## Summary

This note describes a reversible dispersion matching system with arbitrary phase advance per cell. "Reversible" means that the match is independent of the sign of the quadrupoles. The method includes several well-known missing-magnet and reduced-magnet schemes as special cases.

## Method

## Requirements

1. In general it may be desired to match the periodic dispersion function ( $n$ ) of the standard cells to arbitrary $7, \eta^{\prime}$ in the insertion. Usually the desired condition is $\eta=\eta^{\prime}=0$ in the insertion.
2. The system must be reversible - i.e., it must work for either sign of the quadrupoles with the same bending geometry.
3. The system should be applicable to arbitrary phase advance per cell.
4. The system should be tunable over small range of phase advances and/or a small range of $\eta$ and $\pi^{\prime}$ in the insertion.

## -Description

For convenience the strengths of the quadrupoles of the $F 000$ array are kept at the standard cell values through the matching section. The matching is accomplished by varying the lengths of selected bending magnets, keeping the bending field fixed. One might expect that four variables would be required to match $\eta$ and $\eta^{\prime}$ simultaneously with either polarity of the quadrupoles; however (at least in the zerodispersion case) it turns out that because of symmetry only two independent variables are required.

As an additional constraint the varied lengths are kept less than or equal those of the standard cell magnets.

It is suggested that small trim quadrupoles or trim uindings on some of the standard quadrupoles be added to permit a certain range of tunability.

## Results

The TRANSPORT program has been used to obtain solutions for variety of cell tunes. Figure 1 shows the standard cell design used in all the following examples. Figure 2 depicts dispersion functions and magnet modifications for a typical case ( $\mu_{h}=\mu_{v}=80^{\circ}$ ). Table l lists solutions expressed as reletive magnet lengths $\left(\ell_{\square} / \ell_{b}\right)$ for a number of cell tunes. In all these examples the matching condition is $\boldsymbol{\eta}=\boldsymbol{\eta}^{\prime}=0$ in the insertion.

## Discussion of Results

Reversible dispersion suppression can be provided for arbitrary phase advance per cell by modifying lengths of the bending magnets in a few cells near the insertion. For phase advance $\mu 260^{\circ}$ the first two cells are sufficient. For $\mu \leqslant 60^{\circ}$ three or more cells may need to be modified.
if $\mu$ is a submultiple of $\pi$, it always works to reduce all the bend magnets by half over the first $\pi$ of phase.

In general the solutions are not unique; the results given are merely selected examples.

The solutions are essentially independent of vertical cell tune (see the examples of $\mu_{h}=\mu_{v}=80^{\circ}$ and $\mu_{h}=80^{\circ}, \mu_{v}=70^{\circ}$ ). However in order to make the match tunable in $\mu$ h one would have to allow some of the quadrupoles to vary.


Figure 1. Standard half-cell.

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Figure 2. Dispersion functions in matching section; $\mu_{h}=\mu_{v}=80^{\circ}$.

Table l. Relative lengths of bending magnets in reversible m-matehing section for various cell tunes.

| $\mu \mathrm{h}$ | $\mu_{v}$ | Half-cell number |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $90^{\circ}$ | $90^{\circ}$ | 0.0 | 0.5000 | 0.5000 | 0.5000 | 0.5000 | 1.0000 | 1.0000 | 1.0000 |
| $80^{\circ}$ | $80^{\circ}$ | 0.0 | 0.6050 | 0.6050 | 0.3948 | 0.3948 | 1.0000 | 1.0000 | 1.0000 |
| $80^{\circ}$ | $70^{\circ}$ | 0.0 | 0.6051 | 0.6051 | 0.3948 | 0.3948 | 1.0000 | 1.0000 | 1.0000 |
| $70^{\circ}$ | $70^{\circ}$ | 0.0 | 0.7599 | 0.7599 | 0.2399 | 0.2399 | 1.0000 | 1.0000 | 1.0000 |
| $60^{\circ}$ | $60^{\circ}$ | 0.0 | 1.0000 | 1.0000 | 0.0000 | 0.0000 | 1.0000 | 1.0000 | 1.0000 |
|  |  | 0.0 | 0.5000 | 0.5000 | 0. 5000 | 0.5000 | 0.5000 | 0.5000 | 1.0000 |
| $50^{\circ}$ | $50^{\circ}$ | 0.0 | 0.6776 | 0.6776 | 0.6776 | 0. 3249 | 0.3249 | 0.3249 | 1.0000 |
| $40^{\circ}$ | $40^{\circ}$ | 0.0 | 1.0000 | 1.0000 | 0.9449 | 0.0580 | 0.0000 | 0.0000 | 1.0000 |


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