SLAC-PUB-3279
January 1984
(N)

SUPERSYMMETRY BREAKING AT FINITE TEMPERATURE AND THE EXISTENCE OF THE GOLDSTONE FERMION*

D. BOYANOVSKY
Stanford Linear Accelerator Center
Stanford University, Stanford, California 94305

Invited talk presented at the
Workshop on Supersymmetry in Physics
Los Alamos, New Mexico
December 14-21, 1983

* Work supported by the Department of Energy, contract DE-AC03-76SF00515.

1. Introduction

Supersymmetry (SUSY) is a very interesting and aesthetically beautiful theoretical concept which is playing an important role in constructing realistic models in particle physics [1]. It is hoped that it might provide for solutions to outstanding problems, for example naturalness and gauge hierarchy because it provides a natural mechanism for cancelling divergences.

However despite these appealing features, SUSY must be broken in nature since we do not observe any bosonic partners of fermions [2].

If it is incorporated in the framework of Grand Unified Theories (SUSY GUT's) it is necessary to understand the effect of finite temperature in this theories.

The behavior of regular symmetries at finite temperatures is by now fairly well understood [3]. While most symmetries – with few exceptions [4] – if broken at zero temperature are restored at sufficiently high temperature, if has been realized that SUSY behaves differently: unbroken SUSY at $T = 0$ breaks at high temperature [5].

In a more recent article, Girardello et al. [6] have studied some aspects of SUSY at finite temperature and concluded that its breaking is a natural consequence of the fact that bosons and fermions have different quantum statistics. This can be understood from the following argument: one of the consequences of unbroken SUSY is that fermions and bosons belonging to the same multiplet have the same mass. At finite temperature, particules move in a plasma of excitations *the heat bath* with an "effective mass" resulting from the interactions of the particles with the medium. However bosons and fermions are treated differently by the heat-bath (due to different statistics) and therefore they will *not* be related as they would if SUSY were unbroken.

In zero temperature field theory symmetry breaking is generally catalogued in two classes: explicit or spontaneous. In the first case there is a term in the Hamiltonian that describes the system that **is not** invariant under the symmetry transformation. In the second case, the Hamiltonian **is** invariant under this symmetry transformation but the lowest energy state **is not**. It is possible to show that in the second case there is a massless particle in the spectrum of the theory that carries the same quantum numbers as the charges that generate this symmetry (Goldstone excitation). In the case of SUSY this particle is a fermion: the Goldstone Fermion [1,7].

We have argued above that at finite temperature SUSY is broken in the sense that physical properties of bosons and fermions are no longer related. However, this breaking *cannot* be explicit, there is no term in the Hamiltonian that can break SUSY explicitly. Rather it is the interaction of the particles with the heat-bath the responsible for SUSY breaking. Can it be spontaneous? This question has been investigated in ref. [6] were it was concluded that there is no Goldstone Fermion in the spectrum, however these authors used a non-covariant formalism.

Van Hove [8] has recently argued that there is a subtlety in the definition of the thermal averages of variations of operators under a SUSY transformation. Several authors have investigated this point further and they conclude that a "graded" thermal average has to be introduced [9,10,11]. However in this formalism bosons and fermions obey the **same** quantum statistics leading to the conclusion that if SUSY is unbroken at zero temperature it stays unbroken at any finite temperature. Since bosons and fermions have *unphysical* statistics in this formalism nothing can be said about the physical spectrum of excitations using these arguments.

In this article we attempt to understand the nature of SUSY breaking at finite temperature and whether or not there is a Goldstone Fermion associated with this phenomenon.

Unlike ref. [6] we will use a manifestly Lorentz covariant formalism to study the theories in consideration, this is necessary to fully expose the dynamical properties of physical observables [12]. It will be shown that in this way a Goldstone excitation is found as a consequence of the breaking at finite temperature, an it will become clear how this excitation decouples at zero temperature. The rest of the article is devoted to the technical details of the investigation and is divided as follows: in section 2 the standard Ward identities at zero temperature are briefly reviewed. In section 3 the finite temperature formalisms are reviewed exposing the differences between the covariant (real time) and non-covariant (imaginary-time) formulations. The results of section 2 will be extended to finite temperature in the covariant approach.

Section 4 is devoted to the computation of the fermion thermal Green's functions exposing the mechanism that gives rise to the Goldstone pole and the way it decouples at zero temperature. Some conclusions are summarized at the end of the article.

2. Zero Temperature Ward Identities

The phenomenology of symmetry breaking is usually understood by means of certain identities among Greens's functions of these fields, the Ward identities, that stem from the invariance properties of the Lagrangian. Here we will review these identities and their relation to well known results in current algebra and Goldstone's theorem.

In the theories to be considered there is a supermultiplet (ϕ, ψ, F) of spinless bosons, Majorana fermions and auxiliary fields.

The SUSY transformations are written as:

$$\delta\phi(x) = \delta\bar{\epsilon}\ \psi(x) \tag{2.1a}$$

$$\delta\psi(x) = [-i\,\not{\partial}\,\phi(x) - F(x)]\delta\epsilon \tag{2.1b}$$

$$\delta F(x) = \delta\bar{\epsilon}\, i\,\not{\partial}\,\psi(x) \tag{2.1c}$$

where $\delta\epsilon$ is a constant Grassman (Majorana) parameter. These relations can be generalized to more complicated theories. Under the transformations (2.1a)-(2.1c) the action changes by a total derivative.

$$\delta \int d^d x \, \mathcal{L} = \int \delta\epsilon \, \partial_\mu S_\mu(x) d^d x \tag{2.2}$$

where $S_\mu(x)$ is the super current. The broken symmetry Ward identities can be obtained as a response to a SUSY transformation with $\delta\epsilon(x)$ a space-time dependent parameter. Define

$$\langle \varphi_1(x_1) \dots \varphi_n(x_n) \rangle_J = \frac{\int \mathcal{D}\varphi_a \dots \varphi_1(x_1) \dots \varphi_n(x_n) \exp\left(\int d^d x [\mathcal{L}[\varphi] + J_i \varphi_i]\right)}{(\text{Same for } J_i = 0)} \tag{2.3}$$

where $\varphi_a(x_i)$ stands for either bosonic of fermionic fields. The transformation (2.1a)-(2.1c) in the numerator of (2.3) amounts to a change of variables in the functional integral under which it is invariant, therefore

$$\frac{\delta}{\delta \bar{\epsilon}(x)} \langle \varphi_1(x_1 \dots \varphi_n(x_n) \rangle_J = 0 \tag{2.4}$$

writing $\delta\varphi_i(x) = (\partial\varphi_i / \partial\epsilon)\delta\epsilon(x)$, eq. (2.4) reads

$$\begin{aligned} &\partial_{\mu_z} \langle S_\mu(z) \varphi_1(x_1) \dots \varphi_n(x_n) \rangle_J + \delta(x_i - z) \left\langle \varphi_1(x_1) \dots \frac{\partial\varphi_i(x_i)}{\partial\epsilon(x_i)} \dots \varphi_n(x_n) \right\rangle_J \\ &+ J_\phi(z) \langle \psi(z) \varphi_1(x_1) \dots \rangle_J + \langle (i \not{\partial} \phi(z) - F(z)) \varphi_1 \dots \rangle_J J_\psi(z) + \\ &J_F(z) \langle i \not{\partial} \psi(z) \varphi_1 \dots \rangle_J = 0 \end{aligned} \tag{2.5}$$

This is the most general form of the Ward identities [13]. Consider the case $n = 1$, $\varphi_1 = \bar{\psi}$ and $J_i = 0$ in eq. (2.4). This gives

$$\partial_{\mu_z} \langle S_\mu(z) \bar{\psi}(x) \rangle + \delta(z - x) \langle i \not{\partial} \phi - F \rangle = 0 \tag{2.6}$$

or

$$\int d^d z \, \partial_{\mu_z} \langle S_\mu(z) \bar{\psi}(x) \rangle = \langle F \rangle \tag{2.7}$$

where we have assumed that ϕ and F can acquire position independent vacuum expectation values.

The current algebra relation (2.7) implies that if $\langle F\rangle \neq 0$ there is a Goldstone fermion in the spectrum. Another interesting relation can be derived. Consider eq. (2.4) with $n=0$ but $J_i \neq 0$.

$$\partial_{\mu_z}\langle S_\mu(z)\rangle_J + J_\phi(z)\langle\psi(z)\rangle_J + \langle i\,\partial\!\!\!/\,\phi(z) - F(z)\rangle_J J_\psi(z) + J_F\langle i\,\partial\!\!\!/\,\psi(z)\rangle_J = 0 \qquad (2.8)$$

Perform a Legendre transformation and integrate over z, this leads to

$$\int dZ\left\{\frac{\delta\Gamma}{\delta\phi}\langle\psi\rangle_J + \langle i\,\partial\!\!\!/\,\phi - F\rangle_J\frac{\delta\Gamma}{\delta\,\bar{\psi}} + \frac{\delta\Gamma}{\delta F}\langle i\,\partial\!\!\!/\,\psi\rangle_J\right\} = 0 \qquad (2.9)$$

In this expression take the functional derivative $\delta/\delta\psi(x)$ and set the sources to zero assuming $\langle\phi\rangle \underset{J\to 0}{\to} v$, $\langle F\rangle \underset{J\to 0}{\to} f$ we find

$$0 = \left.\frac{\delta V[F,\phi]}{\delta\phi}\right|_{v,f} = f\, S_\psi^{-1}\,(p_\mu = 0) \qquad (2.10)$$

where $V[F,\phi]$ is the effective potential as a function of F,ϕ and $S_\psi^{-1}(p_\mu = 0)$ is the inverse of the full fermion propagator at zero momentum. Therefore if $f \neq 0$ there is a pole at zero momentum in the fermion propagator, the Goldstone pole.

3. Finite Temperature Formalisms

A. Imaginary time (Matsubara)

In this formalism the quantum theory is studied in Euclidean time with the (imaginary) time variable restricted to the internal $0 \leq \tau \leq \beta = 1/T$ (T = temperature). This formalism is not Lorentz covariant, momenta are continous but frequencies are discrete variables: $(2n+1)\pi T$ for fermions and $2n\pi T$ for bosons. At finite temperature the fields must obey periodic (antiperiodic) boundary conditions for bosons (fermions) in imaginary time [3,12,14].

$$
\begin{aligned}
\phi(\beta,\vec{x}) &= \phi(0,\vec{x}) \qquad &\text{bosons} \\
\psi(\beta,\vec{x}) &= -\psi(0,\vec{x}) \qquad &\text{fermions}
\end{aligned}
\tag{3.1}
$$

Since SUSY is deeply related to Lorentz covariance, and the Matsubara formalism is *not* covariant, it is not suited to the study of SUSY at finite T.

B. Real Time

Although at finite temperature a field theory looses its Lorentz invariance because the heat-bath defines a reference frame (its center-of-mass), the theory can still be quantized in a fully covariant fashion [15].

A covariant density matrix operator $\hat{Z}_G$ can be defined and thermal averages of physical operators are written as $\langle O\rangle = (Tr\, O\ \hat{Z}_G)/Tr\, \hat{Z}_G$. In the rest frame of the heat-bath

$$\hat{Z}_G = Tr\, e^{-\beta H} \tag{3.2}$$

$$\langle O\rangle = \frac{Tr\, O\, e^{-\beta H}}{Tr\, e^{-\beta H}} \tag{3.3}$$

The Minkowski space propagator is

$$D(x,t) = \frac{Tr\{e^{-\beta H}\, T\ \hat{\varphi}(x,t)\ \hat{\varphi}(x,0)\}}{Tr\, e^{-\beta H}} \tag{3.4}$$

where $\hat{\varphi}(x,t) = e^{iHt}\ \varphi(x,0)\, e^{-iHt}$ is the Heisenberg operator.

The Matsubara formalism is best suited for the study of the perturbative expansion of the theory. However to study the response of the system to external perturbations and dynamical quantities in general one has to examine the real-time linear response functions [12,16]. This fact and the explicit Lorentz covariance of the real time approach indicate that the study of finite temperature SUSY should be done using this formalism. The real-time (Minkowski space) propagator $D(\vec{x}, t)$ is the analytical continuation of the imaginary-time (Euclidean) propagator $D(\vec{x}, \tau)$ to $-\infty \leq t = i\tau \leq +\infty$. As has been pointed out in refs. [3, 12, 14, 16] the Fourier transform $D(k_o, \vec{k})$ is **not** the continuation of $D(w_n, \vec{k})$.

$D(w_n, \vec{k})$ has to be continued to arbitrary Euclidean energy w (this continuation is unique) [14, 16], $D(w, \vec{k})$ is analytic in the right and left complex w plane with possible discontinuities across the imaginary axis that yield the spectral density

$$\rho(k_o, \vec{k}) = D(ik_o - \epsilon, \vec{k}) - D(ik_o + \epsilon, \vec{k}) \tag{3.5}$$

and finally

$$D(k_o, \vec{k}) = D(i\ (k_o + i\epsilon),\ \vec{k})\ +\ \frac{\rho\,(k_o, \vec{k})}{(e^{\beta k_o} - 1)} \tag{3.6}$$

Expression (3.6) is written in the rest frame of the heat-bath. The poles of $D(k_o,\ \vec{k})$ define the energy of excitations of momentum $\vec{k}$ in a definite reference frame.

With eq. (3.6) the fields are guaranteed to fullfil the conditions (3.1) in imaginary time (Kubo-Martin-Schwinger condition). The real-time free propagators (in the heat-bath rest frame) read [3, 15]:

$$\begin{aligned} D_\beta\,(k) &= \frac{i}{k^2 - m^2 + i\epsilon} + 2\pi\,\frac{\delta\,(k^2 - m^2)}{e^{\beta E} - 1} && \text{(bosons)} \\ S_\beta\,(k) &= \frac{i}{\not{k} - m + i\epsilon} - 2\pi\,(\not{k} + m)\frac{\delta(k^2 - m^2)}{e^{\beta E} + 1} && \text{(fermions)} \\ E &= (\vec{k}^2 + m^2)^{1/2} \end{aligned} \tag{3.7}$$

In order to perform calculations we need an expression for the density matrix. This is given in Euclidean time as a functional integral [12].

$$Z_G = \int\ D\phi \ldots \exp\Big[-\int_o^\beta d\tau \int\ d^3x\, L[\varphi(x, \tau)]\Big] \tag{3.8}$$

where the fields obey the condition (3.1) the result of the calculation can then be continued to real-time as indicated above. We want to extend the results of section 2 to finite temperature. It has been recognized by Girardello et al. [6]. that the SUSY transformations (2.1a)-(2.1c) with constant $\delta\epsilon$ are incompatible with the conditions (3.1) therefore one must impose

$$\delta\,\epsilon\,(\tau=0) = -\delta\,\epsilon\,(\tau=\beta) \tag{3.9}$$

The transformations (2.1a)-(2.1c) are generalized with $\delta\epsilon(\vec{x},\tau)$ satisfying (3.9). The thermal Green functions in Euclidean time are defined as

$$\langle\varphi_1(x_1,\tau_1)\ldots\varphi_n(x_n,\tau_n)\rangle_\beta^J = \frac{\int D\varphi_a\,\varphi_1\ldots\varphi_n\,\exp\Big(-\int_o^\beta d\tau\;dx\;(\mathcal{L}+J_i\varphi_i)\Big)}{(\text{Same with }\;J_i=0)} \tag{3.10}$$

The steps leading to eqs. (2.6) and (2.9) and can be followed leading to:

$$\partial_{\mu_{z,\tau}}\langle S_\mu(\vec{z},\tau)\,\bar{\psi}\,(\vec{x},\tau')\rangle_\beta + \delta(\vec{z}-\vec{x})\delta(\tau-\tau')\langle i\,\not{\partial}\,\phi(\vec{z},\tau)-F(z,\tau)\rangle_\beta = 0 \tag{3.11}$$

In this expression we can continue analytically to real-time and integrate over $\vec{z}$ and t leading to

$$\int dz\,dt\;\partial_{\mu_{z,t}}\langle S_\mu(z,t)\;\bar{\psi}\;(x,t')\rangle_\beta = \langle F\rangle_\beta \tag{3.12}$$

and by the same token the finite T version of eq. (2.10) reads

$$\frac{\partial V^\beta\,[F,\phi]}{\partial\,\phi}\bigg|_{\substack{\nu\\f}} = f\;S^{-1}_{\beta\psi}\;(k_o=0\,,\,\vec{k}=0) \tag{3.13}$$

where we assumed $\langle\phi\rangle_\beta=\nu$, $\langle F\rangle_\beta=f$ and V^β is the temperature dependent effective potential [3]. Therefore if $f\neq 0$ a long wavelength fermionic excitation can be created with zero energy, this is the extension of Goldstone's theorem.

It should be mentioned in passing that there is another formalism close in spirit to the real-time approach [17]. It allows to avoid ambiquities in high order calculations in perturbation theory with the real-time propagators, in this formalism the same set of identities can be found [18]. The content of the last two sections is formal, in the next section calculations in specific models will be carried out in real-time approach.

4. Explicit Computations

We have learned in the last section that the criteria for SUSY breaking is the same one as $T = 0$, namely that $f \neq 0$ and that this implies the existence of a Goldstone fermion. In this section we will compute the effective potential for the scalar fields and the real-time fermion propagator in some models and relations (3.12) and (3.13) will be checked. We study examples where SUSY is unbroken at $T = 0$.

A. Wess-Zumino in $D = 4$ [19]

The model is defined by the supermultiplet $\Phi = (Z, \psi, \mathcal{H})$ where $Z = \frac{1}{\sqrt{2}}(\mathcal{A} + i\mathcal{B})$, ψ a Majorana spinor and $\mathcal{H} = \frac{1}{\sqrt{2}}(\mathcal{F} + i\,\mathcal{G})$ an auxiliary field. $\mathcal{A}$ and $\mathcal{F}$ are scalar and $\mathcal{B}$ and $\mathcal{G}$ pseudoscalar fields. The Lagrangian is

$$
\begin{aligned}
\mathcal{L} =& \partial_\mu Z \partial_\mu Z^* + \frac{1}{2}\bar{\psi}\, i\, \partial\!\!\!/\, \psi + \mathcal{H}^*\mathcal{H} + \mathcal{H}P'(Z) + \mathcal{H}^* P'(Z^*) \\
& - \frac{1}{2}\bar{\psi}\left[\gamma_+ P''(Z) + \gamma_- P''(Z^*)\right]\psi
\end{aligned}
\tag{4.1}
$$

where $\gamma_\pm = \frac{1}{2}(1 \pm \gamma_5)$ and $P(Z) = -\ell Z + \frac{g}{6} Z^3$ with $\underline{\ell}$ and $\underline{\mathrm{g}}$ positive constants. The effective potential is calculated as usual [3], shifting the fields $\mathcal{A} = \mathcal{A}' + A$, $\mathcal{F} = \mathcal{F}' + F$, the induced masses for the particles are

$$
m_A^2 = \frac{g}{\sqrt{2}}\left(\frac{g}{\sqrt{2}} A^2 - F\right) \tag{4.2a}
$$

$$
m_B^2 = \frac{g}{\sqrt{2}}\left(\frac{g}{\sqrt{2}} A^2 + F\right) \tag{4.2b}
$$

$$
m_\psi = \frac{g}{\sqrt{2}} A \tag{4.2c}
$$

Following the methods of Dolan and Jackiw [3] the one loop effective potential can be written as $V_{eff} = V_{T=0} + V_\beta$ where

$$V_{T=0}[A,F] = V_{tree}[A,F] + \frac{1}{2}\int \frac{d^4k}{(2\pi)^4}\left\{\ell n[k^2+m_A^2] + \ell n[k^2+m_B^2] - 2\ell n[k^2+m_\psi^2]\right\}$$

$$V_{tree}[A,F] = -\frac{1}{2}F^2 - \sqrt{2}\ F\left[\frac{g}{4}A^2 - \ell\right] \tag{4.3}$$

and

$$V_\beta[A,F] = \frac{1}{\beta}\int \frac{d^3k}{(2\pi)^3}\left\{\ell n[1-e^{-\beta E_A}] + \ell n[1-e^{-\beta E_B}] - 2\ell n[1+e^{-\beta E_\psi}]\right\} \tag{4.4}$$

$$E_i = (k^2 + m_i^2)^{1/2}$$

To check eq. (3.13) we need $\frac{\partial V_{eff}}{\partial A}$. From (4.3) and (4.4) we find

$$\frac{\partial V_{eff}}{\partial A} = -Fm_\psi + Fm_\psi\frac{g^2}{2}\int\frac{d^4k}{(2\pi)^4}\left\{\frac{1}{(k^2+m_A^2)(k^2+m_\psi^2)} - \frac{1}{(k^2+m_B^2)(k^2+m_\psi^2)}\right\}$$
$$+\frac{1}{2}g^2A\int\frac{d^3k}{(2\pi)^3}\left\{\frac{1}{E_A(e^{\beta E_A}-1)} + \frac{1}{E_B(e^{\beta E_B}-1)} + \frac{2}{E_\psi(e^{\beta E_\psi}+1)}\right\} \tag{4.5}$$

It is interesting to notice that the first two terms (the $T=0$ contributions) vanish for $F=0$, thus there is a supersymmetric solution at $T=0$, this is in agreement with no-renormalization results. However the finite temperature contribution [third term in (4.5)] does not vanish at $F=0$, therefore there is no solution for the set of equations $\partial V_{eff}/\partial A = 0$ and $\partial V_{eff}/\partial F = 0$ with $F=0$ at $T \neq 0$. This result can be traced back to the fact that bosons and fermions obey different statistics in agreement with conclusions of Girardello et al. [6]. The solution of $\partial V_{eff}/\partial A = 0$ gives rise to $F \sim C_\beta$ where C_β is the finite temperature contribution to (4.5), at very low temperatures we find ($T \ll m$)

$$F \approx m\left(\frac{T}{m}\right)^{3/2} e^{-m/T} \tag{4.6}$$

where m is the common mass of the multiplet at $T = 0$. We also need to compute the real-time inverse fermion propagator at $p_o = 0$, $\vec{p} = 0$

$$
\begin{aligned}
S^{-1}(p) &= S_o^{-1}(p) - \Sigma(p) \\
S_o^{-1}(p) &= -i(\not{p} - m_\psi)
\end{aligned}
\tag{4.7}
$$

using the real-time propagators quoted in eq. (3.7), the one-loop self-energy is $\Sigma = \Sigma_{T=0} + \Sigma_\beta$

$$
\Sigma_{T=0}(p_\mu = 0) = i\, g^2 \frac{m_\psi}{2} \int \frac{d^4k}{(2\pi)^4} \Big[\frac{1}{(k^2 + m_A^2)(k^2 + m_\psi^2)} - \frac{1}{(k^2 + m_B^2)(k^2 + m_\psi^2)} \Big]
$$

$$
\Sigma_\beta(p_o = 0, \vec{p} = 0) = \frac{i\, g^2 A}{2F} \int \frac{d^3k}{(2\pi)^3} \Big[\frac{1}{E_A(e^{\beta E_A} - 1)} + \frac{1}{E_B(e^{\beta E_B} - 1)} + \frac{2}{E_\psi(e^{\beta E_\psi} + 1)} \Big]
\tag{4.8}
$$

where we have used the relations (4.2a)-(4.2c). Comparing (4.5) and (4.8) we find that indeed:

$$
\frac{\partial V_{eff}}{\partial A} = F\Big(i S_{\psi\beta}^{-1}(p_o = 0\,, \vec{p} = 0) \Big) = 0 \tag{4.9}
$$

Therefore since $F \neq 0$ [see eq. (4.6)], $S_{\beta\psi}^{-1}(p_o = 0\,, \vec{p} = 0) = 0$, thus there is a pole at $p_o = 0\,, \vec{p} = 0$ in the fermion propagator. Although we have not mentioned the renormalization procedure for this theory, it will not affect our results since renormalization can be performed at $T = 0$ with wavefunction renormalization for F as usual, the finite temperature corrections are finite. This analysis can be carried out in some other models with similiar results [20].

At this point two interesting questions arise: How is it that at $T = 0$ SUSY is explicit and the fermion is massive and at $T \neq 0$ this fermion acquires a massless pole? The second question is: How is the current algebra relation (3.12) realized? To answer these questions we will study a simple model in $D = 2$.

The Lagrangian is

$$L = \frac{1}{2}\left[(\partial_\mu \phi)^2 + \bar{\psi}\; i\; \not{\partial}\; \psi + F^2 - 2mF(\phi^2 - b) - 2m\phi\,\bar{\psi}\,\psi\right] \tag{4.10}$$

To calculate the effective potential we shift the fields $\phi \rightarrow \phi + \varphi, F \rightarrow F + f$. Following the same steps as above we find

$$0 = \frac{\partial V}{\partial \phi} = f\; S^{-1}_{\beta\psi}\left(p_o = 0\,,\, \vec{p} = 0\right) \tag{4.11}$$

where at low temperatures

$$F \simeq -m_\psi\, e^{-m_\psi/T} \times \text{(powers of } T/m_\psi) \quad ; \quad m_\psi = 2m\varphi \tag{4.12}$$

Since the inverse propagator vanishes at $p_o = 0\,,\, \vec{p} = 0$, it can be written at small p_o, $\vec{p}$ as:

$$S^{-1} \simeq\; -i\; \not{p}\, A(p^2 = 0) - i\,\gamma_o p_o\, B_\beta\; - i\vec{\gamma}\vec{p}\; D_\beta \tag{4.13}$$

where B_β (D_β) is the derivative of the finite temperature correction to the self-energy with respect to p_o $(\vec{p})$ evaluated at $p_o = 0$, $\vec{p} = 0$. Since at $T \neq 0$ there is no Lorentz invariance $B_\beta \neq D_\beta$. Calculating the one-loop self-energy in real-time we find at $\vec{p} = 0$

$$B_\beta\; \approx\; e^{m_\psi/T} \times \text{(powers of } T/m_\psi) \tag{4.14}$$

Therefore we see that as $T \rightarrow 0$ this term dominates over the $T = 0$ contribution to $S^{-1}(p)$, hence

$$S\,(p_o\,,\, \vec{p} = 0) \underset{\substack{p_o \to 0 \\ T \to 0}}{\longrightarrow} \frac{e^{-m_\psi/T}}{\gamma_o\, p_o} \approx \frac{f}{\gamma_o\, p_o} \times \text{(powers of } m_\psi/T) \tag{4.15}$$

This expression indicates that the residue of the Goldstone pole vanishes as $T \rightarrow 0$ (and it is positive) cleary exposing the fact that the Goldstone contribution should vanish as $T \to 0$. A straightforward calculation indicates that the imaginary part of the finite T contribution to the self-energy vanishes at $p_o = 0$, $\vec{p} = 0$ [21], this fact is crucial and necessary to check the Ward identities (there is no imaginary contribution to $\partial V_{eff}/\partial\phi$) and indicates that this massless excitation does not decay on mass-shell.

Although we call this massless pole a Goldstone excitation, we have to show that it couples to the supercurrent, namely that the Ward identity (3.12) is satisfied. The supercurrent of the model defined by (4.10) is

$$S_\mu = (\not{\partial}\phi + iF)\gamma^\mu\psi + if\gamma^\mu\psi \tag{4.16}$$

where we have shifted the fields. Since $\langle\bar{\psi}\,\psi\rangle$is proportional to $e^{-m_\psi/T}$ as T, $p_o \to 0$ the second term in (4.16) contributes to higher order in $e^{-m_\psi/T}$. The first term gives:

$$\partial\mu S_{\mu_q} = \Big(\Box\phi + i\,\not{\partial}F\Big)\psi + \Big(\not{\partial}\phi + iF\Big)\not{\partial}\psi \tag{4.17}$$

up to one loop we use the linearized equation of motion for F

$$F = m_\psi\phi + \ldots \tag{4.18}$$

and adding and substracting the mass terms for ϕ and ψ m_B^2 and m_ψ^2 respectively and using

$$m_B^2 - m_\psi^2 = 2mf \tag{4.19}$$

eq. (3.12) can be written as:

$$I(p) = -2im\int\frac{d^2k}{(2\pi)^2}\Big\{(-2mf) - (k^2 - m_B^2) + (\not{k} + m_\psi)(\not{p} + \not{k} - m_\psi)\Big\} \\ D_\beta(k)\,S_\beta(p+k)\,\times\,S(p)\,\delta(p) \tag{4.20}$$

where D_β and S_β are the real-time boson and fermion propagators respectively.

After some tedious but straightforward algebra it can be shown that the p_μ independent contribution cancels out and as $p_o \to 0$ ($\vec{p} = 0$), $T \to 0$ [20]

$$I(p) = - \lim_{p_o \to 0} f\, i\, \gamma_o\, p_o\, B_\beta\, S(p) = f \tag{4.21}$$

where we have used the relations (4.13) and (4.14). Therefore this excitation couples to the current in the way predicted by the Ward identities and corresponds to the Goldstone fermion. This Goldstone fermion appears in the spectrum as a fermion-boson bound state and is similar in nature to the "plasmon" excitation in gauge theories [12]. Its interpretation is simple: at very low temperatures the only states that contribute to the partition function are the ground state and the first excited states corresponding to a boson and a fermion of common mass **m** [22] this states appear in the heat-bath with probability $e^{-m/T}$ (thus the residue is proportional to $e^{-m/T}$). The Goldstone fermion thus appears as a fermion-boson oscilation that couples to the current. Since the imaginary part of the self energy vanishes on mass-shell, this oscilation is not damped.

Although we have not calculated the full structure of the fermion propagator, we expect at low temperatures that besides the Goldstone pole at $p_\mu = 0$ there is another pole close to $p^2 = m^2$ (with corrections of order $e^{-m/T}$). Therefore we expect exponential fall-off of the fermionic correlation functions for distances $x \ll 1/T$ and algebraic decay (long range) for distances $x \gtrsim 1/T$.

5. Conclusions

In this article we have shown that in theories with unbroken SUSY at $T = 0$, the symmetry is broken at any $T \neq 0$ due to different statistics for bosons and fermions. Furthermore looking at the real-time thermal Green's functions it was established that the breaking of SUSY is associated to a massless fermionic particle that couples to the super-current in the way predicted by the current algebra relations. This Goldstone mode arises as a fermion-boson oscilation with probability proportional to $e^{-m/T}$ where m is the common mass of the supermultiplet at $T = 0$, this translates to the fact that the residue of the Goldstone pole is proportional to $e^{-m/T}$ and therefore vanishes as $T \to 0$. It is argued that fermionic correlation functions should fall-off exponentially at distances $x \ll /T$ while they should decay algebraically for $x \gtrsim 1/T$.

References

1. For reviews in Supersymmetry see: P. Fayet and S. Ferrara Phys. Rep. 32C (1977) No. 5.
2. See for example, S. Samuel's contribution to this workshop.
3. L. Dolan and R. Jackiw, Phys. Rev. D9 (1974) 3320; S. Weinberg, Phys. Rev. D9 (1974) 3357.
4. S. Weinberg, Phys. Rev. D9 (1974) 3357; R. N. Mohapatra and G. Senjanovic, Phys. Rev. Lett. 42 (1979) 1651, Phys. Rev. D20 (1979) 3390.
5. A. Das and M. Kaku, Phys. Rev. D18 (1978) 4540.
6. L. Girardello, M. T. Grisaru and P. Salomonson, Nucl. Phys. B178 (1981) 331.
7. E. Witten, Nucl. Phys. B202 (1982) 253, Nucl. Phys. B185 (1981) 513.
8. L. Van Hove, Nucl. Phys. B207 (1982) 15.
9. M. B. Paranjape, A. Taormina and L. C. R. Wijewardhana Phys. Rev. Lett. 50 (1983) 1350.
10. D. A. Dicus and X. R. Tata, University of Texas preprint (1983) (unpublished).
11. T. E. Clark and S. T. Love, Nucl. Phys. 217 (1983) 349.
12. D. J. Gross, R. Pisarski and L. G. Yaffe, Revs. of Mod. Phys., Vol. 53, No. 1 (1981) 43.
13. This derivation is due to S. Coleman, see his lectures "The Uses of Instantons" in "Whys of Subnuclear Physics," Proceedings of the 1977 International School of Subnuclear Physics Ettore Majorana.
14. A. L. Fetter and J. D. Walecka, "Quantum Theory of Many Particle Systems" (McGraw Hill, New York, 1971).
15. H. A. Weldon, Phys. Rev. D26 (1982) 1394, and references therein.

16. L. Kadanoff and G. Baym, "Quantum Statistical Mechanics" (Benjamin, New York, 1962).

17. A. J. Niemi and G. W. Semenoff (to be published in Annals of Physics) and references therein.

18. See for example, M. Nakahara's contribution to this volume.

19. This is the model studied in ref. [6].

20. D. Boyanovsky, "Supersymmetry Breaking at Finite Temperature: the Goldstone Fermion," SLAC-PUB-3228 (1983) to be published in Phys. Rev. D.

21. H. Aoyama, private communication.

22. The spectrum is supersymmetric.