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# ASPECTS OF THE DYNAMICS OF HEAVY-QUARK SYSTEMS

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Contents:



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## 1. Introduction

The analysis of bound states composed of heavy quarks and antiquarks has provided a window into the structure of the strong interactions which has offered a view at the same time brilliantly clear and tauntingly ambiguous. These systems contain our best evidence that the quarks from which the hadrons are built are ordinary fermions which obey the Dirac equation and which couple to electromagnetism just as electrons do. The spectrum of bound states and the systematics of its quantum numbers make clear that the same basic principles that lead to the spectrum of the hydrogen atom or positronium also govern the behavior of quarks. However, the heavy-quark systems which have been studied to date, the systems of c-$\bar{c}$ and b-$\bar{b}$ bound states, seem to be bound by forces which bear no obvious relation to the gluons which we expect are the fundamental mediators of the strong interactions. The spectrum of bound states can be explained by insisting that the quark and antiquark interact through a phenomenologically determined potential. This phenomenological picture, however, seems very difficult to connect to an underlying description in terms of a color gauge theory.

The essential difficulty in understanding this connection arises from the fact that the c-$\bar{c}$ and b-$\bar{b}$ systems occupy an intermediate regime in the behavior of the gauge theory. At very small distances, comparable to those probed in deep inelastic scattering, the q-$\bar{q}$ potential is expected to become a Coulomb potential, directly reflecting one-gluon exchange. At very large distances, if the notion of quark confinement is a correct one, the potential should become simply proportional to the q-$\bar{q}$ separation, reflecting the formation of confining strings of color flux. To understand the transition region, however, a qualitative picture does not suffice; for a proper understanding, one would need to see precisely how the collective behavior of gluons modifies and alters single gluon effects. At the present time, we seem very far from such a detailed understanding. It is possible, however, to gain some insight into the nature of this intermediate regime by considering the behavior of q-$\bar{q}$ systems from a broad perspective, assembling a variety of distinct aspects of these systems which are sensitive to the properties of gauge theories at intermediate distances.

In these lectures, I will present the theory of three different facets of this behavior by the use of a unified mathematical formalism. My goal will be to clarify the interrelation of these phenomena and, more importantly, their connection to the properties of an underlying gauge theory. The plan of these lectures is as follows: In § 2, I will present some basic theoretical orientation, setting out the formalism which will be the basis of our discussion. I will also discuss the application of this formalism to the static potential; I will discuss the foundation of the static potential approximation and justify the general shape of the potential as emerging from a gauge theory. In § 3, I will discuss the theory of spin-dependent forces in heavy-quark systems, presenting a connection of these forces to gauge-theory amplitudes discovered by Eichten and Feinberg.[1] In § 4, I will discuss the theory of hadronic transitions between q-$\bar{q}$ states, following the approach of Yan.[2] This discussion will not deal with detailed phenomenological theories or extensive comparison with experiment. My intent is, rather, to make clear what gauge theories have to say about these topics. My presentation will be somewhat formal, but, as is appropriate to a summer school, the formalism will be built from the ground up and kept as comprehensible as possible. I should also make clear that none of the work to be discussed is new; these are classic topics in the theory, but ones not sufficiently widely appreciated and so most deserving of review and explication.

In the remainder of this section, I will remind you of a few of the basic features of heavy quark meson spectroscopy. The spectrum of levels of the c-$\bar{c}$ and b-$\bar{b}$ systems are generally well known, and are discussed in many reviews,[3] so I can be brief. Let me begin by simply presenting, in Fig. 1, the observed level spectrum of these heavy-quark systems, together with that of positronium. I use the notational convention, often used in q-$\bar{q}$ spectroscopy but unusual in

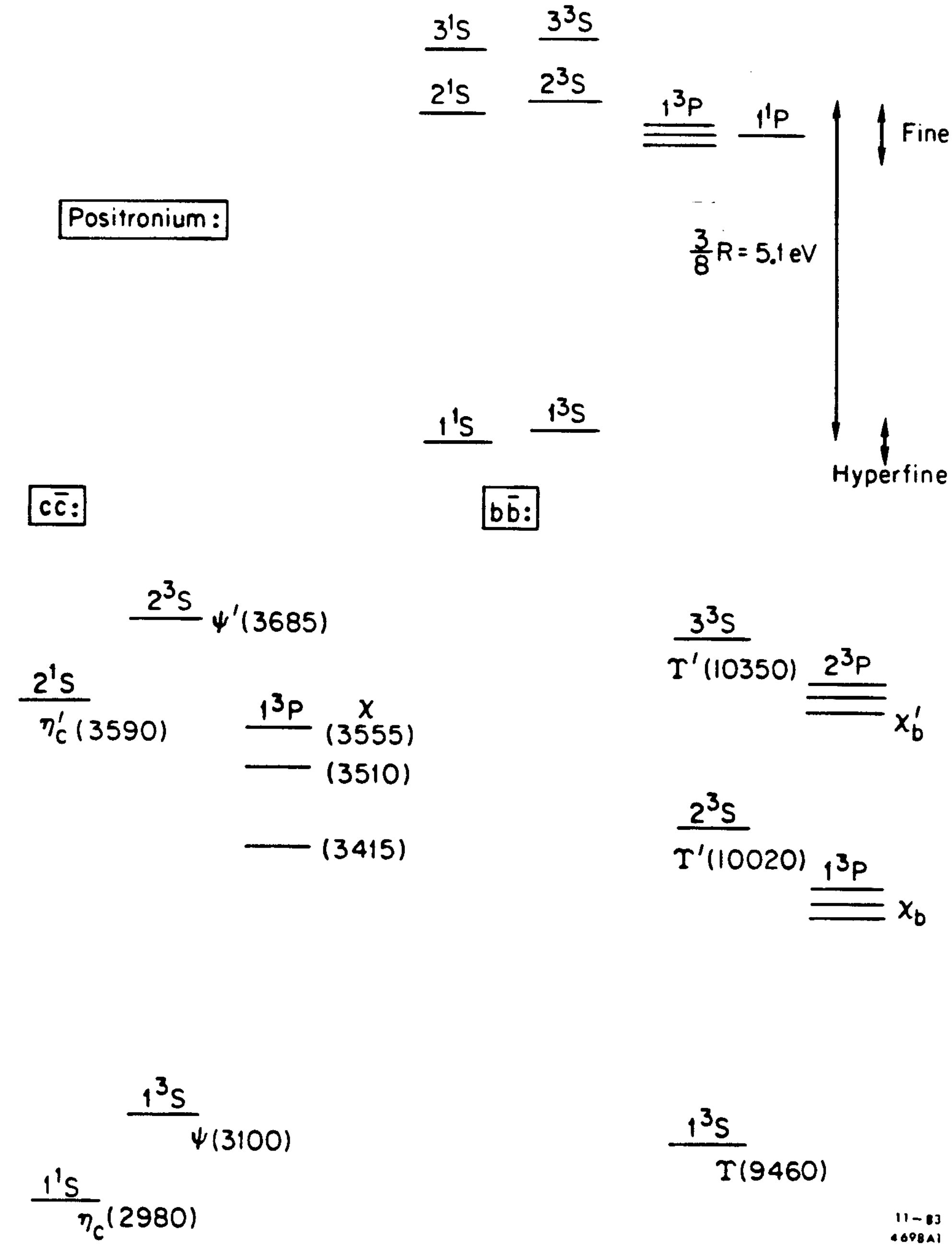


FIG. 1. The spectrum of observed levels of three anti-fermion bound-state systems, the positronium ($e^+$-$e^-$),$\psi$ (c-$\bar{c}$), and $\Upsilon$ (b-$\bar{b}$) systems.

atomic systems, of labelling the lowest-lying P states as '1P'. This difference in nomenclature emphasizes the only significant qualitative difference between the heavy-quark and atomic spectra: The degeneracy of states with the same principal quantum number, a special property of the Coulomb potential, is lost in the quark systems, to which this potential does not directly apply.

The c-$\bar{c}$ and b-$\bar{b}$ level spectra shown in Fig. 1 are well known to be accurately described by a model in which the quarks are treated as nonrelativistic fermions interacting through a simple potential, if one allows the potential to be determined phenomenologically. A typical procedure is to choose a potential with a small number of parameters which may be fit, for example, to the $\psi$, $\psi'$, and $\Upsilon$ masses and the ratio of the $\psi$ and $\psi'$ leptonic widths. The merits of various fitting procedures are debated in References 4-5. A number of different functional forms for the potential have been used in the literature. Some typical forms are those of Eichten *et. al.*,[4] who use a linear combination of a linear potential and a Coulomb potential, Richardson,[6] who uses the Fourier transform of a Coulomb potential with a running coupling constant

$$V(r) = \int d^3q e^{iq\cdot r}\left[\frac{\alpha_s(q^2)}{4\pi q^2}\right], \tag{1.1}$$

and Martin,[7] who uses a simple power law $V(r) \sim r^{0.1}$ for the shape of his potential. All of these potentials are quite successful; in the sense that they all give the locations of the $\Upsilon'$,$\Upsilon''$, $\Upsilon'''$, and the $\chi_b$ to within about 10 MeV. This striking convergence of predictions is easy to understand by plotting these potentials against one another, as is done in Fig. 2. Also plotted in this figure are the mean radii of the various $\psi$ and $\Upsilon$ states. It is clear that our restricted experimental information–most especially, the insensitivity of accessible states to the short-distance behavior of the potential–leave many possibilities open. The discovery of the top quark and the associated t-$\bar{t}$ mesons should provide an important constraint on the phenomenological potential. Until this discovery

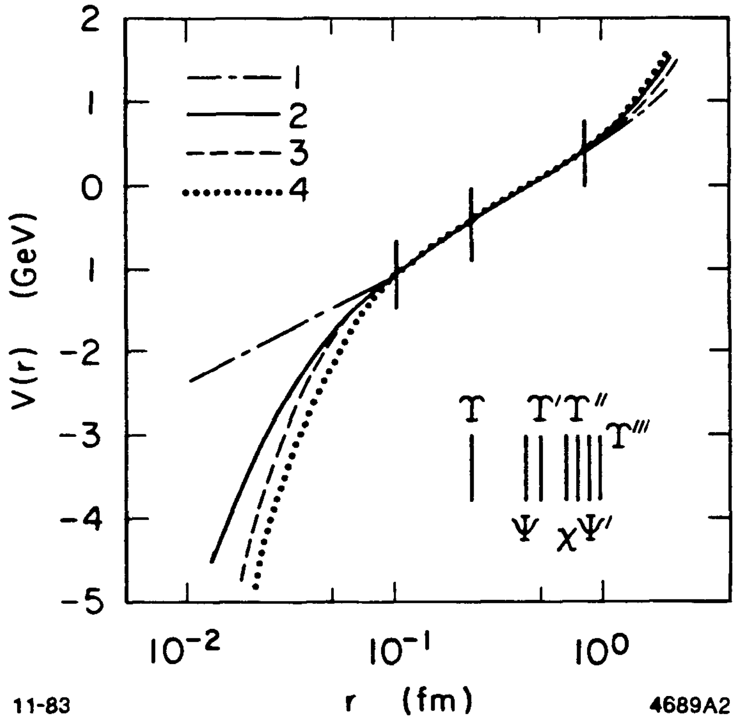


FIG. 2. A comparison of four different phenomenological q-q̄ potentials, from Ref. 5. The potentials represented are those proposed in (1) Ref. 7, (2) Ref. 5 (a variant of that of Ref. 6), (3) Ref. 8, (4) Ref. 4.

arrives, we can learn more about heavy-quark interactions only by extending the theory which includes this potential, and applying it more widely.

## 2. The Wilson Line and the Static Potential

In order to argue precisely about the intricacies of heavy-quark behavior, we need a mathematical description of heavy-quark systems which is at the same time powerful and physically transparent. In this section, I would like to introduce you to such a formalism. I will explain how to represent the motion of heavy quarks directly in space-time, using the Feynman path integral approach to quantum mechanics. We will see that the static potential description of the q-$\bar{q}$ interaction emerges as an obvious first approximation to this exact representation. This observation will allow us to discuss the conditions for the validity of the static potential picture, to understand when this description is valid, and to account which quantum effects are subsumed and which omitted by this approach. I will also use this picture to explain what the gauge theories predict about the general shape of this potential.

In fermion-antifermion systems bound by electrical forces, such as positronium, it is possible to discuss the applicability of the static potential picture, and the shape of this potential, without needing to rely on any special theoretical tools. One needs only to expand systematically in powers of $\alpha$. This is, of course, not completely straightforward, because a large class of Feynman diagrams–the set, shown in Fig. 3, which one would normally associate with fermion-antifermion binding–all have values of order $\alpha^0$. But one can show[9] that the leading terms in these diagrams have the form of the solution to a nonrelativistic Schrödinger equation. The various corrections to the static picture appear as successive powers of $\alpha$ are uncovered: For positronium, the fundamental scale is the reduced mass $\mu = \frac{1}{2}$m. The binding energy is then $R = \frac{1}{2}\alpha^2\mu$. That the lowest bound state is nonrelativistic follows from the fact that $\alpha$ is small–the Bohr radius is equal to $(\alpha\mu)^{-1}$ and the fermion velocities are of order

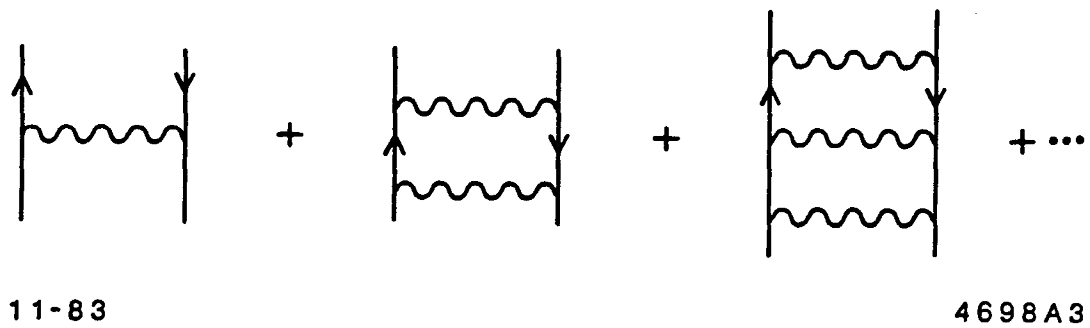


FIG. 3. Graphs whose sum produces the nonrelativistic positronium spectrum.

$\alpha$. The spin-orbit splittings in the spectrum appear in order $\alpha^2 \cdot R$; the Lamb shift appears in order $\alpha^3 \log(\alpha^{-1}) \cdot R$.

In quark-antiquark systems, however, the situation is rather different. As long as the quark-antiquark binding forces remain constant in magnitude as the quark mass is increased, one will eventually reach a point at which the motion of quarks in their bound states is nonrelativistic. The phenomenological success of the potential models for the c-$\bar{c}$ system tells us that this point has been reached already for the charmed quark. However, this certainly does not imply that the strong binding forces are described by a weak-coupling approximation. All that should actually be necessary is that the bound quarks should have masses much larger than the energy scale of quark-binding in the gauge theory. This observation, though, raises a question about one's theoretical description of the heavy-quark system: We must describe the system in such a way that we can expand directly in the velocity of the quark and antiquark, without needing to make any additional approximation in describing the gauge-field dynamics.

It is clear that such an approximation scheme requires an ability to visualize the motion of the quark and antiquark in space-time, as they participate in the bound state. Such a scheme of visualization would also be interesting and informative in its own right. Let me, then, set up such a description, based on Feynman's description of quantum mechanics in terms of paths in space-time.[10]

To begin, compare the structure of the Schrödinger equation

$$i \frac{\partial}{\partial t} \psi = \Big(-\frac{1}{2m} \nabla^2 + V(x)\Big) \psi \tag{2.1}$$

to that of the diffusion equation:

$$-\frac{\partial}{\partial \tau} \phi = \Big(-D\nabla^2 + V(x)\Big) \phi \, . \tag{2.2}$$

I have added to (2.2) a term which allows the diffusing particle to be destroyed with probability $V(x)$, to make the analogy with (2.1) more precise. The process of diffusion is apparently described by quantum mechanics, with the replacement

$$it \rightarrow \tau\ , \tag{2.3}$$

or

$$e^{-iHt} \rightarrow e^{-H\tau}, \tag{2.4}$$

and by setting $D = \frac{1}{2m}$. This relationship works also in the other direction. In principle, then, we can construct a precise representation of quantum mechanics by simulating the diffusion process and then reversing the transposition of Eq. (2.3). This program is actually quite easy to carry out in practice.

Consider first carrying out the diffusion process over a small increment of time $\epsilon$. The probability of diffusion from $x_i$ to $x_f$ in time $\epsilon$ is given by

$$\phi(x_f;x_i;\epsilon) = \frac{1}{(4\pi D\epsilon)^{3/2}}\exp\left[-\frac{(x_f-x_i)^2}{4D\epsilon} - \epsilon\, V\Big(\frac{x_i+x_f}{2}\Big)\right]. \tag{2.5}$$

The probability of diffusion from $x_i$ to $x_f$ in a finite time $\tau$ can then be constructed by dividing the interval from 0 to $\tau$ into a large number of intervals of size $\epsilon$ and then integrating over the particle positions at these intermediate times:

$$\phi(x_f;x_i;\tau) = \int \frac{dx_1\ldots dx_{n-1}}{(m/2\pi\epsilon)^{(n-1)\frac{3}{2}}}\prod_i \exp\left[-\epsilon\left\{\frac{m}{2}\left(\frac{x_{i+1}-x_i}{\epsilon}\right)^2 + V\Big(\frac{x_{i+1}+x_i}{2}\Big)\right\}\right]. \tag{2.6}$$

We may then let $\epsilon \rightarrow 0$; the points $x_n$ become a continuous path $x(\bar{\tau})$ ;then the expression (2.6) can be represented as follows:

$$\phi(x_f;x_i;\tau) = \int_{\substack{\text{paths from} x_i \\ \text{to } x_f}} Dx \exp\left[-\int_0^\tau d\bar{\tau}\left[\frac{m}{2}\dot{x}^2(\bar{\tau}) + V(x(\bar{\tau}))\right]\right]. \tag{2.7}$$

Formally replacing $\tau$ by $it$, we find a special case of Feynman's formula for the quantum mechanical transition amplitude from $x_i$ to $x_f$ in time $t$:

$$\phi(x_f,x_i;t) = \int Dx\ e^{i\int dt L} \tag{2.8}$$

where $L$ is the classical Lagrangian.

We will find it more useful, however, to apply Eq. (2.7) directly. To see why this is so, consider the visualization of this equation presented in Fig. 4. Consider the evolution in $\tau$ induced by a potential of the shape shown on a wave function sharply peaked about $x = 0$. The waveform diffuses into an equilibrium shape, which then decreases exponentially in magnitude under the action of the term which includes $V(x)$. Because $\tau$ is related to physical time $t$ through (2.4), this exponential decay must be of just the form

$$\phi(x_f, \tau) = e^{-E_0 \tau} \cdot \psi(x_f), \tag{2.9}$$

where $E_0$ is the ground-state energy in the potential $V(x)$. The equilibrium shape $\psi(x_f)$ is just the ground-state wave-function. Thus, the paths contributing to (2.7) for large $\tau$ may be thought of as particle motions in the ground state of the Schrödinger equation.

Let us now make this description relativistic. Instead of solving the Schrödinger equation, we would like to represent the solution of the Klein-Gordon equation:

$$(\partial^2 + m^2)\psi(x) = \delta^{(4)}(x - x_i) \ . \tag{2.10}$$

I have written explicitly a point source of $\psi$; this is implicit in our treatment of Eqs. (2.1) and (2.2). If we replace, as before, $it = x_0$, this equation becomes

$$(-\partial_0^2 - \nabla^2 + m^2)\psi = \delta^{(4)}(x - x_i) \ . \tag{2.11}$$

We may solve this equation by the trick of introducing a fictitious additional time $T$. Then we must solve

$$\left(\frac{\partial}{\partial T} - \frac{1}{2}\left[(\partial_0^2 + \nabla^2) - m^2\right]\right)\hat{\psi} = \delta^{(4)}(x - x_i)\delta(T) \ . \tag{2.12}$$

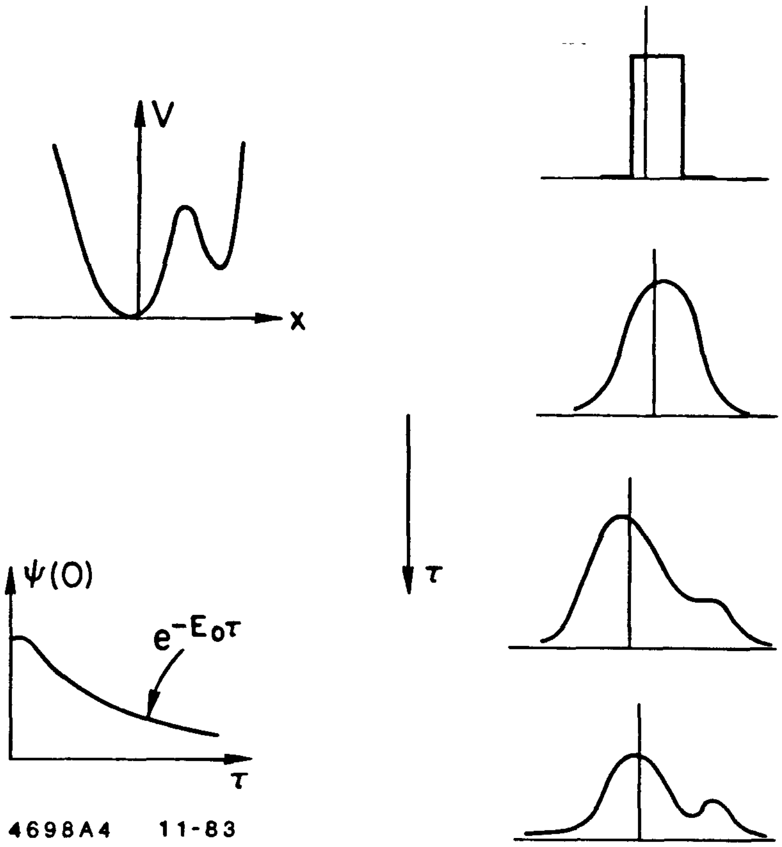


FIG. 4. Evolution on an initial waveform into the ground-state wavefunction, as the result of integrating Eq. (2.2).

But this is just a diffusion equation of the form (2.2), in 4 space dimensions, and so we can recognize immediately that the solution is

$$\hat{\psi}(x,T) = \int \mathcal{D}x \exp\left[-\int_0^T d\tau \, \frac{1}{2}(\dot{x}^2 + m^2)\right]. \tag{2.13}$$

But now note that, if we have found a $\hat{\psi}(x,T)$ which solves (2.12), we can readily construct a $\psi(x)$ which solves (2.10) by writing

$$\psi(x) = \int_0^\infty dT \quad \hat{\psi}(x,T). \tag{2.14}$$

as one can see simply by integrating Eq. (2.12) over $T$. This gives us a path representation of the propagator for the Klein-Gordon equation of the following form:

$$\psi(x_f; x_i) = \int_0^\infty dT \int \mathcal{D}x \exp\left[-\int_0^T d\tau \, \frac{1}{2}(\dot{x}^2 + m^2)\right]. \tag{2.15}$$

The integral $\int \mathcal{D}x$ runs over paths $x(\tau)$ which run through 4-dimensional space in a fairly arbitrary way from $x_i$ to $x_f$; a typical path is shown in Fig. 5. I might note parenthetically that if this integral over $x(\tau)$ is replaced, as would be proper, by the solution to the diffusion equation, one finds

$$\psi(x_f; x_i) = \int_0^\infty dT \frac{1}{(2\pi T)^2} \exp\left[-\frac{(x_f - x_i)^2}{2T} - \frac{m^2}{2}T\right] \tag{2.16}$$

which is actually a standard integral representation of the free Klein-Gordon propagator.

We can extract the nonrelativistic limit of the Klein-Gordon theory directly from (2.15) in the following way: Let us choose limit points which are separated by a nonrelativistic trajectory:

$$x_i = (0, \vec{0}), \qquad x_f = (t_f, \vec{0}) \tag{2.17}$$

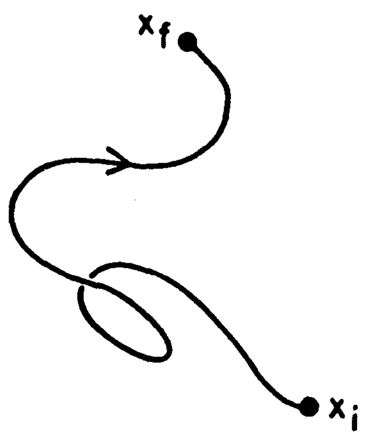


FIG. 5. A typical space-time trajectory included in the integral of Eq. (2.15).

and send $t_f \to \infty$. (To extract the propagation of a nonrelativistic antiparticle, for which the quantum numbers of $\psi$ flow from $x_i$ to $x_f$, just reverse these assignments.) In this limit, the integral over $x(\tau)$ is dominated by the path which maximizes the integrand of (2.15), or which minimizes

$$\int_0^T d\tau \, \frac{1}{2}\left[\dot{x}^2(\tau) + m^2\right] . \tag{2.18}$$

Setting $\delta/\delta x(\tau)$ of the above equal to 0 yields the condition

$$\ddot{x}_\mu = 0 \quad \to \quad x(\tau) = \left(t_f \cdot \frac{\tau}{T}, \vec{0}\right) . \tag{2.19}$$

If we insert (2.19) into (2.18), this exponent becomes

$$\frac{t_f^2}{2T} + m^2 \frac{T}{2} . \tag{2.20}$$

The minimum with respect to $T$ is found at

$$T = \frac{t_f}{m} , \quad \text{or} \quad \tau = \frac{x_0}{m} . \tag{2.21}$$

Now examine the collection of paths near this joint minimum with respect to $T$ and $x(\tau)$. We can recognize the following properties: First, in the vicinity of (2.19) and (2.21) the exponential of (2.18) becomes

$$\exp\left[-\left(\int_0^{t_f} dt \left[\frac{m}{2} \dot{x}^2\right]\right) - m\, t_f\right] \tag{2.22}$$

where now $\dot{x}$ represents the derivative of $x$ with respect to $t = x_0$. This is just (2.7) with an extra exponential decay representing the rest energy of the heavy particle. For an antiparticle, we would find the same result, but with the dominant paths formally running backwards in time. Secondly, paths longer than the minimal path length between $x_i$ and $x_f$ are suppressed by the weighting factor

$$e^{-m\Delta s} \; . \tag{2.23}$$

where $\Delta s$ is the excess path length. This means, in particular, that paths which bend backwards in time, in the manner shown in Fig. 6, are suppressed by the factor

$$\int_0^\infty d\Delta t \, e^{-2m\Delta t} = \frac{1}{2m} \; . \tag{2.24}$$

By making a constant-time slice through Fig. 6, you may recognize that the back-bend represents a component of the state which contains an extra particle-antiparticle pair. The factor (2.24) is just the energy denominator associated with this higher-energy state.

To couple the Klein-Gordon equation to electromagnetism, we need only generalize (2.10) by the replacement

$$\partial_\mu \to D_\mu = (\partial_\mu - igA_\mu) \; ; \tag{2.25}$$

this prescription makes the Klein-Gordon equation properly gauge-invariant. Let us, for the moment, consider the motion of a Klein-Gordon particle in a fixed A field. We then need to solve

$$\left[ -(\partial_\mu - igA_\mu)^2 + m^2 \right] \psi = \delta^{(4)}(x - x_i) \; . \tag{2.26}$$

To solve this equation, we can again add a fictitious time:

$$\left( \frac{\partial}{\partial T} + \frac{1}{2} \left[ (\partial_\mu - igA_\mu)^2 - m^2 \right] \right) \hat{\psi} = \delta^{(4)}(x - x_i) \; . \tag{2.27}$$

The solution to (2.27) for propagation over a short time $\epsilon$ can be found by taking $A_\mu$ approximately constant. Then one can check that

$$\hat{\psi}(x_f; x_i; \epsilon) = \frac{1}{(2\pi\epsilon)^2} \exp \left[ -\frac{(x_f - x_i)^2}{2\epsilon} + ig(x_f - x_i) \cdot A - \frac{\epsilon}{2} \, m^2 \right] \tag{2.28}$$

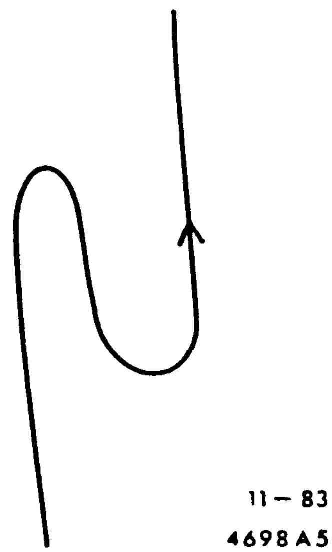


FIG. 6. A path which bends backwards in time.

by noting that

$$D_\mu(exponent) = -\frac{(x_f - x_i)_\mu}{\epsilon} \tag{2.29}$$

as before. Assembling the increments of time $\epsilon$ into a finite time $t$, taking the limit of continuous paths as above, and then integrating over $T$, we find for the solution to (2.26):

$$\psi(x_f; x_i) = \int_0^\infty dT \int \mathcal{D}x \exp\left[-\int_0^T d\tau \frac{1}{2}\left[\dot{x}^2 + m^2\right]\right] \exp\left[ig \int d\tau\, \dot{x} \cdot A(x)\right]. \tag{2.30}$$

We have shown, then, that a charged particle is described by a sum over space-time paths in which each path P is assigned the phase factor

$$W(x_f, x_i) = e^{ig \int_P dx \cdot A}. \tag{2.31}$$

This is a famous phase factor, whose relevance to the study of gauge theories has been emphasized by Schwinger,[11] Mandelstam,[12] and Wilson,[13] among others. I will discuss its beautiful properties in a moment. First, however, I should indicate how Eq. (2.31) generalizes when the Klein-Gordon equation is coupled to a non-Abelian gauge theory. In this case, the number $A_\mu$ must be replaced by a matrix in the color space

$$\mathbf{A}_\mu = A_\mu^a t^a \tag{2.32}$$

where $t^a$ is a generator of the color gauge group. Then the various factors (2.28) which depend on $\mathbf{A}$ must be arranged and multiplied out in order. The ordered product of these phase factors may be written as follows:

$$\prod_{\substack{\text{increments } \Delta \\ \text{of path}}} \left(e^{ig \int_\Delta d\tau\, \dot{x}(\tau) \cdot \mathbf{A}(x(\tau))}\right) = P\left(e^{ig \int_{x_i}^{x_f} dx \cdot \mathbf{A}}\right). \tag{2.33}$$

This equation defines the path-ordering operator $P$. For antiparticles, since the path runs backward, the path-ordering also runs backwards in time, one finds

$$P\left(e^{ig \int_{x_f}^{x_i} dx \cdot \mathbf{A}}\right). \tag{2.34}$$

In the same way that we saw (2.7) related to the classical action, we can recognize the exponent of the phase factor (2.31) as the classical action for the coupling of the electromagnetic field to a point particle:

$$\int \mathcal{L}\, d^4x = \int d^4x\, j_\mu A^\mu = \int d^4x \left[ \int d\tau\, g\dot{x}^\mu \delta^{(4)}(x - x(\tau)) \right] A_\mu(x)\,. \qquad (2.35)$$

Let us now discuss the properties of $W(x, y)$. We should first note that $W$ has a very simple transformation law under gauge transformations. Under a gauge transformation in electrodynamics,

$$\psi(x) \rightarrow e^{i\alpha(x)}\psi(x)\;; \quad W(x, y) \rightarrow e^{i\alpha(x)} W(x, y)\, e^{-i\alpha(y)} \qquad (2.36)$$

so it is easy to build gauge-invariant quantities from $W$. In a non-Abelian gauge theory, $W$ transforms as in (2.36) with the phase factor $e^{i\alpha}$ replaced by a unitary transformation in the color group. The second property of $W(x, y)$ is that it depends on the path chosen to connect $x$ to $y$. It is instructive to compare the values of $W$ evaluated over two paths $P$ and $P'$ which differ only by a small detour, as shown in Fig. 7. If the small square is taken to lie in the $\hat{1} - \hat{2}$ plane and to have sides of length $\epsilon$, we can evaluate:

$$\begin{aligned} W[P'] &= W(x, z)\Big\{\Big(1 - ig\epsilon\, \mathbf{A}_2(z)\Big)\Big(1 - ig\epsilon\, \mathbf{A}_1(z + \epsilon\hat{2})\Big) \\ &\qquad \cdot \Big(1 + ig\epsilon\, \mathbf{A}_2(z + \epsilon\hat{1})\Big)\Big(1 + ig\epsilon\, \mathbf{A}_1(z)\Big)\Big\} W(z, y) \qquad (2.37) \\ &= W(x, z)\Big\{1 + ig\epsilon^2\Big[\partial_1 \mathbf{A}_2 - \partial_2 \mathbf{A}_1 - ig[\mathbf{A}_1, \mathbf{A}_2]\Big]\Big\} W(z, y)\,. \end{aligned}$$

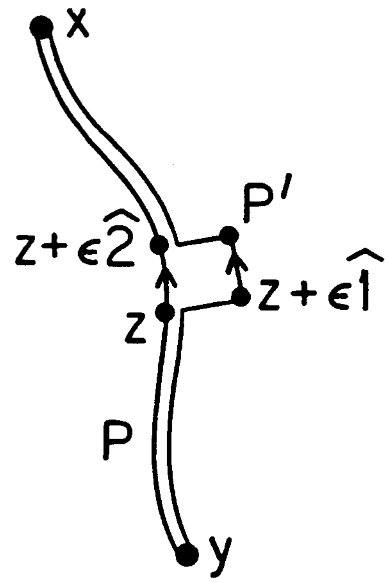


FIG. 7. Computation of the difference between two nearby Wilson lines.

Thus,

$$W[P'] - W[P] = W(x,z)\,(ig\Delta\sigma^{\mu\nu}\mathbf{F}_{\mu\nu})\,W(z,y) \tag{2.38}$$

where $\mathbf{F}_{\mu\nu}$ is the gauge field strength tensor, where $\Delta\sigma^{\mu\nu}$ is the increment of area by which $P$ and $P'$ differ. We can use this relation to derive a formula for the derivative of $W$ which will be useful to us in the next section. Let $W(x,-\infty)$ represent the phase factor for a line which runs forward in the time direction from $-\infty$ and ends at $x$, as shown in Fig. 8. We can differentiate this quantity with respect to $x$ by performing the motion indicated in Fig. 8 and breaking up the area between the two paths into small rectangles. This gives:

$$\epsilon^{\mu}D_{\mu}W(x,-\infty) = \int_{-\infty}^{x} d^2k\,W(x,z)\,(ig\,\epsilon^{\mu}\mathbf{F}_{0\mu}(z))\,W(z,-\infty)\,. \tag{2.39}$$

It is thus quite natural that the effect of a gauge field on a charged or colored particle should be accounted by the phase factors (2.31) or (2.33). In a quark-antiquark system, the effect of a color gauge field on the quark-antiquark state is represented by including in the path sum (2.15) for the quark and the antiquark the phase factor

$$\left[P\left(e^{i\int_{x_i}^{x_f} dx\cdot A}\right)\right]_{a_i a_f}\cdot\left[P\left(e^{ig\int_{y_f}^{y_i} dx\cdot \mathbf{A}}\right)\right]_{b_f b_i} \tag{2.40}$$

where $x_i$,$x_f$ and $y_i$,$y_f$ are the initial and final positions of the quark and antiquark, respectively, and $a_i$,$a_f$ and $b_i$, $b_f$ are their initial and final colors. We can insist that (2.40) contains only color singlet initial and final states by bringing $x_i$ and $y_i$, $x_f$ and $y_f$ to the same point, as indicated in Fig. 9, and then summing over colors $a_i = b_i$, $a_f = b_f$. This prescription associates to the evolving color singlet q-$\bar{\text{q}}$ state a factor

$$\text{trace}\left[P\exp\left(ig\oint_C dx\cdot\mathbf{A}\right)\right], \tag{2.41}$$

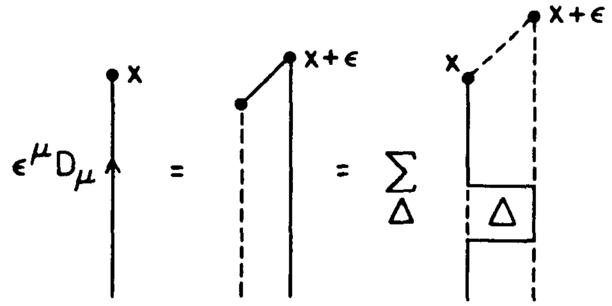


FIG. 8. Computation of the derivative of the Wilson line with respect to its endpoint.

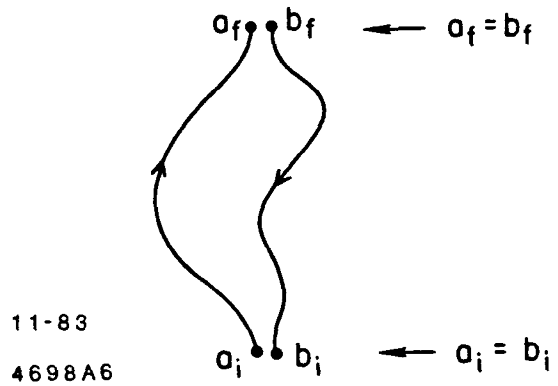


FIG. 9. Wilson lines associated with a propagating quark-antiquark pair.

where the integral is taken over the closed curve C indicated in Fig. 9, on which the quark moves forward and the antiquark backward in time.The factor (2.41) is often referred to as the **Wilson loop**; I will refer to an individual amplitude (2.33) as a **Wilson line**.

Now let us make the **A** field a dynamical entity also. According to Feynman's path integral prescription, we can turn the expression (2.30), valid for a fixed **A** field, into a description of mutually interacting Klein-Gordon and gauge fields by simply integrating the expression (2.30)over possible **A** field configurations, with a weight given by the classical Lagrangian for the gauge field (or, rather, because of (2.3), its continuation to imaginary time.) If we call the proper expression with a fixed **A** field $\mathcal{W}(\mathbf{A})$, then for a dynamical **A** field, we would write

$$\int \mathcal{D}\mathbf{A}\, \mathcal{W}(\mathbf{A}) \exp\left[-\int \mathcal{L}\, d^4x\right]. \tag{2.42}$$

For small values of **A**, the exponent of (2.42) is approximately a quadratic form in its components $A^a_{\mu\nu}$, so that (after fixing the gauge) we can rearrange it into the form

$$\int \mathcal{L}\, d^4x = \int d^4x\, \frac{1}{4}(\mathbf{F}_{\mu\nu})^2 \approx \frac{1}{2}\int d^4x\, A^a_\mu\, \delta_{ab}\Delta^{-1\mu\nu} A^b_\nu \tag{2.43}$$

where $\Delta^{\mu\nu}(x,y)$ is the usual gauge field propagator

$$\Delta^{\mu\nu}(x,y) = \int \frac{d^4k}{(2\pi)^4}\, e^{-ik(x-y)}\, \frac{g^{\mu\nu}}{k^2} = \frac{g^{\mu\nu}}{2\pi^2(x-y)^2}\,. \tag{2.44}$$

To understand what kind of forces the quark and antiquark experience as a result of their gauge interactions, it suffices to study the behavior of the Wilson loop (2.41) when averaged with the weight (2.42). To obtain an idea of how to treat these forces, let us evaluate the average of (2.41) in three model situations in which this average can be computed easily.

The simplest case in which to examine the behavior of the Wilson loop is that of pure electrodynamics. In that case, the expectation value of the Wilson loop is given exactly by

$$\langle e^{ig\oint_c dx\cdot\mathbf{A}}\rangle = Z^{-1}\int D\mathbf{A}\, e^{-\frac{1}{2}\int A\Delta^{-1}A} e^{ig\oint_c dx\cdot\mathbf{A}} \tag{2.45}$$

where $Z$ is the indicated Gaussian integral without the factor (2.41). The integral is readily evaluated by completing the square in the exponent; then the integral over $A$ is an identical overall factor in numerator and denominator and cancels. The result is

$$\exp\left[+\frac{1}{2}\Big(ig\oint dt_1\dot{x}^\mu(t_1)\Big)\Big(ig\oint dt_2\dot{x}^\nu(t_2)\Big)\quad\Delta_{\mu\nu}(x(t_1)-x(t_2))\right] \tag{2.46}$$

where I have let $t_1$ and $t_2$ be the time coordinates of points on the curve $C$ and have written:

$$dx^\mu = dt\frac{dx^\mu(t)}{dt} = dt\,\dot{x}^\mu\ . \tag{2.47}$$

Note that

$$dt\,\frac{dx^\mu}{dt} = dx^0\,\frac{dx^\mu}{dx_0} = dx^0(1,\vec{v}) \tag{2.48}$$

so that (2.46) has precisely the form of the electromagnetic interaction of charges and currents.

Now consider the behavior of the exponent of (2.46) for nearby pieces of path with zero relative velocity. There are three contributions, shown in Fig. 10. The contributions (a) and (b) are independent of the particle-antiparticle separation; these contribute only to the particle self-energies and do not affect the potential. The contribution (c) is well approximated by

$$-2\cdot\frac{g^2}{2}\int_{-\infty}^{\infty}dt_1\int_{\infty}^{-\infty}dt_2\,\frac{1}{2\pi^2[(t_1-t_2)^2+R^2]} = g^2\cdot(\text{time of interaction})\cdot\frac{1}{4\pi R}\ . \tag{2.49}$$

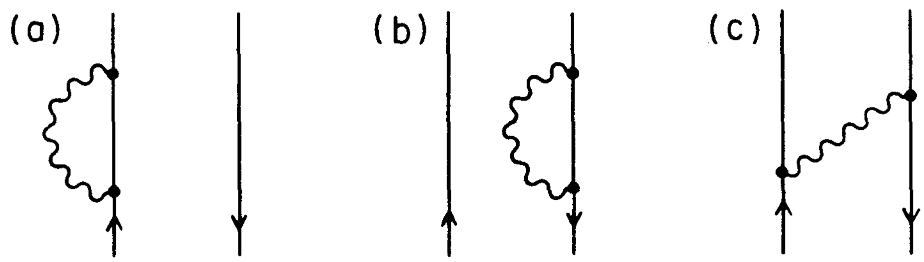


FIG. 10. Contributions from QED to the expectation value of the Wilson loop for a static q-$\bar{q}$ system.

But note also that the integral over relative times in (2.49) is highly convergent and so is insensitive to relative times much larger than the distance $R$ between them. Thus, this formula is also approximately correct for slowly moving particles, as long as the separation between the particles varies slowly compared to $R$. If we call this time-varying separation $R(t)$, we can write

$$\langle e^{ig\oint dx\cdot A}\rangle \simeq \exp\left[-\int dt\left(\frac{-g^2}{4\pi R(t)}\right)\right]; \tag{2.50}$$

this result is illustrated by Fig. 11.

Now consider what happens if we give the photon a mass $\mu$. This could be done, for example, through the Higgs mechanism, after the model of weak-interaction gauge theories. The expression (2.43) would be changed only by the replacement

$$\Delta^{\mu\nu}(x-y) \to \int \frac{d^4k}{(2\pi)^4}\, e^{-ik\cdot(x-y)}\, \frac{g^{\mu\nu}}{(k^2+\mu^2)}\,. \tag{2.51}$$

Then all of the manipulations which we carried out for the case of pure electrodynamics go through here too, and we can derive for the case of static particles an expression of precisely the form of (2.49), with the replacement

$$V(R) = -\int_{-\infty}^{\infty} dx^0\, \Delta(x^0, R) = \frac{-g^2 e^{-\mu R}}{4\pi R}\,. \tag{2.52}$$

In this case, the integral over relative times is even more convergent than before. As long as the time required to change $R(t)$ significantly is much greater than *either one* of the two distance scales $R$, $\mu^{-1}$, the expression

$$\langle e^{ig\oint dx\cdot A}\rangle \simeq \exp\left[-\int dt\, V(R(t))\right] \tag{2.53}$$

gives a good approximation to the expectation value of the Wilson loop.

For my final example, let me consider again the situation of a photon field given a mass by the Higgs mechanism, but let me alter the Wilson loop slightly

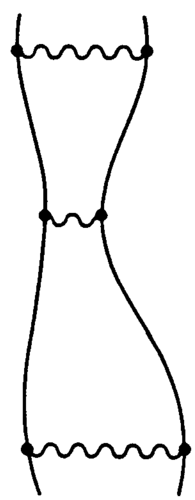


FIG. 11. Calculation of the expectation value of the Wilson loop for a slowly moving quark and antiquark, according to the approximation of Eq. (2.50).

so that it describes the interaction not of electric charges but, rather, of magnetic monopoles. This can be done straightforwardly by rewriting the integral in (2.41) by a 2-dimensional integral over a surface $S$ which spans $C$:

$$\exp\left[ig \oint_c dx \cdot A\right] = \exp\left[ig \int_S d\sigma^{\mu\nu} F_{\mu\nu}\right] \tag{2.54}$$

and then replacing electric by magnetic fields in (2.54); this is done relativistically by the replacement

$$F_{\mu\nu} \rightarrow \tilde{F}_{\mu\nu} \quad \text{where} \quad \tilde{F}_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\lambda\sigma}F^{\lambda\sigma} . \tag{2.55}$$

(Actually, one must make this replacement in the gauge field Lagrangian (2.43) [14–16] by replacing

$$\begin{aligned} \int d^4x \, \frac{1}{4}(F_{\mu\nu})^2 = d^2x \, \frac{1}{4}(\tilde{F}_{\mu\nu})^2 \rightarrow \\ \int d^4x \, \frac{1}{4}\Big(\tilde{F}_{\mu\nu}(x) - g_m \int d^2\sigma_{\mu\nu}(y)\delta^{(4)}(x-y)\Big)^2 ; \end{aligned} \tag{2.56}$$

the second term added allows $\tilde{F}_{\mu\nu}$ to have sources and sinks. Multiplying out the square gives (2.54) with the replacement (2.55) (up to a factor of $i$), plus an extra singular term).

Now we can integrate over $A$ by completing the square as before; for a loop which lies in a plane, the result is:

$$\langle e^{ig\oint_c dx \cdot A}\rangle = \exp\left[+\frac{g_m^2}{2}\int d^2\sigma_x \int d^2\sigma_y \Big(\frac{\partial}{\partial \vec{x}_\perp}\cdot\frac{\partial}{\partial \vec{y}_\perp}\Delta(x-y) - \delta^{(4)}(x-y)\Big)\right] ; \tag{2.57}$$

the delta function comes from the singular term identified above. This expression can be rearranged using the identity

$$\begin{aligned} \frac{\partial}{\partial \vec{x}_\perp}\cdot\frac{\partial}{\partial \vec{y}_\perp}\Delta(x-y) = -\frac{\partial^2}{\partial x_\perp^2}\Delta(x-y) = \Big(-\nabla^2 + \frac{\partial^2}{\partial x_\parallel^2}\Big)\Delta(x-y) \\ = \Big(-\frac{\partial}{\partial x_\parallel}\cdot\frac{\partial}{\partial y_\parallel} + (-\nabla^2+\mu^2) - \mu^2\Big)\Delta(x-y) \end{aligned} \tag{2.58}$$

The term including $\partial/\partial\vec{x}_\parallel \cdot \partial/\partial\vec{y}_\parallel$ can be integrated by parts to give just the result of the Higgs case above. The second term forms the equation for the massive $A$ field and yields a delta function when acting on $\Delta(x-y)$; this cancels the singular term. The third term yields a term in the exponent of (2.57)

$$-\frac{g_m^2}{2}\int d^2\sigma_x \int d^2\sigma_y\, \mu^2 \Delta(x-y) = -\frac{g_m^2}{2}\int dt \int_0^R dx\, dy\, \frac{\mu^2 e^{-\mu|x-y|}}{4\pi|x-y|}\,. \quad (2.59)$$

For $R \gg \mu^{-1}$, this becomes

$$\langle e^{ig\oint_c dx\cdot A}\rangle \sim \exp\left[-\int dR \quad K\cdot R(t)\right] \quad (2.60)$$

where

$$K = \frac{g^2 m}{2}\left(\frac{1}{2\pi}\int_0^\infty dy\,\frac{\mu^2}{y}\,e^{-\mu y}\right). \quad (2.61)$$

If the Higgs field dynamics is treated more correctly, the lower limit of integration becomes $m_{\mathrm{Higgs}}^{-1}$. This behavior corresponds to a potential $V(R)$ with the asymptotic form

$$V(R) \sim KR; \quad (2.62)$$

the particle and antiparticle represented by the Wilson loop are permanently confined. As in the previous example, the static approximation to the potential is a good one as long as $R$ changes slowly in time on the scale either of $R$ or of $\mu^{-1}$.

I should note that it is not hard to understand in this case why the magnetic charges ought to be permanently confined. When the $A$ field acquires a mass through the Higgs mechanism, the homogeneous Maxwell equations are unchanged, but the inhomogeneous equations are changed by new $\mu$-dependent terms. Thus, we have

$$\vec{\nabla}\cdot\vec{E} = \mu^2 A^0 \quad \text{but} \quad \vec{\nabla}\cdot\vec{B} = 0. \quad (2.63)$$

The flux of $\vec{E}$ is no longer conserved. This allows the electric field to disappear into the vacuum, and it does so in a distance $\mu^{-1}$ from the charge (hence the form of (2.52)). Magnetic flux is still conserved. However, correlations of the magnetic fields must still fall off in a distance $\mu^{-1}$. This is possible only if the magnetic field needs information only from within a distance $\mu^{-1}$ to tell it how to stay conserved. It must then form a configuration of the form of Fig. 12, in which the magnetic field forms a tube of width $\mu^{-1}$ which carries a flux corresponding to 1 unit of charge.[16–17] This mechanism has been shown to be the explanation for the permanent confinement of charge seen in the strong-coupling limit of lattice gauge theories, at least for the case of Abelian gauge groups. [18–21] Unfortunately, physical quark-antiquark systems are described, not by any of these idealized limiting cases, but rather by the $SU(3)$ color gauge theory QCD. We do not know how to compute the Wilson loop expectation value in QCD except by rather cumbersome numerical methods which are only recently beginning to show results. [22–24] However, we do have some analytical control over the limiting behavior of the potential $V(R)$. For small $R$, perturbative QCD is applicable, and the potential has been computed to the two-loop order. [25–27] The leading behavior for small $R$ is just a Coulomb potential

$$\frac{-4\alpha_s(R)}{3\pi R,} \tag{2.64}$$

with $\alpha_s(R)$ the running QCD coupling constant. At large distances, our best information comes from a recent paper of Tomboulis,[28] who has proven a bound on the Wilson loop expectation value in lattice gauge theories which implies that the behavior (2.62) is true for some value of $K$ for any finite lattice spacing. Combining these two limiting cases, we might make a sketch of the potential in QCD; it has the general form shown in Fig. 13. The principal question which remains concerning this potential is that of what sets the scale $\mu^{-1}$ which characterizes the scale over which the static gauge fields can be deformed. This quantity bears no obvious relation to the value of the constant $K$ which appears

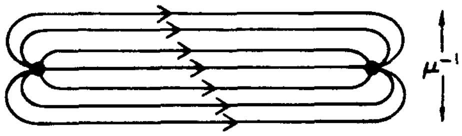


FIG. 12. Configuration of magnetic flux around a pair of magnetic sources in a superconductor.

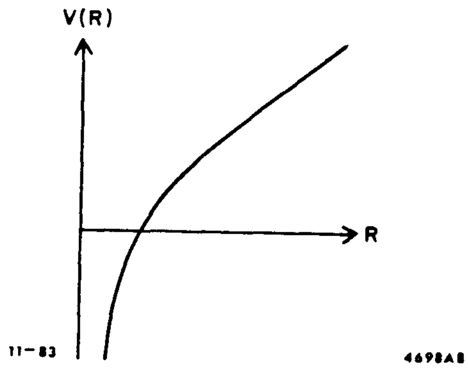


FIG. 13. Sketch of the form of V(R) in QCD.

in the potential, as Eq. (2.61) makes clear. Presumably, this scale is a mass scale of the gluon sector. But, unfortunately, the gluon dynamics is the aspect of QCD which we understand least well how to connect with phenomenology. I believe it not unreasonable to assume that the effective value of $\mu$ is of order 1 GeV; however, we should really hope that the study of heavy-quark systems can give us more precise information on this size of this quantity.

## 3. Spin-Dependent Forces

In the previous section, we saw how to use the Feynman path integral formalism to visualize the dynamics of nonrelativistic particles in space-time. We derived theFormula(2.22) which confirmed our intuition that such particles could be viewed as travelling on space-time trajectories which were close to being straight lines in the time direction. We saw, further, that such particles could be coupled to gauge fields by associating a phase factor (2.33) with the trajectory. These two ingredients give us a complete picture of particle-antiparticle dynamics at the leading order of a nonrelativistic expansion. In this lecture, I would like to study the simplest relativistic corrections to this picture, the spin-dependent forces in fermion-antifermion bound states. As in our study of the static potential, I will try as far as possible not to make any approximation other than the nonrelativistic limit, in order to clarify what structure for the spin-dependent forces follows directly from the gauge-theory structure of the underlying interactions. In taking this point of view, I follow the work of Eichten and Feinberg.[1] The bulk of my analysis in this section will be a derivation of a general formula for the spin-dependent forces first presented in their paper.

Before we can do any detailed analysis, we must first recall that we performed the analysis leading to equation (2.22) only for the case of the Klein-Gordon equation. We must generalize this analysis to apply to the Dirac equation. This generalization was first presented by, Feynman[29] I learned of it from the presentation of Halpern and Siegel.[30]

We need, then, to represent the solution to

$$\left[\gamma^\mu(\partial_\mu - ig\mathbf{A}_\mu) - m\right]\psi = \delta^{(4)}(x-y) \; . \tag{3.1}$$

Again, we will work with imaginary time ($it = x_0$); the appropriate Dirac matrices satisfy

$$\{\gamma^\mu, \gamma^\nu\} = 2\delta^{\mu\nu} \; . \tag{3.2}$$

It is convenient to choose the following representation of the gamma matrices:

$$\gamma^\mu = \left(\begin{array}{c|c} 0 & \sigma^\mu \\ \hline \bar{\sigma}^\mu & 0 \end{array}\right) \tag{3.3}$$

where

$$\sigma^\mu = (1, i\vec{\sigma}), \qquad \bar{\sigma}^\mu = (1, -i\vec{\sigma}) \; . \tag{3.4}$$

In this basis, $\gamma^5$ is diagonal

$$\gamma^5 = \left(\begin{array}{c|c} 1 & 0 \\ \hline 0 & -1 \end{array}\right) \tag{3.5}$$

and the spin matrices $\Sigma^{\mu\nu}$ take the form

$$\Sigma^{\mu\nu} = \frac{-i}{2}\left[\gamma^\mu, \gamma^\nu\right] = \left(\begin{array}{c|c} \eta_{a\mu\nu}\sigma^a & 0 \\ \hline 0 & \bar{\eta}_{a\mu\nu}\sigma^a \end{array}\right) \tag{3.6}$$

where $\eta_{a\mu\nu}$ is a numerical tensor defined[31] by

$$\eta_{ai0} = 1, \quad \eta_{aij} = \epsilon_{aij}, \quad \eta_{a\mu\nu} = -\eta_{a\nu\mu} \; . \tag{3.7}$$

Keeping this notation in mind, we can write the solution to (3.1) in the form

$$\psi(x;y) = \left[\gamma^\mu D_\mu + m\right] G(x;y) \tag{3.8}$$

where $G(x;y)$ is the solution to

$$[\gamma^{\mu} D_{\mu} - m]\,[\gamma^{\nu} D_{\nu} + m]\, G(x;y) = \delta^{(4)}(x-y)\,. \tag{3.9}$$

Of course, it is not clear that this helps enormously: We now see that $\psi(x;y)$ can be written in the form

$$\psi(x;y) = \left(\begin{array}{c|c} m & \sigma^{\mu} D_{\mu} \\ \hline \bar{\sigma}^{\mu} D_{\mu} & m \end{array}\right) G(x,y)\,. \tag{3.10}$$

But what is $G$?

Actually, it is not hard to construct $G$ explicitly using the set of tricks introduced in the previous section. Using the identity

$$\gamma^{\mu}\gamma^{\nu} = \frac{1}{2}\{\gamma^{\mu},\gamma^{\nu}\} + i\left(\frac{-i}{2}[\gamma^{\mu},\gamma^{\nu}]\right) \tag{3.11}$$

and (3.4), we can cast (3.9) into the form

$$\begin{aligned} &\left(D^2 - m^2 + i\Sigma^{\mu\nu}\Big[\partial_{\mu} - ig\mathbf{A}_{\mu}, \partial_{\nu} - ig\mathbf{A}_{\nu}\Big]\right) G(x;y) \\ &\qquad = -\left(-D^2 + m^2 - \frac{g}{2}\Sigma^{\mu\nu}\mathbf{F}_{\mu\nu}\right) G(x;y) = \delta(x-y)\,. \end{aligned} \tag{3.12}$$

This equation is of the same form as those we considered earlier, and so we may immediately write down a functional integral representation for $G$

$$G = \int_0^{\infty} dT \int \mathcal{D}x\, e^{-\int_0^T d\tau\, \frac{1}{2}\dot{x}^2 + m^2} P\!\left(e^{i\int \dot{x}\cdot\mathbf{A} + \int d\tau\, \frac{g}{4}\Sigma^{\mu\nu}\mathbf{F}_{\mu\nu}}\right). \tag{3.13}$$

Note that, since the various components of $\Sigma^{\mu\nu}$ do not commute with one another, we must path-order these matrices just as we path-order the $\mathbf{A}_{\mu}$. Since the $\Sigma^{\mu\nu}$ are block-diagonal in the basis we have chosen, $G$ also falls into the block form

$$G(x;y) = \left(\begin{array}{c|c} G_R(x;y) & 0 \\ \hline 0 & G_L(x;y) \end{array}\right). \tag{3.14}$$

Within each block, we may take the nonrelativistic limit as we did above for the Klein-Gordon propagator; for $G_R$ the result is

$$G_R = \int \mathcal{D}\vec{x}\, e^{-\int dt\, \frac{1}{2}\dot{\vec{x}}^2} \cdot P\left(e^{i\int dx\cdot \mathbf{A} + \int dt\, \frac{g}{4m}\eta_{a\mu\nu}\sigma^a \mathbf{F}_{\mu\nu}}\right) . \tag{3.15}$$

The representation (3.15) differs from the corresponding expression (2.22) for the Klein-Gordon equation by the addition of the path-ordered factor containing the coupling of the spin to $\mathbf{F}_{\mu\nu}$. Written more explicitly, this new term has the form $(-\int dt \Delta V)$, where

$$\Delta V = -\frac{g}{2m}\vec{\sigma}\cdot(\vec{\mathbf{B}} - \vec{\mathbf{E}}) . \tag{3.16}$$

The first part of this expression is obviously the magnetic-moment interaction of the fermion; note that it has just the right magnitude. The second part looks rather more peculiar. To explain its presence, I must apologize for the one awkward feature of the formalism I use in these lectures: In order to consistently work with Euclidean time ($x_0 = it$), we must also work with Euclidean $\vec{\mathbf{E}}$ fields. In particular, the $\vec{\mathbf{E}}$ fields which appear in the expression (2.42) should, like $x_0$, be properly considered imaginary quantities analytically continued to real values. The same goes for the $\mathbf{A}^0$ and $\vec{\mathbf{E}}$ fields in (3.13). If we introduce an external, physical $\vec{\mathbf{E}}$ field, then, we must orient it properly relative to the fluctuating $\vec{\mathbf{E}}$ by supplying a factor of $i$:

$$\vec{\mathbf{E}}_{\text{ext}} = -i\vec{\mathbf{E}}; \quad \vec{\mathbf{B}}_{\text{ext}} = \vec{\mathbf{B}} . \tag{3.17}$$

In the same way, if we try to connect Euclidean time derivatives to physical quark position operators, we again need to supply this factor:

$$\vec{\mathbf{p}} = im\frac{d\vec{x}}{dx_0} . \tag{3.18}$$

Using (3.17), we can see that an external $\vec{\mathbf{B}}$ field causes an exponential growth or decay in the magnitude of the wave function, indicating, through comparison with the form of (2.9), that the energy of the quantum state is shifted. An external $\vec{\mathbf{E}}$ field, however, introduces only an inconsequential phase. Let me warn the reader to watch out (or not, as he prefers) for sly factors of $i$ throughout the arguments of this section; fortunately, these factors are the only subtle feature of this formalism.

In any event, we would like to use the Formula (3.13) (or, better, its nonrelativistic reduction) to compute bound state energies in fermion-antifermion systems. Any such bound state energy shows up as a term in the functional integral which decays in amplitude as

$$\exp\left(-E_b x_0\right); \tag{3.19}$$

the lowest bound state (or the lowest such state with specified quantum numbers) dominates the functional integral expression as $x_0$ is taken large, as long as the initial conditions for the particle paths have overlap with this state. In particular, we might note that nonrelativistic fermions are approximately equal admixtures of $\gamma^5 = +1$ and $\gamma^5 = -1$ states, so that we can, if we wish, limit our attention to $\gamma^5 = +1$ components of the Green's function $\psi(x; y)$ and still retain the full information available on bound state energies. Let us, then, disregard most of (3.10) and concentrate on the upper left-hand corner of this matrix. This element has the nonrelativistic expansion:

$$\int D\vec{x}\, e^{-\int dt\, \frac{1}{2}\dot{\vec{x}}^2} P\left(e^{ig\int d\mathbf{x}\cdot\mathbf{A}+\int dt\, \frac{g}{2m}\vec{\sigma}\cdot(\hat{\mathbf{B}}-\hat{\mathbf{E}})}\right). \tag{3.20}$$

We can learn whatever we might wish to know about the nonrelativistic fermion-antifermion spectrum by isolating the exponentially decaying amplitudes of the form of (3.19) contained in (3.20).

The explicitly spin-dependent term in the exponent of (3.20) contains a factor $m^{-1}$ and is therefore of higher order in a nonrelativistic expansion. As we will see in a moment, spin-dependent effects contribute to the energy of the q-$\bar{q}$ state for the first time in order $m^{-2}$ relative to the nonrelativistic binding energy. Since any such terms contain at least one power of the term involving $\vec{\sigma}$, the corrections to (3.23), which are by themselves of order $m^{-2}$, do not contribute to this order. Let us now try to account the set of leading spin-dependent terms systematically. In order to do this, it will be convenient to introduce one more piece of notation. I will use brackets to denote the expectation value of gauge field operators on the quark trajectory and path-ordered along with the Wilson line. For example,

$$\langle \vec{\mathbf{B}}(t, x_+) \rangle = \frac{1}{Z} \int \; \mathcal{D}x \, \mathcal{D}\mathbf{A} \, e^{-\int \mathcal{L}} e^{-\int \frac{1}{2}\dot{x}^2} P\left[ e^{-\int dx \cdot \mathbf{A}} \vec{\mathbf{B}}(t, x_+) \right] . \qquad (3.21)$$

In Eq. (3.21), $Z$ denotes the value of the indicated integral without the operator $\vec{\mathbf{B}}$ and $x_+$ indicates the quark position; I will denote the antiquark position by $x_-$. This expectation value is indicated graphically in Fig. 14.

To warm up for this accounting, let us examine the various spin- dependent terms of order $m^{-1}$. To this order, we should bring down from the exponent one factor containing $\vec{\sigma}$; then we can ignore all other corrections to the static limit. In this approximation, the correction to the wavefunction is given by

$$\delta\psi = \langle \oint \; dt \Big( \frac{g}{2m} \vec{\sigma} \cdot \vec{\mathbf{B}} - \frac{g}{2m} \vec{\sigma} \cdot \vec{\mathbf{E}} \Big) \rangle \; . \qquad (3.22)$$

The piece involving $\vec{\sigma} \cdot \vec{\mathbf{B}}$ gives no contribution, since

$$< \mathbf{B}^i(x_+, t) > = 0 \qquad (3.23)$$

by symmetry for a static $q - \bar{q}$ pair. The piece involving $\vec{\sigma} \cdot \vec{\mathbf{E}}$ can be evaluated by noting that the only component of $< \mathbf{E}^i >$ which does not vanish by symmetry

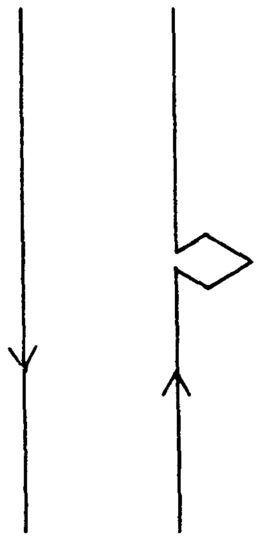

FIG. 14. Graphical representation of the expectation value shown in Eq. (1.21).

is the component in the direction of the quark-antiquark separation $\vec{R}$. This component can be evaluated by comparison with Eq. (2.39):

$$
\begin{aligned}
\langle\int dt\, g\mathbf{E}^i(t,x_+)\rangle &= \frac{-i}{\epsilon}\langle\int dt\,\epsilon(ig\mathbf{F}_{oi})\rangle \\
&= -i\frac{1}{Z}\frac{\partial}{\partial R^i}Z = +i\int dt\,\frac{\partial}{\partial R^i}V(R)\,.
\end{aligned}
\tag{3.24}
$$

Exponentiated, this factor gives just a time-dependent phase and not an exponential decay; thus, it does not contribute to the bound state energy.

Now let us move on to the terms of order $m^{-2}$. These terms are of three types. The first arises from two insertions of $\vec{\sigma}$ terms. We might, first, bring down two factors of $\vec{\sigma}\cdot\vec{\mathbf{B}}$ on the same quark line. This gives:

$$
\delta\psi = \langle\frac{1}{2}\int dt\,\frac{g}{2m}\vec{\sigma}\cdot\vec{\mathbf{B}}(t,x_+)\int dt'\,\frac{g}{2m}\vec{\sigma}\cdot\vec{\mathbf{B}}(t',x_+)\rangle\,. \tag{3.25}
$$

But this expression is symmetric in the two $\vec{\sigma}$'s, so we may use the identity

$$
\sigma^i\sigma^j+\sigma^j\sigma^i = 2\delta^{ij} \tag{3.26}
$$

to see that (3.25) is not in fact spin-dependent. We can also bring down factors of $\vec{\sigma}\cdot\vec{\mathbf{B}}$ on both the quark and antiquark lines. This produces a change in the wavefunction of the form:

$$
\delta\psi = \langle\int_{-\infty}^{\infty} dt\quad \frac{g}{2m}\vec{\sigma}_+\cdot\vec{\mathbf{B}}(t,x_+)\int_{-\infty}^{\infty} dt'\quad \frac{-g}{2m}\vec{\sigma}_-\cdot\vec{\mathbf{B}}(t',x_-)\rangle\,. \tag{3.27}
$$

I use $\vec{\sigma}_+$ and $\vec{\sigma}_-$ to represent the spin matrices acting on the quark and antiquark lines, respectively. The expectation value indicated in the second line of (3.27)is represented graphically in Fig. 15. In the models which I presented at the end of the previous section, the correlation function of two operators $\vec{\mathbf{B}}$ is short-ranged, so that the integral over $t'$ falls off rapidly if the separation of the two $\vec{\mathbf{B}}$

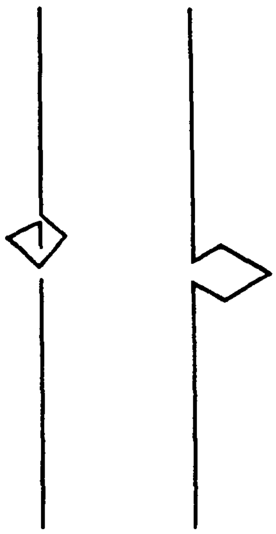


FIG. 15. Graphical representation of the expectation value appearing in Eq. (1.27).

operators is greater than $R$ or $\mu^{-1}$. It is then appropriate to ignore the relative motion of the quark and antiquark during the time interval $(t'-t)$ and represent this integral as some new type of static potential. Since the expectation value contains two tensor structures, we can represent it in the form:

$$\langle g^2 \int dt' \mathbf{B}^i(t, x_+) \mathbf{B}^j(t', x_-) \rangle = \left( \hat{R}^i \hat{R}^j - \frac{1}{3} \delta^{ij} \right) V_3(R) + \frac{1}{3} \delta^{ij} V_4(R) . \quad (3.28)$$

Then Eq. (3.27) becomes

$$\delta\psi = -\int dt \frac{1}{4m^2} \left( \left[ \hat{R} \cdot \vec{\sigma}_+ \hat{R} \cdot \vec{\sigma}_- - \frac{1}{3} \vec{\sigma}_+ \cdot \vec{\sigma}_- \right] V_3 + \frac{1}{3} \vec{\sigma}_+ \cdot \vec{\sigma}_- V_4 \right) . \quad (3.29)$$

It is appropriate to view this as the first term in the expansion of a factor $exp(-\int dt \Delta V)$; thus, Eq. (3.29) gives our first spin-dependent correction to the bound-state energy. The terms involving $\vec{\sigma} \cdot \vec{\mathbf{E}}$ do not contribute to this order: The only term which is not a pure phase is the second-order term arising from the exponentiation of (3.24).

The second type of contribution comes from bringing down one $\vec{\sigma}$ term and then expanding to first order in the quark or antiquark velocity. For definitiveness, let us consider first the case in which we bring down a $\vec{\sigma} \cdot \vec{\mathbf{B}}$ term on the quark line and consider the quark velocity to be nonzero. We can represent the term of first order in the quark velocity in the manner indicated in Fig. 16, by considering it as displaced from a line aligned precisely in the time direction and expanding for small displacements as we did in Eq. (2.39). This procedure gives, for the term in $\delta\psi$ of first order in the quark velocity,

$$\begin{aligned} \delta\psi &= \int \langle dt \frac{g}{2m} \vec{\sigma} \cdot \vec{\mathbf{B}}(t, x_+) \int dt' ig \frac{d\vec{x}}{dt'} \cdot (t'-t) \cdot \vec{\mathbf{E}}(t', x_+) \rangle \\ &= \int dt \frac{1}{2m^2} \sigma^i \mathbf{p}^j \langle \int dt' g^2 \mathbf{B}^i(t, x_+) t' \mathbf{E}^j(t+t', x_+) \rangle \end{aligned} \quad (3.30)$$

In the last line, we have used Eq. (3.18) to replace the line velocity by the quark momentum operator $\vec{\mathbf{p}}$; this introduces a factor of $i$. The indicated expectation

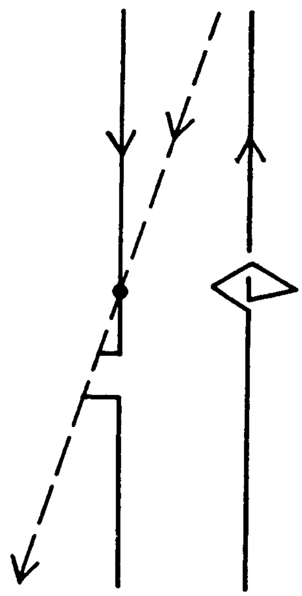

FIG. 16. Representation of the expansion of a finite- velocity Wilson line about its zero-velocity limit.

value is again highly restricted by symmetry; we may represent it in the form:

$$\langle \int dt' g^2 \mathbf{B}^i(t, x_+) \mathbf{E}^j(t', x_+) \rangle = \epsilon_{ijk} \hat{R}^k \frac{\partial}{\partial R} V_1(R) \,. \tag{3.31}$$

Then (3.30) becomes

$$\sigma\psi = -\int dt \frac{1}{2m^2} \vec{\sigma}_+ \cdot \vec{\mathbf{L}}_+ \frac{1}{R} \frac{\partial}{\partial R} V_1(R) \tag{3.32}$$

where $\vec{\mathbf{L}}_+$ denotes the angular momentum of the quark. The correction arising from a field insertion on a finite-velocity antiquark line is of almost the same form:

$$\delta\psi = +\int dt \frac{1}{2m^2} \vec{\sigma}_- \cdot \vec{\mathbf{L}}_- \frac{1}{R} \frac{\partial}{\partial R} V_1(R) \,. \tag{3.33}$$

The change in sign comes from $g \to -g$, or,equivalently, from the change in direction of the antiquark line. The corresponding terms in which $\vec{\sigma} \cdot \vec{\mathbf{E}}$ is inserted can be seen to yield only phases.

In a similar way, we can reduce the term arising from an insertion on one line and finite velocity on the other to the form:

$$\delta\psi = -\int dt \frac{1}{2m^2} (\vec{\sigma}_- \cdot \vec{\mathbf{L}}_+ - \vec{\sigma}_+ \cdot \vec{\mathbf{L}}_-) \frac{1}{R} \frac{\partial}{\partial R} V_2 \,, \tag{3.34}$$

where we have represented

$$\langle g^2 \int dt' t' \mathbf{B}^i(t, x_-) \mathbf{E}^j(t + t', x_+) \rangle \frac{\partial}{\partial R^k} V_2 \,. \tag{3.35}$$

Actually, though, $V_2$ is not independent of the potentials we have defined previously. Consider the effect of taking the divergence of Eq. (3.35). We can take $\partial / \partial R^k$ of this object by considering the derivative to act on the line with the $\vec{\mathbf{E}}$ insertion and using (2.39). There are two contributions, represented graphically in Fig. 17. The first contribution, in which the derivative acts on the Wilson

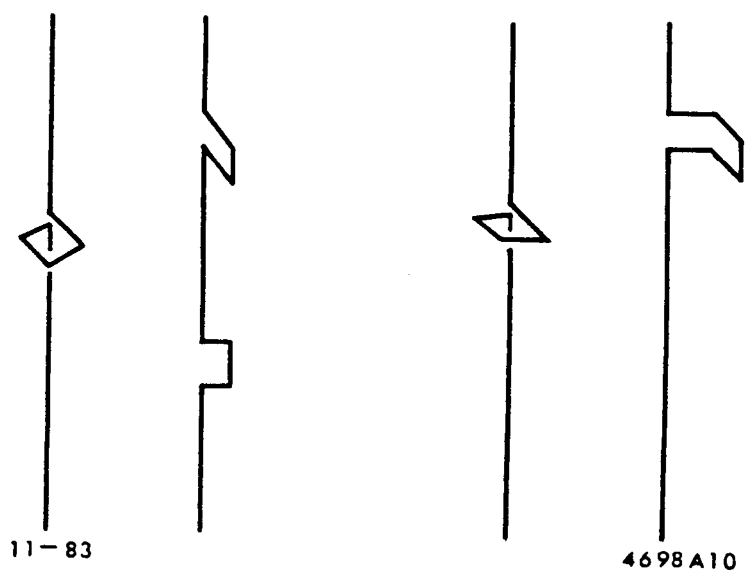


FIG. 17. Representation of the two contributions to $\nabla^2 V_2$.

line, gives zero by the antisymmetry of the $\epsilon_{ijk}$ symbol. In the second term, we can use the non-Abelian Maxwell equation

$$\vec{D} \times \vec{\mathbf{E}} = D_0 \vec{\mathbf{B}} \tag{3.36}$$

and the relation for the path-ordering operator

$$\langle D_0 \vec{\mathbf{B}}(t) \rangle = \frac{\partial}{\partial t} \langle \vec{\mathbf{B}}(t) \rangle \tag{3.37}$$

to write this term as

$$-\frac{g^2}{2} \int dt' t' \frac{\partial}{\partial t'} \langle \mathbf{B}^i(t, x_-) \mathbf{B}^j(t, t', x_+) \rangle = \frac{g^2}{2} \int dt' \langle \mathbf{B}^i(t, x_-) \mathbf{B}^j(t', x_+) \rangle \ . \tag{3.38}$$

Thus, we have the relation

$$2\nabla^2 V_2(R) = V_4(R) \ . \tag{3.39}$$

The third type of term comes from introducing the correct relation between the spin matrices $\vec{\sigma}$ appearing in (3.20) and the quark and antiquark spins. The $\vec{\sigma}$ matrices are the matrix elements of $\Sigma^{\mu\nu}$; these are connected to the spin via the relation

$$\xi^+ S^k \xi = \frac{1}{2} \epsilon_{ijk} \bar{u}(p) \Big( \frac{1}{2} \Sigma^{ij} \Big) u(p) = \xi^+ \Big( \frac{\sigma^k}{2} - \Big( \frac{d\vec{x}}{dt} \times \frac{\vec{\sigma}}{2} \Big)^k + \cdots \Big) \xi \tag{3.40}$$

where $\xi$ is a nonrelativistic spinor and $u(p)$ is the corresponding Dirac spinor. If we correct $\vec{\sigma}$ to $\vec{S}$ in our evaluation of the order $m^{-1}$ terms, and use (3.18), we obtain a correction of order $m^{-2}$ which is *real* and contributes to the energy shift:

$$\mathrm{Re}\,(\delta\psi) = \int dt \, \frac{g}{m} \frac{\vec{\sigma}_+}{2} \cdot \frac{\partial}{\partial \vec{R}} V(R) \ . \tag{3.41}$$

This is precisely the classical spin-orbit contribution, familiar from first-year quantum mechanics. It is equally familiar that, since the spin has been referred to an accelerating frame of reference, this contribution to the energy must be decreased by half because of Thomas precession. A similar contribution arises from correcting the order $m^{-1}$ term involving $\vec{\sigma}_-$.

Summing the various contributions of these three types, we find a general representation for the leading spin-dependent contributions to the q-q̄ bound state energy. This expression, first, derived by Eichten and Feinberg,[1] is the following

$$\begin{aligned} V_{\text{spin-dep}} = & \left(\frac{\vec{S}_+ \cdot \vec{\mathbf{L}}_+ + \vec{S}_- \cdot \vec{\mathbf{L}}_-}{2m^2}\right) \frac{1}{R} \frac{dV}{dR} \\ & + \left(\frac{\vec{S}_+ \cdot \vec{\mathbf{L}}_+ + \vec{S}_- \cdot \vec{\mathbf{L}}_-}{m^2}\right) \frac{1}{R} \frac{dV_1}{dR} \\ & - \left(\frac{\vec{S}_+ \cdot \vec{\mathbf{L}}_- + \vec{S}_- \cdot \vec{\mathbf{L}}_+}{m^2}\right) \frac{1}{R} \frac{dV_2}{dR} \\ & - \frac{2}{3m^2} \vec{S}_+ \cdot \vec{S}_- \nabla^2 V_2(R) \\ & - \frac{1}{m^2} (\vec{S}_+ \cdot \hat{R}\, \vec{S}_- \cdot \hat{R} - \frac{1}{3} \vec{S}_+ \cdot \vec{S}_-) V_3(R) \, . \end{aligned} \tag{3.42}$$

The first three terms of (3.42) are spin-orbit interactions. The fourth is a hyperfine interaction. The fifth is a tensor force. To evaluate the matrix elements of (3.42) for two-body q-q̄ bound states, one should set

$$\vec{\mathbf{L}}_+ = \vec{\mathbf{L}}_- = \vec{\mathbf{L}}.$$

In order to use Eq. (3.42), however, we need to know the form of the various potentials $V_i$ which appear in it. $V$, of course, is the same static potential used to represent the nonrelativistic bound state spectrum, but the other potentials are not otherwise accessible phenomenologically. We can get a first idea of their form by evaluating them in QED. In this case, the expectation value of two field operators in the presence of a Wilson loop is just equal to the same expectation value in free space; for example,

$$\langle \mathbf{B}^i(t,x_+)\mathbf{B}^j(t,x_-)\rangle = -\epsilon_{ikl}\,\epsilon_{jmn}\partial_k\partial_m\delta_{\ell n}\Big(\frac{1}{2\pi^2(x_+ - x_-)^2}\Big)\,. \tag{3.43}$$

One can easily show, from the definitions (3.28), (3.31), (3.35), that

$$V = -\frac{e^2}{4\pi R}\,,\quad V_1 = 0 \quad V_2 = \frac{e^2}{4\pi R}\,,\quad V_3 = -\frac{3e^2}{4\pi R^3}\,. \tag{3.44}$$

Inserting these results into (3.42), we find for the spin-dependent forces in QED the expression

$$\begin{aligned} V_{s-d} = {} & \frac{\vec{\mathbf{L}}\cdot(\vec{S}_+ + \vec{S}_-)}{m^2}\frac{3}{2}\frac{\alpha}{R^3} + \frac{2\alpha}{3m^2}\vec{S}_+\cdot\vec{S}_-\,4\pi\delta(\vec{R}) \\ & + \frac{1}{m^2}(3\vec{S}_+\cdot\hat{R}\vec{S}_-\cdot\hat{R} - \vec{S}_+\cdot\vec{S}_-)\frac{\alpha}{R^3}\,. \end{aligned} \tag{3.45}$$

The corresponding formula in perturbative QCD, to leading order in perturbation theory, is obtained by replacing

$$\alpha \rightarrow \frac{4}{3}\alpha_s \tag{3.46}$$

in this equation.

However, at least for the $\psi$ and $\Upsilon$ systems, we need to deal with real QCD, which does not stay entirely within the perturbative regime. The only way to proceed from here is to guess forms for the various functions $V_i$ which have appropriate small-distance limits. These potentials should obey one general constraint which follows from the considerations at the end of the previous section. In the models we discussed there, one can readily show that gauge field correlations of the type involved in the definitions of the $V_i$ fall off as $R^{-3}$ (the QED result) in the massive gauge model for $R < \mu^{-1}$, but then fall off much more rapidly, as

$$\frac{\mu^2}{R} e^{-\mu R} \tag{3.47}$$

for $R > \mu^{-1}$. I expect a similar behavior to appear in confining gauge theories, since the flux tube which binds quark and antiquark cannot carry information. Let me remark again, however, that I do not know precisely what mass scale plays the role of $\mu$. In any event, if this is so, the first term of (3.42) gives rise to a long-ranged force, since it contains the factor

$$\frac{1}{R}\frac{dV}{dR} \rightarrow \frac{K}{R} \quad \text{as} \quad R \rightarrow \infty, \tag{3.48}$$

but the other terms decrease exponentially. Perhaps it is, then, not unreasonable to approximate these terms by their perturbative values; this is the approximation scheme chosen by Eichten and Feinberg.[1] I should, however, note two minor difficulties with this hypothesis. The first is that $\nabla^2 V_2$ should contain an exponentially decaying term of the form of (3.47), whereas in leading order of perturbation theory it is completely local, proportional to $\delta(\vec{R})$. The second is that Kogut and Parisi [32] have found a contribution to the tensor force of order $m^{-4}$ which decays only algebraically (as $R^{-5}$) as $R \rightarrow \infty$; thus (3.48) is not the only long-range spin-dependent effect in the theory. A better approximation might be to evaluate the complete expression (3.42) in an explicit model of confinement such as the magnetic superconductor model of the previous section. A somewhat incomplete study of this approximation has been carried out by Banks and Spiegelglas.[33]

Despite the comment I have just made about the hyperfine interaction, it is certainly true that perturbation theory gives an extremely local interaction. It is tempting to compute this interaction more seriously in perturbative QCD and take seriously the corrections to the leading-order result. The hyperfine term in

(3.42) does not contribute to the splittings of the P-wave q-$\bar{q}$ states, but it can be used to predict the splitting between the singlet and triplet S states. This calculation has been carried out by Buchmüller, Ng, and Tye,[34] who computed the one-loop radiative correction to the expression for the hyperfine interaction obtained from (3.45) and (3.46). They find

$$\Delta E(^3S -^1 S) = \frac{32\pi}{9}\frac{\alpha_{\overline{MS}}}{m^2}|\psi(0)|^2\left[1+\left(0.563+0.375\frac{\langle log\frac{Q^2}{m^2}\rangle}{|\psi(0)|^2}\right)\frac{\alpha_{\overline{MS}}}{\pi}\right]. \tag{3.49}$$

This result is numerically quite reasonable; the $\psi - \eta_c$ mass splitting of 120 MeV would correspond, using (3.49), to a value of $\Lambda_{\overline{MS}}$ of about 300 MeV.

Let me now discuss briefly, and in very general terms, the application of Eq. (3.42) to the the splittings of the $^3P$ states. In this case the hyperfine term does not contribute, so we must evaluate the matrix elements of

$$\vec{\mathbf{L}}\cdot\vec{S} \quad \text{and} \quad \hat{R}\cdot\vec{S}_+\,\hat{R}\cdot\vec{S}_- \,. \tag{3.50}$$

This is easily done; let me provide you with a table of the relevant diagonal matrix elements of spin operators:

| | $^3P_0$ | $^3P_1$ | $^3P_2$ | |
|---|---|---|---|---|
| $\langle\vec{\mathbf{L}}\cdot\vec{S}\rangle$ | $-2$ | $-1$ | $+1$ | |
| $\langle\hat{R}\cdot\vec{S}_+\hat{R}\cdot\vec{S}_-\rangle$ | $-\frac{1}{4}$ | $\frac{1}{4}$ | $\frac{1}{20}$ | (3.51) |

The mass splittings of the P states are most often characterized by the value of a parameter $r$, defined by

$$r = \frac{M(^3P_2) - M(^3P_1)}{M(^3P_1) - M(^3P_0)}. \tag{3.52}$$

Using the elements of the Table (3.51), one can see that, in the limits in which the spin-orbit and tensor terms, respectively, dominate the mass splitting, $r$ takes the following values:

| | | |
|---|---|---|
| spin-orbit dominance | $\rightarrow$ | $r = 2$ |
| tensor dominance | $\rightarrow$ | $r = -0.4$ |
| QED | $\rightarrow$ | $r = 0.8$ . |

The third value of $r$ is that obtained from Eq. (3.45). The experimental values of $r$ for the c-$\bar{c}$ and b-$\bar{b}$ systems are

| | | |
|---|---|---|
| $c\bar{c}$ | $1^3P$ | $r = 0.48$ |
| $b\bar{b}$ | $1^3P$ | $r \sim 1.0$ |
| $b\bar{b}$ | $2^3P$ | $r \sim 0.9$ . |

It is quite reasonable that these numbers indicate the approach of the spin-dependent potentials to their perturbative form.

Clearly, there is much more that one could do to compare Eq. (3.42) to the experimental data on spin-dependent forces. One further application is given in Estia Eichten's lecture in this volume. I do not have space in the scope of this review to survey the variety of forms which have been postulated for the potentials $V_i$ and the experimental success of these forms. But recently Rosner[35] has compiled a detailed survey of such models; I recommend his review to the interested reader.

Thus ends our formal discussion of spin-dependent forces, as they are described by gauge theories. What do we learn from this analysis? First of all, the spin-dependent interactions of heavy quarks can be analyzed systematically and represented in terms of more fundamental expectation values of the gauge fields. The classical spin-orbit interaction still appears, and this term is of long range in a confining theory. But the short-ranged part of the spin-dependent potential is a new object, a new probe of gauge field correlations in the region between confinement and perturbation theory.

## 4. Hadronic Transitions Between Heavy-Quark States

In this section, I would like to examine another, more dynamical aspect of the behavior of heavy quark-antiquark systems, the theory of transitions between heavy q-$\bar{q}$ states mediated by the strong interactions. Again, our analysis will be guided by its use of the Feynman path formalism, and this discussion will provide another illustration of the power of this formalism in allowing one to visualize the basic processes of the gauge theory. To begin, let me pose the problem more precisely. The c-$\bar{c}$ and b-$\bar{b}$ quark-antiquark bound states are observed to have very small widths for decay into ordinary hadrons and seem otherwise to be very weakly coupled to ordinary hadrons. Still, though, excited states of the $\psi$ and $\Upsilon$ are observed to decay (albeit slowly) via:

$$\psi' \rightarrow \psi + 2\pi, \quad \psi + \eta, \quad \ldots$$

$$\Upsilon' \rightarrow \Upsilon + 2\pi, \quad \ldots \,. \tag{4.1}$$

The fact that these transitions are slow should encourage us, if we remember the Golden Rule of Perturbative QCD: That which is small is calculable (especially if it is too small to be measured). Let us explore what QCD has to say about the mechanism and rates of these transitions.

Before we enter our detailed analysis, let me pause to note that it is already remarkable that the processes (4.1) should be suppressed at all, given that they are mediated by the strong interactions acting at low momentum transfers and therefore, presumably, at full strength. Further, the observed suppression is much greater for the $\Upsilon'$ than for the $\psi'$. It was Gottfried[36] who realized that these features of the hadronic transitions are quite naturally understood within a gauge theory of the strong interactions and, indeed, display clearly the nature of the fundamental strong-interaction coupling. To see this, think about the process obtained from (4.1) by crossing, the scattering of an ordinary hadron by a heavy q-q̄ state. This situation is shown, in a somewhat idealized fashion, in Fig. 18. The heavy bound state is very small in its spatial extent. If we imagine taking the radius of this state to zero, the color charges of the quark and antiquark would overlap in space and precisely cancel; then this state would produce no gauge fields to interact with those of the larger light hadron. It therefore makes sense to compute the scattering amplitude as an expansion in the size of the heavy-quark system. In QED, such an expansion is well-known; it is just the multipole expansion of the charge density:

$$\int j_\mu A^\mu = QA^0(0) + \vec{d} \cdot \vec{E}(0) + \ldots \tag{4.2}$$

The first term is the only one independent of the size of the system. This vanishes for a particle-antiparticle system; thus, the leading contribution must come from the dipole interaction. Please note that for a hypothetical scalar gluon, the monopole term would be nonzero and the scattering amplitude would not decrease with the size of the heavy-quark system. Conversely, a gluon of spin greater than one would not couple to the dipole moment of the q-q̄ state, but only to a higher multipole moment. Any indication we can find that the dipole interaction does dominate is evidence that the gluon is a particle of spin 1.

Still, one must realize that QCD is more subtle than its Abelian counterpart. It is, unfortunately, not quite correct in QCD simply to write down the analogue of Eq. (4.2); one must perform some more careful analysis. Let me now lead you through this analysis in several stages. First, I will discuss the idealized situation of pure perturbative QCD. After studying this system in some detail, I will outline a phenomenological generalization of this discussion, due to Kuang and Yan, [37] to the intermediate regime of distances described by the potential models. Finally, I will briefly indicate how hadronic transitions are described in the magnetic superconductor model of confinement discussed at the end of § 2.

Let us, then, attempt to derive a multipole expansion for heavy q-q̄ bound states in QCD. My discussion here follows the work of Yan.[2] At first sight, we

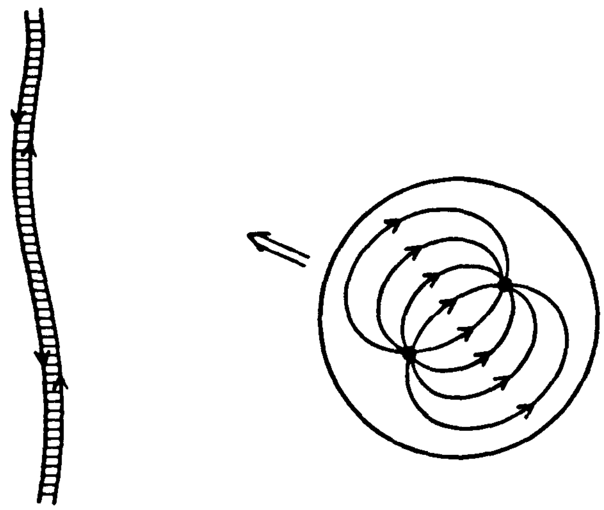

FIG. 18. Scattering of a light hadron from a heavy q-$\bar{q}$ bound state.

may produce this expansion simply by expanding the Feynman path representation of the heavy-quark bound state about the limit of zero q-q̄ separation. The resulting expression is indicated graphically in Fig. 19. Consider first the situation in which the antiquark sits at $\vec{R}=0$ and the quark moves freely. Using the method of Eq. (2.39), we can represent the excursions of the quark by insertions of field operators in a vertical Wilson line. In this way, we can expand:

$$e^{ig\oint dx\cdot\mathbf{A}} \simeq P\Big[e^{ig\oint dx\cdot\mathbf{A}}|_0 \cdot \Big(1+\int dt\, ig\vec{R}_+\cdot\vec{\mathbf{E}}+\frac{1}{2}\Big(\int dt\, ig\vec{R}_+\cdot\vec{\mathbf{E}}\Big)^2 + \int dt\,\frac{\vec{v}_+\times\vec{R}_+}{2}\cdot ig\vec{\mathbf{B}}+\cdots\int dt\,\frac{g}{2m}\vec{\sigma}_+\cdot\vec{\mathbf{B}}+\cdots\Big)\Big] \tag{4.3}$$

where

$$e^{ig\oint dx\cdot\mathbf{A}}|_0 \tag{4.4}$$

represents a Wilson loop with the quark and antiquark at zero separation, and the P operator indicates that the whole expression is to be path-ordered together. The last term listed in (4.3) arises from the spin-dependent term in the exponent of (3.20) and not, strictly speaking, from the Wilson line itself. Assembling the contributions from both the quark and antiquark, we find for the expansion of the Wilson loop:

$$e^{ig\oint dx\cdot\mathbf{A}} = P\Big[e^{ig\oint dx\cdot\mathbf{A}}|_0\cdot\exp\Big[\int dt\, ig\vec{R}\cdot\vec{\mathbf{E}}-\int dt\,\frac{(\vec{\mathbf{L}}_+-\vec{\mathbf{L}}_-)}{2m}\cdot g\vec{\mathbf{B}}+\cdots\Big]\Big]\,. \tag{4.5}$$

The two terms listed in the exponent are the color electric and magnetic dipole interactions.

The expansion indicated in Eq. (4.5) has the general form of a series of local gauge field operators inserted into an $\vec{R}=0$ Wilson loop:

$$P\Big[e^{ig\oint dx\cdot\mathbf{A}}|_0\exp\Big[\int dt\sum_i C_i(\vec{R},\vec{\mathbf{p}})O_i(t)\Big]\Big]\,. \tag{4.6}$$

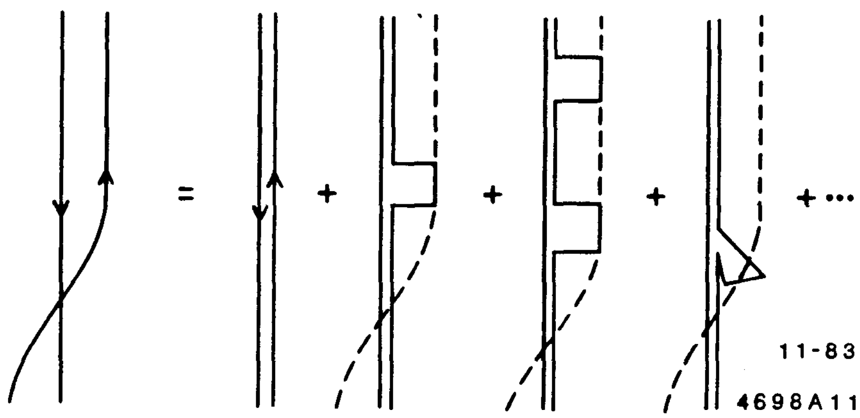


FIG. 19. Representation of the expansion of a typical quark trajectory about $\vec{R} = 0$.

It is natural to try to use this expansion to represent the emission of ordinary hadrons from the heavy q-q̄ system by considering the gauge-field operators $\mathcal{O}_i$ as gluon creation operators. Then, for example, the amplitude for the q-q̄ system to radiate a single gluon would be given by

$$< P[e^{ig\int dx\cdot\mathbf{A}}\int_{-\infty}^{\infty} dt\, g\vec{R}\cdot\vec{\mathbf{E}}_{\rm ext}(t)] > . \tag{4.7}$$

This formula looks innocuous, and perhaps even correct, but it contains some physics which is seriously wrong. The multipole operators of Eq. (4.6) will, in general, change the color of the q-q̄ state, and the emitted gluons will carry off color. Something that we have left out must impose quark confinement and assure that the q-q̄ state remains a color singlet.

In principle, it might have been that we could see this color singlet restriction only by working through the physics of confinement in some detail. For this problem, however, this restriction can be found, and its effects can be studied, simply by performing QCD perturbation theory with more care. [38] We must first realize that the formal manipulations of Eq. (4.3) omit an important effect of perturbation theory–the attractive Coulomb potential between the quark and antiquark. This effect, which arises from the class of Feynman graphs shown in Fig. 20(a). produces an contribution to the amplitude for a Wilson loop with small $\vec{R}$ of the form of Eq. (3.19):

$$\exp\left[-\int dt\left(-\frac{4}{3}\frac{\alpha_s}{R}\right)\right] . \tag{4.8}$$

This Coulomb potential indicated in (4.8) is that for the attractive interaction of a quark and antiquark in a color singlet state. This contribution is nonanalytic and certainly not negligible as $\vec{R}\to 0$. Now consider the class of diagrams shown in Fig. 20(b). If the q-q̄ system was in a color singlet state before the emission of the gluon, then afterwards it must be in a color octet state (to conserve color);

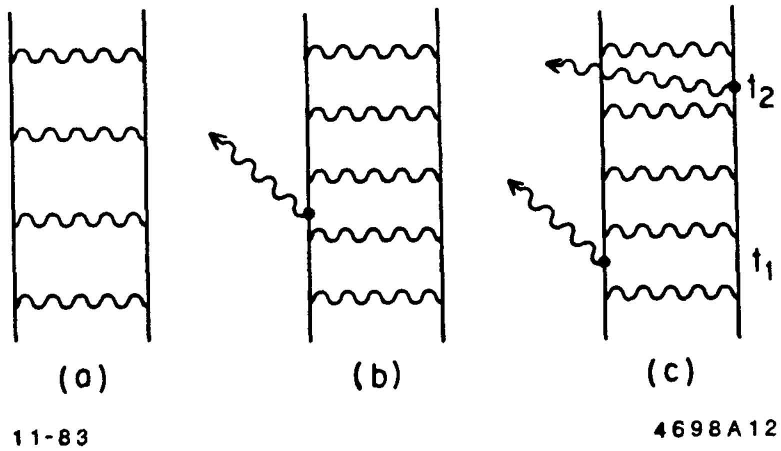


FIG. 20. Feynman graphs building up the Coulomb potential between a quark and an antiquark (a) for an undisturbed Wilson loop, (b) for a loop emitting one gluon, and (c) for a loop emitting two gluons.

the exponential factor is thus changed to the Coulomb potential appropriate to this state, which is smaller in magnitude and repulsive:

$$\exp\left[-\int dt\left(+\frac{\alpha_s}{6R}\right)\right] . \tag{4.9}$$

Eventually, this color octet state might radiate another gluon and return to a color singlet configuration (Fig. 20(c)). In this case, the exponential factor is modified only for the length of time for which the q-$\bar{\text{q}}$ system stayed a color octet. If the times of the two gluon emissions are labelled $t_1$ and $t_2$, respectively, this factor is given by

$$\exp\left[-\int dt\left(-\frac{4\alpha_s}{3R}\right)\right]\exp\left[-(t_2-t_1)\left(\frac{3\alpha_s}{2R}\right)\right] . \tag{4.10}$$

The higher energy of the color octet configuration provides an exponentially decaying term which restricts the size of $(t_2 - t_1)$ and thus binds together the two gluon emissions. This effect of the color Coulomb potential was first noticed by Appelquist, Dine, and Muzinich.[26]

Let us now ask how this Coulomb term–and, especially, the exponentially decaying factor displayed in Eq. (4.10)–influence the multipole expansion presented in Eq. (4.3). Clearly, it restricts the amount of time the heavy-quark system can be in a color octet state; thus it ties together two or more gluon creation operators into a cluster of emissions which is an overall color singlet. The leading term in the expansion (4.5) for the process shown in Fig. 20 is then:

$$\exp\left[-\int dt\, V_0(R)\right]\int dt_1 dt_2 ig\vec{R}\cdot\vec{\mathrm{E}}(t_2)e^{-(t_2-t_1)(H_q-E_0)}ig\vec{R}\cdot\vec{\mathrm{E}}(t_1) . \tag{4.11}$$

If we perform the integral over $(t_2 - t_1)$ for radiated gluons of fixed energy, including as well (by hand) the quark kinetic energy, we find a transition amplitude proportional to

$$\langle f|R^i\frac{1}{H_q-E_0}R^j|i\rangle\cdot\langle\text{hadrons}|\frac{1}{N}\,\mathrm{tr}\,\mathbf{E}^i\mathbf{E}^j|0\rangle \tag{4.12}$$

where now $H_q$ is the complete energy of the octet $q-\bar{q}$ system. This equation represents the leading term in the multipole expansion relevant for QCD, since it is the term of lowest order which allows the heavy-quark system to begin and end in a color singlet state. More careful derivations of this formula have been given by Voloshin [39] and myself.[38]

The double-dipole Formula (4.12) will be the starting point for my discussion of the phenomenology of hadronic transitions. Before beginning this discussion, however, I would like to discuss briefly two more theoretical topics. The first is to note that, whereas the Formula (4.11) for double gluon emission does arise quite readily from the Wilson line picture, it is not quite so straightforward to derive as a sum of Feynman diagrams. One might think that it is necessary only to sum diagrams of the form of Fig. 21(a); however, these diagrams alone give a result of the form

$$g^2(R^i\partial^i\mathbf{A}^0)e^{-(3\alpha_s/2R)(t_2-t_1)}(R^j\partial^i\mathbf{A}^0) \tag{4.13}$$

which is not gauge-invariant. One must remember that the integral over $(t_2-t_1)$, since its range is controlled by the exponential factor in Eq. (4.10), has an extent of order $\alpha_s{}^{-1}$. This means that diagrams which are apparently of different orders in $\alpha_s$ can be comparable in size if they receive a large weight from this integral. This actually happens in the diagrams of Fig. 21(b) and (c). The diagrams of the form of Fig. 21(b) pick up one extra factor of $\alpha_s{}^{-1}$ and yield the result

$$(-R^i\partial^0\mathbf{A}^i)e^{-(3\alpha_s/2R)(t_2-t_1)}(R^j\partial^i\mathbf{A}^0)+(R^i\partial^i\mathbf{A}^0)e^{-(3\alpha_s/2R)(t_2-t_1)}(-R^j\partial^0\mathbf{A}^j)\,, \tag{4.14}$$

the diagrams of Fig. 21(c) yield

$$(R^i\partial^0\mathbf{A}^i)e^{-(3\alpha_s/2R)(t_2-t_1)}(R^j\partial^0\mathbf{A}^j)\,. \tag{4.15}$$

The sum of these contributions is, of course, the gauge-invariant Formula (4.11).

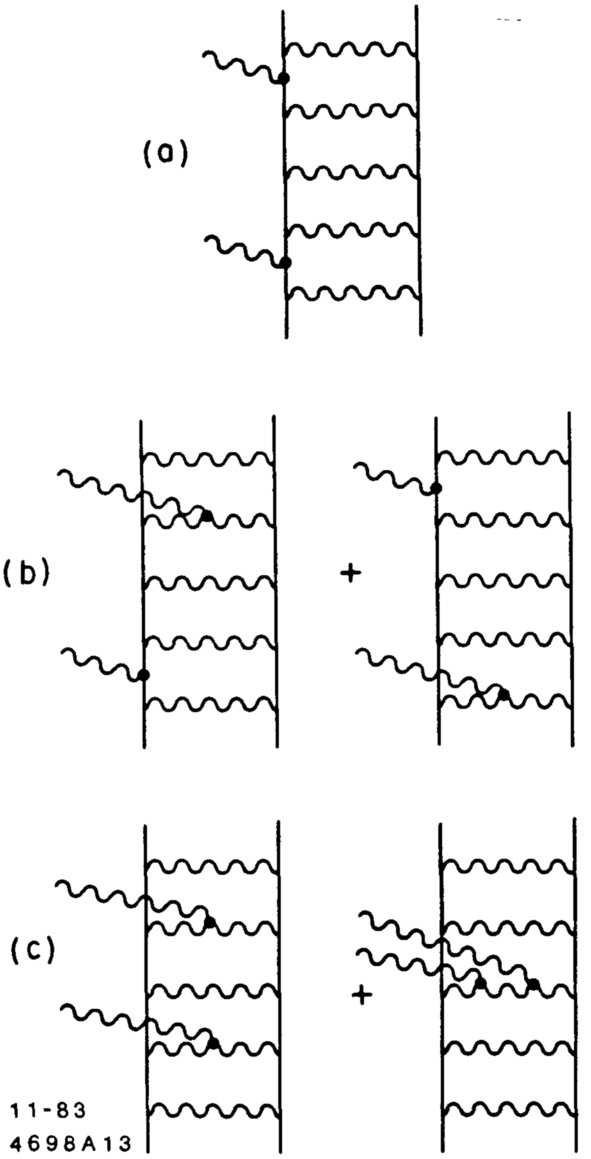


FIG. 21. Feynman diagrams contributing to the double-dipole formula for gluon emission from a heavy q-$\bar{q}$ state.

I would also like to give a first illustration of the physics of the Formula (4.12) in a context which is relatively simple, though somewhat artificial. Let me discuss the long-ranged interaction of two heavy-quark bound states, and, in particular, the question of the existence of hadronic van der Waals forces.[40] I will argue toward this point by presenting three pictures of increasing sophistication. These three viewpoints are illustrated in Fig. 22. Figure 22(a) shows the most naive picture, a dipole-dipole interaction based on the static potential $V(R)$. For a confining potential, this gives a long-ranged potential between heavy hadrons of the form

$$V_{h-h} \sim \left(\frac{\partial}{\partial R^i}\frac{\partial}{\partial R'^j}V(|\vec{R}-\vec{R}'|)\right)^2 \sim \frac{1}{|\vec{R}-\vec{R}'|^2}. \tag{4.16}$$

We can make a slightly more sophisticated picture by representing the strong-interaction fields connecting the two hadrons as gluons and using the double-dipole vertex to describe the coupling of the gluons to the heavy state. This picture, represented in Fig. 22(b), leads to a potential

$$V_{h-h} \sim \frac{1}{|\vec{R}-\vec{R}'|^7}\ . \tag{4.17}$$

This picture, with photons substituted for gluons, is actually the correct one for the very long range interactions of atoms; the Formula (4.17) is the color analogue of the Casimir-Polder effect[41,9] . In QCD, however, one should properly replace the gluons in the intermediate state by the various color-singlet hadron states created by the operator $tr(\mathbf{E}^i\mathbf{E}^j)$, as indicated in Fig. 22(c). The lightest such state dominates the long-range behavior. In a pure color gauge theory, this would be some glueball state; in a realistic version of QCD, it is the 2-pion state. Thus, we find at last the expression

$$V_{h-h} \sim |\langle G|\mathrm{tr}\,\mathbf{E}^i\mathbf{E}^j|\,0\rangle|^2 e^{-M_G|\vec{R}-\vec{R}'|}\ . \tag{4.18}$$

Let us now move on from these idealized considerations toward the problem of computing the rates of hadronic transitions. We would like to perform this

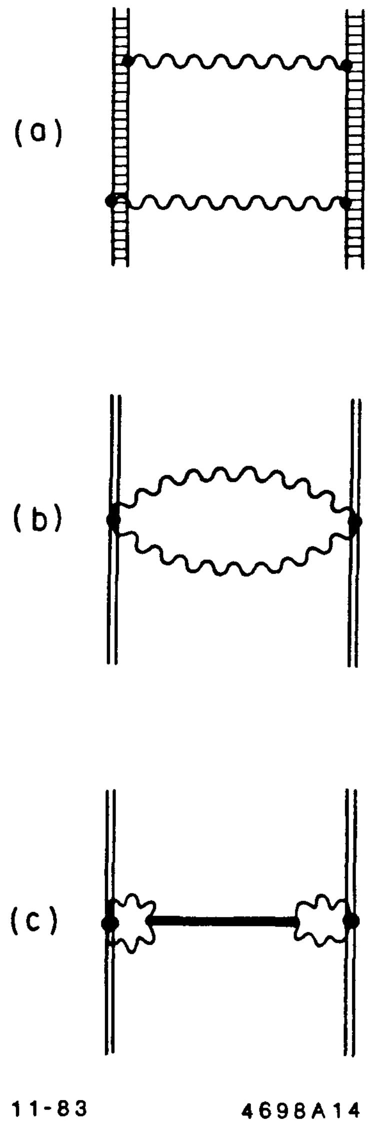


FIG. 22. Three pictures of the long-ranged interactions of two heavy-quark bound states.

computation by the direct use of the double-dipole Formula (4.12). However, perturbative QCD does not give us a precise understanding of two elements which enter into this formula–the spectrum of levels of the Hamiltonian $H_8$ describing a q-$\bar{q}$ pair in a relative color octet configuration, and the effective value of $\alpha_s$ with which the two emitted soft gluons couple. It would be valuable, then, to identify tests of this formula which are relatively independent of these two ingredients. One such test was suggested by Gottfried in his original paper.[36] Gottfried noted that the spectrum of low-lying energy levels is almost exactly the same in the $\psi$ and $\Upsilon$ systems; in particular, the excitation energies of the $\psi'$ and $\Upsilon'$ are almost identical:

$$m(\Upsilon') - m(\Upsilon) = 560\ MeV \quad \simeq \quad m(\psi') - m(\psi) = 585\ MeV\ . \tag{4.19}$$

Thus, the hadronic systems created in the hadronic transitions from $\psi'$ to $\psi$ and from $\Upsilon'$ to $\Upsilon$ are the same. It is now unreasonable that the low-lying spectrum of $H_8$ should also be the same in these two systems. But then, the only difference in the amplitude for the hadronic transition in these two systems comes from the dependence of the matrix element (4.12) on the size of the q-$\bar{q}$ bound states. Specifically, one should expect that these amplitudes are in the ratio

$$\frac{\mathcal{M}(\psi \to \psi + \text{hadrons})}{\mathcal{M}(\Upsilon' \to \Upsilon + \text{hadrons})} = \frac{\langle R^2\rangle_\psi}{\langle R^2\rangle_\Upsilon} \simeq 3 \tag{4.20}$$

so that the decay rates should be in the ratio

$$\frac{\Gamma(\psi' \to \psi + \pi^+\pi^-)}{\Gamma(\Upsilon' \to \Upsilon + \pi^+\pi^-)} \sim 10\ . \tag{4.21}$$

The experimental values of these rates are

$$\begin{aligned} \Gamma(\psi' \to \psi + \pi^+\pi^-) &= 71 \pm 14\ keV \\ \Gamma(\Upsilon' \to \Upsilon + \pi^+\pi^-) &= 6 \pm 2\ keV\ ; \end{aligned} \tag{4.22}$$

their ratio is in quite reasonable agreement with (4.21). Let me recall that, if the gluon were a spin zero object, the ratio of these rates should be 1. The results (4.22) should thus put to rest all doubts about the spin of the gluon.

A second such scaling law has been proposed by Yan[2] for hadronic transitions producing a single $\eta$. Since the $\eta$ is a pseudoscalar, the matrix element for the process $\psi' \to \psi + \eta$ must have the form

$$\mathcal{M} \sim \vec{\epsilon}(\psi') \times \vec{\epsilon}(\psi) \cdot \vec{p}(\eta), \tag{4.23}$$

where the $\vec{\epsilon}$'s denote the polarization vectors of the $\psi$ mesons and $\vec{p}(\eta)$ is the momentum of the produced $\eta$. The spin-dependence indicated in (4.23) is not present in the leading-order double-dipole formula; to find such spin-dependence, it is necessary to go to the double-magnetic-dipole and electric dipole-magnetic quadrupole terms. In either case, one finds

$$\mathcal{M} \sim \frac{1}{m^2}, \tag{4.24}$$

where $m$ is the mass of the heavy quark. Thus, Yan expects

$$\frac{\Gamma(\psi' \to \psi + \eta)}{\Gamma(\Upsilon' \to \Upsilon + \eta)} = \left(\frac{m_b}{m_c}\right)^4 \cdot \mathcal{R} \simeq 400 \ . \tag{4.25}$$

$\mathcal{R}$ is the ratio of phase space, which is a large factor ($\approx 6$) for this process. Unfortunately, the decay $\Upsilon' \to \Upsilon + \eta$ has not yet been observed. The scaling law (4.25) would predict

$$BR(\Upsilon' \to \Upsilon + \eta) \simeq 5 \times 10^{-4} \ . \tag{4.26}$$

In order to go beyond these scaling laws to make more detailed predictions, it is necessary to make more dynamical assumptions; in particular,one must explicitly resolve the two issues I had noted at the start of our discussion of scaling. The most complete attempt to give a detailed evaluation of the rates of hadronic transitions within this formalism has been made by Kuang and Yan[37]. These authors resolve the question of the value of $\alpha_s$ appropriate to Eq. (4.12) in the most straightforward way, by simply taking this coupling constant, times the amplitude for the two gluons to materialize as two pions, as a parameter and fixing it from the rate for $\psi' \to \psi + 2\pi$. The question of how to represent the spectrum of $H_8$ is considerably more puzzling. Kuang and Yan chose to identify the spectrum of color octet q-q̄ states with the spectrum of q-q̄ states in the potential corresponding to a vibrational excitation of the confining flux tube, a set of states which Giles and Tye [42] have insisted should occur in heavy quark-antiquark systems. I should note that the lowest such vibrational state has been searched for unsuccessfully in the b-b̄ system,[43] though one should keep in mind that this search assumed a substantial coupling of this state to a single photon. In any event, Kuang and Yan have predicted a substantial number of rates for hadronic transitions in the $\Upsilon$ system. Let me present their predictions for the branching ratios of these transitions and compare them, where possible, to experimental results:

| BR : | theory | expt.(CLEO, Ref.44) |
|---|---|---|
| $\Upsilon' \to \Upsilon\pi\pi$ | $25-29\%$ | $32 \pm 1\%$ |
| $\Upsilon'' \to \Upsilon\pi\pi$ | $2-5\%$ | $7 \pm 1\%$ |
| $\Upsilon'' \to \Upsilon'\pi\pi$ | $2-3\%$ | |
| | | |
| $2^3P_0 \to 1^3P_0\pi\pi$ | $0.05-0.06\%$ | |
| $2^3P_1 \to 1^3P_1\pi\pi$ | $0.3\%$ | |
| $2^3P_2 \to 1^3P_2\pi\pi$ | $0.2\%$ | |
| $2^3P_2 \to 1^3P_1\pi\pi$ | $0.01-0.02\%$ | |

The predictions for the rates of the $2^3P \to 1^3P$ transitions are small, but these transitions are particularly interesting in displaying additional symmetry constraints following from the double-dipole form of the amplitude (4.12). Yan[2] has noted that there are more $2^3P \to 1^3P$ transitions than there are independent invariant amplitudes in (4.12). To see this explicitly, note that the double-dipole operator

$$R^i \frac{1}{H_8 - E_0} R^j \tag{4.27}$$

transforms under spatial rotations as a reducible tensor; it can be decomposed into spin 0, spin 1, and spin 2 components. We may associate with each of these components, taken between $2^3P$ and $1^3P$ states, a single invariant matrix element. The transition amplitude from a given $2^3P$ state to a given $1^3P$ state may be found in terms of these invariant matrix elements through some accomplished Clebsch-ology. One finds

$$\frac{d\Gamma}{d\mathcal{M}^2_{\pi\pi}}\Big((J'L'S) \to (JLS) + 2\pi\Big) = (2J+1)\sum_{k=0}^{2}\left(\begin{Bmatrix} k & L' & L \\ S & J & J' \end{Bmatrix}\right)^2 A_k \tag{4.28}$$

where the expression in brackets is the 6-j symbol. $\mathcal{M}_{\pi\pi}$ denotes the invariant mass of the two-pion system. As Eq. (4.28) indicates, this symmetry decomposition holds for each fixed value of $\mathcal{M}_{\pi\pi}$. To express the relations between various transition rates, I will use the notation:

$$d\Gamma(J' \to J) = \frac{d\Gamma}{d\mathcal{M}_{\pi\pi}{}^2}(2^3P_{J'} \to 1^3P_J). \tag{4.29}$$

Since there are 9 such transition rates and only 3 invariant amplitudes, there must be 6 symmetry relations, obtainable by eliminating the $A_k$ from among the formulae (4.28). Yan thus finds the relations:

$$\begin{aligned} d\Gamma(0 \to 1) &= 3d\Gamma(1 \to 0) \\ d\Gamma(0 \to 2) &= 5d\Gamma(2 \to 0) \\ 3d\Gamma(1 \to 2) &= 5d\Gamma(2 \to 1) \\ d\Gamma(1 \to 1) &= d\Gamma(0 \to 0) + \frac{1}{4}\, d\Gamma(0 \to 1) + \frac{1}{4}\, d\Gamma(0 \to 2) \\ d\Gamma(1 \to 2) &= \frac{5}{12}\, d\Gamma(0 \to 1) + \frac{3}{4}\, d\Gamma(0 \to 2) \\ d\Gamma(2 \to 2) &= d\Gamma(0 \to 0) + \frac{3}{4}\, d\Gamma(0 \to 1) + \frac{7}{20}\, d\Gamma(0 \to 2) \ . \end{aligned} \tag{4.30}$$

Yan notes, further, that the transition rates $d\Gamma(0 \to 1)$ and $d\Gamma(1 \to 0)$ should be small because of a soft-pion suppression; ignoring these quantities, (4.30) is a set of 5 relations among 7 small but finite rates.

To complete this discussion of hadronic transitions, I would like to look at the computation of such transition amplitudes from a somewhat different perspective, that of the magnetic superconductor model of confinement introduced at the end of § 2. This model contains an explicit mechanism of quark confinement, but it also replaces the gluons by a phenomenological massive gauge field. In terms of this new gauge field, the dipole operator is given by

$$g_m \vec{R} \cdot \vec{B}. \tag{4.31}$$

There is no color in this phenomenological model, so the emission of a single one of the new gauge bosons is not forbidden by symmetry. However, there is no light axial-vector boson in the hadronic spectrum (especially if it is required to be mostly glueball). Thus, for a realistic value of the mass $\mu$ of this state, the decay of $\psi'$ to $\psi$ plus one of these bosons should be forbidden by energetics. The simplest process allowing a hadronic transition from $\psi'$ to $\psi$ is one in which two gauge bosons are produced, and these two bosons recombine into two pions, as indicated in Fig. 23. This amplitude for this process still involves a heavy-quark matrix element of the double-dipole form, so that the scaling laws and Clebsch-ology discussed above apply equally well to this model.

In this model, we might also try to understand the properties of the 2-pion state produced in the hadronic transition. To do this, of course, we must assume some form for the 2 gauge boson $\rightarrow$ 2 pion transition amplitude. However, this amplitude is restricted by the requirement from current algebra that it vanish at zero pion momentum. The simplest amplitude consistent with this requirement is

$$\delta \mathcal{L} = \frac{1}{8}\left(a\, \partial_\mu \pi^i \partial^\mu \pi^i (F_{\lambda\sigma})^2 + b\, \partial_\mu \pi^i \partial_\nu \pi^i F^{\mu\lambda} F^{\nu}{}_{\lambda}\right). \tag{4.32}$$

If one uses this expression to evaluate the amplitude of Fig. 23, one must obtain a result containing one power of each pion's momentum. Thus, the amplitude of Fig. 23 must be of the form

$$\mathcal{M}(nS \rightarrow n'S + 2\pi) \sim (A q_1^\mu q_{2\mu} + B q_1^0 q_2^0). \tag{4.33}$$

This restriction from current algebra is of course more general than the specific model we consider here; it was first pointed out some time ago by Brown and Cahn[46] and was applied also in the analysis of Yan.[2]

As a simple way of approaching the process of Fig. 23, let me approximate the amplitude shown by considering the intermediate two-vector state to be replaced by a set of bound states; this approximation is indicated diagrammatically in Fig. 24. These bound states should have the quantum numbers of a pair of axial vectors combined symmetrically; that is, they should be scalars and tensors. The simplest case is the one in which the only intermediate state is a scalar with a large mass or a large width: In this case, the amplitude $\mathcal{M}$ has just the form of (4.33) with $B = 0$. This limit yields a decay spectrum of the form:

$$\frac{d\Gamma}{d\mathcal{M}_{\pi\pi}} \sim k \cdot (\mathcal{M}_{\pi\pi}^2 - 4m_\pi^2)^{\frac{1}{2}} \cdot (\mathcal{M}_{\pi\pi}^2 - 2m_\pi^2)^2 \tag{4.34}$$

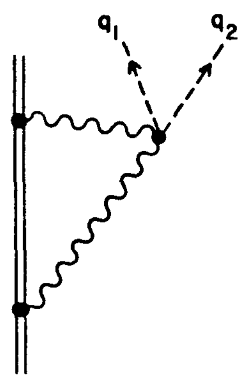


FIG. 23. The dominant process mediating hadronic transitions between heavy-quark states in the magnetic superconductor model of quark confinement.

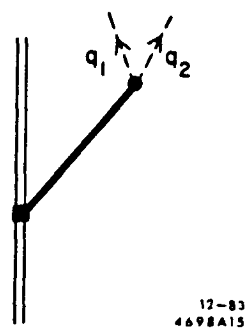


FIG. 24. An approximation to the amplitude shown in Fig. 23.

where $k$ is the recoil 3-momentum of the final q-$\bar{q}$ state. This expression peaks for large values of $\mathcal{M}_{\pi\pi}$. This shape for the 2-pion mass spectrum agrees strikingly with that observed in the hadronic transitions from the $\psi'$ and $\Upsilon'$. The agreement for the $\Upsilon'$ transition is shown in Fig. 25(a); Eq. (4.34), represented by the dashed curve, is compared there to recent results of the CLEO experiment.[46] Figure 25(b), however, indicates that this simple picture is not at all in agreement with the spectrum obtained in the transition from the $\Upsilon''$ to the $\Upsilon$.

To understand this discrepancy, let us write a slightly more sophisticated rendering of Fig. 24. Let me included both spin-0 and spin-2 bound states, each coupling to a double-dipole heavy-quark matrix element. These bound states are closely analogous to glueball states, and I encourage you to think of them in that way. One should represent these states with the following propagators:

$$\text{spin}-0 \quad \Delta(p) = \frac{i}{p^2 - M_G^2 + iM_G\Gamma_G} \quad ,$$

$$\begin{aligned} \text{spin}-2 \quad \Delta^{\mu\nu;\lambda\sigma} = \frac{1}{2}\Big[&\Big(g^{\mu\lambda} - \frac{p^\mu p^\lambda}{M_G^2}\Big)\Big(g^{\nu\sigma} - \frac{p^\nu p^\sigma}{M_G^2}\Big) + (\mu \leftrightarrow \nu) \\ &- \frac{2}{3}\Big(g^{\mu\nu} - \frac{p^\mu p^\nu}{M_G^2}\Big)\Big(g^{\lambda\sigma} - \frac{p^\lambda p^\sigma}{M_G^2}\Big)\Big] \cdot \Delta(p) \end{aligned} \quad . \tag{4.35}$$

The horrible spin factor in the second line arises in the following way: The polarization of the spin-2 state is characterized by the values of pair of indices $(\mu\nu)$. The propagator must contain projectors so that, first, $\mu$ and $\nu$ can take only space-like values when the meson is on shell and at rest ($p = (M_G, \vec{0})$), and, second, that the space-space components of in this frame form a traceless tensor. The latter condition can be written:

$$\delta_{ij}\,\Delta^{ij;\lambda\sigma}\big|_{p=(M_G,\vec{0})} = 0 \tag{4.36}$$

where $i, j$ run over spatial indices only. These two conditions give the projector in (4.35) uniquely; the second condition produces the factor $\frac{2}{3}$ in the last line.

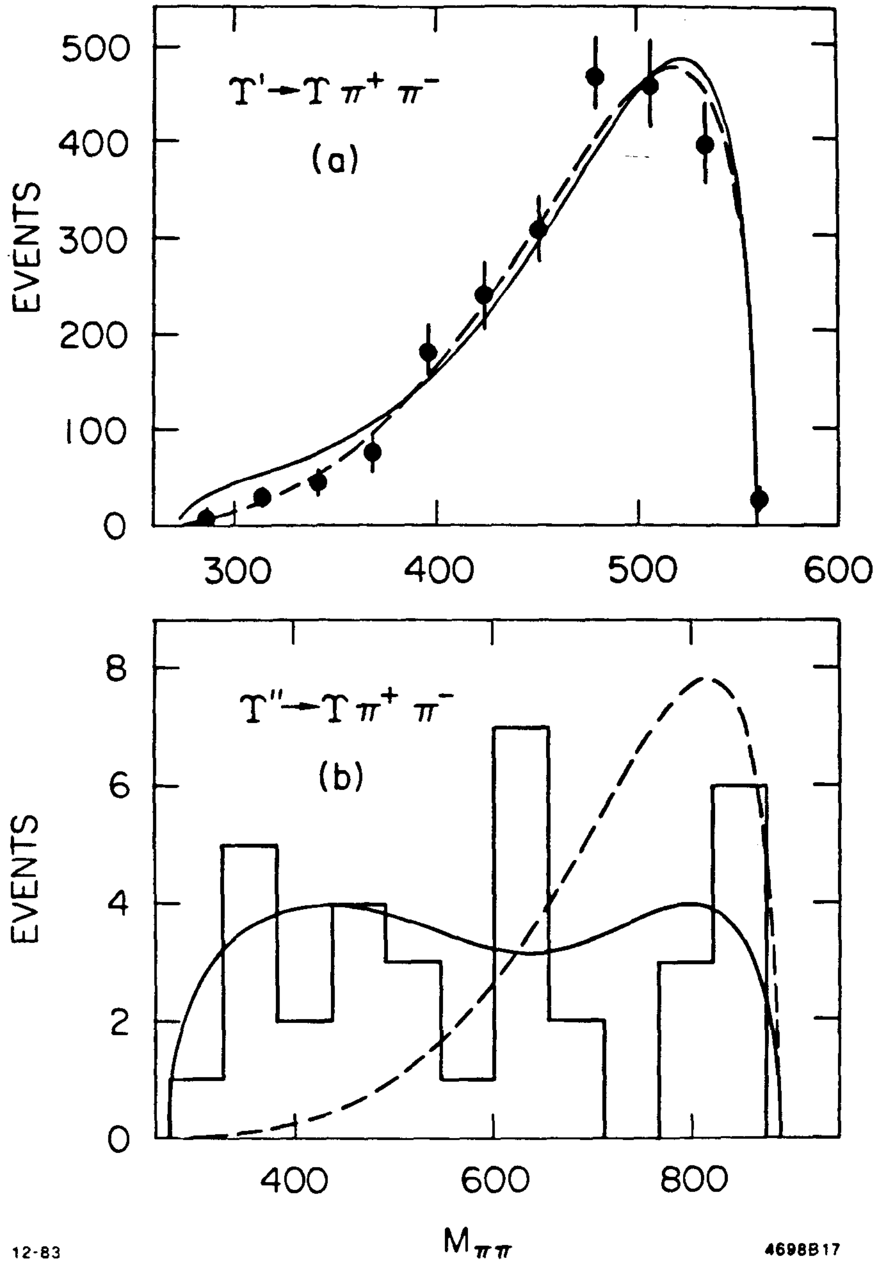


FIG. 25. Distribution of events containing hadronic transitions between $\Upsilon$ states according to the mass $M_{\pi\pi}$ of the 2-pion system, for (a) $\Upsilon' \to \Upsilon + 2\pi$ and (b) $\Upsilon'' \to \Upsilon + 2\pi$. The data are taken from Refs. 46 and 44, respectively. The dashed curve is the prediction of Eq. (4.34); the solid curve corresponds to Eq. (4.37) with the parameters given in the text. The overall normalization of each curve is arbitrary, and adjusted.

Using these propagators, we can write the amplitude of Fig. 24 as

$$\mathcal{M} \sim \alpha \cdot \Delta(Q)\, q_1 \cdot q_2 + (\frac{1}{3}\delta_{ij})\, \Delta^{ij;\lambda\sigma}(Q)\, q_{1\lambda} q_{2\sigma} \tag{4.37}$$

where $Q = q_1 + q_2$.

From Eq. (4.37), it is straightforward to generate an expression for the dipion spectrum; the formula of Yan given in Footnote 6 of Ref. 44 is useful in performing this calculation. The resulting formula has enough parameters that one has wide latitude in curve-fitting, but it is still not trivial that, using the same amplitude, one can produce pions in the low mass region for the $\Upsilon''$ transition but not for the $\Upsilon'$. The solid curves in Fig. 25 show a reasonable fit: In order to obtain the right behavior from the spin projectors in (4.35), it is necessary to take $M_G$ to be very low (600 MeV). One must then insist that these states are very wide; I have set $\Gamma_G = M_G$. The remaining parameter is given by $\alpha = -2$. It is amusing that this simple model requires a set of glueball states which are at very low mass but, at the same time, are too broad to show themselves clearly as $\pi\pi$ resonances.

In this section, then, we have seen that the Feynman path formalism leads directly to multipole expansions for the interaction of heavy q-$\bar{\text{q}}$ states with external probes or with ordinary hadrons. The systematics of the rates of hadronic transitions between q-$\bar{\text{q}}$ states seems to accord well with this picture. But again, as with the topics treated earlier in these lectures, the imperfections in our knowledge of the transition region between gluons and hadrons eventually catches up with us. Can we, eventually, understand quantitatively what goes on in this intermediate regime, where perturbative QCD no longer applies but confinement is not yet an absolute constraint? I hope I have enticed you to ponder this problem and, perhaps, to find a way to solve it.

## ACKNOWLEDGEMENTS

The perceptive reader should by this point appreciate the extent of my debt to all of my colleagues from Cornell, theorists and experimenters, for teaching me so much of what I know about heavy-quark systems.

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