) 1.

Table Captions

1. All possible continuous family symmetries $H \subseteq O(8)$ that protect some light families from acquiring mass at the grand unified scale M_{GUT}.

2. The charges of various O(8) representations under the two U(1) symmetries which leave four light left-right families. The values of a and c are determined by the vacuum expectation values of certain Higgs fields. We will not address the question of what Higgs potential is required to break O(8) down to one of the U(1)'s. However, any such Higgs potential will restrict a and c to be integers. Note that it is also possible for the left- and right-handed charges to be interchanged.

3. Decompositions of the $8'$ and $8''$ under the maximal little groups of the 56_V.

Table 1

Family Group H	Representations of Light Fermions	
O(8)	$8'$	$8''$
SU(3) $\times$ U(1)	$(3, a),\ (\bar{3}, -a)$	$(3, -a),\ (\bar{3}, a)$
SU(2) $\times$ U(1)	$(2, a+b),\ (2, -a-b),$ $(1, a-2b),\ (1, -a+2b)$	$(2, -a+b),\ (2, a-b),$ $(1, -a-2b),\ (1, a+2b)$
U(1)	$\pm(a+b),\ \pm(a-b),$ $\pm(c+d),\ \pm(c-d)$	$\pm(b+c),\ \pm(b-c),$ $\pm(a+d),\ \pm(a-d)$
U(1)	$\pm\frac{1}{2}(3a+c),\ \pm(c+d),$ $\pm(c-d)$	$\pm\frac{1}{2}(3c+a),\ \pm(a+d),$ $\pm(a-d)$
U(1)	$\pm\frac{1}{2}(3a+c),\ \pm\frac{1}{2}(a-3c)$	$\pm\frac{1}{2}(3a-c),\ \pm\frac{1}{2}(a+3c)$
U(1)	$\pm\frac{1}{2}(5a+3c),\ \pm\frac{1}{2}(5a-c)$	$\pm\frac{1}{2}(a+3c),\ \pm\frac{1}{2}(7a+c)$

Table 2

O(8) Representation	U(1) Charges	
$8'$	$\pm\frac{1}{2}(3a+c),\ \pm\frac{1}{2}(a-3c),$ $\pm\frac{1}{2}(a+c),\ \pm\frac{1}{2}(a-c)$	$\pm\frac{1}{2}(5a+3c),\ \pm\frac{1}{2}(5a-c),$ $\pm\frac{1}{2}(3a+c),\ \pm\frac{1}{2}(a-c)$
$8''$	$\pm\frac{1}{2}(3a-c),\ \pm\frac{1}{2}(a+3c),$ $\pm\frac{1}{2}(a+c),\ \pm\frac{1}{2}(a-c)$	$\pm\frac{1}{2}(a+3c),\ \pm\frac{1}{2}(7a+c),$ $\pm\frac{1}{2}(3a+c),\ \pm\frac{1}{2}(a-c)$
8_V	$\pm a,\ \pm c,\ \pm(a+c),\ \pm(a-c)$	$\pm 2a,\ \pm(a+c),\ \pm(a-c),\ \pm(3a+c)$
$35'$	$0,\ \pm a,\ \pm c,\ \pm 2a,\ \pm 2c,$ $\pm(a+c),\ \pm(a-c),\ \pm(2a+c),$ $\pm(2a-c),\ \pm(a+2c),\ \pm(a-2c),$ $\pm(3a+c),\ \pm(a-3c)$	$0,\ \pm 2a,\ \pm 2c,\ \pm 4a,$ $\pm(a+c),\ \pm(a-c),\ \pm(3a+c),$ $\pm(3a-c),\ \pm(5a+c),\ \pm(5a-c),$ $\pm(5a+3c),\ \pm 2(a+c),\ \pm 2(2a+c)$
$35''$	$0,\ \pm a,\ \pm c,\ \pm 2a,\ \pm 2c,$ $\pm(a+c),\ \pm(a-c),\ \pm(2a+c),$ $\pm(2a-c),\ \pm(a+2c),\ \pm(a-2c),$ $\pm(3a-c),\ \pm(a+3c)$	$0,\ \pm 2a,\ \pm 2c,\ \pm 4a,$ $\pm(a+c),\ \pm(a-c),\ \pm(3a+c),$ $\pm(3a-c),\ \pm(5a+c),\ \pm(7a+c),$ $\pm(a+3c),\ \pm 2(a+c),\ \pm 2(2a+c)$
56_V	$0,\ \pm a,\ \pm c,\ \pm 2a,\ \pm 2c,$ $\pm 3a,\ \pm 3c,\ \pm(a+c),\ \pm(a-c),$ $\pm(a+2c),\ \pm(a-2c),\ \pm(2a+c),$ $\pm(2a-c),\ \pm 2(a+c),\ \pm 2(a-c)$	$0,\ \pm 2a,\ \pm 2c,\ \pm 4a,\ \pm 6a,$ $\pm(a+c),\ \pm(a-c),\ \pm(3a+c),$ $\pm(3a-c),\ \pm(5a+c),\ \pm 2(3a+c),$ $\pm 2(a+c),\ \pm 2(a-c),\ \pm 3(a+c),$ $\pm 2(2a+c)$

Table 3

Maximal Little Group	Decomposition of $8'$	Decomposition of $8''$	Complex?
SU(3)	8	8	no
G(2)	7+1	7+1	no
SU(2) $\times$ Sp(4)	(2,4)	(2,4)	no
SU(3) $\times$ U(1)	$(3,a)+(\bar{3},-a)+(1,a)$ $+(1,-a)$	$(3,a)+(\bar{3},-a)+(1,a)$ $+(1,-a)$	no
SU(3) $\times$ U(1)	$(3,a)+(\bar{3},-a)+(1,a)$ $+(1,-a)$	$(3,-a)+(\bar{3},a)+(1,a)$ $+(1,-a)$	yes
U(1)	$\pm(a+b),\ \pm(a-b),$ $\pm(c+d),\ \pm(c-d)$	$\pm(b+c),\ \pm(b-c),$ $\pm(a+d),\ \pm(a-d)$	yes

Figure Captions

1. A schematic picture illustrating the hierarchy of symmetry breakings. The vacuum expectation value $\langle\chi\rangle$ breaks U(1) to Z_n at a scale greater than 10^5 GeV.

2. Half of the left-right generations join together and gain masses at the grand unified scale M_{GUT}.

3. The remaining right-handed families acquire masses at the weak scale $\langle\phi\rangle \simeq M_W$. The Weinberg-Salam Higgs ϕ_W does not couple directly to left-handed fermions.

4. The left-handed families obtain masses through mixings with the right-handed families. Dominant contributions include the loop graphs illustrated here.

5. The predicted mass fractions $X(^2\mathrm{H})$ and $X(^4\mathrm{He})$ for 2, 4 and 8 light neutrinos. The curves vary with the baryon-to-photon ratio η. The popular limits for primordial helium and deuterium production indicated. These graphs are compiled from the figures in Ref. [7].

6. The left-handed doublet neutrinos get masses through radiative corrections. The left-handed masses are suppressed by $(\alpha_{GUT}/2\pi)\langle\psi\rangle^2/\langle\omega\rangle^2$. The electron and muon neutrino masses must be further suppressed by powers of the U(1) or Z_n breaking scales.

7. The heavier right-handed neutrinos should decay into photons and lighter right-handed neutrinos through magnetic-moment couplings like these. There are also one-loop Higgs diagrams whose magnitudes depend on the parameters of the Higgs potential.

8. Dramatic signatures of O(18) include (*a*) the production of $\phi^{++}\phi^{--}$ pairs in high energy e^+e^- interactions, and (*b*) $u\bar{d} \to \phi^{++}\phi^{=}$ in high energy hadronic colliders.

9. Depending on their masses, the right-handed charged leptons decay into right-handed neutrinos and either real or virtual W's.

10. The lightest right-handed quarks decay into left-handed quarks and either scalars or fermions via dimension five or six operators. Diagram (*a*) dominates for intermediate masses greater than about 10^8 GeV.

11. A typical diagram mixing left- and right-handed neutrinos.

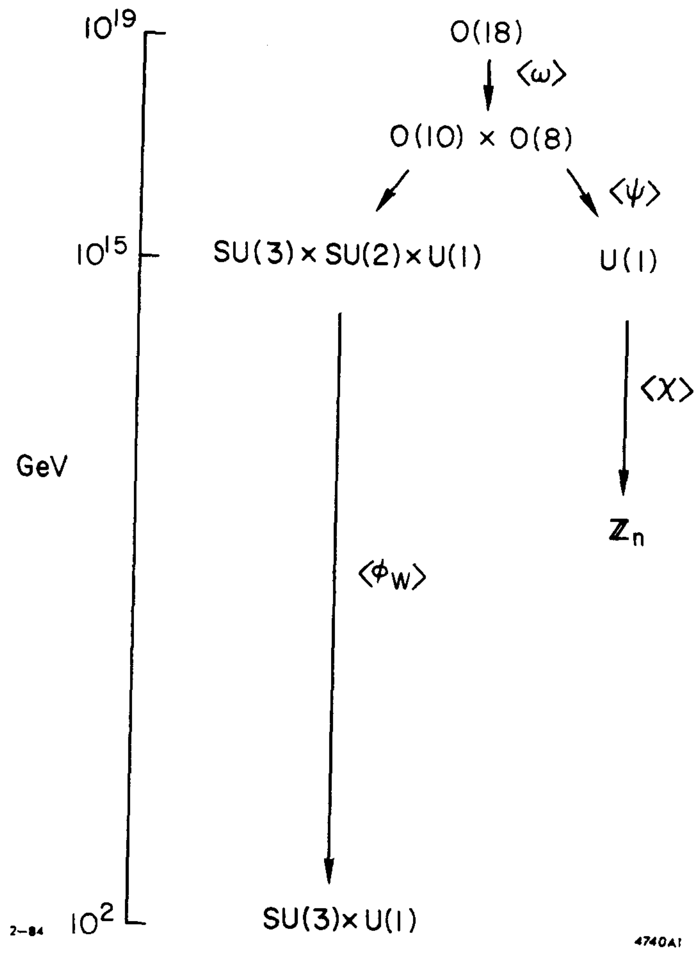

10^{19}
10^{15}
GeV
10^2
2-84
O(18)
$\langle\omega\rangle$
O(10) × O(8)
$\langle\psi\rangle$
SU(3) × SU(2) × U(1)
U(1)
$\langle\chi\rangle$
$\mathbb{Z}_n$
$\langle\phi_W\rangle$
SU(3) × U(1)
4740A1

Fig. 1

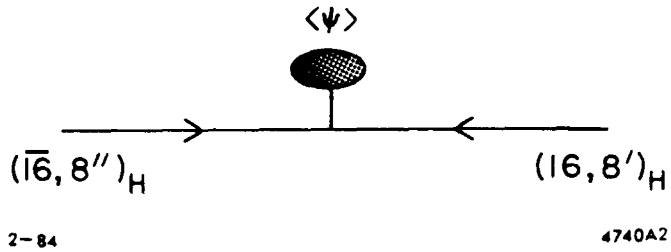

⟨ψ⟩
$(\overline{16}, 8'')_H$
$(16, 8')_H$
2-84
4740A2

Fig. 2

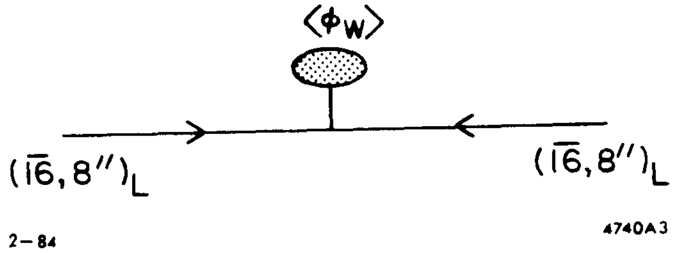

⟨ϕ_W⟩
$(\overline{16}, 8'')_L$
$(\overline{16}, 8'')_L$
2-84
4740A3

Fig. 3

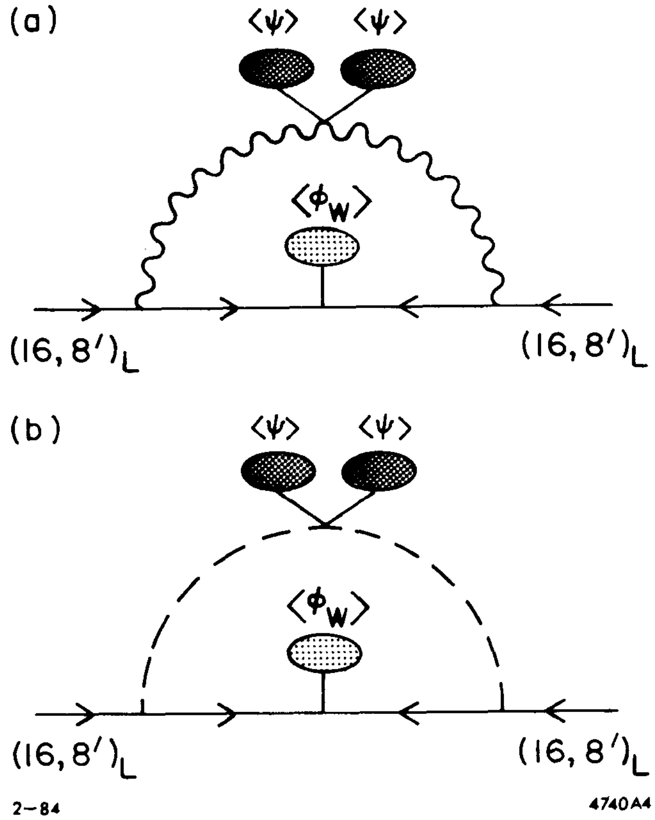
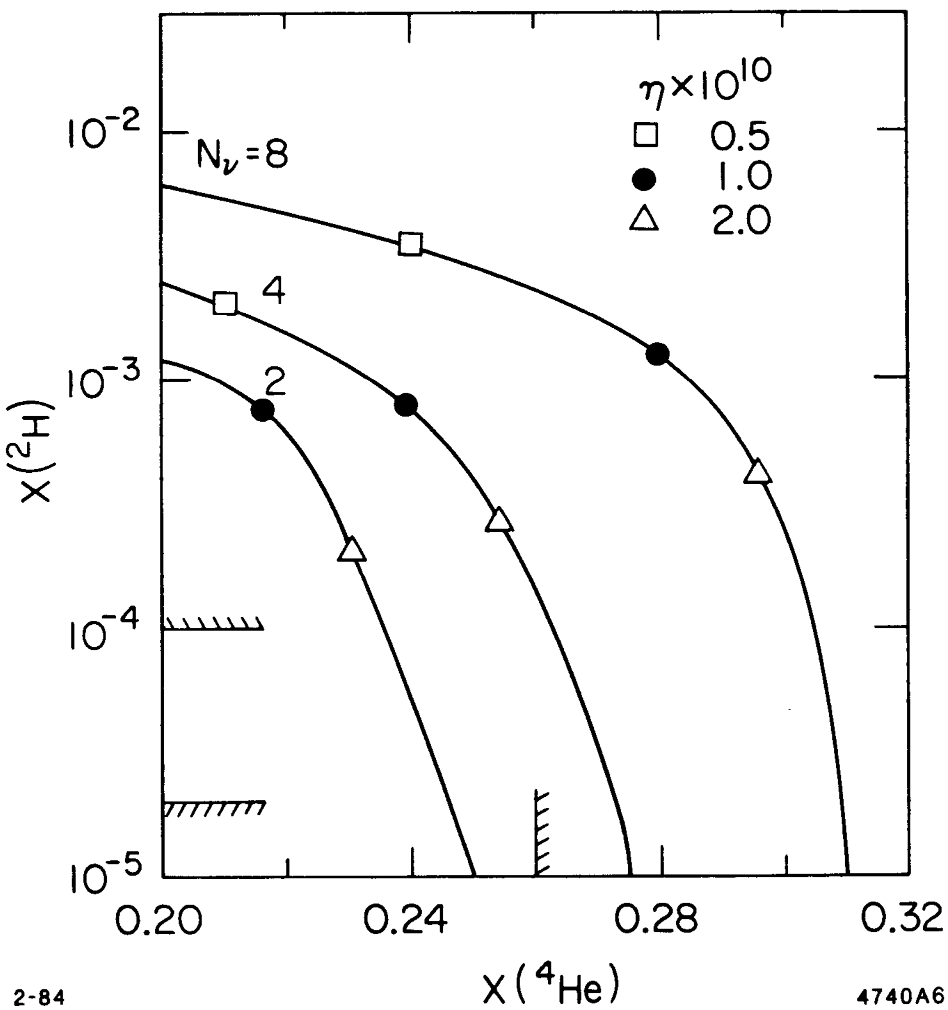


Fig. 4



$\eta \times 10^{10}$
0.5
1.0
2.0
$N_\nu = 8$
4
2
10^{-2}
10^{-3}
10^{-4}
10^{-5}
$X(^2H)$
0.20
0.24
0.28
0.32
$X(^4He)$
2-84
4740A6

Fig. 5

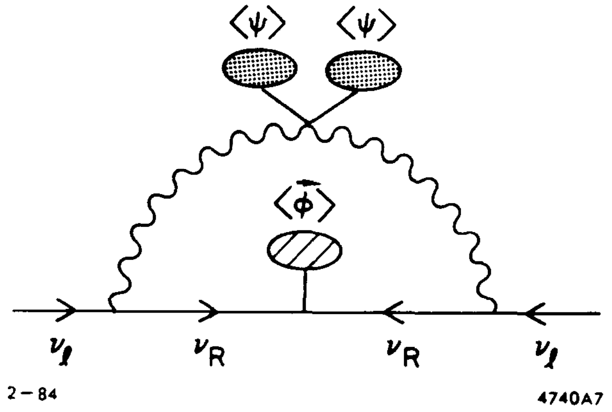

$\langle\psi\rangle$
$\langle\psi\rangle$
$\langle\vec{\phi}\rangle$
ν_ℓ
ν_R
ν_R
ν_ℓ
2-84
4740A7

Fig. 6

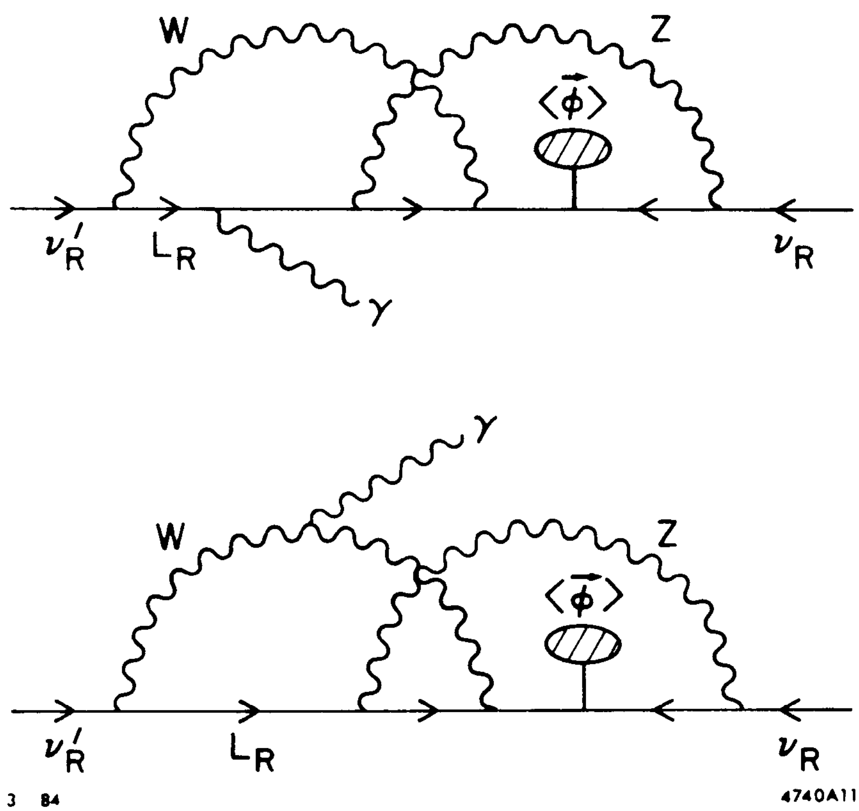

W
Z
$\langle\vec{\phi}\rangle$
ν_R'
L_R
ν_R
γ
γ
W
Z
$\langle\vec{\phi}\rangle$
ν_R'
L_R
ν_R
3 84
4740A11

Fig. 7

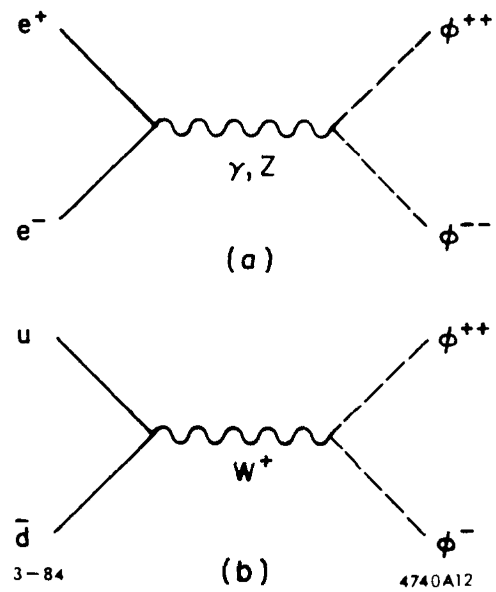
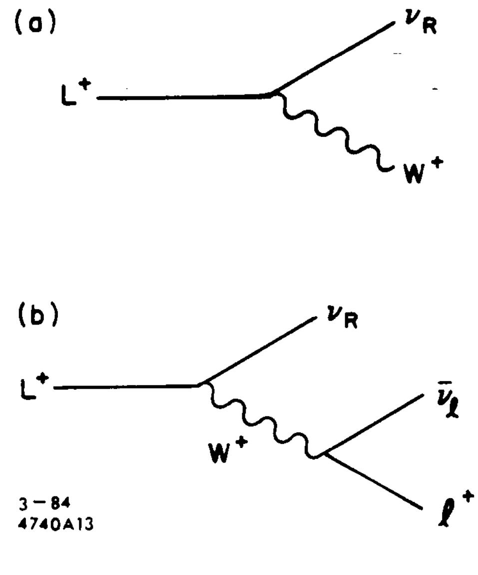
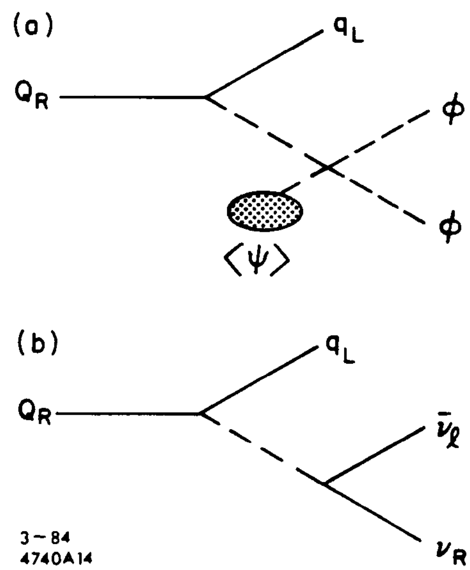


Fig. 8



(a)
ν_R
L^+
W^+
(b)
ν_R
L^+
$\bar{\nu}_\ell$
W^+
ℓ^+
3-84
4740A13

Fig. 9



(a)
q_L
Q_R
ϕ
ϕ
$\langle\psi\rangle$
(b)
q_L
Q_R
$\bar{\nu}_\ell$
ν_R
3-84
4740A14

Fig. 10

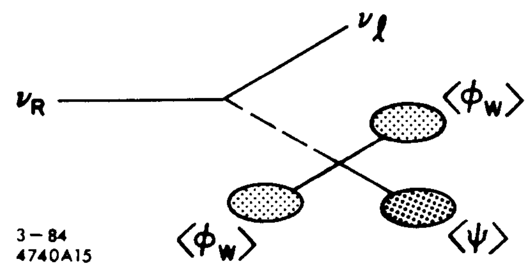

ν_ℓ
ν_R
$\langle\phi_W\rangle$
$\langle\phi_W\rangle$
$\langle\psi\rangle$
3 – 84
4740A15

Fig. 11