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EXCITATION OF SIMPLE ATOMS BY SLOW MAGNETIC MONOPOLES*

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ABSTRACT

We present a theory of excitation of simple atoms by slow moving massive monopoles. Previously presented results for a monopole of Dirac strength on hydrogen and helium are reviewed. The hydrogen theory is extended to include arbitrary integral multiples of the Dirac pole strength. The excitation of helium by double strength poles and by dyons is also discussed. It is concluded that a helium proportional counter is a reliable and effective detector for monopoles of arbitrary strength, and for negatively charged dyons.

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I. INTRODUCTION

As discussed by Drell et al.[1] (hereafter referred to as DKMPR), massive Dirac monopoles have a large effect on atomic energy levels, and can cause degeneracy or near degeneracy between the ground state and excited states of the atom. This phenomenon leads to greatly enhanced excitation cross sections for slow moving monopoles. For simple atoms the effect can be reliably calculated, thus providing the possibility of reliable low $\beta = v/c$ detection. What is believed to be a quite accurate calculation has been carried out for a minimum strength monopole in hydrogen, and a calculation of uncertain accuracy has been carried out for such a monopole in helium.

In the following we shall review the method and results of DKMPR, discuss some further investigation of the hydrogen case which includes an extension to monopoles of arbitrary mutiples of the Dirac charge, and describe work in progress which should lead to comparably accurate predictions for helium. We shall also discuss the interaction of double strength poles and dyons with helium. Finally, we shall conclude with a few comments about the potential utility of other noble gas atoms as monopole detectors.

II. THE BASIC MECHANISM AND COMPUTATIONAL STRATEGY

In the following we treat the monopole as infinitely massive, and, until we discuss recoil effects, the nucleus as infinitely massive. We use

non-relativistic theory and assume the normal Dirac magnetic moment for the electrons. The Hamiltonian for an atom may then be written

$$H = \sum_{i=1}^{z} \frac{(p_i - A_i)^2}{2m} - \frac{e}{2m} \underset{\sim}{\sigma}_i \cdot \underset{\sim}{B}_i + U(r_1 \ldots r_z) \tag{2.1}$$

In (2.1) we have

$$\underset{\sim}{B}_i = \frac{g \hat{r}_i}{r_i^2} \tag{2.2}$$

$$\underset{\sim}{A}_i = g \hat{\phi}_i \frac{(\pm 1 - \cos \theta_i)}{r_i \sin \theta_i} \tag{2.3}$$

The two signs in (2.3) refer to alternate gauges[2] and (r_i, θ_i, ϕ_i) are spherical coordinates for the i^{th} electron relative to the pole. Finally, U represents the electrostatic interaction between the electrons and between the electrons and nucleus.

The angular momentum operator for this system may be written[3]

$$J = \sum_i \left(\underset{\sim}{r}_i \times (\underset{\sim}{p}_i - e\underset{\sim}{A}_i) + \frac{1}{2} \underset{\sim}{\sigma}_i - q\hat{r}_i \right) \tag{2.4}$$

where $q = eg$ must be an integer or half integer. The term $\underset{\sim}{r}_i \times (\underset{\sim}{p}_i - e\underset{\sim}{A}_i)$ $= m\underset{\sim}{r}_i \times \dot{\underset{\sim}{r}}_i$ is the mechanical orbital angular momentum while $-q\hat{r}_i$ represents the angular momentum associated with the electron's electrostatic field crossed with magnetic field of the pole.[4]

Because of the coordinates we have used in the above equations, and also because the pole is assumed to be much more massive than the atom,

it is convenient to work in a reference frame in which the pole is at rest. If the atom impinges on the monopole with zero impact parameter so that the monopole passes through it (i.e. the nuclear coordinate z_N goes from $-\infty$ to $+\infty$) J_z is conserved. Since, however, the z component of field angular momentum changes from $+qZ$ to $-qZ$ in this process, the mechanical angular momentum and spin, which we identify with the atomic angular momentum when the atom is outside the range of the pole's magnetic field, must change so as to compensate. Thus J_z (atom) increases by $2qZ$ as a result of the collision. Let us suppose that the incident atom has atomic spin zero, that is to say the atomic ground state has zero angular momentum (as would be the case for a noble gas atom). Then no matter how slowly the collision takes place, after it is over the atom must be in a state of angular momentum greater or equal (typically equal) to $|2qZ|$, and hence in an excited state. Even if J (atom) is not zero for the ground state, some of the magnetic substates must become excited. For example, if the incident atomic spin is 1/2 then half the atoms are excited for $|2qZ| = 1$, and all for any larger value.

To see what happens when the impact parameter is nonzero, we first note that when the pole and nucleus coincide, all three components of J commute with the Hamiltonian and hence the states may be characterized as eigenstates of $\underset{\sim}{J} \cdot \underset{\sim}{J}$ with eigenvalue $\sqrt{J(J+1)}$ and a $2J+1$ degeneracy. Thus the atomic spin zero incident atom finds itself in a state of degeneracy of at least (typically, precisely) $|2qZ| + 1$ as it passes over the nucleus. In the case of nonzero impact parameter it is convenient to use a time dependent coordinate

system in which the nucleus is on the negative z axis. In that case the instantaneous states are still characterized by the eigenvalues of J_z. Furthermore, for adiabatic motions transition between states of different J_z will be very improbable unless degeneracy or near degeneracy occurs. (Note that constant J_z here means that the atom remains in its ground state. This differs from our previous discussion because here J_z refers to a z axis which has a different direction before and after the collision.) The previous discussion, however, tells us that for small impact parameters the spin zero incoming state will become nearly degenerate with the $|2qZ|$ partners which it would have at the center. Quasi-adiabatic transfers to these states become probable and lead to excitation. Additional degeneracies will occur if the $z_N = 0$ state to which the incident ground state connects is not the ground state of the $z_N = 0$ Hamiltonian. These additional degeneracies, which we refer to as off center level crossings, occur for a range of impact parameters, and can provide an additional source of excitation. They may enhance the excitation cross sections for double strength poles on He, and they are expected to be important for the heavier noble gas atoms. For non-zero incident spin the situation is similar but different magnetic substates must be treated separately as they connect with different $z_N = 0$ states, and the degree of degeneracy of the $z_N = 0$ state varies with magnetic substate.

With the above picture in mind, a general strategy for calculating excitation cross sections may be described.

(1) The $z_N = 0$ low lying energy level system must be established. This is a relatively simple task for one and two electron atoms. A

substantial effort would be involved to do it reliably for such things as Ne and A. Connections between these states and the states at $z_N = \pm \infty$ can be established by assuming that states having the same eigenvalues of J_z do not cross. These connections are sufficient to establish whether or not there are off center crossings and to determine which states will be excited.

(2) Excitation transitions via the central multiplet can be calculated by means of a simple extension of the Landau-Zener theory.[5] To obtain quantitative results for a given distance of closest approach, and velocity at that distance, one requires only a knowledge of the central multiplet splitting which occurs at distances of that order. We assume here that these transitions are probable only if the splitting is small, and hence linear in the separation. It can therefore be obtained by applying first order perturbation theory to the central multiplet, provided a sufficiently accurate form for the wave function of the central multiplet can be obtained.

(3) Excitation via off center crossings are significant only if $\Delta J_z = \pm 1$ for the crossings, a circumstance which has, among the cases we have studied, has occurred only for double strength poles on He. Such crossings are also expected to occur in more complex systems such as Ne and A. They are most likely to lead to transitions when the distance of closest approach is equal to the crossing distance. Hence the determination of this distance and the wave function is needed, as well as an appropriate treatment of the differential equation which couples the states.

(4) Since we are treating the orbit of the incoming atom classically, we can use the dependence of the energy level on z_N to determine an

effective potential between the atom and the pole. For excitation via the central multiplet, one is primarily interested in relating the distance of closest approach and the atomic velocity at that distance to the velocity and impact parameter at $z_N = -\infty$. This can be determined to a good approximation from the information already determined in (1) and (2), and together with (1) and (2) provides a practical procedure for determining the excitation cross sections. More detailed information about the energy as a function of z_N is likely to be required to discuss excitation via off center crossings.

The general procedure described above will be illustrated and further explained in the applications to be described in subsequent sections. Before doing so, however, we explain the $\Delta J_z = \pm 1$ selection rule for off center crossings. We first imagine the eigenvalues and wave functions of the Hamiltonian with the nucleus on the negative z axis to be known. The states are also eigenstates J_z, and as mentioned before, states of the same J_z will not cross so that a unique energy $E_n(z_N)$ can be defined for each state n. We next consider the case of the nucleus moving in some orbit in the $x = 0$ plane, and define a primed coordinate system whose $x' = 0$ plane coincides with the $x = 0$ plane (i.e. $x' = x$) and whose z axis points from the pole to the nucleus. The Hamiltonian in these primed coordinates has the same form as (2.1) with nuclear coordinate $\underset{\sim}{r}'_N = (0, 0, -z_N)$. In these variables the Hamiltonian has an explicit time dependence due to the variation of z_N with time and an implicit time dependence due to the time dependence of y' and z' with reference to a fixed y, z. A similar remark applies to the wave function. Now write $\psi = \Sigma C_m(t)\psi_n(\underset{\sim}{r}', z_N)e^{-i\int E_n\,dt}$, and insert in the time

dependent Schroedinger equation to obtain[6]

$$\frac{dC_n}{dt} = -\sum_m \left(\psi_n, \frac{d\psi_m}{dt}\right) e^{-i\int (E_m - E_n)dt} C_m \qquad (2.5)$$

The quasi adiabatic approximation[7] consists of restricting the values of n, m to refer to the pair of levels which are crossing. With the natural choice of phase $(\psi_n, d\psi_n/dt)$ always vanishes so that the sum in (2.5) reduces to a single term, and we have a pair of coupled equations for C_n and C_m. Next we observe that

$$\frac{d\psi_m}{dt} = \left(-i\frac{\underset{\sim}{v}\cdot\hat{y}'}{z_N} L'_x - \hat{z}'\cdot\underset{\sim}{v}\frac{d}{dz_N}\right)\psi_m$$

where

$$L'_x = \sum_j^z \underset{\sim}{r}'_j \times \underset{\sim}{p}'_j \cdot \hat{x}'$$

Now $(\psi_n, d\psi_m/dz_N)$ vanishes unless $\Delta J'_z = 0$, which never holds for crossing states, while $(\psi_n, L_x\psi_m)$ vanishes unless $\Delta J'_z = \pm 1$. The selection rule for off center crossings is thus explained.

III. THE HYDROGEN ATOM

The simplest illustration of the preceding discussion is provided by the hydrogen atom. For notational simplicity, we assume the pole strength parameter q to be positive, and, of course, Z = 1.

a) The energy levels at $z_N = 0$, where the pole and charge coincide, is given by,[8] $E = Ry/n^{*2}$ where $n^* = 1 + n_r + \sqrt{(J+1/2)^2 - q^2}$, $J = q - 1/2$, $q + 1/2$, ... and a linearly independent set with $n^* = n_r + \sqrt{(J+1/2)^2 - q^2}$,

$J = q + 1/2,\ q + 3/2, \ldots$. In both cases, $n_r = 0, 1, \ldots$ is the number of radial nodes. Since for large J

$$\sqrt{(J + 1/2)^2 - q^2} \approx J + \frac{1}{2} - \frac{q^2}{2J + 1}$$

and thus n^* depends primarily upon $n_r + J$, the pattern of J degeneracy and multiplet structure is similar to that of the pole free case.

The ground state, with $E = -Ry$, $J = q - 1/2$; and the first excited state, with $E = -Ry/(2q + 1)$, $J = q + 1/2$ are the two $z_N = 0$ states which connect to the ground state at $z_N = -\infty$, and correspond to the central multiplets referred to in section II. The state with $J_z(\text{atom}) = -1/2$ at $z_N = -\infty$ has $J_z = q - 1/2$. If the collision is at zero impact parameter, it connects to the lowest state with $J_z(\text{atom}) = 2q - 1/2$ at $z_N = +\infty$. For $q = 1/2$ this is simply the $+1/2$ component of the ground state and no excitation is involved. For larger q excitation must occur. For collisions in which the impact parameter is nonzero but sufficiently small, the $2q$ components of the ground state never actually cross but come sufficiently close together to allow quasi adiabatic transfer to occur among them, leading to a distribution of excited final states with $2q - 1/2 \geq J_z(\text{atom}) \geq 3/2$. The state with $J_z(\text{atom}) = +1/2$ at $z_N = -\infty$ has $J_z = q + 1/2$ and hence must connect to the $J = q + 1/2$ state. Thus as z_N varies from $-\infty$ to zero, its energy increases from $-Ry$ to $-Ry/(2q + 1)$. At the same time there are $2q - 1$ states which descend from $z_N = -\infty$ excited states to the $z_N = 0$ ground state with energy $-Ry$ and hence experience off center crossings with the $J_z = q + 1/2$ state. These off center crossings are with states having

$q - 3/2 \geq J_z \geq -(q - 1/2)$. Thus a transfer to any of these states involves $\Delta J_z \geq 2$, and hence do not occur in the quasi adiabatic regime. Hence we confine our attention to excitation via the central multiplet.

In order to discuss excitation via the central multiplet, we first write the Hamiltonian as

$$H = H_0 + H_e$$

where

$$H_0 = \frac{(p - eA)^2}{2m} - \frac{q}{2m} \frac{\underset{\sim}{\sigma} \cdot \hat{r}}{r^2} - \frac{e^2}{r} \tag{3.1}$$

$$H_e = \frac{-e^2}{r^2} \hat{r} \cdot \underset{\sim}{r}_N \tag{3.2}$$

We are assuming the pole to be fixed at the origin and the distance r_N of the proton from the origin to be sufficiently small to allow us to represent the change in the electrostatic potential by the electric dipole approximation.[9] Furthermore, we write

$$\underset{\sim}{r}_N = v_0 t \hat{z} + b_0 \hat{x} \tag{3.3}$$

where b_0 is the distance of closest approach and v_0 is the velocity of the nucleus when it is at that distance. Thus we are neglecting orbit curvature and velocity variation near the point of closest approach. Degenerate first order perturbation theory applied to the subspace formed by the central multiplet can be written

$$i \frac{d\psi}{dt} = \gamma (v_0 t J_z + b_0 J_x) \psi \tag{3.4}$$

where

$$\gamma = -\frac{e^2}{J(J+1)} \left\langle \frac{\hat{r} \cdot \underset{\sim}{J}}{r^2} \right\rangle$$

$$= -\frac{e^2}{J(J+1)} \left\langle \left(\frac{1}{2} \hat{r} \cdot \underset{\sim}{\sigma} - q \right) / r^2 \right\rangle \tag{3.5}$$

In (3.4) ψ is a $2J+1$ component column matrix whose components are the amplitudes of the J_z eigenstates, and J_z and J_x are angular momentum matrices in the standard J representation.

Setting

$$\psi = \begin{pmatrix} C_J \\ \vdots \\ C_M \\ \vdots \\ C_{-J} \end{pmatrix} \tag{3.6}$$

we obtain

$$i\dot{C}_M = \gamma v_0 t M C_M + \frac{\gamma b_0}{2} \left(C_{M+1} \sqrt{(J-M)(J+M+1)} + C_{M-1} \sqrt{(J+M)(J-M+1)} \right)$$

$$-J \leq M \leq J \tag{3.7}$$

The Ansatz

$$C_M = \left[\frac{(2J)!}{(J-M)!\,(J+M)!} \right]^{1/2} u^{J+M} v^{J-M} \tag{3.8}$$

solves these equations provided

$$\dot{u} = -\frac{1}{2} i \gamma v_0 t\, u - \frac{1}{2} i \gamma b_0 v \tag{3.9}$$

$$\dot{v} = \frac{1}{2} i \gamma v_0 t\, v - \frac{1}{2} i \gamma b_0 u \tag{3.10}$$

Equations (3.9) and (3.10) are equivalent to Zener's (eq. 4).[5] We seek a solution for which $C_J = 1$, $C_M = 0$ $(M \neq J)$ at $t = -\infty$. This is obtained by solving (3.9) and (3.10) with $u(-\infty) = 1$, $v(-\infty) = 0$. These are just the Zener boundary conditions, so that we can adopt his solutions. Since we only need the the $t = +\infty$ values, we write

$$|u^2(+\infty)| = \exp(-\pi\gamma b_0^2/(2v_0)) \tag{3.11}$$

$$|v^2(+\infty)| = 1 - |u^2(+\infty)| \tag{3.12}$$

which yields at $t = +\infty$

$$|C_M|^2 = \frac{(2J)!}{(J-M)!\,(J+M)!} \exp\frac{-\pi\gamma b_0^2(J+M)}{2v_0} \left(1 - \exp\frac{-\pi\gamma b_0^2}{2v_0}\right)^{J-M} \tag{3.13}$$

It will be shown below that the impact parameter $b = \lambda b_0$ and the incoming velocity $v = v_0/\lambda$, where λ is a function of v which we shall determine. Assuming this to be the case here, we obtain the partial excitation cross sections

$$\sigma_M = \pi \int db^2 |C_M|^2 = \lambda^3 \frac{2v}{\gamma} \int dx\, e^{-(J+M)x} (1-e^{-x})^{J-M} \frac{(2J)!}{(J-M)!\,(J+M)!}$$

$$= \frac{2v\lambda^3}{\gamma(J+M)}, \quad J \geq M \geq -J+1 \tag{3.14}$$

It should be clear that the above theory of excitation via a central multiplet applies to any atom. The quantitative problem in the case of complex atoms arises in the evaluation of γ and λ. For the case of hydrogen, we find

$$\gamma = 4\alpha/(2q+1) \tag{3.15}$$

for the $J = q - 1/2$ state, and

$$\gamma = 4\alpha q \Big/ \left[(2q+3)(2q+1)^2\left(2q+1 - \frac{1}{2}\sqrt{2q+1}\right)\right] \tag{3.16}$$

for the $J = q + 1/2$ state. Here α is the fine structure constant and in our units γ is an inverse length squared, so that the units are inverse Bohr radii squared.

To determine λ, we assume the motion of the proton may be described classically and that it is determined by studying motion in the potentials $V_- = E(q - 1/2, \underset{\sim}{r}_N)$ and $V_+ = E(q + 1/2, \underset{\sim}{r}_N)$ where $E(q + 1/2, \underset{\sim}{r}_N)$ is the minimum eigenvalue of the Hamiltonian with $\underset{\sim}{r}_N$ held fixed and J_z with eigenvalue $q \pm 1/2$. Here J_z refers to the component of angular momentum along an axis directed from the pole to the proton. Since the potential depends only on $|\underset{\sim}{r}_N|$, angular momentum is conserved in the motion, yielding $vb = v_0 b_0$. The ratio of v to v_0 is given by energy conservation. In order to obtain a b independent form for λ, we have assumed $V_\pm(b_0) = V_\pm(0)$. With these assumptions, $\lambda = 1$ for the lower state and

$$\lambda = \left(1 - \frac{2q\alpha^2}{(2q+1)v^2}\right)^{1/2} \tag{3.17}$$

for the upper state. Evidently λ depends only upon the excitation energy of the central multiplet state so that its evaluation for complex atoms depends only upon a determination of the excitation energies.

The total excitation cross section is thus given

$$\sigma = \frac{1}{2}(\sigma_{-1/2} + \sigma_{1/2})$$

$$\sigma_{-1/2} = (q+1/2)\frac{\beta}{\alpha}\left(1 + \frac{1}{2} + \cdots + \frac{1}{2q-1}\right) a_0^2 , \quad q > 1/2 \qquad (3.18)$$

$$\sigma_{1/2} = \frac{\beta}{2\alpha q}\left(1 - \frac{2q\alpha^2}{(2q+1)\beta^2}\right)^{3/2} (2q+3)(2q+1)^2\left(2q+1 - \frac{1}{2}\sqrt{2q+1}\right)\left(1 + \frac{1}{2} + \cdots + \frac{1}{2q+1}\right) a_0^2 \qquad (3.19)$$

These last expressions have been written in standard units so that v has been replaced by $\beta = v/c$ and the Bohr radius factors have been put in explicitly. The factors $(1 + 1/2 + \ldots)$ come from the sum over partial cross-sections, and in the form written it is assumed that all are above threshold. For example, for $q = 1$, $\sigma_{-1/2}$ has only one term and represents excitation to the $n = 2$ level with $(v/c) = (\alpha/2)\sqrt{3m_e/m_p}$ at threshold. $\sigma_{1/2}$ has three terms, representing excitation to an $n = 2$ level and two excitations to $n = 3$ levels with a slightly higher threshold. For fixed n, λ determines the threshold when the energy of the relevant central multiplet exceeds that of the final state, that is when $2q+1 > n^2$. The excitation cross sections increase quite rapidly with q, and our approximations lose their validity as cross sections become of order a_0^2. As was emphasized in DKMPR, the cross section is quite large even for $q = 1/2$.

IV. THE HELIUM ATOM

The simplest system of practical interest to which our theory applies is the helium atom. We obtain results in this section for the $q = 1/2$ and $q = 1$ case.

(a) The $q = 1/2$ case

The lowest two hydrogenic states for $z_N = 0$ have $j = 0$ and 1. Thus in the shell model approximation the ground state will be an antisymmetrized product of $j = 0$ and 1 orbitals yielding a ground state of $J = 1$. This state, which provides the central multiplet through which transitions occur, has a binding energy which we have variationally estimated to be 8.49 eV, so that the He atom sees a repulsive barrier of height 16.09 eV. This leads to a threshold factor $\lambda = (1 - \beta_c^2/\beta^2)^{1/2}$ with $\beta_c = 9.29 \times 10^{-5}$. The excitations are to the lowest $^3S_1$ and $^3P_2$ states. These occur in the ratio 2:1 with thresholds at $\beta \times 10^4 = 1.03$ and 1.06, respectively. Applying eq. (3.14), the excitation cross section is given by $\sigma = \frac{2v\lambda^3}{\gamma}(1 + 1/2)$. The critical problem remaining is the determination of γ.

A preliminary determination has been made using a product wave function whose spin and angle dependence is the same as that of the hydrogenic orbitals and whose radial parts take the form

$$f_0 = e^{-\alpha_0 r/a_0}$$

$$f_1 = r^{\sqrt{2}-1} e^{-\alpha_1 r/a_0 \sqrt{2}} \left(1 - t\frac{r}{a_0} e^{-w r/a_0}\right)$$

In these expressions, α_0, α_1, t, and w are parameters determined by minimizing the energy subject to the constraint[9] that the electric and magnetic dipole forms of the perturbation energy give the same result. While they are guaranteed to give the same result if the perturbation is computed using exact wave functions of the Hamiltonian, they are in general not the same for approximate wave functions. The value obtained for γ is $4.66\ \gamma_H$ where $\gamma_H = \alpha/4(4-\sqrt{2})a_0^2$. The result reported in DKMPR, $2.35\ \gamma_H$, was obtained from the magnetic dipole form, using a cruder wave function, namely (4.2) with t set equal to zero. The cruder wave function with the electric dipole form yields $3.53\ \gamma_H$. While we believe the new value to be the more reliable, the results above have convinced us that a more elaborate and systematic approach is desirable. The computational problem is quite similar to that involved in computing the fine and hyperfine (He_3) structure of the helium P states, which has been done to great accuracy, and we are in the process of adopting the methods used there[10] to this problem. It should be possible to assess the accuracy of a given level of approximation for the wave function by comparison with the helium hyperfine structure results and by comparing its results for the electric and magnetic dipole forms of the interaction. For the present, however, we consider our best value to be

$$\sigma = 2.5 \times 10^{-18}\ (\beta/10^{-4})(1 - \beta_c^2/\beta^2)^{3/2}\ \text{cm}^2$$

with $\beta_c = 9.29 \times 10^{-5}$.

(b) The $q = 1$ case

The lowest hydrogenic states for $z_N = 0$ have $j = 1/2$ and $3/2$. Thus the shell model ground state is obtained by putting both electrons in the ground state, yielding a $J = 0$ ground state. This state connects with the lowest $^3P_2$ state at $z_N = \pm \infty$. Because the ground state at $z_N = -\infty$ has $J_z = 2$, the central multiplet which it connects is the $J = 2$ level formed by the $(1/2)(3/2)$ configuration. This configuration also has a $J = 1$ level and it is important to know which of the two lies lower. A simple variational calculation seems to provide convincing evidence that the $J = 1$ level is lower by about 0.9 eV. There are therefore four off center crossings by the $J_z = 2$ state as z_N varies from $-\infty$ to zero. The first, which occurs at $z_N \approx a_0$, is with the $J_z = 0$ state which connects the $^3P_2$ state at $z_N = -\infty$ to the $J = 0$ ground state at $z_N = 0$. There is no quasi adiabatic transfer at this crossing because $\Delta J_z = 2$. The other three crossings are with the $J_z = 1, 0, -1$ components of the $J = 1$ central multiplet. The $J_z = 1$ crossing, which satisfies the $\Delta J_z = \pm 1$ rule, occurs at $z_N \approx 0.1\, a_0$ and is a potential additional source of excitation. More careful examination of the spectrum at small z_N ($z_N \approx 0.5\, a_0$ and less) reveals a more complex situation than we have discussed previously. The coupling of the electric dipole perturbation to the inner electron is so strong that it breaks the quartet-doublet coupling for $z_N \gtrapprox 0.003\, a_0$. Since such distances make a negligible contribution to the cross section, one can confine one's attention to the "Pashen-Bach" region in which the inner electron has $j_{1z} = 1/2$ and the outer electron has $j_{2z} = 3/2$. The levels

which contribute to the excitation process are just the $j_z = -3/2, -1/2, 1/2$ levels of the outer electron. The electrostatic interaction between the inner and outer electrons does, however, reduce the spacing between levels for $z_N \gtrsim 0.08\ a_0$, with the reversal of level order taking place at $z_N \approx 0.08\ a_0$. We have not worked out the details of the excitation process for this more complex situation, but it is clear that application of the central multiplet theory to this excited electron will yield a lower limit for the cross sections. Comparing γ values for $q = 1/2$ and $q = 1$ and using (3.16), we see that the $q = 1$ monopole is at least a factor 2.3 more effective than the $q = 1/2$ monopole. This factor will be enhanced not only by the electrostatic narrowing of the level separations but also by the fact that the screening will be stronger and the fact that there is an additional level. The excitations are to $^3S_1$, $^3P_2$, and $^3D_3$ with $\beta \times 10^4$ thresholds at 1.03, 1.11, and 1.11, respectively.[11] While one might imagine that the threshold factor λ^3 might be smaller at threshold due to the fact that the central barrier is somewhat higher, it is likely that the tighter binding of the inner $j_{1z} = 1/2$ electron at small negative z_N overcomes this effect. We conclude, therefore, that He is a very effective detector for $q = 1$ monopole and it is very likely to be so for higher q's as well.

As discussed in ref. 1 and by several speakers at this conference, the preferred method of detecting the excitation is by collisional ionization of the $^3S_1$ states with a doping gas such as CO_2 or CH_4. All of the higher triplet states quickly decay either directly or by cascade to the metastable $^3S_1$ state.

V. DYON INTERACTIONS WITH HELIUM

In this section we consider the interaction of a dyon with helium. The dyon is assumed to have $q = 1/2$ and plus or minus one unit of electric charge. Such a charge on a monopole could arise during the production mechanism in the early universe or in the case of positive charge by the subsequent capture of a proton. The size of the monopole-proton bound state is approximately 10 fm and is therefore very small compared to atomic dimensions and hence will be considered point like.

The positively and negatively charged dyons need to be considered separately, and we first consider the positive charged case. Following the discussion of sections II and III, the eigenstates of the dyon-helium atom system are first identified at large separation. Apart from the usual states of the helium atom, an electron can form bound states with the positively charged dyon with energies $-\frac{1}{2}m\alpha^2/(n+\mu)^2$ where $n = 0, 1, \ldots$ and $\mu = \sqrt{j(j+1)}$; $j = 0, 1, 2, \ldots$ being the angular momentum. The ground state of the dyon is 13.6 eV below the continuum and thus the dyon will pick up an electron while travelling through matter. Metal surrounding or making up a detector would be an ideal source of such electrons. Therefore we need to

consider a dyonic atom and a helium atom colliding, i.e. a three electron system.

Next, we consider the dyon and helium nucleus on top of one another. In doing this, we must ignore for the time being the Coulomb repulsion between the dyon and the nucleus. This interaction, which turns out to be quite important, will be taken into account later.

The relevant $z_N = 0$ central multiplet of the three electron system is in a $(0)(1)^2$ configuration formed from the two lowest hydrogenic states and because of the Pauli principle, must have $J = 1$. Because one of the electrons is already bound to the dyon (with $J = 0$) when the collision begins, the J_z value which connects to the $z_N = -\infty$ configuration is $J_z = 1$. This shows that the ground state $z_N = 0$ central multiplet described above is indeed the relevant central multiplet and there are no off center crossings. The states which connect to the $J_z = 1$ and $J_z = 0$ states of the central multiplet when $z_N \to +\infty$ are, as discussed in section III, the minimum energy $J_z = 1$ and $J_z = 0$ states with $z_N = +\infty$. One sees by inspection that these are states with He in its ground state and with one electron bound to the dyon and in excited $J = 2$ or $J = 1$ states with excitation energies of 11.3 and 6.8 volts, respectively.

Following the method of section III, we may calculate the transition probabilities as a function of distance of closest approach b_0 (eq. 3.13). Again the interaction parameter γ (call it γ_D here) must be calculated. Following the previous calculation for helium, this is performed variationally

using the same wave functions as in section IV but with the extra electron in a $j = 1$, $m_j = 0$ monopole harmonic. The parameters of the variational wave functions are determined so as to minimize the energy subject to the constraint that the electric and magnetic dipoles as discussed earlier give the same result. The interaction parameter γ_D is found to be approximately 20 times the hydrogen value due mainly to the extra charge (partially shielded) at the origin, which comes in as the third power. Thus we find $\gamma_D \approx 1.4 \times 10^{-2}/a_0^2$.

In order to compute the partial cross sections we use eqs. (3.11), (3.12), and (3.13). For this case, we obtain

$$|C_0|^2 = 2(e^{-\Gamma} - e^{-2\Gamma})$$

$$|C_1|^2 = e^{-2\Gamma}$$

where

$$\Gamma = \pi\gamma_D b_0^2/2\beta_0 = \pi\gamma_D b_0^3/2b\beta$$

We have used angular momentum conservation of the nuclear orbit in the last equality. In order to compute a cross section, we need to express b_0 in terms of b and β. This can easily be done taking account of the Coulomb repulsion between the dyon and the nucleus, the attractive force of the electrons, and angular momentum conservation. Thus we have

$$\frac{1}{2} M v^2 = \frac{1}{2} M \frac{b^2 v^2}{b_0^2} + \frac{2\alpha^2}{b_0} - \Delta \qquad (5.1)$$

where Δ is the net increase in binding energy of the electrons when the dyon and nucleus are separated by distances small compared to $\bar{a}_0$. Here b_0 and b are in units of a_0 and $\Delta \approx 180$ eV. The cross sections may be evaluated by computing

$$\sigma_M = 2\pi \int |C_M^2(b_0)|\, bdb$$

numerically, where b_0 is computed from (5.1). It is convenient to express the result as

$$\sigma(6.8) = (\beta/10^{-4})\,\Lambda_1\ 2.0\times10^{-19}\ cm^2$$

$$\sigma(11.5) = (\beta/10^{-4})\,\Lambda_2\ 1.0\times10^{-19}\ cm^2$$

Here $\sigma(6.8)$ represents the excitation cross section for the $J=1$ excited state of the dyon, which has an excitation energy of 6.8 eV. It corresponds to $J=1$, $M=0$ in (3.13). Similarly, $\sigma(11.5)$ corresponds to $J=1$, $M=1$ in (3.13) and represents excitation of the dyon to its $J=2$ state with excitation energy 11.5 eV. The parameters Λ_1 and Λ_2 represent threshold factors, analogous to the λ^3 which appears in (3.14), and are determined by the numerical integration described above. Representative values are:

$\beta/10^{-4}$	Λ_1	$\beta/10^{-4}$	Λ_2
2	1.4×10^{-3}	3	7.1×10^{-4}
3	8.5×10^{-2}	4	4.4×10^{-2}
4	.48	5	.236
5	.86	6	.48
6	1.02	8	.77
10	1.04	10	.88
↓	1.00	20	.99
		↓	1.00

The difference in the behavior of Λ_1 and Λ_2 arises from the fact that $|C_1^2|$ peaks at $b_0 = 0$ where the Coulomb suppression is complete, while $|C_0^2|$ vanishes at b_0 and peaks at a finite value. The fact that Λ_1 slightly exceeds unity is a consequence of the fact that the force becomes attractive at large distances.

The excitation may be detected via the 6.8 eV and 4.5 eV photons which would be emitted by these states. The small cross section, the strong Coulomb suppression for $\beta < 4\times10^{-4}$, and the requirement that one detect photons rather than ionization makes He rather unattractive as a detector of positively charged dyons.

For a negatively charged dyon the eigenstates for a widely separated dyon and atom are just the eigenstates of the atom. When the dyon and helium atom are on top of one another we are trying to bind two electrons to a charge

one center with a monopole. This situation has similarities to the cases of a hydrogen minus ion which has one bound state with energy -0.75 eV and a helium minus ion which does not exist as a bound state. Let us compare the dyon of charge one and the hydrogen minus ion. The first electron goes into a state with energy -13.6 eV in both cases, whereas the second electron for the dyon case goes into a triplet state with energy -6.8 eV (ignoring shielding) and into the $n = 1$ state with energy -13.6 eV (ignoring shielding) for hydrogen ion. Also the shielding for the dyon case will be larger than for the hydrogen ion so that the monopole will be bound less than 0.75 eV and there is probably no bound state. The details here are not important as a dyon at a velocity of 10^{-4} c can cause transitions of a few tenths of an electron volt.

Thus the cross section for ionization of helium is just π times the square of the distance at which binding becomes a few tenths of an electron volt. This distance is estimated to be of order $1/3\ a_0$, giving a cross section of 3×10^{-17} cm^2. Of course, the Coulomb attraction between the dyon and the nucleus will enhance this effect by drawing the nucleus towards the dyon. The added energy in the electrons due to the reduction in charge near the center (<55 eV) is less than the Coulomb attraction between dyon and nucleus at a distance of a_0 and is therefore negligible. The energy loss, neglecting the attraction between dyon and nucleus will be of order 100 MeV cm^2 g^{-1}. This is about 50 times minimum ionizing and thus an uncertainty in our estimate of the cross section by a factor of 4 or more is not important. Also there is no need for a quenching gas like CO_2 or CH_4 for this case as the dyon ionizes the helium atom directly. The threshold for this process is

1.2×10^{-4} c and there is no reduction in cross section near threshold. Below threshold, the ionization cross section probably continues to be high but accompanied by capture of the α particle and electron by the dyon. This process ultimately leads to a tightly bound α particle dyon system which behaves like a positive dyon.

In summary, helium would be a very efficient detector for negatively charged dyons down to a β of 1.2×10^{-4}, but appears to be much less promising as a detector for positively charged dyons.

VI. HEAVIER NOBLE GAS ATOMS

The primary limitation of He as a slow monopole detector is its threshold at $\beta = 1.03 \times 10^{-4}$. One would prefer to be sensitive at least to 0.37×10^{-4}, the velocity of escape from the earth. Because our arguments relied heavily upon the spherical symmetry which is obtained when the monopole and nucleus coincide, it is natural to consider heavier noble gas atoms. Such atoms are extensively used in proportional counters and are not subject to the annoying leakiness of He proportional counters. Assuming the threshold to be determined by the excitation energy, which we take to be of the order of the ionization energy, one finds (β_t = threshold β)

	Ne	A	Kr	Xe
$10^4 \beta_t$ =	0.48	0.29	0.19	0.14

It is not, however, clear that we have used a proper measure of the threshold. We recall that the monopole presents a 16 eV central barrier to

the impinging He atom, a fact which gives rise to the λ^3 threshold factor which appears in eq. (3.14). This barrier provides a threshold factor, rather than the principal determinant of the threshold, because it is lower than the required excitation energy. We have already noted that the existence of a Coulomb barrier in the He-positive dyon problem more than doubles the effective threshold.

In order to assess the barrier height question in a preliminary way, we first consider the diamagnetic repulsion which the atom experiences when it is far from the monopole. Using experimental values for the diamagetic susceptibility, we find for $a_0/z_N \ll 1$,

$$E \text{ (diamagnetic)} = \mathcal{K}(a_0/z_N)^4 \tag{6.1}$$

with

	He	Ne	A	Kr	Xe
$\mathcal{K}$ (eV)	1.3	4.8	130	265	423

While (6.1) obviously gives a gross overestimate as $z_N \to 0$, it should be noted (see fig. 1 of DKMPR) that for He at $z_N = a_0$ it is roughly a factor ten too small. While it would be unwarranted to assume ten is a universal factor, it does seem very likely that the actual barrier at $z_N = a_0$ is several times the value given by (6.1). We further comment that an argon atom with $\beta = 1.03 \times 10^{-4}$, the He threshold, cannot penetrate beyond a 200 eV barrier height, and on the basis of the above discussion, seems unlikely to achieve a z_N as small as a_0. It may therefore be out of the

range at which quasi adiabatic excitation takes place. If this is the case, the quasi adiabatic threshold for argon would be higher than that of He.

Once β is sufficiently large to allow barrier penetration, it is certain that degeneracies and near degeneracies will be encountered, and hence quasi adiabatic excitation will take place. The question of the magnitude of γ factors, which depend primarily on the extent to which the last shell electrons are screened for the state which connects to the ground state, is then of crucial importance in determining quantitative cross sections. A contribution from off center crossings may also prove to be important, and distortions of the multiplet structure of the sort found in the $q = 1$ case for He may further complicate matters.

In summary then, the probability that the threshold for quasi adiabatic excitation can be pushed below $\beta = 10^{-4}$ by using heavy noble gas atoms seems to be low. More work is required both to determine what the threshold is and how large the cross sections are. It seems likely, however, that they become satisfactorily large somewhere between $\beta = 10^{-4}$ and $\beta = 10^{-3}$.

VII. CONCLUDING REMARKS

Despite some quantitative uncertainties about cross section magnitudes, it seems certain that He proportional counters will satisfactorily detect massive Dirac monopoles of arbitrary charge in the $\beta = 10^{-4}$-10^{-3} velocity range. The threshold increases slowly with magnetic charge but will remain below the ionization limit at $\beta = 1.2 \times 10^{-4}$. Such counters will also detect negatively charged dyons at this threshold.

APPENDIX

Because our use of the magnetic dipole perturbation has received some criticism,[12] we add some technical comments about the hydrogen energy level problem here.

Considering the energy as a function of z_N, the term linear in z_N near $z_N = 0$ can be obtained from either the point electric or point magnetic dipole. Here we restrict the perturbation Hamiltonian to terms linear in z_N, which means the δA^2 term is omitted in the magnetic dipole case. As explained in footnote 9, the two forms of the Hamiltonian give the same result.

One might also consider carrying out first order perturbation theory with real dipoles, that is two point charges (electric or magnetic) of opposite sign separated by a distance z_N. In this case, our argument for the equivalence of the two cases no longer applies. First order perturbation theory for the real electric dipole can be readily carried out, and one finds a fractional correction to our point dipole result of order z_N/a_0. Higher order perturbation theory would, of course, be expected to yield terms of similar order. First order perturbation theory for the real magnetic dipole case is, however, singular. To be specific, we consider the case in which the dipole is aligned along the z axis, with the original magnetic pole displaced from the origin in the positive direction. Then for the $J_z = 1$ state, first order perturbation theory can be carried out, and a fractional correction to the point dipole result of order $(z_N/a_0)^{2\sqrt{2}-2}$ is found. However, for the $J_z = 0$ and $J_z = -1$ cases, the

perturbation integrals diverge. If one first performs the radial integrals, which do converge, but carries out the small z_N expansion before completing the angular integration, one finds that the linear terms agree with the point dipole result, but the angular integral for the order $(z_N/a_0)^{2\sqrt{2}-2}$ correction diverges. If the poles are displaced to the negative side, it is the $J_z = -1$ state rather than the $J_z = 1$ state which is well behaved. (Note that the difficulty does not depend upon the choice of string direction, which only affects the ϕ dependence of the states.) The formal origin of the difficulty lies in the structure of the differential equation and has been discussed in the context of variational calculations by Tiktopoulos.[6] To explain the origin physically we first recall that the ordinary spherical harmonics vanish along the z axis for $m \neq 0$. This vanishing can be understood as resulting from the effect of the repulsive centrifugal potential associated with the $m^2/r^2 \sin^2\theta$ term in the Hamiltonian. In the case of monopole harmonics, the magnitude of the centrifugal potential changes as one switches from one sign of the z axis to the other, and for the $q = 1/2$ case, may vanish on one side or the other for $|J_z| = 1$ or 0. Accordingly, the wave functions do not vanish for such states along the entire z axis. Shifting the position of the pole without changing the states, which is what one does when one carries out first order perturbation theory, can shift the repulsive and singular centrifugal potential into a region where the wave function does not vanish, leading to the divergent expectation values described above. Thus an attempt to apply first order perturbation theory to a real magnetic dipole using numerical integration techniques[12] is bound to lead to

unreliable and confusing results.

As a final remark, it may be of interest to mention that we have carried out numerical variational calculations for the $J_z = \pm 1$ excited states and the $J_z = 0$ ground state all the way from $z_N = -\infty$ to $z_N = +\infty$ using wave functions which guarantee consistency with perturbation theory at the $|z_N| = 0$ and infinity limits. (Our wave function for the ground state is the same as that used by Tiktopoulos[6] and our results agree with his.) These have been done accurately enough to allow us to numerically confirm the value of the slope at the origin to 0.2 percent. The calculations are useful because they indicate that the departure from linearity is small, linearity being valid within ~ 5 percent out to a distance $0.5\, a_0$. This result supports the approximations we have used in calculating excitation probabilities. Analogous calculations have also been carried out for $q = 1$.

Footnotes

1. S. Drell, N. Kroll, M. Mueller, S. Parke, and M. Ruderman, Phys. Rev. Lett. 50, 644 (1983).

2. T. T. Wu and C. N. Yang, Nuclear Phys. B107, 365 (1976).

3. M. Fierz, Helv. Phys. Acta 17, 27 (1944).

4. For divergence free magnetostatic fields and vector potentials, this field angular momentum is given by $e\underset{\sim}{A}$, so that the canonical angular momentum $\underset{\sim}{r}_i \times \underset{\sim}{p}_i$ represents the sum.

5. C. Zener, Proc. Roy. Soc. A137, 696 (1932).

6. G. Tiktopoulos, Phys. Lett. 125B, 156 (1983).
Compare with eq. (15) of the above paper. We are including our time dependent choice of gauge in the definition of our time dependent Hamiltonian and hence do not have the $d\Omega/dz_0$ term which appears there. While we think that such a term should not be included in our treatment, to be on the safe side we have verified that it also satisfies the selection rule.

7. We are using the phrase "quasi adiabatic" to refer to a situation in which relative velocities are slow enough to allow us to assume that excitations are dominated by transitions to levels whose energy separation from the incident state is small compared to the minimum excitation energy which would be required for the isolated atom. The transitions are actually non-adiabatic.

8. W. V. R. Malkus, Phys. Rev. 83, 899 (1951).

9. In DKMPR we discussed this problem in a reference frame in which the nucleus was fixed at the origin. This made it natural to use as the perturbation Hamiltonian the change in H due to the addition of a point magnetic dipole instead of a point electric dipole. That is to say, in place of H_e we used

$$H_m = -\frac{-q}{2mr^3}\,\underset{\sim}{r}_N \cdot [2\underset{\sim}{r} \times (\underset{\sim}{p} - e\underset{\sim}{A}) - \underset{\sim}{\sigma} + 3\hat{r}\,\underset{\sim}{\sigma} \cdot \hat{r}]$$

That the two must give the same result follows from

$$H_e - H_m = \underset{\sim}{r}_N \cdot \underset{\sim}{\nabla} H = i\,\underset{\sim}{r}_N \cdot [\underset{\sim}{P}, H]$$

and $(\psi_n\,,\ [\underset{\sim}{P}, H]\,\psi_n) = 0$ for any eigenstate of H.

10. C. Schwartz, Phys. Rev. 134, A1181 (1964); J. D. Prestage, E. A. Hinds and F. M. J. Pichanick, Phys. Rev. Lett. 50, 828 (1983).

11. There are four additional excitation states, including the $^3F_4$ state to which all excitation occurs at zero impact parameter, which are not included in the lower Paschen-Bach multiplet discussed in the text. Their total contribution to the cross section is much smaller.

12. R. Hagstrum, Theory of Excitation and Ionization of Matter by Monopoles, paper presented at this conference, Oct. 6, 1983.