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SINGLE WEAK BOSON PRODUCTION IN ELECTRON–POSITRON ANNIHILATION*

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ABSTRACT

The cross sections for the process, $e^+e^- \rightarrow W^\pm$ + hadrons are estimated extensively at the lowest order in the minimal gauge theory of $\mathrm{SU(2)}_L \times \mathrm{U(1)}$. These estimations lead to the rigorous upper bounds for the processes, $e^+e^- \rightarrow W^\pm\pi^\mp, e^+e^- \rightarrow W^\pm\rho^\mp, \ldots$.

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The CERN UA1 and UA2 collaborations have recently reported [1] that they made the observations of weak boson candidates through the analysis of large transverse momentum electrons and muons. The masses reported by UA1 and UA2 [1] are, respectively,

$$M_W = 81 \pm 5\ GeV\ ,\ 80\ {}^{+10}_{-6}\ GeV\quad . \tag{1}$$

After that, the UA1 and UA2 reported [2] that they found also the high mass dimuon events which may come from the Z boson. The reported dimuon masses by UA1 and UA2 [2] are, respectively,

$$M_Z = 95.2 \pm 2.5\ GeV\ ,\ 91.9 \pm 1.3 \pm 1.4\ GeV\quad . \tag{2}$$

Both of these are very similar with the predicted masses [3]. The data recorded at the CERN SPS proton–antiproton collider show also that the topology as well as the number of events fit well the hypothesis that these leptons with the large transverse momentum are produced through the process $p\bar{p} \to W(Z) + X$ and $W^- \to \bar{\nu}_\mu \mu (Z \to \bar{\mu}\mu)$. Here the estimate of the cross sections for $p\bar{p} \to W(Z) + X$ has been given theoretically [4] by using the Drell–Yan model [5].

The present authors would like to study the cross sections of the weak boson production with the hadrons in electron–positron collisions from the Z–pole to the threshold of region for W–pair productions.

There are several articles [6-8] in which the authors studied single weak boson production in electron–positron scatterings in the Weinberg–Salam gauge theory [3]. However, in almost all of these studies, neither the angular and energy distributions of the weak boson have been given, nor the contributions of all the Feynman diagrams have been calculated. It is clear that the trace of the gauge invariance disappears if we neglect even one of the Feynman diagrams as will be discussed below. Neufeld [9] has presented the extensive calculations of the single weak boson production with the leptons in electron–positron scatterings. Then we would like to report on our studies of the single weak boson production with the hadrons which seem to be complementary to the studies published by Neufeld [9]. Here it is assumed that these cross sections in the energy range discussed in this paper could be estimated by the cross sections for the weak boson production with the free u, d and c, s massless quarks which are hadronized with the probability one.

If the massless quarks are assumed, you may deduce that there is no difference between $e^+e^- \to W^+ \bar{\nu}_\mu \mu$ and $e^+e^- \to W^+ \bar{u}\, d$, because u and d quarks are assigned to the SU$(2)_L$ doublet as the neutrino and muon. You may understand, however, that it seems to be too optimistic to deduce so, if you look at the U(1) quantum numbers, or the electric charge of these particles. For example, the diagrams D1 and D2 in Fig. 1 lead to the amplitudes for these processes in the different way, although the diagrams C1 and C2 lead to the same amplitude for these processes.

We have used the REDUCE [10] program for the trace calculations and the VEGAS [11] program for the phase space integrations.* We have checked the results obtained by Neufeld [9] and others [7,8], who used the SCHOONSHIP [12] program for the trace calculation.

Defining g_2 and g_1 as the SU$(2)_L$ coupling and U(1) coupling, respectively, and the polarization vector ϵ_δ for the weak boson, we write the matrix element as follows:

$$M = i(g_2 e^2/\sqrt{2})\, \epsilon_\delta \Big[d_L^1\, D_L^{\delta,1} + d_L^2\, D_L^{\delta,2} + c_L^1\, C_L^{\delta,1} + c_L^2\, C_L^{\delta,2} + L \to R \Big], \tag{3}$$

$$1/e^2 \equiv 1/g_2^2 + 1/g_1^2\ . \tag{4}$$

* VEGAS is employed by choosing 5000 random points (NCALL=5000) and performing 10 iterations (ITMX=10) in this paper.

Here, D_L^δ's and C_L^δ's are defined as

$$D_L^{\delta,1} \equiv \frac{g^{\alpha\beta}}{(W+D)^2(Q+P)^2}[\bar{U}_L(D)\gamma^\delta(\not{W}+\not{D})\gamma_\alpha V_L(U)][\bar{V}_L(P)\gamma_\beta U_L(Q)], \quad (5)$$

$$D_L^{\delta,2} \equiv \frac{g^{\alpha\beta}}{(W+U)^2(Q+P)^2}[\bar{U}_L(D)\gamma_\alpha(\not{W}+\not{U})\gamma^\delta V_L(U)][\bar{V}_L(P)\gamma_\beta U_L(Q)], \quad (6)$$

$$\begin{aligned} C_L^{\delta,1} \equiv & \frac{(W+2U+2D)^\delta g^{\alpha\beta}+2W^\beta g^{\delta\alpha}-2W^\alpha g^{\beta\delta}}{[(U+D)^2-M_W^2+iM_W\Gamma_W](Q+P)^2} \\ & \times [\bar{U}_L(D)\gamma_\alpha V_L(U)][\bar{V}_L(P)\gamma_\beta U_L(Q)] \end{aligned}, \quad (7)$$

$$\begin{aligned} C_L^{\delta,2} \equiv & \frac{g^{\alpha\beta}}{[(U+D)^2-M_W^2+iM_W\Gamma_W](W-P)^2} \\ & \times [\bar{U}_L(D)\gamma_\alpha V_L(U)][\bar{V}_L(P)\gamma^\delta(\not{P}-\not{W})\gamma_\beta U_L(Q)] \end{aligned}, \quad (8)$$

and D_R^δ's, C_R^δ's are defined similarly if we replace $[\bar{v}_L(P)...u_L(Q)]$ by $[\bar{v}_R(P)...u_R(Q)]$, where notations P, Q, W, U and D represent the momentum of initial positron and electron, final weak boson u quark and d quark, respectively.

The coefficients d_L's, d_R's, c_L's and c_R's are represented in Table I, where $s \equiv (P+Q)^2$. These are obtained by dictating the Feynman diagrams $D1, D2, C1$ and $C2$ as shown in Fig. 1, respectively. As the reference, we list the d coefficients also for the process $e^+e^- \to W^+\,\bar{\nu}_\mu\,\mu$, where U and D should be regarded as the momentum for the neutrino and muon. It is needless to say that there is no difference in the coefficients c's between our amplitude and the amplitude for $e^+e^- \to W^+\,\bar{\nu}_\mu\,\mu$ as we discussed above, and we have to take account into the color degree of freedoms for the process $e^+e^- \to W^+\,\bar{u}\,d$ when we estimate the cross sections for $e^+e^- \to W^+ +$ hadrons.

Now we would like to give a comment on the gauge invariance. The gauge invariance is broken spontaneously, however, it should be recovered at the high energy so that the weak boson mass may be neglected. As a trace of the gauge invariance, we can see

$$W_\delta \left[d^1_{L/R}\, D^{\delta,1}_{L/R} + d^2_{L/R}\, D^{\delta,2}_{L/R} + c^1_{L/R}\, C^{\delta,1}_{L/R} + c^2_{L/R}\, C^{\delta,2}_{L/R} \right] = 0, \tag{9}$$

at $M_W^2/S \to 0$, if we shoot a glance at the coefficients in Table I.

Finally, the total cross section versus the center of mass energy is shown in Fig. 2. Figure 3 represents the angular Θ_W distributions at several energies, where Θ_W is the angle between the initial electron (positron) and the produced weak boson $W^-(W^+)$. The weak boson energy distributions are shown at several energies in Fig. 4. The parameters used here are as follows:

$$M_W = 80.0\ GeV \quad , \quad M_Z = 92.0\ GeV \quad , \tag{10}$$

$$\Gamma_W = 2.69\ GeV \quad , \quad \Gamma_Z = 2.74\ GeV \quad , \tag{11}$$

$$tan^2\theta_W = 129/400 \quad . \tag{12}$$

For the decay widths of W and Z bosons, we have used the following formula;

$$\Gamma_W = 4n\ G_F\ M_W^3/(6\pi\sqrt{2}) \quad , \tag{13}$$

$$\Gamma_Z = [1 - tan^2\theta_W\ (1 - 8sin^2\theta_W/3)]\ \Gamma_W/cos\theta_W \quad . \tag{14}$$

Here n is the number of generations ($n = 3$),

Although our calculated cross sections are small as the previous authors [6-8] have pointed out, we could refer some supports for our studies as follows: (1) We can assert safely that our cross sections are regarded as two times of the upper bounds for the processes $e^+e^- \to W^\pm\pi^\mp, W^\pm\rho^\mp \ldots$, as shown in the right titles of Fig. 2 and 3.

(2) Our result seems to be useful in e^+e^- collider experiments of the next generations.

(3) The recent works on the weak boson decays into the supersymmetric gauge particle [13] seem to require more analysis on the weak boson production processes.

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Table I

$e^+e^- \to W^+\,\bar{u}\,d$	
$d_L^1 = \frac{2}{3} + \left[\frac{1}{12}\left(\frac{g_1}{g_2}\right)^2 - \frac{1}{3} + \frac{1}{4}\left(\frac{g_2}{g_1}\right)^2\right]\frac{s}{s-M_Z^2+iM_Z\Gamma_Z}$	$d_R^1 = \frac{2}{3} + \left[\frac{1}{6}\left(\frac{g_1}{g_2}\right)^2 - \frac{1}{2}\right]\frac{s}{s-M_Z^2+iM_Z\Gamma_Z}$
$d_L^2 = \frac{1}{3} + \left[-\frac{1}{12}\left(\frac{g_1}{g_2}\right)^2 - \frac{1}{6} + \frac{1}{4}\left(\frac{g_2}{g_1}\right)^2\right]\frac{s}{s-M_Z^2+iM_Z\Gamma_Z}$	$d_R^2 = \frac{1}{3} + \left[-\frac{1}{6}\left(\frac{g_1}{g_2}\right)^2 - \frac{1}{2}\right]\frac{s}{s-M_Z^2+iM_Z\Gamma_Z}$
$c_L^1 = 1 + \left[-\frac{1}{2} + \frac{1}{2}\left(\frac{g_2}{g_1}\right)^2\right]\frac{s}{s-M_Z^2+iM_Z\Gamma_Z}$	$c_R^1 = 1 + [-1]\frac{s}{s-M_Z^2+iM_Z\Gamma_Z}$
$c_L^2 = \frac{1}{2} + \frac{1}{2}\left(\frac{g_2}{g_1}\right)^2$	$c_R^2 = 0$

$e^+e^- \to W^+\,\bar{\nu}_\mu\,\mu$	
$d_L^1 = \left[-\frac{1}{4}\left(\frac{g_1}{g_2}\right)^2 + \frac{1}{4}\left(\frac{g_2}{g_1}\right)^2\right]\frac{s}{s-M_Z^2+iM_Z\Gamma_Z}$	$d_R^1 = \left[-\frac{1}{2}\left(\frac{g_1}{g_2}\right)^2 - \frac{1}{2}\right]\frac{s}{s-M_Z^2+iM_Z\Gamma_Z}$
$d_L^2 = 1 + \left[\frac{1}{4}\left(\frac{g_1}{g_2}\right)^2 - \frac{1}{2} + \frac{1}{4}\left(\frac{g_2}{g_1}\right)^2\right]\frac{s}{s-M_Z^2+iM_Z\Gamma_Z}$	$d_R^2 = 1 + \left[\frac{1}{2}\left(\frac{g_1}{g_2}\right)^2 - \frac{1}{2}\right]\frac{s}{s-M_Z^2+iM_Z\Gamma_Z}$

Figure Captions

Figure 1 Feynman diagrams contributing to the process $e^+e^- \to W^- +$ hadrons.

Figure 2 $\sigma_{total}(e^+e^- \to W^\pm +$ hadrons) versus center of mass energy, $\sqrt{s}$. The upper bound of the total cross section for $e^+e^- \to W^\pm\pi^\mp$, $W^\pm\rho^\mp$ is shown in the right title.

Figure 3 $d\sigma(e^+e^- \to W^\pm +$ hadrons$)/d(cos\Theta_W)$ at $\sqrt{s} = 92$, 125, 150, 200 GeV. The upper bounds of the differential cross sections for $e^+e^- \to W^\pm\pi^\mp$, $W^\pm\rho^\mp$ are shown in the right title.

Figure 4 $d\sigma(e^+e^- \to W^\pm +$ hadrons$)/dE_W$ at $\sqrt{s} = 92$, 125, 150, 200 Gev. Note that the cross section at $\sqrt{s} = 125$ GeV is below $10^{-38}cm^2$/GeV.

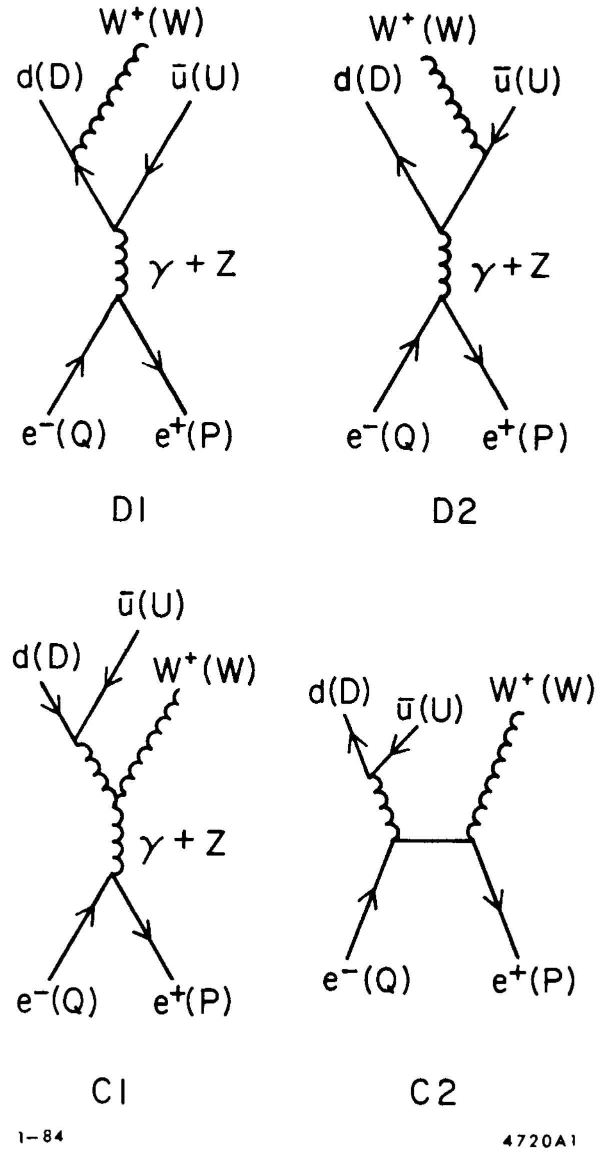
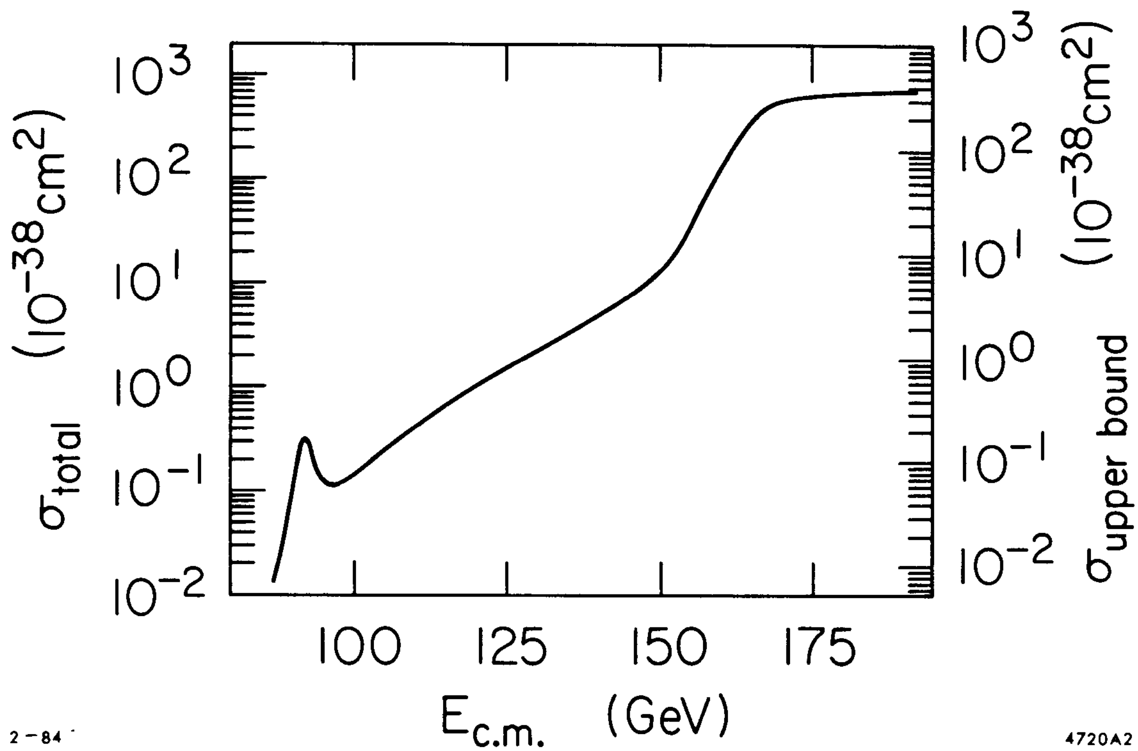
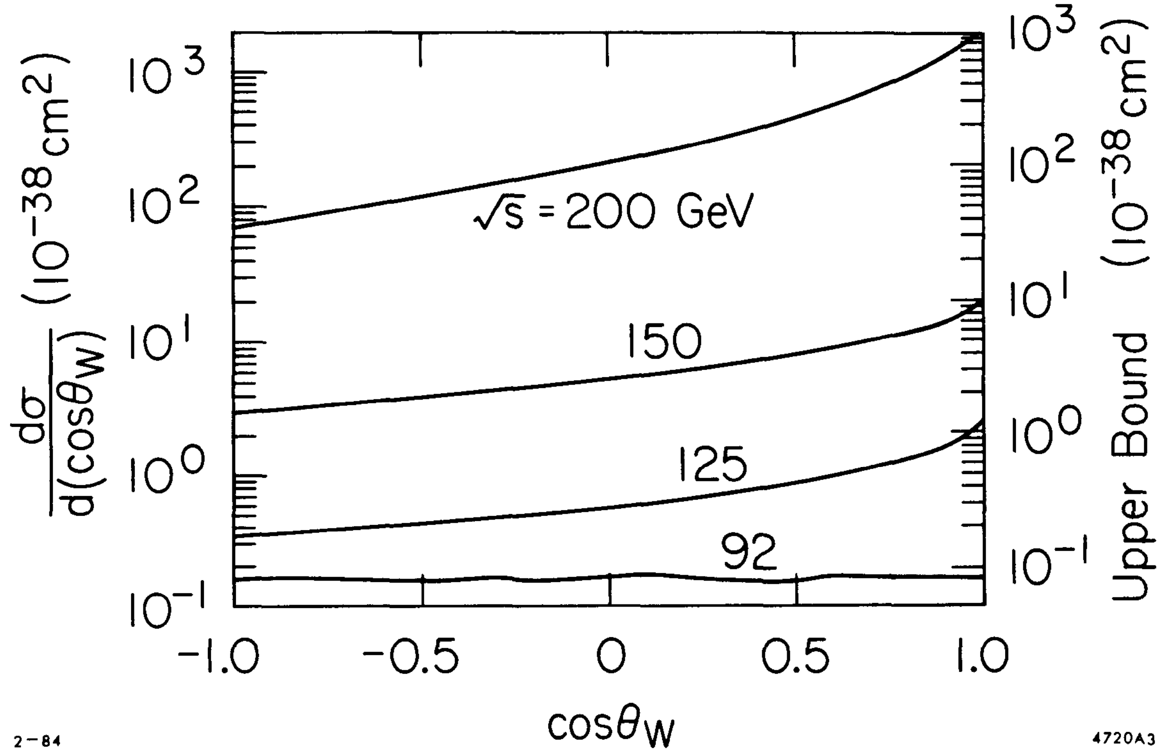

W+(W)
d(D)
ū(U)
γ + Z
e−(Q)
e+(P)
D1
W+(W)
d(D)
ū(U)
γ + Z
e−(Q)
e+(P)
D2
ū(U)
d(D)
W+(W)
γ + Z
e−(Q)
e+(P)
C1
d(D)
ū(U)
W+(W)
e−(Q)
e+(P)
C2
1-84
4720A1

Fig. 1



10^3
10^2
10^1
10^0
10^-1
10^-2
σ_total (10^-38 cm^2)
σ_upper bound (10^-38 cm^2)
100
125
150
175
E_c.m. (GeV)
2-84
4720A2

Fig. 2



$\frac{d\sigma}{d(\cos\theta_W)}$ $(10^{-38} cm^2)$
10^3
10^2
10^1
10^0
10^{-1}
$\sqrt{s}$ = 200 GeV
150
125
92
-1.0
-0.5
0
0.5
1.0
$\cos\theta_W$
Upper Bound $(10^{-38} cm^2)$
2-84
4720A3

Fig. 3

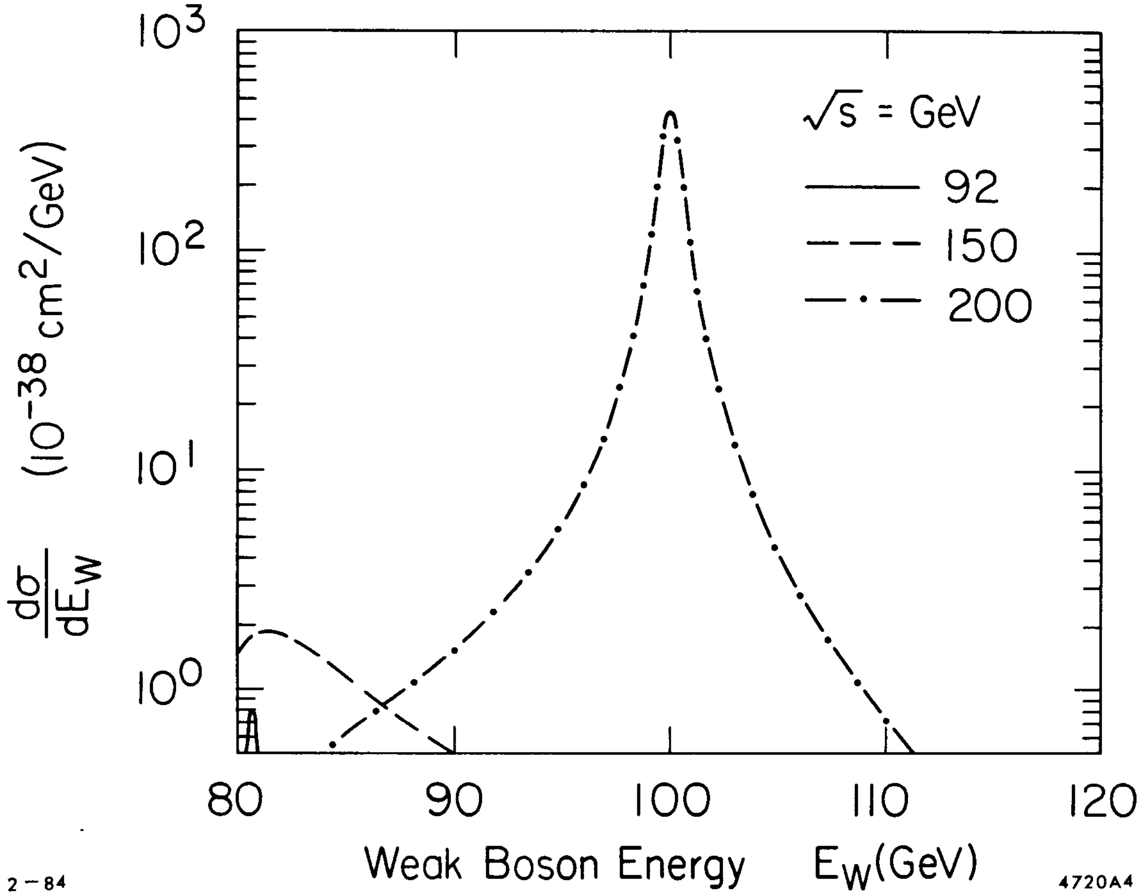

10^3
10^2
10^1
10^0
$\frac{d\sigma}{dE_W}$ $(10^{-38}\ cm^2/GeV)$
$\sqrt{s}$ = GeV
92
150
200
80
90
100
110
120
Weak Boson Energy E_W(GeV)
2-84
4720A4

Fig. 4