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# NONLINEAR REALIZATIONS AND THE PARTIAL BREAKING OF EXTENDED SUPERSYMMETRY*

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**Abstract**

In this paper we use nonlinear realizations to examine the breaking of $N = 2$ extended supersymmetry to $N = 1$. We derive Lagrangians and transformations laws for the generalized $N = 2$ Akulov-Volkov Goldstone field. We analyze the ghost states and show that they may be collected into $N = 1$ supermultiplets. We extend the transformation laws for the $N = 1$ chiral and vector multiplets to $N = 2$. Finally, we give the $N = 2$ generalization of an arbitrary $N = 1$ supersymmetric Lagrangian.

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## Introduction

Within the last few years, many models have been proposed in which $N = 1$ supersymmetry is spontaneously broken at some mass scale $M_1$ less than the Planck mass $M_P$ [1]. It is natural to ask whether these models can arise from a theory with extended supersymmetry, where the extra supersymmetries are first broken to $N = 1$ at some new mass scale $M_2 > M_1$. A model-independent way to investigate this question is through the use of nonlinear realizations. In this paper I shall describe some recent work performed in collaboration with Julius Wess. We have used nonlinear realizations to show that any $N = 1$ supersymmetric theory (with chiral and vector superfields) may be obtained as the "low energy" limit of a corresponding $N = 2$ theory [2].

Nonlinear realizations and effective Lagrangians have played an important role in understanding spontaneously broken symmetries [3]. They were first discussed in the context of chiral dynamics and the phenomenology of the strong interactions [4,5]. They were later used to investigate the consequences of technicolor theories [6] and spontaneously broken supersymmetry [7,8]. The work described here is a natural extension of these ideas to the case of partially broken extended supersymmetry. Before turning to our work, I will briefly review the relationship between nonlinear realizations and spontaneously broken symmetries. Readers familiar with the subject might prefer to skip this section and pass immediately to the next, where we discuss nonlinear realizations of supersymmetric theories.

To illustrate the role of nonlinear realizations, I shall return for a moment to the original sigma model of Gell-Mann and Lévy [4]. In its most simple form, this model contains four fields, three pions $\pi^i$ and one scalar $\sigma$. The $\pi^i$ and $\sigma$ form a vector $\Pi^I = (\pi^i, \sigma)$ of SO(4) $\simeq$ SU(2) $\times$ SU(2). The sigma model Lagrangian

$$\mathcal{L} \;=\; -\,\frac{1}{2}\,\partial_m\pi^i\,\partial^m\pi^i \;-\; \frac{1}{2}\,\partial_m\sigma\,\partial^m\sigma \;-\; V(\Pi\cdot\Pi) \tag{1}$$

is constructed to be invariant under rigid $\mathrm{SU(2)}_L \times \mathrm{SU(2)}_R$ transformations

$$\begin{aligned} \delta^i_L\sigma &= -i\,\pi^i & \qquad \delta^i_R\sigma &= i\pi^i \\ \delta^i_L\,\pi^j &= i\delta^{ij}\sigma + i\epsilon^{ijk}\pi^k & \qquad \delta^i_R\,\pi^j &= -i\delta^{ij}\sigma + i\epsilon^{ijk}\pi^k\,. \end{aligned} \tag{2}$$

The potential $V$ is arranged such that $\sigma$ acquires a vacuum expectation value $v$,

$$V(\Pi \cdot \Pi) = \frac{1}{8}\left[(\pi^2 + \sigma^2) - v^2\right]^2 , \tag{3}$$

and SO(4) is spontaneously broken to SO(3) $\simeq$ SU(2). Expanding $\sigma$ around its vacuum expectation value, $\sigma' = \sigma - v$, one finds that the pions are the massless Goldstone bosons arising from the symmetry breaking, and that the field $\sigma'$ acquires the mass $v$.

The Gell-Mann - Lévy model is instructive because it lets one understand the physics of spontaneous symmetry breaking at different energy scales. At energies $E \gg v$, both $\pi$ and $\sigma'$ appear massless, and the SO(4) symmetry is manifest. At energies $E$ much less than $v$, the $\sigma'$ field is so heavy that it freezes out, and only the Goldstone modes are excited. As shown in Figure 1, the low-energy dynamics are constrained to a three-sphere of radius $v$, $\sigma^2 + \pi^2 = v^2$.

This example illustrates how linear models reduce to nonlinear models at energies far below the scale of spontaneous symmetry breaking. When $E \gg v$, the model is symmetric and the transformations (2) are linear. When $E \ll v$, the $\sigma$ field is frozen out, and the pions lie on the coset space (SU(2) $\times$ SU(2))/SU(2) $\simeq$ $S^3$. The low-energy Lagrangian is given by

$$\mathcal{L} = -\frac{1}{2}\,\partial_m\pi^i\partial^m\pi^i - \frac{1}{2}\,\frac{(\pi^i\partial_m\pi^i)^2}{(v^2 - \pi\cdot\pi)} , \tag{4}$$

and the transformation laws are as follows,

$$\delta^i_{L+R}\,\pi^j = i\epsilon^{ijk}\pi^k \qquad \delta^i_{L-R}\,\pi^j = i\delta^{ij}\sqrt{v^2 - \pi\cdot\pi} . \tag{5}$$

From (5) we see that the pions transform linearly under the unbroken SU(2) generators, and nonlinearly under the broken ones. Nonlinear realizations and effective Lagrangians provide a convenient framework with which to analyze the low-energy dynamics of Goldstone bosons.

The formalism of nonlinear realizations is much more general than the SU(2) case discussed above [5]. In the spontaneously broken regime, the interactions of Goldstone bosons are described by nonlinear models. For example, the strong interactions realize an SU(3) $\times$ SU(3) chiral symmetry in the limit of massless quarks. This symmetry is spontaneously broken to the diagonal SU(3) subgroup. The octet of pseudoscalar mesons contains the Goldstone bosons associated with the symmetry breaking. The spontaneously broken theory is described by an effective Lagrangian where the Goldstone bosons lie on the coset space (SU(3) $\times$ SU(3))/SU(3) $\simeq$ SU(3).

The general case of a theory in which a group $G$ is spontaneously broken to a subgroup $H$ was analyzed in two elegant papers by Callan, Coleman, Wess and Zumino [9]. They showed that the Goldstone bosons which arise from breaking $G$ to $H$ must lie on the coset space $G/H$, and that one can always find a coordinate system on $G/H$ such that the Goldstone bosons transform linearly under generators in $H$, but nonlinearly under those generators in $G$ that are not in $H$. This is the general picture which we wish to carry over into our study of spontaneously broken supersymmetry.

## Nonlinear Realizations of Supersymmetry

In the spirit of the preceeding discussion, we shall treat spontaneously broken supersymmetry from a coset space approach. In this case $G$ is the supergroup generated by the full supersymmetry algebra,

$$\begin{aligned} \{Q_\alpha^L, \bar{Q}_{\dot{\alpha}M}\} &= 2\sigma^m_{\alpha\dot{\alpha}} P_m \,\delta^L_M & [P_m, Q_\alpha^L] &= 0 \\ \{Q_\alpha^L, Q_\beta^M\} &= \epsilon_{\alpha\beta} X^{(LM)} & [P_m, X^{(LM)}] &= 0 \,, \end{aligned} \tag{6}$$

and $H$ is either the usual Poincaré group or the supergroup generated by the unbroken supersymmetry algebra. In Eq. (6), the dotted and undotted Greek indices denote two-component spinors, and the capital letters $A, B, \ldots$ run from 1 to $N$ and count the number of supersymmetry generators. *

Instead of Goldstone bosons, the spontaneous breaking of rigid supersymmetry gives rise to Goldstone fermions. The Goldstone fermions transform nonlinearly under the broken supersymmetry generators, and linearly under the Poincaré and unbroken supersymmetry generators. The first nonlinear realization of $N = 1$ supersymmetry was found by Akulov and Volkov [11,12]. Their transformation law for the Goldstone spinor is given by

$$\delta_\xi \tilde{\lambda}_\alpha \;=\; \frac{1}{k}\xi_\alpha - ik\left(\tilde{\lambda}\sigma^m\bar{\xi} - \xi\sigma^m\bar{\tilde{\lambda}}\right)\partial_m\tilde{\lambda}_\alpha \,. \tag{7}$$

---

* I shall follow the notation of Ref. [10].

The coefficient $k^{-\frac{1}{2}}$ denotes the scale of supersymmetry breaking. It is not hard to show that this transformation closes into the supersymmetry algebra,

$$\left[\delta_\eta, \delta_\xi\right] \tilde{\lambda}_\alpha = -2i\left(\eta\,\sigma^m\,\bar{\xi} - \xi\sigma^m\,\bar{\eta}\right)\partial_m\,\tilde{\lambda}_\alpha \;. \tag{8}$$

A somewhat simpler transformation law was used in Refs. [14,15]:

$$\delta_\xi\lambda_\alpha = \frac{1}{k}\xi_\alpha - 2ik\left(\lambda\sigma^m\,\bar{\xi}\right)\partial_m\lambda_\alpha \;. \tag{9}$$

This transformation also closes into the supersymmetry algebra. The fields $\lambda$ and $\tilde{\lambda}$ are related through a field redefinition,

$$\lambda_\alpha(x) = \tilde{\lambda}_\alpha(y) \;, \tag{10}$$

where

$$y^m = x^m - ik^2\left(\tilde{\lambda}\,(y)\sigma^m\,\bar{\tilde{\lambda}}\,(y)\right) \;. \tag{11}$$

This implicit redefinition may be rendered explicit by expanding (and inverting) Eq. (10) [14]:

$$\begin{aligned}
\lambda_\alpha &= \tilde{\lambda}_\alpha - i\Big[\,\tilde{v}^m - i\,\tilde{v}^n\,\partial_n\,\tilde{v}^m \\
&\quad - \tilde{v}^n\,\partial_n\,\tilde{v}^\ell\,\partial_\ell\,\tilde{v}^m - \frac{1}{2}\tilde{v}^n\,\tilde{v}^\ell\,\partial_n\partial_\ell\,\tilde{v}^m\,\Big]\,\partial_m\,\tilde{\lambda}_\alpha \\
&\quad - \frac{1}{2}\,\tilde{v}^m\,\tilde{v}^n\,\partial_m\partial_n\,\tilde{\lambda}_\alpha \\
\tilde{\lambda}_\alpha &= \lambda_\alpha + i\Big[\,v^m + ik^2v^n\left(\,\partial_n\lambda\sigma^m\,\bar{\lambda} - \lambda\sigma^m\partial_n\,\bar{\lambda}\,\right) \\
&\quad + v^n\partial_n v^\ell\,\partial_\ell v^m + \frac{1}{2}\,v^n v^\ell\,\partial_n\partial_\ell v^m\Big]\,\partial_m\lambda_\alpha \\
&\quad - \frac{1}{2}\,v^n v^m\,\partial_n\,\partial_m\lambda_\alpha \;,
\end{aligned} \tag{12}$$

where $\tilde{v}^m = k^2 \tilde{\lambda} \sigma^m \bar{\tilde{\lambda}}$ and $v^m = k^2 \lambda \sigma^m \bar{\lambda}$. In Eq. (12), all fields are functions of $x^m$.

The transformations (7) and (9) nonlinearly realize the $N = 1$ supersymmetry algebra. They describe $N = 1$ supersymmetry spontaneously broken to $N = 0$. The relation between these nonlinear realizations and the superfield formalism has been worked out in detail [13–15]. As with the ordinary sigma model, one finds that linear representations become nonlinear realizations at energies below the scale of supersymmetry breaking. The nonlinear formalism may be used to explore the phenomenological consequences of spontaneously broken supersymmetry. Some of these consequences are described by Stuart Samuel in his contribution to this volume [16].

Nonlinear realizations of extended supersymmetry have been investigated in Ref. [17]. In these papers all the supersymmetries are realized nonlinearly. This corresponds to completely broken extended supersymmetry. In what follows, I shall describe *partially* broken extended supersymmetry. In this case, the unbroken supersymmetries are represented linearly, and the broken supersymmetries are realized nonlinearly. For simplicity I shall restrict myself to $N = 2$ supersymmetry, spontaneously broken to $N = 1$. The method presented here may be trivially extended to higher $N$ as well.

## Partial Breaking of Extended Supersymmetry

We are now ready to examine the partial breaking of extended supersymmetry. We will start by deriving transformation laws and Lagrangians for the generalized $N = 2$ Akulov-Volkov Goldstone field. Since there is an unbroken $N = 1$ supersymmetry, we will find that the Akulov-Volkov field belongs to an entire $N = 1$ supermultiplet. We will also discover that the Akulov-Volkov Lagrangians contain ghost fields. We will analyze the Fock space and show that the $N = 2$ ghost states may be collected into $N = 1$ supermultiplets. In the next section, we will demonstrate how the transformation laws for $N = 1$ chiral and vector multiplets may be extended to $N = 2$ with the help of the Akulov-Volkov multiplet. Finally, we will conclude by giving the $N = 2$ generalization of an arbitrary $N = 1$ supersymmetric Lagrangian.

The appearance of ghost fields in the "low energy" effective Lagrangian is no surprise in extended supersymmetry. Ghosts are expected because of a general argument based on the supersymmetry algebra [18]. Heuristically, the argument proceeds as follows. Imagine that one has two supersymmetries, generated by the supercharges $Q_\alpha^{(1)}$ and $Q_\alpha^{(2)}$. Suppose that $Q_\alpha^{(1)}$ is unbroken and that $Q_\alpha^{(2)}$ is broken. Then $Q_\alpha^{(1)}$ annihilates the vacuum, $Q_\alpha^{(1)}|0\rangle = 0$, and $Q_\alpha^{(2)}$ carries the vacuum into another state, $Q_\alpha^{(2)}|0\rangle = |G_\alpha\rangle$. Because of the supersymmetry algebra, the state $|G_\alpha\rangle$ has zero norm,

$$\| \, |G_\alpha\rangle \, \| \; = \; \| \, Q_\alpha^{(2)}|0\rangle \, \| \; = \; \| \, Q_\alpha^{(1)}|0\rangle \, \| \; = \; 0 \, . \tag{13}$$

From this we see that partially broken supersymmetry gives rise to ghost states of zero norm. The presence of ghosts forces one to conclude that in flat space the partial breaking of rigid supersymmetry is unphysical.

This need not be the case for *local* supersymmetry in *curved* space. We shall see that the fermionic ghosts are precisely the Akulov-Volkov fields associated with the $N = 2$ supersymmetry breaking. When gravity is included, one combination of the Akulov-Volkov fields will be absorbed by the second gravitino. Because of the unbroken $N = 1$ supersymmetry, the supersymmetric partners of the Akulov-Volkov ghost will also be eliminated. The massless $(\frac{3}{2}, 1)$ gravitino multiplet will eat an entire $(1, \frac{1}{2})$ ghost multiplet, forming one massive $(\frac{3}{2}, 1, \frac{1}{2})$ gravitino multiplet. Our hope is that all the ghosts may be eliminated in this way [19].

It is easy to find transformation laws for the Akulov-Volkov field associated with breaking $N = 2$ supersymmetry down to $N = 1$. One simply recalls the transformations (7) and (9). If $\lambda_\alpha$ and $\tilde{\lambda}_\alpha$ are promoted to $N = 1$ superfields $\Lambda_\alpha$ and $\tilde{\Lambda}_\alpha$, and $\xi_\alpha$ is replaced by ${\xi_\alpha}^{(2)}$, the transformation laws (7) and (9) realize the $N = 2$ supersymmetry algebra (without a central charge). The superfields $\Lambda_\alpha$ and $\tilde{\Lambda}_\alpha$ are Goldstone superfields – because of the unbroken supersymmetry, the Goldstone spinors $\lambda_\alpha$ and $\tilde{\lambda}_\alpha$ are members of $N = 1$ supermultiplets.

An advantage of the transformation law (9) is that the $N = 1$ superfield $\Lambda_\alpha$ may be constrained to either be chiral $\bar{D}\Lambda = 0$ or antichiral $D\Lambda = 0$. To distinguish the two cases, we shall call the antichiral superfield $X_\alpha$, $DX = 0$.

To derive Lagrangians for $\Lambda_\alpha$ and $X_\alpha$, we follow the method of Ref. [14]. We first construct the $N=2$ superfields associated with the transformation laws (7) and (9) [14,20]:

$$\begin{aligned}
\tilde{\boldsymbol{\Lambda}}_\alpha\Big(x,\theta^{(1)},\bar\theta^{(1)},\theta^{(2)},\bar\theta^{(2)}\Big) &= exp\Big[\delta_{\theta(2)}\Big] \times \tilde{\Lambda}_\alpha\Big(x,\theta^{(1)},\bar\theta^{(1)}\Big) \\
\boldsymbol{\Lambda}_\alpha\Big(x,\theta^{(1)},\bar\theta^{(1)},\theta^{(2)},\bar\theta^{(2)}\Big) &= exp\Big[\delta_{\theta(2)}\Big] \times \Lambda_\alpha\Big(x,\theta^{(1)},\bar\theta^{(1)}\Big) \quad . \\
\boldsymbol{X}_\alpha\Big(x,\theta^{(1)},\bar\theta^{(1)},\theta^{(2)},\bar\theta^{(2)}\Big) &= exp\Big[\delta_{\theta(2)}\Big] \times X_\alpha\Big(x,\theta^{(1)},\bar\theta^{(1)}\Big)
\end{aligned} \tag{14}$$

These superfields may also be defined through constraints:

$$\begin{aligned}
D_\beta{}^{(2)}\,\boldsymbol{\Lambda}_\alpha &= \frac{1}{k}\,\epsilon_{\alpha\beta} \qquad\qquad \bar D_{\dot\alpha}{}^{(1)}\,\boldsymbol{\Lambda}_\beta = 0 \\
\bar D^{\dot\beta}{}^{(2)}\,\boldsymbol{\Lambda}_\alpha &= 2ik\,(\bar\sigma^m\boldsymbol{\Lambda})^{\dot\beta}\,\partial_m\,\boldsymbol{\Lambda}_\alpha \\
D_\beta{}^{(2)}\boldsymbol{X}_\alpha &= \frac{1}{k}\,\epsilon_{\alpha\beta} \qquad\qquad D_\alpha{}^{(1)}\,\boldsymbol{X}_\beta = 0 \\
\bar D^{\dot\beta}{}^{(2)}\boldsymbol{X}_\alpha &= 2ik\,(\bar\sigma^m\boldsymbol{X})^{\dot\beta}\,\partial_m\boldsymbol{X}_\alpha \quad .
\end{aligned} \tag{15}$$

The constraints for $\tilde{\boldsymbol{\Lambda}}_\alpha$ may be worked out with the help of Eq. (12) [14]. The constraints (15) are consistent with the $N=2$ D-algebra [21],

$$\begin{aligned}
\Big\{D_\alpha{}^{(A)},\ \bar D_{\dot\beta}{}^{(B)}\Big\} &= -2i\,\delta^{AB}\,\sigma^m_{\alpha\dot\beta}\,\partial_m \\
\Big\{D_\alpha{}^{(A)},\ D_\beta{}^{(B)}\Big\} = \Big\{\bar D_{\dot\alpha}{}^{(A)},\ \bar D_{\dot\beta}{}^{(B)}\Big\} &= 0 \quad .
\end{aligned} \tag{16}$$

From these superfields we can construct manifestly $N=2$ invariant Lagrangians for the Akulov-Volkov field:

$$-\frac{1}{2}k^2\int d^4x d^4\theta d^4\bar{\theta}\,\boldsymbol{\Lambda}\boldsymbol{\Lambda}\bar{\boldsymbol{\Lambda}}\bar{\boldsymbol{\Lambda}} = \int d^4x d^2\theta^{(1)} d^2\bar{\theta}^{(1)}\Big[-\frac{1}{2k^2} - i\Lambda\sigma^m\partial_m\bar{\Lambda} + \ldots\Big] \quad (17a)$$

$$-\frac{1}{2}k^2\int d^4x d^4\theta d^4\bar{\theta}\,\boldsymbol{X}\boldsymbol{X}\bar{\boldsymbol{X}}\bar{\boldsymbol{X}} = \int d^4x d^2\theta^{(1)} d^2\bar{\theta}^{(1)}\Big[-\frac{1}{2k^2} - iX\sigma^m\partial_m\bar{X} + \ldots\Big] \quad (17b)$$

$$-\frac{1}{2}k^2\int d^4x d^4\theta d^2\bar{\theta}^{(2)}\,\boldsymbol{\Lambda}\boldsymbol{\Lambda}\bar{\boldsymbol{X}}\bar{\boldsymbol{X}} + h.c. = \int d^4x d^2\theta^{(1)}\Big[-\frac{1}{2k^2} - i\Lambda\sigma^m\,\partial_m\bar{X} + \ldots\Big] + h.c. \quad (17c)$$

The triple dots $(\ldots)$ stand for nonrenormalizable interactions, suppressed by powers of $k$. Note that the last Lagrangian is $N=1$ chiral. The integral over $d^2\theta^{(1)}d^2\bar{\theta}^{(1)}$ annihilates the constants, so there is no $N=1$ supersymmetry breaking for any of the three cases.

Before exhibiting the ghost structure, we first expand the $N=1$ superfields $\Lambda_\alpha$ and $X_\alpha$ in terms of component fields,

$$\begin{aligned} \Lambda_\alpha(y,\theta^{(1)}) &= \lambda_\alpha(y) + F_\alpha{}^\beta(y)\,\theta_\beta{}^{(1)} + \phi_\alpha(y)\,\theta^{(1)}\theta^{(1)} \\ \bar{X}_{\dot{\alpha}}(y,\theta^{(1)}) &= \bar{\chi}_{\dot{\alpha}}(y) + V_n(y)\,\theta^{\alpha\,(1)}\,\sigma^n_{\alpha\dot{\alpha}} + \bar{\psi}_{\dot{\alpha}}(y)\,\theta^{(1)}\theta^{(1)} \quad , \end{aligned} \quad (18)$$

where $y^m = x^m + i\theta^{(1)}\sigma^m\bar{\theta}^{(1)}$. The fields $\lambda_\alpha$ and $\bar{\chi}_{\dot{\alpha}}$ are the Goldstone spinors. In terms of components, the Lagrangians $(17a)$ - $(17c)$ become

$$\int d^4x\Big[-i\phi\sigma^m\partial_m\bar{\phi} - i\lambda\sigma^m\partial_m\Box\bar{\lambda} - \frac{1}{2}F^{*\,\dot{\alpha}}{}_{\dot{\beta}}\,\bar{\sigma}^{m\dot{\beta}\beta}\,\partial_m\partial_n F_\beta{}^\alpha\sigma^n_{\alpha\dot{\alpha}} + \ldots\Big] \quad (19a)$$

$$\int d^4x\Big[-i\psi\sigma^m\partial_m\bar{\psi} - i\chi\sigma^m\partial_m\Box\bar{\chi} - V^*_m\Big(\Box V^m - 2\partial^m\partial\cdot V\Big) + \ldots\Big] \quad (19b)$$

$$\int d^4x\Big[-i\phi\sigma^m\partial_m\bar{\chi} - i\psi\sigma^m\partial_m\bar{\lambda} - \frac{i}{2}(\sigma^n\bar{\sigma}^m)_\alpha{}^\beta\,V_n\partial_m F_\beta{}^\alpha + \ldots\Big] + h.c. \quad (19c)$$

The first two Lagrangians have higher derivatives. Third is off-diagional. All three contain ghosts.

The Lagrangians (19$a$) - (19$c$) are invariant under two sets of supersymmetry transformations. The first set are nonlinear. They follow from Eq. (9):

$$
\begin{aligned}
\delta_\eta \lambda_\alpha &= \frac{1}{k}\,\eta_\alpha \;-\; 2ik\,(\lambda\sigma^m\,\bar\eta)\partial_m\lambda_\alpha \\
\delta_\eta F_\alpha^{\beta} &= -\,2ik\,(\bar\eta\,\bar\sigma_m\,F)^\beta\partial_m\lambda_\alpha \;-\; 2ik\,(\lambda\sigma^m\,\bar\eta)\partial_m F_\alpha^{\beta} \\
\delta_\eta \phi_\alpha &= -\,2ik\,(\lambda\sigma^m\,\bar\eta)\partial_m\phi_\alpha \;-\; 2ik\,(\phi\sigma^m\,\bar\eta)\partial_m\lambda_\alpha \\
&\quad +\; ik\,\epsilon_{\beta\gamma}(\bar\eta\,\bar\sigma^m\,F)^\beta\partial_m F_\alpha^{\gamma} \\
\delta_\eta \chi_\alpha &= \frac{1}{k}\,\eta_\alpha \;-\; 2ik\,(\chi\sigma^m\,\bar\eta)\partial_m\chi_\alpha \\
\delta_\eta V_n &= -\,2ik\,(\chi\sigma^m\,\bar\eta)\partial_m V_n \;+\; ik\,(\bar\eta\,\bar\sigma^m\,\sigma^p\,\bar\sigma_n\,\partial_m\chi)V_p \\
\delta_\eta \psi_\alpha &= -\,2ik\,(\chi\sigma^m\,\bar\eta)\partial_m\psi_\alpha \;-\; 2ik\,(\psi\sigma^m\,\bar\eta)\partial_m\chi_\alpha \\
&\quad -\; ik\,(\bar\eta\,\bar\sigma^m\,\sigma^n\,\bar\sigma^p\,\epsilon)_\alpha V_n\partial_m V_p \;\; .
\end{aligned}
\tag{20}
$$

The second set of supersymmetry transformations are linear. They follow from the fact that $\Lambda_\alpha$ and $X_\alpha$ are $N=1$ superfields:

$$
\begin{aligned}
\delta_\xi\lambda_\alpha &= F_\alpha{}^{\beta}\xi_\beta \\
\delta_\xi F_\alpha{}^{\beta} &= 2\,\phi_\alpha\xi^\beta \;-\; 2i\;\partial_m\lambda_\alpha(\bar\xi\,\bar\sigma^m)^\beta \\
\delta_\xi\phi_\alpha &= -\,i\;\partial_m F_\alpha{}^{\beta}\sigma^m_{\beta\dot\beta}\,\bar\xi^{\dot\beta} \\
\delta_\xi\,\bar\chi_{\dot\alpha} &= \xi^{\alpha}\sigma^n_{\alpha\dot\alpha}\;V_n \\
\delta_\xi V^n &= -\;\xi\sigma^n\,\bar\psi \;+\; i\;\partial_m\,\bar\chi\,\bar\sigma^n\,\sigma^m\xi \\
\delta_\xi\,\bar\psi_{\dot\alpha} &= i\,(\bar\xi\,\bar\sigma^m\,\sigma^n)_{\dot\alpha}\;\partial_m V_n \quad .
\end{aligned}
\tag{21}
$$

These transformations (20) and (21) also act on the states. Because of the supersymmetry algebra, the unbroken transformations (21) commute with the Hamiltonian.

They map physical states into physical states, and ghosts into ghosts. This is easiest to see for Lagrangian (19c). We shall examine it first.

To untangle the ghost structure, we canonically quantize the fields in (19c). The momentum conjugate to $F_\alpha{}^\beta$ is $-\frac{i}{2}(\sigma^n\bar{\sigma}^o)_\beta{}^\alpha V_n$, while $-i(\phi\sigma^o)_{\dot{\alpha}}$ and $-i(\psi\sigma^o)_{\dot{\beta}}$ are conjugate to $\bar{\chi}^{\dot{\alpha}}$ and $\bar{\lambda}^{\dot{\beta}}$. These lead to the following equal-time commutation relations:

$$\begin{aligned}
\left[V^n(\boldsymbol{x},t),\ F_\alpha{}^\beta(\boldsymbol{y},t)\right] &= -(\sigma^o\bar{\sigma}^n)_\alpha{}^\beta\,\delta^{(3)}(\boldsymbol{x}-\boldsymbol{y})\\
\left\{\bar{\chi}_{\dot{\alpha}}(\boldsymbol{x},t),\ \phi_\alpha(\boldsymbol{y},t)\right\} &= -\sigma^o_{\alpha\dot{\alpha}}\,\delta^{(3)}(\boldsymbol{x}-\boldsymbol{y})\\
\left\{\bar{\lambda}_{\dot{\alpha}}(\boldsymbol{x},t),\ \psi_\alpha(\boldsymbol{y},t)\right\} &= -\sigma^o_{\alpha\dot{\alpha}}\,\delta^{(3)}(\boldsymbol{x}-\boldsymbol{y})\quad .
\end{aligned}\tag{22}$$

All other commutation relations vanish. As usual, the relations (22) can be extended to any two spacetime points,

$$\begin{aligned}
\left[V^n(x),\ F_\alpha{}^\beta(y)\right] &= -(\sigma^m\bar{\sigma}^n)_\alpha{}^\beta\,\partial_m\,\Delta(x-y)\\
\left\{\bar{\chi}_{\dot{\alpha}}(x),\ \phi_\alpha(y)\right\} &= -\sigma^m_{\alpha\dot{\alpha}}\,\partial_m\,\Delta(x-y)\\
\left\{\bar{\lambda}_{\dot{\alpha}}(x),\ \psi_\alpha(y)\right\} &= -\sigma^m_{\alpha\dot{\alpha}}\,\partial_m\,\Delta(x-y)\quad .
\end{aligned}\tag{23}$$

Here $\Delta(x)$ is the usual Lorentz-invariant commutator function normalized such that $(\partial/\partial x^o)\,\Delta(x) = \delta^{(3)}(\boldsymbol{x})$. The Fock space is constructed by expanding the fields in plane-wave solutions

$$\begin{aligned}
F_\alpha{}^\beta(x) &= \int \frac{d^3\boldsymbol{p}}{(2\pi)^{3/2}}\left[G_\alpha{}^\beta(p)e^{ipx} + H^\dagger_\alpha{}^\beta(p)e^{-ipx}\right]\\
V^n(x) &= \int \frac{d^3\boldsymbol{p}}{(2\pi)^{3/2}}\left[U^n(p)e^{ipx} + W^{n\dagger}(p)e^{-ipx}\right]\\
\phi_\alpha(x) &= \int \frac{d^3\boldsymbol{p}}{(2\pi)^{3/2}}\left[A_\alpha(p)e^{ipx} + B^\dagger_\alpha(p)e^{-ipx}\right]\\
\chi_\alpha(x) &= \int \frac{d^3\boldsymbol{p}}{(2\pi)^{3/2}}\left[C_\alpha(p)e^{ipx} + D^\dagger_\alpha(p)e^{-ipx}\right]\quad ,
\end{aligned}\tag{24}$$

and similarly for $\lambda_\alpha$ and $\psi_\alpha$. The operator relations

$$
\begin{aligned}
\left[W^{n\dagger}(p),\, g_\alpha{}^\beta(q)\right] = \left[U^n(p),\, h^{\dagger}_\alpha{}^\beta(q)\right] &= \frac{1}{2}\,(\sigma^m\bar{\sigma}^n)_\alpha{}^\beta\, p_m\, E^{-1}\,\delta^{(3)}(\boldsymbol{p}-\boldsymbol{q}) \\
\left\{A_\alpha(p),\, \bar{C}^\dagger_{\dot{\alpha}}(q)\right\} = \left\{B^\dagger_\alpha(p),\, \bar{D}_{\dot{\alpha}}(q)\right\} &= \frac{1}{2}\,\sigma^m_{\alpha\dot{\alpha}}\, p_m\, E^{-1}\,\delta^{(3)}(\boldsymbol{p}-\boldsymbol{q})
\end{aligned}
\tag{25}
$$

allow us to recover the commutators (23). They also permit us to identify the daggered and undaggered quantities as creation and annihilation operators, respectively. From (25) we see that the Fock space is off-diagonal. The one-particle states have zero norm, and nonvanishing matrix elements with each other. To exhibit the ghost structure, we diagonalize the space of one-particle states. Without loss of generality, we consider positive helicity states moving along the $+z$ axis, $p^m = (E, 0, 0, E)$.

The following linear combinations

$$
\begin{aligned}
A^{(\pm)} &= \frac{1}{\sqrt{2E}}\,(A_1 \pm E\, C_1) \\
B^{(\pm)} &= \frac{1}{\sqrt{2E}}\,(B_1 \pm E\, D_1) \\
U_\parallel{}^{(\pm)} &= \frac{1}{\sqrt{2}}\,(U^o + U^3 \mp H_1{}^1) \\
U_\perp{}^{(\pm)} &= \frac{1}{\sqrt{2}}\,(U^o + iU^2 \mp H_1{}^2) \\
W_\parallel{}^{(\pm)} &= \frac{1}{\sqrt{2}}\,(W^o + W^3 \pm G_1{}^1) \\
W_\perp{}^{(\pm)} &= \frac{1}{\sqrt{2}}\,(W^1 - iW^2 \pm G_1{}^2)
\end{aligned}
\tag{26}
$$

diagonalize the one particle states. The superscripts $(\pm)$ denote positive and negative norm states, as may be seen from the following commutation relations:

$$
\begin{aligned}
\left[U_\parallel{}^{(\pm)}(p),\, U_\parallel{}^{(\pm)\dagger}(q)\right] = \left[W_\parallel{}^{(\pm)}(p),\, W_\parallel{}^{(\pm)\dagger}(q)\right] &= \pm\,\delta^{(3)}(\boldsymbol{p}-\boldsymbol{q}) \\
\left[U_\perp{}^{(\pm)}(p),\, U_\perp{}^{(\pm)\dagger}(q)\right] = \left[W_\perp{}^{(\pm)}(p),\, W_\perp{}^{(\pm)\dagger}(q)\right] &= \pm\,\delta^{(3)}(\boldsymbol{p}-\boldsymbol{q}) \\
\left\{A^{(\pm)}(p),\, A^{(\pm)\dagger}(q)\right\} = \left\{B^{(\pm)}(p),\, B^{(\pm)\dagger}(q)\right\} &= \pm\,\delta^{(3)}(\boldsymbol{p}-\boldsymbol{q}) \;.
\end{aligned}
\tag{27}
$$

It is not hard to show that the supersymmetry transformations (21) preserve the norms of the states. They map physical states into physical states and ghosts into ghosts.The physical states and the ghosts form multiplets under the unbroken $N=1$ supersymmetry algebra.

The Lagrangians with higher derivatives can be reduced to Lagrangians with first and second order derivatives through the introduction of auxiliary fields. This can be done in terms of component fields or in terms of $N=1$ superfields. The quadratic piece of the Lagrangian (17*b*) is reproduced with two extra superfields:

$$\int d^4x d^2\theta^{(1)} \left[ -\frac{i}{2}\, \Gamma\, \sigma^m\, \partial_m \bar{X} \;+\; a\, \Gamma W \;+\; b\, WW \;+\; c\, \Gamma\Gamma \right] + h.c. \tag{28}$$

Here $\Gamma_\alpha$ and $W_\alpha$ are $N=1$ auxiliary superfields, subject to the following constraints:

$$\begin{aligned} \bar{D}_{\dot\alpha}{}^{(1)}\, \Gamma_\beta &= \bar{D}_{\dot\alpha}{}^{(1)}\, W_\beta = 0 \\ \bar{D}^{(1)}\, \bar{W} &= D^{(1)}\, W \quad . \end{aligned} \tag{29}$$

The Lagrangian (28) gives rise to the following superfield equations of motion:

$$\begin{aligned} \bar\sigma^m\, \partial_m\, \Gamma &= 0 \\ -\frac{i}{2}(\sigma^m\, \partial_m\, \bar{X})_\alpha \;+\; a\, W_\alpha \;+ 2c\, \Gamma_\alpha &= 0 \\ 2b\, D^{(1)}\, W \;+\; 2b^*\, \bar{D}^{(1)}\, \bar{W} \;+\; a\, D^{(1)}\, \Gamma \;+\; a^*\, \bar{D}^{(1)}\, \bar{\Gamma} &= 0 \quad . \end{aligned} \tag{30}$$

Eliminating $W_\alpha$ and $\Gamma_\alpha$, and imposing the relation $a^2 = 8c\,\mathrm{Re}(b)$, we find the superfield equation for $X$,

$$\sigma^m_{\alpha\dot\alpha}\, \partial_m\, D^{(1)} D^{(1)}\, \bar{X}^{\dot\alpha} = 0 \quad . \tag{31}$$

In terms of component fields, Eq. (31) takes the following form:

$$\begin{aligned} \sigma^m\, \partial_m\, \bar\psi &= 0 \\ \sigma^m\, \partial_m \Box\, \bar\chi &= 0 \\ \Box\, V^n \;-\; 2\, \partial^n\, \partial \cdot V &= 0 \quad . \end{aligned} \tag{32}$$

These equations are the same as those which follow from Lagrangian (19*b*). Their ghost structure may be analyzed as before. In this case, the positive and negative norm (one-particle) states are not eigenstates of the Hamiltonian, so the norm is not preserved under supersymmetry transformations. Instead, one finds that the one-particle states may be grouped into two physical $N = 1$ supermultiplets, and one unphysical $N = 1$ dipole ghost supermultiplet [22]. This is similar to the ghost structure of conformal supergravity, first analyzed by Ferrara and Zumino [23].

The third Lagrangian, (19*a*), can be analyzed in terms of component fields, leading to analogous results. We have not, however, been able to find a formulation in terms of $N = 1$ superfields.

## Matter Couplings to Broken Supersymmetry

The formalism of nonlinear realizations can be extended to describe the low-energy interactions of matter fields with Goldstone bosons. In the sigma model of Gell-Mann and Lévy, one introduces spinors $\psi_L$ and $\psi_R$ which transform under chiral $SU(2)_L \times SU(2)_R$. At low energies, where the chiral symmetry is strongly broken to diagonal SU(2), the model describes the effective interaction between nucleons and pions. The fields $\psi_L$ and $\psi_R$ represent the unbroken SU(2), and together with the pions, they nonlinearly realize the full chiral symmetry.

In the general case $G/H$, the spinors $\psi$ become spectator fields $\Psi$. Spectator fields transform linearly under the unbroken group $H$. Their coupling to the Goldstone bosons in $G/H$ was considered by Callan, Coleman, Wess and Zumino, who showed that it is always possible to extend a representation of $H$ to a realization of $G$. They gave a prescription for generalizing any Lagrangian invariant under $H$ to a new Lagrangian invariant under $G$. The new Lagrangian contains the spectator fields $\Psi$ and the Goldstone bosons of $G/H$. It describes the low-energy interactions of Goldstone bosons with other matter.

The same procedure may be followed for the case of spontaneously broken supersymmetry. Any Lorentz-invariant Lagrangian may be made invariant under $N = 1$ supersymmetry with the help of the Goldstone fermion. The supersymmetric Lagrangian describes the low-energy interactions of the Goldstone fermion with the spectator fields. In the remainder of this paper we shall generalize this work to show that any $N = 1$ supersymmetric Lagrangian (with chiral and vector superfields) may be extended to $N = 2$ with the help of the Goldstone superfield.

We start by extending the transformation law of an $N = 1$ chiral superfield to $N = 2$,

$$\delta_{\xi^{(2)}} \Phi = -2ik \Lambda \sigma^m \bar{\xi}^{(2)} \partial_m \Phi \quad . \tag{33}$$

This transformation law preserves the $N = 1$ chirality constraint $\bar{D}_{\dot{\alpha}}{}^{(1)} \Phi = 0$. As before, it may be used to construct an $N = 2$ superfield $\boldsymbol{\Phi}$,

$$\boldsymbol{\Phi}\Big(x, \theta^{(1)}, \bar{\theta}^{(1)}, \theta^{(2)}, \bar{\theta}^{(2)}\Big) = exp\,[\delta_{\theta^{(2)}}] \times \Phi\Big(x,\ \theta^{(1)},\ \bar{\theta}^{(1)}\Big) \quad . \tag{34}$$

The superfield $\boldsymbol{\Phi}$ obeys the following constraints:

$$\bar{D}_{\dot{\alpha}}{}^{(1)} \boldsymbol{\Phi} = 0 \tag{35a}$$

$$\bar{D}^{\dot{\beta}\,(2)} \boldsymbol{\Phi} = 2ik\,(\bar{\sigma}^m \Lambda)^{\dot{\beta}}\, \partial_m \boldsymbol{\Phi} \quad . \tag{35b}$$

These constraints are consistent with the $N = 2$ D-algebra (16).

We proceed analogously for the $N = 1$ vector superfield $V$. Its $N = 2$ transformation law is given by

$$\delta_{\xi^{(2)}} V = -ik \Big(\tilde{\Lambda} \sigma^m \bar{\xi} - \xi \sigma^m \bar{\tilde{\Lambda}}\Big)\, \partial_m V \quad , \tag{36}$$

where $\tilde{\Lambda}_\alpha$ is expressed in terms of $\Lambda_\alpha$ as in Eq. (12). This preserves the $N = 1$ constraint $V = V^+$ . The $N = 2$ superfield $\tilde{\boldsymbol{V}}$ obeys the following constraints:

$$\tilde{\boldsymbol{V}} = \tilde{\boldsymbol{V}}^+ \tag{37a}$$

$$D_\alpha{}^{(2)}\, \tilde{\boldsymbol{V}} = ik\, \sigma^m_{\alpha\dot{\alpha}}\, \bar{\tilde{\boldsymbol{\Lambda}}}{}^{\dot{\alpha}}\, \partial_m\, \tilde{\boldsymbol{V}} \quad . \tag{37b}$$

These constraints are also consistent with the $N = 2$ D-algebra.

To generalize the concept of a gauge transformation, we first consider the transformation law of a matter multiplet

$$\Phi' = exp\,(i\Sigma)\;\Phi\;. \tag{38}$$

The gauge parameter $\Sigma$ must satisfy the same constraints as $\Phi$. In analogy to $N=1$, one might expect the transformation on $\Phi$ to be compensated by a transformation on $\tilde{V}$:

$$\tilde{V}' = \tilde{V} + i(\Sigma^+ - \Sigma)\;. \tag{39}$$

However, this does not preserve the constraints (37). To circumvent this difficulty, we must discard (39), and find a function $V(\tilde{V}, \Lambda)$ whose constraints are also satisfied by $\Sigma$. There exists a general procedure for converting a superfield $\tilde{V}$, which satisfies a constraint of type (37*b*), into a new superfield $V$, a function of $\Lambda_\alpha$ and $\tilde{V}$, which satisfies a constraint of type (35*b*). One first performs the $N=2$ chiral projection

$$V^o = -\frac{1}{4}k^2\;\bar{D}^{(2)}\bar{D}^{(2)}\;\bar{\Lambda}\bar{\Lambda}\;\tilde{V}\;, \tag{40}$$

and then constructs the superfield $V$:

$$V = V^o - k\,\Lambda^\alpha\,D_\alpha{}^{(2)}\;V^o - \frac{1}{4}k^2\;\Lambda\Lambda\,D^{(2)}D^{(2)}\;V^o\;. \tag{41}$$

The superfield $V$ obeys the constraint (35*b*). Its lowest component is the same as the lowest component of $\tilde{V}$. Equations (40) and (41) illustrate the general procedure for decomposing a chiral superfield into a standard form [14]. Armed with the superfield $V$, we can now construct a gauge transformation which preserves the $N=2$ constraints. If we take

$$V \;\rightarrow\; V - i\,\Sigma\;, \tag{42}$$

then

$$\Phi^+\;exp\left(\frac{1}{2}(V+V^+)\right)\Phi \tag{43}$$

is both gauge invariant and $N=2$ symmetric.

For $\Lambda_\alpha = 0$, $\theta_\alpha{}^{(2)} = \bar{\theta}_{\dot{\alpha}}{}^{(2)} = 0$, these expressions reduce to the usual $N = 1$ gauge invariant expressions used in supersymmetric gauge theories.

The $N = 2$ generalization of the gauge invariant $N = 1$ superfield $W_\alpha$ is given by

$$\boldsymbol{W}_\alpha = -\frac{1}{8} \bar{D}^{(1)2} D_\alpha{}^{(1)} (\boldsymbol{V} + \boldsymbol{V}^+) \quad . \tag{44}$$

The $N = 2$ superfield $\boldsymbol{W}_\alpha$ is both chiral and gauge invariant. The above construction can be immediately generalized to nonabelian gauge groups.

Having constructed the $N = 2$ superfields corresponding to $N = 1$ chiral and vector superfields, we now proceed to construct an $N = 2$ Lagrangian which reduces to the appropriate $N = 1$ Lagrangian when $\Lambda_\alpha = X_\alpha = 0$. We follow the general procedure given in Ref. [14]. This procedure takes advantage of the fact that the $\theta^{(2)}\theta^{(2)}\bar{\theta}^{(2)}\bar{\theta}^{(2)}$ components of $\boldsymbol{\Lambda}\boldsymbol{\Lambda}\bar{\boldsymbol{\Lambda}}\bar{\boldsymbol{\Lambda}}$, $\boldsymbol{X}\boldsymbol{X}\bar{\boldsymbol{X}}\bar{\boldsymbol{X}}$ and $\boldsymbol{\Lambda}\boldsymbol{\Lambda}\bar{\boldsymbol{X}}\bar{\boldsymbol{X}}$ all contain a constant term. Therefore, these objects pick out the $\theta^{(2)} = \bar{\theta}^{(2)} = 0$ components of anything the multiply. This is just what we need to construct invariant Lagrangians with the correct low energy limits. Since we wish to preserve $N = 1$ chirality properties, we use $\boldsymbol{\Lambda}\boldsymbol{\Lambda}\bar{\boldsymbol{X}}\bar{\boldsymbol{X}}$ for $N = 1$ F-terms. For simplicity, we also use it for $N = 1$ D-terms. Thus an $N = 2$ extension of the $N = 1$ Lagrangian

$$\frac{1}{2}\int d^2\theta^{(1)}\, d^2\bar{\theta}^{(1)}\; \Phi^+\, exp\,(V)\Phi + \frac{1}{4}\int d^2\theta^{(1)}\, WW + \int d^2\theta^{(1)}\, f(\Phi) + h.c. \tag{45}$$

is given by

$$\begin{aligned} &-\frac{1}{2}k^2\int d^4\theta\, d^2\bar{\theta}^{(2)}\; \boldsymbol{\Lambda}\boldsymbol{\Lambda}\bar{\boldsymbol{X}}\bar{\boldsymbol{X}} \\ &+\frac{1}{2}k^4\int d^4\theta\, d^4\bar{\theta}\; \boldsymbol{\Lambda}\boldsymbol{\Lambda}\bar{\boldsymbol{X}}\bar{\boldsymbol{X}}\; \boldsymbol{\Phi}^+\; exp\left(\frac{1}{2}(\boldsymbol{V}+\boldsymbol{V}^+)\right)\boldsymbol{\Phi} \\ &+\frac{1}{4}k^4\int d^4\theta\, d^2\bar{\theta}^{(2)}\; \boldsymbol{\Lambda}\boldsymbol{\Lambda}\bar{\boldsymbol{X}}\bar{\boldsymbol{X}}\; \boldsymbol{W}\boldsymbol{W} \\ &+ k^4\int d^4\theta\, d^2\bar{\theta}^{(2)}\; \boldsymbol{\Lambda}\boldsymbol{\Lambda}\bar{\boldsymbol{X}}\bar{\boldsymbol{X}}\; f(\boldsymbol{\Phi}) + h.c. \end{aligned} \tag{46}$$

Equation (46) reduces to (45) when $\Lambda_\alpha = X_\alpha = 0$. It gives the low energy coupling of the $N = 2$ goldstino to the $N = 1$ effective theory. As discussed earlier, the Lagrangian (46) contains ghost fields. How many of these become gauge degrees of freedom when (46) is coupled to $N = 2$ supergravity is currently under investigation.

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## Figure Caption

1. a) The potential $V$ as a function of the radial field $|\Pi|$. The minimum is at $v$. b) The potential $V$ rescaled by a factor of 100. At low energies the fields are constrained to lie on the sphere $\Pi^2 = \pi^2 + \sigma^2 = v^2$.

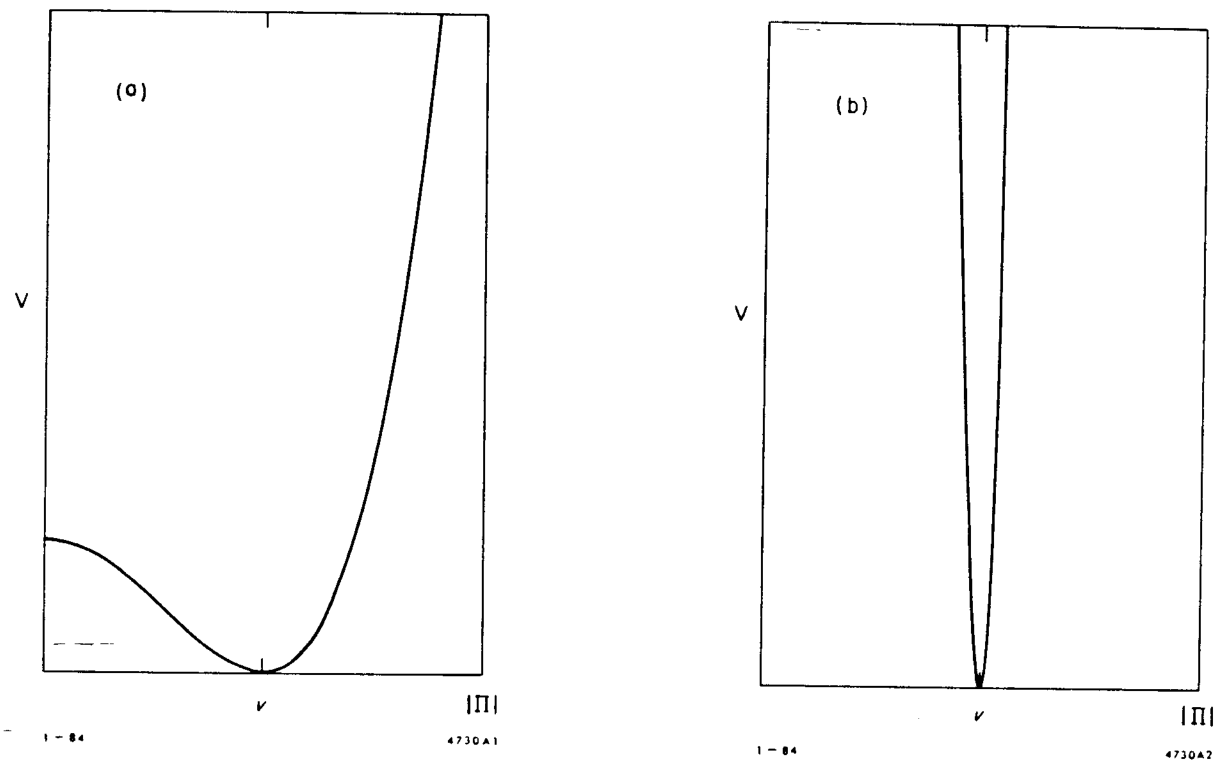


Fig. 1