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**A PARTICIPATOR:  
THE METAPHYSICAL SUBJECT\***

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What is required in the analysis of genesis  
is not private judgment, but public judgment  
– which is to say science.

*John Archibald Wheeler*

## 0 INTRODUCTION

It has often been recognized that problems connected with *meaning* arise in the analysis of many philosophical problems which, superficially, do not belong to the philosophy of language. As examples of apparently non-linguistic problems of this kind we can consider the question whether the physical universe is deterministic or non-deterministic or if it is continuous or discrete. It is characteristic of these problems that they are *not* well-defined in the same sense as is e.g. the question whether quarks are the ultimate building-blocks of the universe. This problem is well-defined in the sense that we know a theoretical framework - quantum chromodynamics - in which to ask questions that are relevant for the task we have set ourselves, i.e. to answer this question. We know, in principle, *what kind* of knowledge it is that is asked for, even if we do not possess it for the moment. It is supposed to be answered by QCD (quantum chromodynamics). Consequently, we have developed a theoretical framework in which we can try to answer these questions.

On the other hand, there are another kind of problems involved in connection with tasks like the one above. They concern problems of *meaning*. It is characteristic of problems connected with meaning that we are often not only lacking some specific theoretical framework, but even more often that we have no clear understanding of what it is that we do not understand. That is to say, part of the problem, and in a concrete situation often the most important part, is to try to *understand* the problem, which brings us to questions concerning meaning. The meaning of an expression, an action etc., is something that we know if we are to act meaningfully, e.g. to prove, to compute or to experiment. Knowing the meaning in an articulated manner is to construct the relevant context, which,

in turn, is to understand. It is important to realize that an articulation may be understood either as an act of articulation or as that which is articulated. The meaning of e.g. an expression, is not something inherent in or connected with the expression as such. It is not connected with that which is articulated. It is no "ghost in the machine" to use a phrase from Cartesian philosophy. No, we take the expression "context" to stand for the articulation of a finite string of rules determining activities of a certain kind. This shows something which is important to realize: meaning has to do with knowledge and understanding.

To articulate the rules by an act of articulation, explicitly determines the activity one is articulating, and fixes the *kind* of activity it is. To correctly perform a mathematical computation is to engage in a practical task of mathematics. This is usually what is required in examinations of mathematics in order to be able to claim the right to understand it. This is to engage in a practical task and by correctly performing it one demonstrates *practical understanding*. The concepts and principles that physicists develop in theoretical physics are what regulate their practical experiments and observations. It is on the basis of these concepts and principles that they *understand* the results of their experiments and these concepts and principles make their observation and experimentation into an intelligent activity. Similarly, the rules of a game make the playing of that game an intelligent activity. Understanding the game in a practical sense is having the ability to follow the rules. Just as the concepts and principles of theoretical physics regulate the practice of physical experimentation and observation, so do the rules of the game regulate the practice of playing the game. The ability to follow the rules and principles that regulate a certain practice or activity, such as using a language, playing a game, or doing mathematics, is to demonstrate

practical understanding.

Nevertheless, one is never explicitly told why e.g. the activity of computing is a *mathematical* activity. This is not explicitly articulated by the activity of computing, requiring only practical understanding. It is presupposed to be implicitly understood. Someone, say A, who correctly carries out computations in an examination, but nevertheless insists, in all sincerity, that he is *not* doing mathematics, is a person that the teacher will have trouble with. How is one to react to the claim by A? Since A is capable of correctly carrying out computations, as is shown by the examination, he meets the required criteria to pass the examination. But, however many computations A is performing, this, by itself, does *not* settle the question: Why is somebody who correctly and successfully engages in performing computations explicitly doing *mathematics*?

One has to realize that a statement like the one above is *not* answered by performing computations, since it is a question of a different kind. It is a question concerning *what* one must know, in order to be able to engage successfully in the activity in question, the activity in this case being computations. It is a question concerning meaning and understanding. It is a *philosophical* question, and the expression "philosophical" stands for the activity of articulating the rules *used* in order for A to successfully perform the calculations. By engaging in a philosophical investigation of articulating the rules one uniquely determines the activity performed by A to be the kind of activity it is - a mathematical activity.

The same goes for other activities as well. E.g. why are the activities of experimenting at *Stanford Linear Accelerator* activities of physics? Or, to put it another way: why does *experimenting*, an activity carried out by individual scientists (usually as a group), provide us with *objective* knowledge of the Uni-

verse? Again, this question is *not* answered by engaging in new and more subtle experiments. It is a question of a philosophical kind. It is a question concerning participatorship in the activity of experimentation. It is a question concerned with the problem how come experiments tells us something about nature? We do not answer this sceptical question by performing a new experiment. To explain questions like these in a way that leaves no room for doubt, i.e. a sceptical attitude, is to engage in a philosophical investigation as a participator: the metaphysical subject.

### 1.1 THE ELUSIVE SUBJECT

In setting out an analogy between Platonism in mathematics and realism about the physical universe Kurt Gödel claimed that the justification for our belief in mathematical *entities* is the same as that for our belief in the theoretical entities of physics.<sup>1</sup> In both cases we adopt such a belief for the sake of its explanatory power in accounting for a raw experiment. It seems as if Gödel here almost adopts an empirical conception of mathematics contrary to the platonistic view usually attributed to him. The philosophical dispute is *not* over whether we are justified in believing in existence of the entities, but what it amounts to *assert* the existence of such things. The problem is the status of the necessities involved in the assertion that such things exist.<sup>2</sup> This is the philosophical problem for entities in mathematics as well as in natural science, e.g. physics.

It is important to realize that the point is not the existence of mathematical objects, but the *objectivity* of mathematical truth as has been emphasized by Georg Kreisel.<sup>3</sup> This dictum bears, according to the philosopher Michael Dummett, directly on the puzzle how we could possible resolve the metaphysical question whether mathematical objects are, as on the view advocated by

L.E.J.Brouwer, creations of the human mind - the creative subject - or, as Gödel states, independently existing abstract objects.<sup>4</sup> What we have here are two *metaphors*: the platonist compares *the mathematician* with an astronomer, or explorer, while the intuitionist, like Brouwer, compares *the mathematician* with an artist, or a writer.<sup>5</sup> In both cases the dispute concerns the *subject*, in this case the mathematician. It can concern a scientist as well. This attitude is clearly shown in discussions concerning the role played by the measuring subject in quantum mechanics ( the measurement problem).

In philosophy a question expressing scepticism of a reality independent of mind is often formulated in an epistemological manner: How can I know that there is something which is independent of my consciousness? How does the world enter *my* consciousness? If the question is formulated in this way, then *perception* is the main philosophical problem. In this mode of thinking an object of perception is a presupposition of the philosophical inquiry. The object of perception is then taken to constitute the *objective* ingredient in philosophy. While modern philosophy has again and again aimed at obtaining the objectivity of the sciences, e.g. physics, its attitude towards that objectivity has always been ambiguous. For it seems clear on reflection that even the most objective account of the world is only a picture constructed by human subjects, in short, a picture that is objective for *us* rather than objective in itself. As soon as we begin to think about the origin and status of any objective description of the world we seem to be thrown back on reflections about the knowing subject. It is this which has led philosophy repeatedly back to the positions of nominalism, realism, dialectical materialism, platonism, radical empiricism, intuitionism, just to mention a few examples. In short, the time of philosophical “-isms” (positions)

had arrived.

By taking a philosophical position we mean something like the following. There are a number of philosophical doctrines to which one is habituated when being taught different "approaches" to philosophy. First, perhaps most deeply rooted, is the following picture (Fig.1). The world consists of all objects, and they are beheld by consciousness, the subject, or the ego. Something like this picture occurs in perceptual psychology or neuro- psychology. There one analyzes the process of perception in terms of e.g. light waves, pressures and so on. These act upon the sense organs of the percipient and excite certain electrical and chemical phenomena within his nervous system. Interpreted in this way, the picture is all right.

In the *philosophical* interpretation of the picture, the subject receives sense impressions from the objects in reality, which are organized and sorted according to the categories of pure reason, paradigms, canons of induction, and others, into iterative complexes through which we can have knowledge of reality. Here, the subject, equated with the Eye of the Mind, is modeled after what was supposed to take place in the act of vision. The objects refract light and are seen; it makes a difference to the eye and to the person having an optical apparatus, but none to the thing seen. Getting to know something, like seeing something, consists in establishing the right relation between the knower and the known. The Eye of the Mind has to be correctly oriented, and a person will "see" the truth. A visual model for knowledge becomes almost irresistible. But it is an interesting fact, as has been pointed out by Richard Rorty, that there is "no particular reason why this ocular metaphor seized the imagination of the founders of Western thought. But it did, and contemporary philosophers are still working on its consequences,



analyzing the problems it created and asking whether there may not be something to it after all".<sup>6</sup>

But the fact is that this picture has some rather unsatisfactory features when it is applied in philosophy. First, it gives rise to an uneasy oscillation between the opposite ends of the picture above. We just tried, briefly, to describe the realist version of the picture. In the idealist version the emphasis is on the left side of the picture. Reality is not so much beheld by the subject as constructed by him, so that it is a product of his consciousness, and depends on him. This is also, partly, correct in a dualistic reading of the picture. E.g. incorporation of the measuring subject in quantum mechanics in von Neumann's projection postulate<sup>7</sup> and the "psychologization" resulting from this is an indicator of an idealistic (dualistic) reading of the picture. Read in this way the reality ostensibly described by quantum mechanics becomes to some degree a subjective state of ourselves. This is the dualistic position adopted by Eugene Wigner and John von Neumann.<sup>8</sup> A similar position, concerning mathematical activity, is adopted by Brouwer, when he says that Intuitionism "completely separates mathematics from mathematical language in particular from the phenomenon of language which are described by theoretical logic and, recognizes that intuitionistic mathematics is an essentially languageless activity of the mind...".<sup>9</sup>

The problem when one adopts the picture is that, however much one assigns priority to one side of it or the other, the picture remains substantially the same. Furthermore, it is *silent* about how the two sides of the picture are connected, how, if one has access to reality only through one's impressions, the connection is set up between these impressions and the objects they are impressions of. It is also silent on the question concerning *what* principles regulate the construction

of these objects, and out of what. The third problem is that in this picture the subject is separated from reality as it were by a pane of glass, to use an analogy by John Wheeler.<sup>10</sup> The subject is a spectator, or observer, watching, perhaps, shadows on a wall or an internal theatre as expressed by Plato.

So, one can ask, is the subject *unreal*? Isn't the subject part of the world? A case in point would be to look at the Anthropic Principle in modern cosmology as put forward by Brendan Carter. The principle says that if the universe around us were hostile to Life, then we could not be observing it. Carter has two formulations of this principle<sup>11</sup> 1) the weak formulation: our location in the universe is *necessarily* privileged to the extent of being compatible with our existence as observers, and (2) the strong formulation: our universe must be such as to admit the creation of observers within it at some stage. So, one can ask, again, isn't the theoretical physicist investigating e.g. the Anthropic Principle as used in cosmology a *part* of the universe? In the formulation of the principle given above, the subject, being an observer, is *not* incorporated in an objective way. The whole picture, and the uneasy oscillation between its two ends, gives an impression of ill health.

One could, of course, make the picture more sophisticated by modifying its subjective aspects, replacing them by an *intersubjectivity* sustained between different subjects through their use of language. We have a *social*, or collective, reading of the first picture. Then, the picture looks roughly like the following (Fig.2). One can, and correctly so, say that this picture manages to explain the objectivity of the world to some extent. The most important result is that it achieves symmetry with respect to the observers. The two main deficiencies are that it still leaves the observers *outside* the world, whereas they ought to be part

of it. Consequently, "(t)o say: in the end, we can only adduce such grounds as we hold to be grounds, is to say nothing at all".<sup>12</sup> And the connection between the language and the world is obscure. Why? If we look for a moment on this last aspect of the picture it consists essentially of three separable parts (Fig.3). We notice immediately that language and reality are separate systems. This indicates problems. First, there is the reality which is independent and prior to language. Secondly, we set up the language to communicate among ourselves, and (as observers) record facts we have discovered about it. And then, at last, we assign nonlinguistic items to linguistic ones as their denotations, thereby showing their meaning and setting up their interpretation.

There is something fundamentally mistaken in this picture, which was present in another form also in the first we considered. The mistake is the sharp separation of the whole system into a linguistic third, a non-linguistic third, and an obscure third part which connects them. The picture is mistaken because it makes any account of this last third obscure in the form of an infinite regress. This can be seen as an instant of what is called Bradley's regress.<sup>13</sup> The regress arises if we e.g. suppose that we have two individuals  $a$  and  $b$ . For them to be related there must be a relation  $R$  which relates them. But now there must be two further relations  $R_1$  and  $R_2$  to relate  $R$  to  $a$  and  $b$ . But how is  $R_1$  related to  $R$  and  $a$ , or  $R_2$  to  $R$  and  $b$ ? We have to assume further relations which connect them and, consequently, we end up in an infinite regress. According to this picture we could not have access to anything *real* which we wished to make the denotation of something in the language. For any means employed to *identify* that real thing would have an ineradicably linguistic aspect. Thus *another* linguistic system would interpose itself between the first and its field of

denotation for which the same problems of interpretation would then arise. It would be a *redundant* explanation vis-a-vis meaning of the field of denotation. According to the pattern of thinking indicated here, the relation between a term  $t$  and an object  $a$  which it denotes, is often thought of as a relation proper, to use a terminology from Wittgenstein's *Tractatus*.<sup>14</sup> The meaning of a statement like: "the term  $t$  denotes the object  $a$ " is, as was said above, determined by three separate things: (1) the meaning of  $t$  as a term, (2) the meaning of the denotation, and (3) the meaning of the expression  $a$ . Now, the expression  $a$  is also a term *belonging to the language*, so in order to *understand* the statement above we must *already* know a statement of the form: "the expression  $a$  denotes the object  $b$ ", where  $b$  is another expression that denotes the object  $a$ . But then, again, this last fact must be known, and we are led to an infinite regress. The root of the paradoxical result is the *assumed* separation of language from reality in the pattern of thinking shown above.

The way that Bertrand Russell appealed to, and applied, the so-called Occam's razor, shows the same pattern of thinking. According to Russell, instead of assuming the existence of *unknown* entities, we should, as far as possible, construct what we need on the basis of known entities. This means, according to Russell, that "...in accordance with Occam's razor, we shall do well to abstain from either assuming or denying points and instants".<sup>15</sup> This statement clearly reveals the misconception mentioned. In philosophical reflection on the external world it is *not* a question of assuming or denying points and instants, as if they were some inaccessible entities, but a question of how to *understand* what our talk about points and instants means. The view concerning existence of objects shown above, is also obvious in many other, and more recent, writings. What

is common to these ways of thinking is that problems of “what there are” (ontology), is a matter of *belief* but not of knowledge. We can, according to this pattern of thinking, have good reasons to assume or adopt a certain ontology (or linguistic framework) but we cannot really know that the assumed objects exist.

Another example of this way of thinking is given by Donald Davidson, when he points out that we are able to refer to particular events by means of definite descriptions, and “this claim is persuasive only if there are such things as events to which singular terms, especially definite descriptions, may refer. But the assumption, ontological and metaphysical, that there are events, is one without which we cannot make sense of much of our most common talk;...”<sup>16</sup>

This way of speaking is very common within a modern philosophical tradition, not only in the case of events, but also in the case of things, like physical objects, numbers, sets, properties, propositions, actions, etc. Furthermore it is quite easy to see that the pattern of thinking which, generally unconsciously, underlies this way of thinking, is the one mentioned above. This view is mistaken, because e.g. the events like “it is raining now”, “the sun is shining”, etc., are nothing that we need to *assume* the existence of, because most of us *know* many events of this kind. The misconception in Davidson’s remark shows itself in his remark that we would have to assume the existence of events in order to make sense of our common talk about events. But the crux of the matter is that our common talk of events, propositions, numbers, etc., *does* make sense to a great extent. What we want, is to understand *how* they do it. The ontological or metaphysical problem of what an event, a proposition, a number, is, are not problems that concern some occult objects. They are not problems of some “ultraphysics” which we come to know by some “ultraexperience”.<sup>17</sup>

Yet another example of the misconception concerning meaning, shown above, are the frequent attempts to make use of an assumed distinction between meta- and object-language when constructing theories of meaning. Some people appeal to the work of Alfred Tarski who showed in an excellent way, how, given a formalized language, i.e. a formal notation for expressing certain statements, employing symbolic logic, one can define 'true' *for that language* in a stronger language, a so-called meta-language.<sup>18</sup> Tarski's work was *itself* based on the equivalence principle, that is, the principle that "to say of a statement that it is true is equivalent to asserting the statement". In other words, to argue that there are *no* real philosophical problems about truth. In fact Tarski's criterion for a successful definition of 'true' was that it should yield all sentences of the form 'P' is true iff P, e.g.

'Snow is white' is true iff snow is white

as theorems of the meta-language, where P is a sentence of the formal notation in question. But the *equivalence principle* is philosophically *neutral* and so is Tarski's work. On *any* theory of truth 'Snow is white' is equivalent to "'Snow is white' is true". Positivist philosophers would reply that if you know Tarski's convention, you *know* what "'Snow is white' is true" means: it means *snow is white*. And if you don't understand 'snow' and 'white', they would add, you are in trouble indeed! But the problem is that we don't understand *what it is to understand* "Snow is white". If we, again, take as an example the statement "the term *t* denotes the object *a*" we notice that this is a statement belonging to the meta-language, and it is in that language that the expression *a* is used, while the term *t* is only mentioned. But in order to be able to use the expression

*a* in explaining the meaning of the term *t*, one must *already* understand the expression *a*. One must know what object it denotes. The *distinction* between object-language and meta-language *itself* gives rise to this difficulty, because it is based on the idea that we can separate a language from that which the language treats without any problems of understanding. This is the philosophical problem. About this Tarski's convention says nothing. One is not to confuse certain issues. What has been said above is *not* to deny the intelligibility of Tarski's convention. It is only to deny that it can cope with *understanding in an objective way*. But once this is taken care of, and we have "succeeded in understanding a rich enough language to serve as a meta-language for some theory T, (we) can define 'true in T' a la Tarski".<sup>19</sup>

## 1.2 THE CREATIVE SUBJECT

One could say that to engage in a philosophical investigation is to be a creative (metaphysical) subject, to use a notion from Brouwer.<sup>20</sup> Despite the usefulness of this notion one must be careful not to misunderstand it. There is no solipsism involved, as it is in Brouwer's case. In a philosophical investigation to codify is to express public knowledge. It is not something that happens in the mind of the creative subject, as Brouwer would have it. The central idea with Brouwer's idea of the creative subject is that of an idealized mathematician who performs his mathematical activities in a certain order. The process of his mathematical activity proceeds in *discrete* stages which incorporates inflections of tense on the predicate "is true".<sup>21</sup> These difficulties raise their heads as soon as one attempts to introduce tense into mathematics, as some interpreters of intuitionism suggest (in the form of choice sequences), because in Brouwer's intuitionism a mathematical statement is rendered true or false by a proof or disproof, that is,

by construction, and constructions are effected in time. This can be seen when we try to understand the theory of the creative subject. Following Kreisel we introduce a basic notion.<sup>22</sup>

$$\Sigma \vdash_n A \quad (1)$$

to be read as “the creative subject  $\Sigma$  has proof of  $A$  at stage  $n$ ”. Note, that this statement is in a subject-predicate form. In accordance with Kreisel we further suppose that the basic notion is a decidable relation:

$$\Sigma \vdash_n A \vee \Sigma \neg(\vdash_n A) \quad (2)$$

which tells us that at stage  $n$  we know if we have evidence for  $A$  or if we do not have evidence for  $A$ . Clearly we can now assert that:

$$\frac{(\exists n)\Sigma \vdash_n A}{A} \quad (3)$$

which, in effect, says that if we know, at stage  $n$ , that  $A$  has been (or will be) proved, then we are entitled to assert  $A$ . In order to simplify the interpretation we also assume that:

$$\frac{\forall m \forall n (\Sigma \vdash_m A) (m \geq n)}{\Sigma \vdash_n A} \quad (4)$$

which is to be read as saying that the evidence at stage  $m$  is also contained in all following stages. But now we face a problem. Ought we to admit the form:

$$\frac{A}{(\exists n)(\Sigma \vdash_n A)} \quad (5)$$

being the inverse of (3)? Can we *equate* truth with the obtaining of a proof at some stage  $n$  in the past or in the future? Or, alternatively, can grasping of



the notion of truth be equated with a temporal relation? Can we, and are we justified, in asserting:

$$A \leftrightarrow (\exists n)(\Sigma \vdash_n A) \quad (6)$$

The answer is a clear no! If we accept both (5) and (6) we run into difficulties. In such a case the operator:

$$(\exists n)(\Sigma \vdash_n) \quad (7)$$

becomes a *redundant* truth-operator,<sup>23</sup> the reason being that we presuppose understanding of the point of *using* the notion of truth in the first place. In order to use (6) for the intended purpose of equating the grasping of truth with a temporal relation, we are forced to presuppose that we already master the use of the notion of *truth* and *then* try to impose a temporal relation. With such a presupposition the problem is that we must leave open the possibility of infinitely many assertions  $A$  for which (1) and hence the cautious formulation *having proof (evidence) for  $A$  at stage  $n$*  holds. But since we also *required* (1) we are left with the genuine problem of finding a satisfactory explanation of the *assertion (judgment)*:

$$\Sigma \vdash_n \quad (8)$$

The problem boils down to the question of how we are to understand an assertion (judgment). If (7) is a redundant truth-operator, how are we to escape the redundancy? We cannot admit a statement like:

$$\frac{(\forall x)Ax}{(\exists n)(\Sigma \vdash_n A(a))} \quad (9)$$

because of the apparently insuperable difficulties, on this reading of the creative subject of making evident the laws of forming propositions by means of quantification over infinite domains, like the domain of natural numbers. The problem

regarding Brouwer's idea of a creative subject arises from a desire to relate a notion of truth (intuitionistic) for a mathematical statement in conjunction with a use of an existential quantifier which presupposes a platonistic adequacy condition for truth. This violates the very point of introducing the notion of a creative metaphysical subject, which is why we are, with this reading, constantly trembling on the edge of absurdity or paradox.<sup>24</sup> We have to come up with a better strategy: we have to realize the difference between judgment as an act of judging and as that which is judged.

### 1.3 SUBJECTIVE AND OBJECTIVE UNDERSTANDING

Philosophy and science, alike, must incorporate the subject in an *objective* way. In order to achieve this we must promote both scientists and philosophers to participatory status (metaphysical subjects) from being mere observers. This is our aim. The insight that is being conveyed here can be expressed by the words of Ludwig Wittgenstein: "If I wrote a book called *The World as I found it*, I should have to include a report on my body, and should have to say which parts were subordinate to my will, and which were not, etc., this being a method of isolating the subject, or rather showing that in an important sense there is no subject; for it alone could *not* be mentioned in that book.- The subject does not belong to the world: rather, it is a limit of the world. Where *in* the world is a metaphysical subject to be found? You will say that this is exactly like the case of the eye and the visual field. But you will *not* see the eye. And nothing *in the visual field* allows you to infer that it is seen by an eye".<sup>25</sup> What is being said here is, essentially, that an assertion (judgment) like "A asserts *p*" is of the form "*p* asserts *p*". The expression "A asserts *p*" is to be understood as an act of asserting, not as that which is asserted. We may say that the predicate is

contained in the subject, provided we take the subject to be the object and not the expression A. This is precisely the characteristic for a statement which is *analytic* in Kant's terminology. This reduces the question as to the analysis of assertion (judgment) to the question: What is it for a proposition to have sense? It is this problem we have to face. In order to understand it we must begin by realizing that it is what we actually *do*, e.g. our concrete scientific activity, that is real or concrete. Our *philosophical* knowledge consists of our capacity to intelligently explain this concrete activity. The difference between being able to do a thing intelligently, and being able to give an account of what one is doing, is notorious. The first shows an implicit understanding of the principles regulating the activity, and is quite compatible with a lamentable performance of the second. And the other way round, the mere ability to state certain principles explicitly, implies no understanding of them, that is, no ability which comes out in putting those principles into practice, or use, in a thoughtful way.

We must realize that to engage in a philosophical explanation is *also* to engage in a concrete activity. It is an act of asserting. A philosophical investigation of a certain practice is itself an activity which is different from the activity on which the investigation is being made. It is different in the sense that we cannot take the rules and principles that govern the primary activity to be the ones that govern our philosophical investigation.

In order to "break in" to the circle of understanding one must grasp how to exercise a certain practice intelligently. This is what we will call *practical understanding*. Practical understanding, knowledge, or meaning, could be called *subjective* understanding in that it consists in the ability or capacity of a *person*. However, we must be aware that the term "subjective" may be misleading. It is

absolutely not something mental going on in the mind of the subject. This is a fallacy. The fact that it consists in the capacity of a person easily misleads us. Practical understanding is, indeed, only partially what we could call subjective. The most important part of practical understanding consists of grasping *objective* knowledge of meaning: grasping the use of expressions and sentences occurring in practical tasks. The *genesis* is to grasp understanding of a practice. It is to grasp what a practice is. It is to engage in Language by an act of the will, and e.g. "(a) child uses such primitive forms of language when it learns to talk. Here the teaching of language is not explanation, but training".<sup>26</sup> A child, or adult, engaging in learning a practical task is a metaphysical subject - a participator. One can say that anyone breaking into the circle of understanding, be it practical or philosophical, grasps objective knowledge of meaning. To be a participator is always to grasp objective knowledge of meaning. This is the case as well for practical understanding as for philosophical understanding: "And hence also 'obeying a rule' is a practice. And to *think* one is obeying a rule is not to obey a rule. Hence it is not possible to obey a rule 'privately': otherwise thinking one was obeying a rule would be the same thing as obeying it".<sup>27</sup> This is, as Saul Kripke observes, the core of Wittgenstein's argument against the possibility of a "private language".<sup>28</sup> This argument contains (among other) the genuine insight that the role accorded to agreement of judgments in communication is a constitutive one: it is *essential*, if we are to share a common Language that we agree in certain judgments. This agreement is *not* open to debate: one grasps a form, and debate in virtue of the form adopted. A debate is, by necessity, always carried out in a Language, entailing adoption of a form. To *participate* in a Language necessarily entails adoption of a form. Thus, as Andre Maury expresses

the point: “(t)he notion of the limits of language has a general philosophical point only if we consider that *the form of language* excludes the possibility that linguistic structure can be *justified* by reference to fact. Its form appears as a limit for what can be done (and has unwittingly been done) in *philosophy* (not just in the theories of logic and semantics)”.<sup>29</sup>

One cannot become a participator by being a “private entrepreneur”. A participator is always a “public entrepreneur”: he engages in grasping the limit of Language. To grasp the limit of Language as a result of recursive training is like programming a computer by the keyboard in order for it to be able to perform certain tasks, to use an analogy. Here one cannot say that the computer is being “unjustly” treated vis-a-vis the tasks it is expected to solve in virtue of the form of the Language it is using when programmed. If the programming is not sophisticated enough to cope with the tasks assigned to it, the computer fails in its task. Similarly, a participator is always secured from being unjustly treated. One can say that the participator is in the ultimate democratic situation when he learns the concepts because : “...if a person has not yet got the *concepts*, I shall teach him to use the words by means of *examples* and by *practice*. - And when I do this I do not communicate less to him than I know myself”.<sup>30</sup> To grasp the concepts is to grasp a form, and this can only be done in an objective way: in fact the meaning of the expression “objective” is so determined. One must be aware of the distinction being made here in the use of the expression “objective”. This might easily mislead someone who does not grasp how it is used here. On one hand, there is the expression “objective” as grasped by a participatory activity in practice, e.g. when one learns to use this expression as a child, or as an adult in a new situation. In a participatory activity the *sense*

is the activity of repeating. On the other hand, this expression as standing for the knowledge conveyed and codified by philosophical investigations. Here one applies the expression "objective". It is important to realize that this applies to every expression, e.g. "subjective", "practice", "participator", "observer"... We *use* them in philosophical investigations, but this use is *founded* on a canonical instantiation of the expressions. A canonical instantiation is an act of judging (asserting), and "(w)hat *must* come out is a foundation of judgment, which (one cannot) touch".<sup>31</sup> To grasp in virtue of an act of canonical instantiation is to break into the circle of understanding *sui generis*.

One must not confuse these two distinctions. Not that they convey any different kind of knowledge: they both show *understanding* on the part of the participator. In both cases the metaphysical subject is a public entrepreneur. Having practical understanding does *not* consist in the ability to imitate or reproduce mechanically a certain overt behaviour. In such a case one has not grasped the concept in question. To grasp a concept is to be able to *use* it in practical application. A person does not understand a word, or expression, simply because he can pronounce it correctly, and happens to use it correctly on some occasions. If understanding consisted in the ability to imitate, it would be difficult to distinguish between a mistake in the sense of a slip of the tongue and a mistake in the sense of a misunderstanding. Although most understanding begins with imitation it does *not* consist in the ability to imitate. Someone who has practical understanding *must be able to exercise the practice by his own force*. Having practical understanding of e.g. an expression consists *neither* in the ability to state nor to describe verbally how the expression is used, no more than the ability to drive a car consists in the capacity to *describe* how car driving is done. As

Martin-Löf would say: there are certain limits to what verbal explanation can do. In the end everybody must understand for himself.<sup>32</sup> We shall below give an example of a simple arithmetical task which, when completed, shows that a person who has completed it has practical understanding.

The starting point of a philosophical investigation is the realization that *genesis*, be it subjective (practical) or objective (philosophical), is activity in a concrete sense. Accordingly, it is actions, not objects, which are real, actual, or concrete. The conception of a concrete object is a misconception. An object is what it is in virtue of always being the outcome of concrete metaphysical will. There are no things-in-themselves, to use a Kantian formulation. And so the search for the “furniture of the Universe” will have ended with the realization that the Universe is not a furnished room. There are mathematical and physical objects, but only mathematical and physical activities are concrete. The nature of an object, what it is, depends on the *context* of the practice one is engaged in.<sup>33</sup> The context is the Language in question. To understand a Language is to engage in creating it by an act of the Will. As Wittgenstein’s says: “Strangely enough, the problem of *understanding* language is connected with the problem of the Will”.<sup>34</sup> When somebody is engaged in a certain contextual practice we mean something like the following:

<i>PRACTICE</i>	<i>CONTEXT</i>
calculating and proving theorems	mathematics
experimenting, observing and calculating	physics
speaking, reading and writing	language

Here, we have pulled practice and context apart. In practice they are never separated. To believe the opposite is to adhere to an “observer metaphysic” in Wheeler’s terminology.<sup>35</sup> A practice is a practice of a certain context, and the context is the context of that practice. A practice is an *a priori* activity vis-a-vis meaning. We understand a practice always as a practice with a certain purpose, and to *see* the activity is to *see* the way that purpose is fulfilled. It is important to realize that e.g. observations are acts, or activities and not just passive reception of impressions. For instance, to observe something visually is not just having the eyes in a certain state or position. It is to look, or search, *for* something and find it, to use an expression by Jaakko Hintikka.<sup>36</sup> Even failure can only be understood in relation to the purpose that one fails to realize. A practice cannot be separated from the purpose. If we take that purpose away, the act, so to speak, goes flat. Kant makes a similar point when he maintains that, with respect to empirical knowledge, we cannot separate our sensible faculty from our faculty of understanding: “These two powers or capacities cannot exchange their functions. The understanding can intuit nothing; the senses can think nothing. Only through their union can knowledge arise”.<sup>37</sup>

When we e.g. engage in a certain simple task like  $2+1$ , we say that it is an arithmetical task, but more importantly, we are able to successfully compute the addition, precisely because there are certain rules one has to employ in order to compute it. It is these rules, which, when being explicitly codified by philosophical activities, constitute the context “arithmetic”. Here one cannot separate practice and context. This is *not* to say that in a practical situation when engaging in a task of e.g. performing a simple addition, one must explicitly formulate every rule which is involved in order to successfully terminate the task. This is



easily realized by looking at children learning arithmetic. They are taught to perform a task like:

$$\begin{array}{r} 2 \\ + 1 \\ \hline \end{array}$$

To perform this task, successfully, is a practical task, which gives subjective understanding. To codify the result:

$$\begin{array}{r} 2 \\ + 1 \\ \hline 3 \end{array}$$

is to fulfill the task. What is important to recognize is *the codified pattern of symbols on the paper showing the termination of the task*. Nowhere do we see any codifications of the rules *used* in order to perform the task. As Wittgenstein said: "If the result is the result of the calculation, I have already fixed what I call 'obeying the rules' by my calculation. The calculation gives me a form of expression now: and *now* I say he gets either the right or the wrong result".<sup>38</sup>

The most important step for the child is when it has got the computation right for the first time. The computation has terminated for the first time. This is a canonical termination (step), since from now on the child has acquired the capability to *repeat* the addition. The child has grasped the *form* of ordinary arithmetical language. The child has grasped *the point* of engaging in a task of addition. A teacher can only *repeat* the computation until the child can perform the task on his own. By doing this the child shows that it can engage in a task of performing an addition. At the same time the child has achieved a

criterion of identity, *as the sense is the activity of repeating*, and, consequently the canonical instantiation of the expression "to repeat". This is called Wittgenstein's principle.<sup>39</sup> We shall return to it elsewhere in more detail.<sup>40</sup> Successively, the child can widen its capacity to perform novel tasks. Undoubtedly, the child already at this point has grasped an enormous number of practical skills, say, like speaking a certain language, by canonical steps. Every time the child has succeeded in a novel task he or she has taken a canonical step. In principle, it is always possible to repeat the step, but in that case it is no longer a canonical step, i.e. it does not count as a *criterion*. A necessary condition for a canonical step is that the task is *surveyable*. The task must always involve a finite number of steps, perhaps of increasing complexity, but it must always be surveyable. If this is not the case, a child, or an adult learning more sophisticated skills, could never grasp the point (essence) of the task: *the result*.

In this way a child, but also an adult, acquires more sophisticated skills. One learns the point of speaking, reading, writing, calculating and experimenting, to mention just a few: "(c)hildren are taught their native language by means of such games...when the boy or grown-up learns what one might call special technical languages...he learns more language-games".<sup>41</sup> On reflection, we can say that carrying on such things in an intelligent way implies the possession of certain concepts peculiar to each activity. There is implicit in such intelligent activity an understanding of certain principles which regulate it. The interrelationship between act and end is what connects together practice and context. In short, a person has acquired practical skill. He has acquired subjective understanding. But some of us get interested in the question of *objective* understanding, that is, of philosophy, and then the confusion concerning the part played by the sub-

ject starts. We somehow believe, perhaps in virtue of the earlier and traditional canons of philosophy, that philosophy is an *essentially* different activity in contrast to e.g. physics. A not uncommon view is that physics gives us contingent truths, whereas philosophy (logic) gives us *a priori* truths. This is partly an illusion as we shall see. To engage in e.g. physics or mathematics, as well as philosophy, is essentially like engaging in a computation by a canonical step. In both cases the outcome is a codified result. In both cases the metaphysical subject is like a black box. In both cases, what is important, real, and concrete, is the output from the box: the successful termination and codification of a task. It is important to realize that there is no difference between them. The engagement in a task is like codifying the function of a large extraterrestrial computer, to use an analogy. To engage in a philosophical task is like codifying the function of a smaller computer, being a part of a bigger one, the output of which contains the output of the smaller one. The computers are not built by humans, but by another, more advanced civilization. We do not understand the function of the machines, which has come down through the millenia, but we *do grasp the form* of the language printed by the printer. We do have subjective understanding. But the important point to realize, is, that the metaphysical subject (black box/computer), "...is not the human being, not the human body, or the human soul, with which psychology deals, but rather the metaphysical subject, (is) the limit of the world - not a part of it".<sup>42</sup> The aim of a philosophical investigation (output of the large computer) is to achieve understanding of a certain practice (output of the small computer). The output of the large computer provides objective understanding. It provides the *meaning* of the Language put out by the computers.

#### 1.4 MEANING AND OBJECT

In a series of interesting articles Michael Dummett discusses the problem concerning meaning and truth in mathematics. His approach is intrinsically a *pragmatic* approach, in the sense that it stresses the ability to theoretically explain mathematical practice. The theory for this practice is not to be found in mathematical practice at all, but in the philosophical reflection over what we do in mathematical practice. It has affinities with the pragmatic theories of truth by Peirce, James and Dewey.<sup>43</sup> Dummett studies two important arguments in favour of the view that intuitionistic logic rather than classical logic describes the mode of argumentation used within mathematical praxis. Note, that Dummett differs from the strategy adopted by Martin-Löf, which is a non-logical system, where classical and intuitionistic logic are special cases of a common grammatical structure. For Dummett, on the other hand, the two logics represent conflicting views of argumentation. Consequently, the philosophical problem for Dummett is: How is one to argue in favour of an intuitionistic logic in contrast to classical logic, if by *logic* is to be understood codification of correct argumentation? Although we do not adopt this strategy, it is very close to ours in certain respects.

The first of Dummett's arguments rests upon the traditional, ontological, thesis, that mathematical statements do not refer to a reality existing independently of us, but to objects created by us.<sup>44</sup> The other argument is based upon Wittgenstein's general considerations according to which the meaning of a sentence must be understood in terms of its *use*.<sup>45</sup> We shall, here, concentrate on the first argument, and return to the second in more detail later.<sup>46</sup> According to W.V.O. Quine "...ontology is basic to the conceptual scheme by which (one) interprets all experiences...standing in need of no separate justification at all".<sup>47</sup>

If we construct a theory, this implies, according to Quine, that "... a theory is committed to those and only to those entities to which the bound variables of the theory must be capable of referring in order that the affirmations made in the theory be true".<sup>48</sup> *Existence* can therefore be defined as "to be the value of a (bound) variable".<sup>49</sup> This thesis functions like a criterion of existential (or 'ontological') commitment. A sentence carries *direct* ontological commitment iff it is existential in form, i.e. its main operator is an existential quantifier. Thus we arrive at the extremely influential and important ontological thesis which says that *if a language admits that there are objects, this thesis cannot be straightforwardly denied in the Language in which it can be formulated*. If it can be formulated at all, it will come out as a truth of logic. It belongs to the limit of the Language.

If a logic is supposed to be a *codification* of correct argumentation of the ontological thesis regarding existence of objects, we therefore face a problem, regarding classical vs. intuitionistic logic. The problem is that if we adopt a standard classical logic, it is impossible to draw a line of distinction *between logical and ontological theses*. In standard classical predicate logic one can formulate theses like the following ones:<sup>50</sup>

$$(\exists a)(a = a)$$

$$(\exists a)(Fa \vee \neg Fa)$$

$$(\exists a)(Fa \rightarrow (\forall b)Fb)$$

Theses like these are *not* ontologically neutral, and hence logic should, according to Dummett, be purified of the principles which permit their derivation, if we are to be able, by an ontological mode of reasoning, to *decide* in favour of two competing logics. What we must require of logic is that "all ontological

theses, whether affirmative or negative, can be formulated within it; but logic itself should be ontologically neutral, not allowing the (formulation) of any such theses".<sup>51</sup> But as long as we admit a logic where we can derive statements like the ones above, this logic does *not*, by itself, allow us to derive the truth of the ontological thesis regarding "on what there is", i.e. the thesis that there are objects (individuals).

It appears as if the meaning of the statements above is determined by at least two independent things, namely the meaning of *a*, and the meaning of a certain predicate. Believing this, however, is a fallacy concerning their grammatical form. They *cannot* be understood as conveying some knowledge about the object *a*. The best way to see this is to try to deny or negate them, to try to imagine what it would be like if what they express was wrong. If they purport to express some truth about the object *a*, it would, in principle, be possible for someone who understands the expression *a*, not to be informed about that truth. But negating them would lead to such questions as: What is the thing *a* which is not a thing? What is the object *a* that does not exist? Saying about the thing *a* that it is not a thing, or about the object *a* that it does not exist, is self-contradictory. It may therefore be tempting to say that negating the three existential statements above, leads to statements which are *false*. This is a fallacy. If we mean by the *truth* of a statement, its expressing a fact about objects, and by its *falsehood* that it purports to do so, but does not, then the negations of our statements are not false, but *meaningless*. E.g. to say that "7 is not a prime number" is to say something false. To say of the number seven when one engages in arithmetic, that "the number seven does not exist" is not to make a statement about the number seven at all. Negating this statement gives something meaningless: to

ask whether a formal concept exists is nonsensical.<sup>52</sup> Someone who *understands* the expression:

$$(\exists a)...$$

already knows *what* the statement expresses since he could not negate it without contradicting what he knows. We have arrived in the position that we cannot have a *criterion* expressing existential commitment, for when a sentence

$$\neg(\exists a)...$$

should be regarded as expressing a *negative* ontological thesis. From this it follows that "the route to a defence of an intuitionistic interpretation of mathematical statements which begins from the ontological status of mathematical objects is closed".<sup>53</sup> Preliminary reflection suggests that we cannot *first* decide the ontological status of mathematical objects, and *then*, with that as premiss, deduce the character of mathematical truth or the correct theory of meaning. Rather we have first to adopt a theory of meaning and *then* an ontology will force itself on us. According to this view, ontological disputes are *metaphors* whose stricter formulation can be given in terms of the validity of *tertium non datur*. The point with this strategy, is to break "the false dichotomy between the platonist and the constructivist pictures which surreptitiously dominates our thinking about the philosophy of mathematics"<sup>54</sup> What we have to do, in order to raise the subject to a participator, is to adopt a strategy which differentiates between knowledge of meaning and knowledge of facts (truth) in a philosophical investigation.

### 1.5 KNOWLEDGE AS UNDERSTANDING

To engage in a philosophical investigation is, as we have said earlier, to engage in a concrete, creative, activity of codification, the codified canonical outcome, or

termination, which dissolves problems concerning meaning and understanding.<sup>55</sup> It is like formulating principles and rules about the running of a machine (black box) based on the output from it. To engage in a philosophical investigation is to engage in a task of providing *objective* understanding. By doing this successfully, when we engage in a philosophical task, we dissolve problems of meaning and understanding, – problems which arise in practice due to the fact that our natural language (any natural language) permits us to ask questions which are meaningless in that they allow an infinite regress of questions. To engage in a philosophical investigation *is to terminate any possibility of asking further, meaningful, questions in the Language*. To engage in it is to provide understanding. E.g. the language in which we speak, read or write this text (English), allows us to ask questions like: What is a number? And, it allows us to answer e.g. like Frege: A number is an extension of a property. And, again, we may ask: What is the extension of an object? And, perhaps, answer: The extension of an object is... And so on, in an endless regress. This is clearly a problem of meaning. In such a case we can say that Language has gone on holiday, to use an expression by Wittgenstein.

According to the way of thinking we are criticizing (observer-based metaphysics), each meaningful sentence, at least in science, purports to express a fact and is *prima facie* capable of being proved or empirically verified. This way of thinking is roughly what underlies the notion of factual meaning. In what follows we shall give an example (there are many other) of statements which we use, not only in everyday language, but also in e.g. mathematics (or science), and which can in no way be explained according to observer-based metaphysics. We shall discuss the example in terms of a distinction which will be called a distinction



between:

*knowledge of meaning*  
*and*  
*knowledge of facts*

There are many statements which we understand, but which we do not know. We do understand the statements that it is raining at a certain time and place, and that 221 is a non-prime number, even if we do not know that they express facts. Someone who understands a Language has knowledge of the meaning of a large part of its expressions but he does not have knowledge of all facts that can be expressed by means of the expressions that he understands. What we know, when we have verified or proved a statement, is that it expresses a fact, and what we know, when we have understood a statement, is its meaning. The distinction between knowledge of meaning and knowledge of facts is *not* a distinction between two kinds of knowledge. At least as far as the kinds of knowledge we shall deal with below, *there is basically only one kind of knowledge, namely understanding.* In other words, the concept of knowledge of facts is a derivative notion. This seems to indicate a connection to the hermeneutical program for the human sciences (*Geisteswissenschaften*) set up by Hans-Georg Gadamer, which “is not a methodology of the human sciences, but an attempt to understand what the human sciences truly are, beyond their methodological self-consciousness, and what connects them with the totality of our experience of the world”.<sup>56</sup>

In linguistic terms we may formulate the distinction between knowledge of meaning and knowledge of facts as the distinction between (1) statements that

express something which it is for us only to understand, or more briefly, statements that require only understanding (knowledge *what* to do), and (2) statements which we prove or verify (knowledge *how* to do it). Statements of the latter kind which express facts like the statements:

221 is a non-prime number

It is raining

are statements whose truth is established by means of proof or observation. In the case of the first statement this is done by exhibiting two numbers and a computation which shows that their product is equal to 221. The second statement is confirmed by making an observation. This is a characteristic feature of statements that express facts: in order to know them we have to engage in a certain primary activity (practice) and complete a certain *task*, such as giving a proof, doing a computation, engaging in observation, or carrying out an experiment. The task having been completed, the statement is true; it expresses a fact; it corresponds to reality. This, in turn, means that we have performed a task (x) with the result X and have understood something which can be expressed by the following statements:

(x) verifies the statement that X

(x) shows that X corresponds to reality

(x) computes the task X

(x) proves X

(x) solves the problem X

Statements that express facts are therefore like Kant's synthetic (and *a priori*

since they are not dependent on experience) judgments in the following respect: *understanding is not sufficient even if necessary in order to know their truth.* What we need, to use Kant's way of speaking, is something else (x) outside the concepts involved in order to confirm them.<sup>57</sup> In the case of mathematical statements this (x) is a proof (computation). In the case of a statement of physics this (x) is an experiment. And in the case of a statement like "It is raining" this (x) is observation. In all cases (x) is essentially treated like a black box. One can say that (x) *belongs* to X, or

$$(x) \in X$$

but, as far as meaning is concerned, (x) is not an *essential* part of the *completed* task X. If we take (x) to stand for a proof one can, like Wittgenstein, say that "(a) mathematical proposition is related to its proof as the outer surface of a body is to the body itself. We might talk of the body of proof belonging to the proposition".<sup>58</sup> What makes such a terminated task into something objective, something independent of the observing person, is *what* the person is doing. Understanding a statement like e.g. "(x) proves X", does not consist in the ability to complete the task of proving X, *but in the objective task of knowing when the subjective task would be completed.* This is to understand a statement "(x) proves X" in an *objective* way. It is something one only has to grasp by recursive steps like:

$$x_1 \in X_1, \dots, x_n \in X_n$$

This can never be proved: it can only be understood. For instance, understanding a mathematical statement X does *not* consist in the ability to give a proof, but in the ability to recognize something as being or not being a proof of it. What we need in this case is knowledge of meaning, not knowledge of facts. That

a linguistic expression is a proof of a certain mathematical proposition is not something that we can again set about to prove; it has to be understood. When we reach the words Q.E.D. at the end of a proof of a theorem, we are supposed to have *understood* that it is a proof of the theorem in question. We do just understand that an arrangement ending with the words Q.E.D. is a proof: it *shows* itself.<sup>59</sup> Contrary to what is usually said, we can realize that a person *first* grasps the objective part of a Language (knowledge of meaning) and in virtue of this knowledge he can engage in expressing subjective knowledge (knowledge of facts).

Another example of a statement that requires only understanding is afforded by the sentence:

7 is a natural number

This sentence, considered as an English sentence, can have several uses. We shall be concerned with this sentence, only insofar as it purports to express some knowledge of the true nature of seven as an object dealt with in arithmetic. The fact that it is expressed in English, or in any natural language at all, is only accidental. If we try to understand this sentence as expressing a mathematical fact or some knowledge of the mathematical object seven, we run into difficulties. Could we imagine someone who *knew the meaning* of the expression 7, but who did not have the knowledge which the statement expresses? Could we imagine someone saying: "I understand the statement '7 is a natural number' perfectly well, but I am not quite sure that it is true?". We would rather take this statement as evidence that he does *not* understand. Consider, on the other hand, the statement:

7 is a prime number

It is perfectly conceivable that someone has learned *what* it is for a number to be prime and that he understands the expression 7, but that he does not happen to know that 7 is one of the prime numbers. It is, of course, an almost trivial task for him to find out the facts, but the point is that he needs something *more* than understanding if he is to say that he knows the statement. He has to check or verify it. He has to complete a certain simple *arithmetical task*. He has to convince himself that 7 is not divisible by a number between 1 and 7. But there is clearly no *arithmetical* task the completion of which leads to the conclusion that:

7 is a natural number

On the contrary, the latter is something which anybody must know in order to understand the statement that "7 is a prime number". A statement like the one above is analytic *a priori* in Kant's terminology. More generally, in order to be able to understand and try to solve arithmetical problems at all, one would have to know what is expressed by statements of the form:

x is a natural number

One could say that numbers can only be defined from propositional *forms*, independently of the question which propositions are true or false, as stated by Wittgenstein.<sup>60</sup> Truth entails (logical) form. This can be formulated, according to Martin-Löf so that an expression, in whatever notation (form), is *canonical* or *normal* if it is already fully evaluated, which is to say that it has itself as value.

E.g. in “decimal arithmetic

$0, 1, 2, 3, \dots, 9, 10, 11, \dots$

are canonical expressions, whereas

$2 + 2, 2 \times 2, 2^2, 3!, 10^{10^{10}}$

are not”.<sup>61</sup> Consequently, a statement like “x is a natural number” does not express arithmetical facts in the sense in which the statement “7 is a prime number” does, even though they have the same grammatical (logical) form. So a philosophical investigation of the arithmetical Language must treat these two kinds of statements differently. The difference between the statements is *not* a psychological one. It is *not* that the statement “7 is a natural number” has a little less trivial proof than the statement “7 is a prime number”. The difference would be even more clear, if we replaced 7 by a large prime number. The difference between them is that the former statement is nothing that we prove at all, but something that we have only to understand. Understanding the arithmetical Language involves, as an essential ingredient, the understanding of statements of the form “x is a natural number”. To know that 7 is a natural number is to know the meaning of the expression 7. Despite its grammatical form, the statement expresses *knowledge how* an expression is to be understood, rather than *knowledge that* a mathematical object has a certain property. Someone’s understanding of an expression in the *practical sense* consists in his ability to *use* the expression correctly. Someone’s understanding of the expression 7 consists in his ability to correctly use the expression 7 in arithmetical computations to compute the value of recursively defined functions for the argument 7. Having the understanding

expressed by the statement "x is a natural number" thus consists in the *ability* to do something in a correct way.

To formulate and explain the rules that regulate this kind of activity is to answer the question as to wherein this understanding consists. It is to answer the philosophical problem:

What is a natural number?

Someone who is thinking in terms of factual meaning may think that the nature and the meaning of the statement "7 is a natural number" depends on how we "explicate" it. He might argue that if we explicate it in *set theory*, using von Neumann's device, the statement takes, say, the form:

$$\{0, 1, \dots, 5, 6\} \in \omega$$

which is an ordinary set theoretic statement that differs in no essential way from other set theoretic statements, and, in particular, not from the corresponding translation of the statement "7 is a prime number". But the statement above is *a set theoretic statement and not an arithmetic statement*. And it is the meaning of the latter kinds of statements that we have been talking about. As Michael Beeson puts it: "Do we really mean that a function *is* a set of ordered pairs (its graph), or is it just convenient to identify a function with its graph for formal purposes? Do we mean to say that the number 2 *is* the set  $\{\emptyset\{\emptyset\}\}$ ? I personally think not, yet the formal axiomatics have so influenced modern thought that young children are now taught that this set *is* 2, not just that the von Neumann integers form a useful *copy* of the natural numbers".<sup>62</sup> What has been said above about explications of the concept of a natural number also holds

for other explications. E.g. the difficulty with the way of thinking that a function *is* a set of ordered pairs, is not only that we have obtained several *new* problems. It is *exactly* this mistake that allows one to ask questions like:

What is an ordered pair?(Quine)

What is a set?(von Neumann)

What is an object?(Frege)

What is a class?(Russell)

The worst is that we have not escaped the philosophical problem at all: How do they have meaning? The representation of arithmetic in set theory does not involve any philosophical explanation of the meaning of the arithmetical Language, and, in particular, not of the statement "7 is a natural number" because the arithmetical Language is used in an essential way *in* the reduction. The reduction is a mathematical result. It is carried out within an extension of the arithmetical Language, and it *presupposes* in an essential way that statements of the form "x is a natural number" are *already* understood. To be more explicit, in order *to be able to recognize* the series of sets:

$$\{\}, \{\{\}\}, \{\{\}, \{\{\}\}\}, \dots$$

or

$$0, \{0\}, \{0, 1\}, \dots$$

as a possible candidate for representing the natural numbers, one must already have the understanding, or knowledge of meaning, which is expressed by statements of the form "x is a natural number". In order to *know that* the series



of sets, shown above, can fulfill the functions of the natural numbers, one must already have practical understanding of *what* function that is; knowing that is to understand statements of the form "x is a natural number". Or, again, expressing it otherwise, in order to know that the above representations (sets, objects, classes) of the natural numbers are possible "explications" of them, one must already understand the adequacy conditions of the explication. An explanation like the one e.g. using von Neumann's device, only reduces the question of what a natural number is, to the more refractory question:

What is an extension of a concept?

An explication makes use, in an essential way, of our practical understanding of the original concept *in* the reduction. The way in which the understanding of the original concept is used in an explication is in the formulation of the adequacy conditions. This is analogous to understanding if a formula in logic is well-formed. We call propositions which are well-formed logically adequate. Logical adequacy is a precondition of truth. In order to understand an explication, it is necessary to understand the adequacy conditions. The explanation of what this understanding consists in, or comes from, is usually given only informally, and not as a part of the investigation, when it ought to be the *most important* part of it. The motivation for certain adequacy conditions for certain notions is often given only on subjective or practical grounds. Arguments such as: "the conditions reflect our intuitions about this concept", or, "they express the most evident properties of this notion" are not unusual. The so-called paradox of analysis has been regarded as an objection against such a requirement on philosophical analysis. Quine expresses it by saying that philosophical analysis cannot

make synonymy claims, and does not aim at finding hidden meanings. He states the paradox as follows: "How can correct analysis be informative, since to understand it we must already know the meanings of its terms, and hence already know that the terms which it equates are synonymous?"<sup>63</sup> This is a paradox, however, only for someone who does not realize that engaging in a philosophical investigation of the meaning of a term is to do something different from employing the term in its normal use. It is a paradox for someone who does not realize the difference between knowledge of meaning and knowledge of facts.

#### 1.6 PHILOSOPHY AS PERSON PROGRAMS

It is important to realize that we do *not* deny that there are objects; we only maintain that it is concrete activity, not objects, which are real, actual, or concrete. Alternatively one can state this, like Dummett, by saying that we do not *make* objects but must accept them as we find them; but they were not *already* there for our statements to be true or false of.<sup>64</sup> We maintain that objects only occur in virtue of a will, so to speak. To engage in a philosophical investigation is not to ask for some kind of objects or entities called meanings. By engaging in a philosophical investigation one wants to acquire understanding. Understanding is the correlate of explanation. To engage in the task of constructing a depth grammar is therefore not to construct some objects that can function as the "meanings" of the expressions of that Language. Saying that meaning has to do with knowledge and understanding is not to say that the meaning of an expression is some mental object: "...when I think in language, there aren't 'meanings' going through my mind in addition to the verbal expressions: the language is itself the vehicle of thought".<sup>65</sup> It is rather to say *what* it is to understand the Language, or, in other words: to engage in a philosophical investigation pro-

vides understanding of a Language by providing a codified depth grammar. This depth grammar shows us how to use the expressions and sentences of the Language in question. It shows the meaning of the expressions and sentences being used. As Gordon Baker puts it: "The meaning of a word, phrase or sentence is what is explained in explaining it. Ordinary explanations of meaning are to be taken at face value; they have many different forms, and they function in many ways. Explanations are *rules* for the correct use of the expressions explained. Understanding an expression is to know how to use it".<sup>66</sup> It is when we don't understand a practice that we need philosophical therapy in order to dissolve problems of understanding: "(t)he fundamental fact is here that we lay down rules, a technique,... and when we follow the rules, things do not turn out as we had assumed. That we are therefore as it were entangled in our own rules. This entanglement in our rules is what we want to understand (i.e. get a clear view of). It throws light on our concept of *meaning* something".<sup>67</sup> What we want is to be able to see the philosophical problem corrected. And we see it by engaging in a philosophical investigation of constructing the depth grammar of the troubled activity: we see *how* the activity is to be understood or used. Remember that to *see*, in a philosophical sense, is to see *how* the purpose is fulfilled i.e. "if we wish to know what a proposition means, we can always ask, 'How do I know it?'. Its meaning is determined by the answer to this question".<sup>68</sup> *The whole* activity of codifying a practice formulated as rules is a philosophical investigation. The whole system of rules (context) codified by canonical steps is called the *code* of the practice. Alternatively, it will also be called the *depth grammar* of the practice. A philosophical investigation is the output of the *whole* activity of codification by canonical steps. A philosophical investigation is a closed whole vis-a-vis meaning

and understanding. We call the canonical, or normal form, the *formal* part of the philosophical investigation and the rules necessary to successfully terminate the practice, we call the *non-formal* part of the philosophical investigation. We will return to these rules in more detail elsewhere.<sup>69</sup> Consequently, a Language always consists of a formal and non-formal part. To engage as a participator in a philosophical investigation is, then, to actually construct a depth grammar which regulates a practice, and determines it to what it is. The meaning of a practice is grasped in the very act of abstraction. It is not something over and above this. Understanding of a practice is completely expressed in the philosophical explanation one gives.<sup>70</sup> This is, in constructivism, also a reaffirmation of the Kantian doctrine of the priority of judgments over concepts. Our use of concepts presupposes *unity* of judgment: "The only use which understanding can make of these concepts is to judge by means of them".<sup>71</sup>

The output of a philosophical investigation, consequently, consists of, on the one hand, of a string of codified rules (propositions), and, on the other hand, by the collection of those rules constituting the Language of the investigation. When the last non-formal rule is codified the philosophical investigation terminates. A philosophical investigation terminates when the concrete context, or *person program* (depth grammar), constituting a Language has been codified. When we set up a Language (person program) by the canonical steps of codifying propositions (as rules), we give at the same time a manual for translating between it and the ordinary forms of expression used in practice. The same proposition of the Language may translate several different sentences of the informal language. That is, there are often many different ways in which a single thing is expressed in practice. The different expressions are then synonymous and count as repe-

titions of the proposition set up by the canonical step in the Language. What is important to realize, is that as far as meaning is concerned the synonymous expressions are redundant. A Language (person program) which we set up in a philosophical investigation has no redundancies in that there are no two different synonymous expressions: a co-ordinate of nature can only be determined *once*.<sup>72</sup> As an example we can give an expression from the traditional logical language. It achieves this to some extent. Thus are rendered by the same formula:

$$(\exists x)F(x)$$

such different informal sentences as:

for some x, F(x)

there exists an x such that F(x)

These, latter, informal sentences are redundant vis-a-vis meaning. They count as repetitions of the formula. A Language, or person program, set up in a philosophical investigation by canonical steps, eliminates, besides such redundancies others of a more striking kind.<sup>73</sup> The translation manual, or logical grammar, enables one to grasp how far one's ordinary forms of expression may be formulated within the Language one sets up intended as the code of the practice in question. It is important to realize that it is *not* meant as an explanation of how that Language is to be understood. This would presuppose it was already known how the ordinary forms of expression are to be understood. The aim of a philosophical investigation is precisely to explain how the forms of expression are to be understood. It is to construct the Language in question. It ought to be clear that someone who wants to know the meaning of certain expressions or of

certain activities is *not* asking for some kind of mental objects or entities called "meanings" which are associated with the expressions, or activities, respectively. This would presuppose understanding of a form of Language using the expression "objects". But, as we said above, a philosophical investigation cannot presuppose knowledge of how the form of the expression "object" is to be understood. It is precisely this kind of understanding we want to acquire. One can say that philosophy is an attempt to understand nature as it reveals itself in different forms of life, which sometimes takes the form of science, both experimental and theoretical. These forms of life are what must be accepted: they are given.<sup>74</sup> They cannot be justified. Every justification presupposes acceptance of a form. But as long as we have not achieved understanding of the different Languages we use, e.g. in mathematical and physical practice, that is, we have not incorporated the subject as a participator, so long will our understanding of nature be imperfect. Consequently, one can say that, "...it is incontrovertible that the observer is participator in genesis".<sup>75</sup>

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completely expressed in the explanations that I could give?...”.

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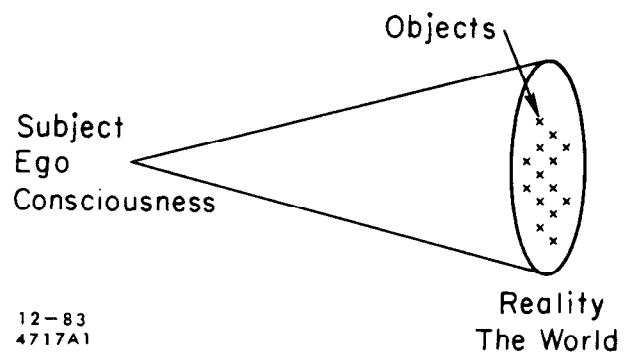


Fig. 1

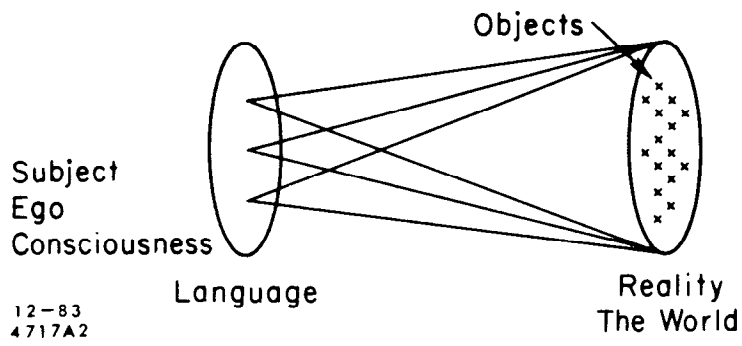


Fig. 2

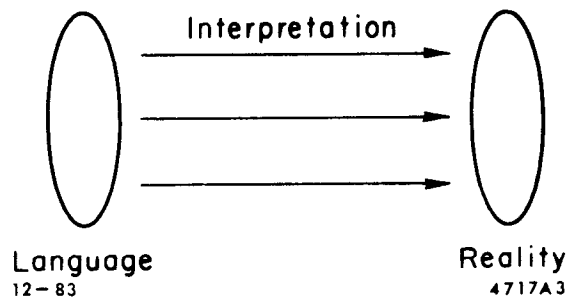


Fig. 3