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MODELS WHICH ALLOW A "NEW INFLATIONARY" UNIVERSE HISTORY*

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ABSTRACT

We analyse a class of Grand-Unified Theories and show that, if one accepts severe fine-tuning of many parameters, it is possible to satisfy all the physical constraints that one would like to impose for an Inflationary Universe scenario. We include some post-inflation constraints, such as the usual zero-temperature heirachy, as well as the constraints that arise directly from the inflationary period. The new feature of our analysis is that we study a slow-roll-over transition between two broken-symmetry minima, rather that the usual Coleman-Weinberg case of a symmetric to broken symmetry transition.

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1. Introduction

The idea of inflation as a solution to the cosmological problems of the flatness and homogeneity of the observed universe, as well as the problem of the density of monopoles from the breaking of the grand-unified symmetry which first led to its suggestion by Guth and Tye,[1] is very attractive. A particularly exciting development in this subject is the observation that these models present the first possibility of understanding the development of the Zeldovich[2] scale-invariant spectrum of density fluctuations[3] and estimating the amplitude of this effect. However, as is usually the case, the more physical effects one seeks to explain with a mechanism, the harder it becomes to find a model where all these effects have their correct (that is observed) properties. We will review what we consider the essential requirements for a successful model and discuss previous attempts to satisfy some or all of these requirements.

We then analyse a class of grand-unified field theories and show that, if one accepts severe fine-tunings of parameters, these models can have sufficient flexibility to accomodate all the constraints. The new feature of our analysis is that we assume the inflation takes place by a slow-roll-over type of transition between two symmetry-broken minima, rather than the usual Coleman-Weinberg[4] scenario of a roll-over from a symmetric configuration to a symmetry broken vacuum.[5,6] We will show that this avoids some of the problems of the Coleman-Weinberg scenario, while retaining the slow-roll-over advantages. We study an SU(5) grand-unified theory with a Higgs multiplet Φ in the adjoint and F in the fundamental representation. The idea is, however, quite general and could equally well be realized in any grand-unified model with a sufficiently rich Higgs sector.

Section 2 summarizes the physical features that one would like to fit in an inflationary universe model and the requirements on the model that arise from each of these features. We make no new contributions to the estimation of these effects. We then briefly discuss previously suggested models. Section 3 presents the analysis of the SU(5) theory. Section 4 presents our conclusions.

2. Review of Physical Requirements

The essential features of an inflationary universe scenario are summarized as follows:

(1) The zero temperature vacuum state of the universe [or the present metastable state in some models] is assumed to have zero cosmological constant. This is achieved by a fine tuning of parameters in most models.

(2) The expectation value of the Higgs fields in this state is nonzero. We will denote this by

$$\langle\{\phi\}\rangle = \{\sigma_0\} \quad . \tag{1}$$

Statement (1) above requires

$$V_{\rm eff}(\{\sigma_0\}\,;\,T=0) = 0 \quad . \tag{2}$$

(3) At early times the universe can be described by a temperature T and values for

$$\langle\{\phi\}\rangle_T = \{\sigma(T)\} \neq \{\sigma_0\} \quad . \tag{3}$$

The Hubble parameter of the theory for any temperature T and field configuration $\{\sigma\}$ is given by

$$\frac{2G\,V_{\text{eff}}(\{\sigma\}\ ;\ T)}{3} = H^2(T) \quad .$$

(For very high temperatures typically one finds $\{\sigma(T)\} \equiv 0$ that is the universe is in a symmetric phase for $T > T_c$. This fact is not essential to our analysis.)

(4) At some temperature T_B there appears a region of the universe, which we will refer to as a bubble, within which the scalar fields have values $\{\hat{\sigma}\,(T_B)\}$ such that the classical field equations predict a smooth evolution from this configuration to the global minimum at $T = 0$. The region inside this bubble constitutes the present universe. This evolution of the scalar fields in this region is the slow-roll-over transition. We are following the ideas of Witten,[7] Lindè,[4] Steinhardt and Albrecht[5] rather than the original first order transition inflationary universe idea of Guth and Tye[1] because of the problem of percolation of bubbles.[8]

(5) The effective potential of the theory controls the roll-over transition as well as the nature of the zero-temperature vacuum. It must be such as to satisfy a number of constraints. These are:

A. Correct zero temperature physics

The physics of the zero-temperature vacuum must be an effective SU(3) $\times$ U(1) theory with the usual Glashow-Weinberg-Salam weak interactions. The heirachy problem of SU(5) – the fact that $m_W \ll m_X$ where X is the heavy boson responsible for proton decay in this theory – requires fine tuning of the zero temperature effective potential. This tuning must not be in conflict with any of the other requirements on the effective potential. We remark here that we chose to study a model with both adjoint and fundamental Higgs multiplets in order to be able to achieve this zero temperature physics correctly.[9]

B. Sufficient inflation

In order to explain the flatness and homogeneity of the observed universe as well as the lack of monopoles from the grand-unified theory we need[10]

$$a(t_f) \lesssim e^{60}\, a(t_o) \tag{4}$$

where a is the scale factor in the Friedman, Robertson-Walker metric

$$ds^2 = dt^2 - a^2(t)\, d\,\vec{x}^2 \quad . \tag{5}$$

The time t_f is the end of the exponential expansion period and t_o is some time during the slow roll-over, at the earliest it is the time of bubble formation. This requires that the potential $V(\{\sigma\};T)$ is very flat over a large range of $\{\sigma\}$, for T near zero.

C. Sufficient reheating

In order that the baryon to photon ratio of the present universe can be explained by the CP-violation inherent in a grand-unified theory,[11] the universe must reheat after the expansion to a temperature high enough to give baryon number generation by this mechanism. The reheating is governed by the shape of the potential $V(\{\sigma\})$ near $\{\sigma_0\}$ [12] and requires a sufficiently deep minimum with large curvature.

D. Limited Fluctuations

A remarkable feature of the new inflationary universe is that it allows us for the first time to envisage a mechanism that generates the Zeldovitch scale invariant spectrum of density fluctuations $\delta\rho/\rho$ in the universe. These fluctuations are responsible for the condensation of matter into galaxies and clusters of galaxies. In such a model these fluctuations in density are a direct result of fluctuations in $\{\hat{\sigma}\}$ during the roll-down period. For a detailed analysis of the fluctuation development, we refer the reader to the work of Brandenberger and Kahn.[13] Although the mechanism is natural, the scale of the resulting $\delta\rho/\rho$ provides a problem. Very crudely this scale is controlled by the value of $(d\hat{\sigma}/dt)^{-1}$ at an early stage of the inflation.[2] Guth and Pi have shown that, under certain conditions, the requirement

$$\frac{\delta\rho}{\rho} < \delta \tag{6}$$

can be translated into a requirement

$$\left.\frac{d\hat{\sigma}}{dt}\right|_{t=t_0} > \frac{H^2}{\delta} \quad . \tag{7}$$

Typical galaxy formation models in conjunction with the observed homogeneity of the $3°K$ background radiation suggest $\delta \sim 10^{-4\pm1}$. The large value of $d\hat{\sigma}/dt$ at t_0 (in H^2 units) and the long expansion time required by Eq. (4) can only be achieved if the effective potential is essentially flat over a very large range of $\hat{\sigma}/H$. It is the combination of Eqs. (4) and (7) that causes the principal problem in achieving a successful scenario, and leads, in our example, to extreme fine-tuning requirements on the parameters of the theory.

E. No domain walls

The requirement of a homogeneous and uniform universe after expansion cannot be met if the model has a degenerate set of inequivalent $T = 0$ minima for V_{eff} and if, at any stage during the inflation, fluctuations can create regions of space in these different vacua. This problem has been pointed out by Breit, Gupta and Zaks.[14] For the Coleman-Weinberg potential the degeneracy is between symmetry breaking vacua with opposite signs of TR Φ^3. This problem is simply resolved in the model we discuss – there is a unique $T = 0$ vacuum.

F. Natural evolution along the desired classical path

Another problem for the Coleman-Weinberg scenario that was pointed out by Breit, Gupta and Zaks[15] is the fact that the classical path from the symmetric point to the SU(3) $\times$ U(1) minimum corresponds to a ridge rather than a valley of V_{eff}. Any small departure from this path could grow – giving a rapid descent to an SU(4) $\times$ U(1) minimum instead of the desired slow roll-over to SU(3) $\times$ U(1). In general, one is studying the evolution of a bubble in a multidimensional group space. One can tune the potential so that particular classical path which leads smoothly to the correct zero-temperature minimum is flat. However it can be a nontrivial problem to assure that this path would be followed by a finite fraction of the bubbles starting from a given initial configuration. In our model the relevant classical path lies at the bottom of a deep valley in V_{eff}, and hence is not unstable in this way.

To summarize the inflationary period requirements in a more quantitative fashion we discuss the case of a single scalar field variable ϕ. Later we will show how the more complicated grand-unified theory analysis can be related to the constraints on this $V(\phi)$. Similar constraints have been given by Ovrut and Steinhardt[16] who study such a potential in the context of a supersymmetric "inflaton" model (their paper also contains a review of such models). Let us write

$$V(\phi) = \sum_{n=0}^{\infty} a_n \phi^n$$

$$\left.\frac{dV}{d\phi}\right|_{\phi=\sigma} = 0 \quad ; \quad V(\sigma) \equiv 0 \quad . \tag{8}$$

Here we assume that we have chosen variables such that the starting configuration is near to $\phi = 0$ and that the roll-down takes us to $\phi = \sigma$.

The constraints are derived as follows. Define

$$H^2 = \frac{8\pi G\, V(0)}{3} \quad . \tag{9}$$

The classical equation of motion for the spatially constant background ϕ field in a Freedman-Robertson-Walker metric are

$$\ddot{\phi} + 3\left(\frac{\dot{a}}{a}\right)\dot{\phi} = -\frac{\partial V}{\partial \phi} \tag{10}$$

and

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{2G}{3}\left(\frac{\dot{\phi}^2}{2} + V(\phi)\right) \quad . \tag{11}$$

One can solve these equations under a number of simplifying assumptions. Consider the evolution of the ϕ field from some time t_o at which [by Eq. (7)]

$$\dot{\phi}\,(t_o) \lesssim \frac{H^2}{\delta} \tag{12}$$

till some time t_f such that $(t_o - t_f)/H \approx 60$ [from Eq. (4)] and

$$\phi(t_f) = \kappa\sigma \quad , \quad \kappa < 1 \quad . \tag{13}$$

Assuming during this period

(i) $\frac{2G}{3}\left(V(\phi)+\frac{\dot{\phi}^2}{2}\right)\approx H^2$.

(ii) $\frac{dV}{d\phi}$ is dominated by a single term $a_n\phi^n$.

(iii) $\ddot{\phi}$ is negligible compared to $3H\,\dot{\phi}$.

After making these approximations, one can solve Eqs. (10) and (11) and find constraints on the a_n such that Eqs. (12) and (13) can be satisfied. One can then check the consistancy of the approximations.

For example if the evolution is dominated by the ϕ^2 term of $V(\phi)$ in this period one finds the requirements

$$a_2 = \frac{H^2}{40} \quad , \quad \frac{H}{\sigma} < \frac{\kappa\delta}{60e} \tag{14}$$

$$a_n < \sigma^{4-n}\cdot\frac{1}{4}\left(\frac{\delta}{60e}\right)^2\kappa^{4-n} \qquad \text{all } n>2 \quad .$$

Similarly, dominance by the n*th* term requires

$$a_n < \sigma^{4-n}\cdot\frac{1}{20n(n-2)}\left(\frac{\delta}{60(n-2)}\right)^2\rho^{4-n} \tag{15}$$

for

$$\frac{H}{\sigma} = \frac{\rho\delta}{60(n-2)} \quad \text{and} \quad a_m(\kappa\sigma)^m < a_n(\kappa\sigma)^n \quad \text{all } m\neq n \quad .$$

In Eq. (14) assumption (ii) gives the constraint on the a_n for $n>2$; one can readily verify that assumptions (i) and (iii) are valid. For $\delta\simeq 10^{-4}$ these requirements can be written

$$\begin{aligned} a_0 &\leq \sigma^4\cdot\frac{m_p^2}{\sigma^2}\cdot 10^{-13}\,\kappa^2 \\ a_n &< (\kappa\sigma)^{4-n}\cdot 10^{-14} \quad . \end{aligned} \tag{16}$$

In a theory where natural scale of a_n is σ^{4-n} this requires tuning of all coefficients up to some power n for which $\kappa^{4-n} \simeq 10^{14}$. For example is we choose $\kappa = 10^{-4}$ then a_0 through a_7 need to be artifically small.[17] The most severe tuning is of a_2, which is this case is required to be less than $\sigma^2 \times 10^{-22}$. (This is however certainly no more of an adjustment than the choice zero which is made in the Coleman-Weinberg case.)

The reason that the fine tuning is so severe is quite obvious. We require that $\dot{\phi}(t_0)$ is large in H-units. This means that, following a classical evolution which naturally begins with both ϕ/H and $\dot{\phi}/H^2$ of order 1, ϕ/H is already large at $t = t_0$. To get a further 60 e-foldings of inflation after $t = t_0$ thus requires an effective potential for which reheating occurs at an extremely large value of ϕ/H. The potential must be approximately flat over a range from 1 to $\kappa\sigma/H \simeq 10^7$. (To achieve 60 e-foldings of inflation starting from ϕ/H and $\dot{\phi}/H^2$ of order 1, would require much less fine-tuning. However one then finds that the fluctuations are too large, in fact so large that the validity of the classical analysis of the expansion is questionable in this case.[3,18])

The requirement of Eq. (12), which is based on a number of assumptions about the formation and development of the fluctuations in the ϕ field within the bubble, was given by Guth and Pi. Brandenberger and Kahn[13] have presented some discussion of cases where the approximations leading to Eq. (12) are invalid. Their criticisms do not apply to a potential $V(\phi)$ such that the restrictions of Eq. (14) are met. In such a case the approximations made by Guth and Pi are reproduced by the more detailed analysis of the evolution given by Brandenberger and Kahn. Perhaps the most debatable point in all analyses is the assumption that the fluctuations in ϕ in the initial bubble are given by the statistical fluctuations at a temperature $H/2\pi$, which is the Hawking temperature of the interior of the bubble due to its exponential expansion. We have nothing new to add to the discussion of this point, we will assume that Eq. (12) is required. We find that drastic fine tuning is needed to satisfy Eqs. (12) and (13) simultaneously.

We now turn to a brief summary of models which have been previously studied for a slow-roll-over transition. We remark that most of these studies do not attempt to discuss all the requirements A through F above. Most discuss only B, C and D. References to the literature on these models can be found in Refs. 13 and 16.

(1) Coleman-Weinberg Models

The idea of a slow-roll-over transition was first analysed in the context of these models,[5,6] but they have not been successful in achieving the desired scenario. They suffer from problems E and F, they give too large fluctuations and, because of this, also insufficient expansion. Generally no attempt is made in discussing these models to include a fundamental Higgs multiplet and achieve the correct zero-temperature heirachy.

(2) Supersymmetric Inverse Heirachy Models

These models founder on requirement C – they cannot be arranged to give sufficient reheating. For these models the assumption that $\ddot{\phi}$ is small compared to $\dot{\phi}$ during the inflationary period is not valid, and the analysis of constraints for a power law potential given above does not apply. In general such models also give large fluctuations.

(3) Supersymmetric Inflaton Models

In these models a gauge singlet scalar supermultiplet ϕ is added to the theory and $V_{\text{eff}}(\phi)$ fine-tuned to satisfy the constraints discussed above. It is assumed that the inflation and reheating is all due to the inflaton potential and that the SU(5) breaking transitions occur at some subsequent time. (If the order is reversed then one loses the ability to generate baryon number via the CP-breaking part of the grand-unified theory.) However if the SU(5) $\to$ SU(3) $\times$ U(1) transitions occur after the slow-roll-over and are first order transitions then new inhomogeneities of the universe are generated in these transitions and the percolation problem of the old inflationary universe scenario is resurrected. These problems appear to us to be very severe. They have not been addressed at all by the proponents of inflaton theories.

3. A Model

Let us review some properties of the effective potential for an SU(5) theory with an adjoint Higgs multiplet Φ and a fundamental Higgs multiplet F. The most general renormalizable scalar interactions for such a theory can be written

$$\begin{aligned} P(\Phi, F) = &-\frac{\mu^2}{2}\text{TR}\,\Phi^2 + \frac{a}{4}(\text{TR}\,\Phi^2)^2 + \frac{b}{2}\text{TR}\,\Phi^4 + \frac{c}{3}\text{TR}\,\Phi^3 \\ &- \frac{\nu^2}{2}F^+F + \alpha F^+F\ \text{TR}\,\Phi^2 + \beta F^+\Phi^2 F + \gamma F^+\Phi F \\ &+ \frac{\lambda}{4}(F^+F)^2 \quad . \end{aligned} \tag{17}$$

We allow the following scalar vacuum expectation values

$$\Phi = \frac{\Lambda}{\sqrt{20}}\begin{pmatrix} 1 & & & & \\ & 1 & & & \\ & & 1 & & \\ & & & 1 & \\ & & & & -4 \end{pmatrix} + \frac{\Sigma}{\sqrt{12}}\begin{pmatrix} 1 & & & & \\ & 1 & & & \\ & & 1 & & \\ & & & -3 & \\ & & & & 0 \end{pmatrix}$$

$$F = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ \eta \end{pmatrix} . \tag{18}$$

This allows us to study breaking of SU(5) to

$$\begin{aligned} SU(4) \times U(1) &\qquad (\eta = 0;\ \Sigma = 0;\ \Lambda \neq 0) \\ SU(4) &\qquad (\eta \neq 0;\ \Sigma = 0;\ \Lambda \neq 0) \\ SU(3) \times SU(2) \times U(1) &\qquad \left(\eta = 0, \frac{\Sigma}{\sqrt{12}} = -\frac{5}{3}\frac{\Lambda}{\sqrt{20}}\right) \\ \text{or} \qquad SU(3) \times U(1) &\qquad (\Sigma \neq 0, \Lambda \neq 0, \eta \neq 0) \ . \end{aligned} \tag{19}$$

In terms of these variables the effective potential is given by

$$\begin{aligned} V = &-\frac{\mu^2}{2}(\Lambda^2 + \Sigma^2) + \frac{a}{4}(\Lambda^4 + 2\Lambda^2\Sigma^2 + \Sigma^4) \\ &+ \frac{b}{2}\left(\frac{13}{20}\Lambda^4 + \frac{3\Lambda^2\Sigma^2}{10} - \frac{2\Sigma^3\Lambda}{\sqrt{15}} + \frac{7}{12}\Sigma^4\right) \\ &- c\left(\frac{\Lambda^3}{2\sqrt{5}} - \frac{\Sigma^2\Lambda}{2\sqrt{5}} + \frac{\Sigma^3}{3\sqrt{3}}\right) - \frac{\nu^2}{2}\eta^2 \\ &+ \left(\alpha + \frac{4}{5}\beta\right)\eta^2\Lambda^2 + \alpha\eta^2\Sigma^2 - \frac{2}{\sqrt{5}}\gamma\Lambda\eta^2 + \frac{\lambda}{4}\eta^4 \ . \end{aligned} \tag{20}$$

We will discuss the slow-rollover transition as if it occurs at $T = 0$. More precisely it should be regarded as occuring at a fixed temperature $H/2\pi$, which is the Hawking temperature due to the expansion. This effect can be included by replacing ν^2 and μ^2 by the temperature dependant mass in the constraints that come from the roll-over period, but the correction is negligible.

We choose the parameters $(a, b, \ldots)$ so that at $T = 0$ there is a global minimum which is as SU(3) $\times$ U(1) minimum $\Lambda = \Lambda_3$, $\Sigma = \Sigma_3$, $\eta = \eta_3$. We choose

V_0 so that $V(\Lambda_3, \eta_3, \Sigma_3) = 0$. Furthermore this minimum must have the property $\eta_3 \ll \Lambda_3,\ \Sigma_3$. In fact we need

$$\begin{aligned} \eta_3 &\sim 300 \ \text{GeV} \\ \Lambda_3 &\sim 10^{15} \ \text{GeV} \\ \Sigma_3 &\sim 10^{15} \ \text{GeV} \end{aligned} \tag{21}$$

in order to give a viable zero temperature theory. This is the usual heirachy problem. One finds

$$\eta_3^2 = \frac{1}{\lambda}\left[\nu^2 - 2\Big(\alpha + \frac{4}{5}\,\beta\Big)\Lambda_3^2 - 2\alpha\,\Sigma_3^2 + \frac{4\gamma\Lambda_3}{\sqrt{5}}\right] \quad . \tag{22}$$

The condition (21) requires a fine tuning of parameters. [We note however that it does not require ν^2 to be of order η_3^2.] This fine-tuning can only be achieved at one temperature, the value of $\eta_3^2(T)$ will be sensitively temperature dependent.

The sequence of global minima of $V_{\text{eff}}(T)$ as the temperature decreases depends on the parameters. However, if parameters are chosen to give the SU(3) $\times$ U(1) global minimum at $T = 0$, there is a wide range of such choices for which there is an intermediate temperature range where either an SU(4) minimum on a different SU(3) $\times$ U(1) minimum is the global minimum. We will choose to discuss the first possibility, although the second could equally well be treated in the same way. We assume, for the sake of definiteness, that at some high temperature T (of order 10^{15} GeV) some region makes a first order phase transition to an SU(4) symmetric global minimum of the effective potential. Since the region in this minimum has $V(\{\phi\}) \neq 0$, the bubble so formed expands exponentially and cools rapidly to its Hawking temperature. We then consider the subsequent development of some region interior to this bubble as this interior region makes a slow-roll-over transition to the global SU(3) $\times$ U(1) minimum configuration.

We view this region as a bubble within the bubble – a local fluctuation away from the metastable SU(4) minimum which subsequently expands and evolves (following the classical equations of motion in the fixed temperature effective potential) to become our universe. It has been suggested that a slow-roll-over transition should occur globally rather than locally – in this scenario this would mean the "interior" bubble fills the SU(4) region, but would not otherwise alter our discussion. We will use the term bubble, very loosely, to describe whatever the region may be in which the classical evolution equations apply. It does not imply a thin-walled bubble of the Coleman-De Luccia type.[7,19]

It is convenient to make a shift of variables to study the slow roll-over. We define the SU(4) minimum by

$$\frac{\partial V}{\partial \Lambda} = 0 \, , \, \frac{\partial V}{\partial \Sigma} = 0 \, , \, \frac{\partial V}{\partial \eta} = 0 \text{ at } \begin{cases} \Lambda = \Lambda_4 \\ \eta = \eta_4 \\ \Sigma = 0 \end{cases} . \tag{23}$$

Now let

$$\Lambda = \Lambda_4 + \tilde{\Lambda} \qquad \eta = \eta_4 + \tilde{\eta} \quad . \tag{24}$$

Then

$$
\begin{aligned}
V = {} & V(\Lambda_4\, ;\, 0\, ;\, \eta_4) \\
& + \frac{1}{2}m_3^2\Sigma^2 + \frac{1}{2}m_\Lambda^2\Lambda^2 + \frac{1}{2}m_\eta^2\eta^2 + m_{\eta\Lambda}^2\Lambda\eta \\
& - \Sigma^3\left(\frac{b\Lambda_4}{\sqrt{15}} + \frac{c}{3\sqrt{3}}\right) + \frac{\Sigma^4}{4}\left(a + \frac{7b}{6}\right) \\
& + \Sigma^2\tilde{\Lambda}\left[\left(a + \frac{3}{10}b\right)\Lambda_4 + \frac{c}{2\sqrt{5}}\right] + \frac{\Sigma^2\tilde{\Lambda}^2}{2}\left(a + \frac{3}{10}b\right) \\
& + \Sigma^2\,\tilde{\eta}\,(2\alpha\eta_4) + \Sigma^2\,\tilde{\eta}^2\,\alpha \\
& + \tilde{\Lambda}^3\left[\left(a + \frac{13b}{10}\right)\Lambda_4 - \frac{c}{2\sqrt{5}}\right] + \frac{\tilde{\Lambda}^4}{4}\left(a + \frac{13b}{10}\right) \\
& + \tilde{\eta}\,\tilde{\Lambda}^2\left[2\left(\alpha + \frac{4}{5}\beta\right)\eta_4\right] + \tilde{\eta}^2\,\tilde{\Lambda}\;2\left(\alpha + \frac{4}{5}\beta\right)\Lambda_4 \\
& + \tilde{\eta}^2\,\tilde{\Lambda}^2\left(\alpha + \frac{4}{5}\beta\right) - \frac{2\gamma}{\sqrt{5}}\tilde{\eta}^2\,\tilde{\Lambda} + \lambda\eta_4\,\tilde{\eta}^3 + \frac{\lambda}{4}\tilde{\eta}^4
\end{aligned}
\tag{25}
$$

where

$$
\begin{aligned}
\frac{1}{2}m_\Sigma^2 &= -\frac{\mu^2}{2} + \left(a + \frac{3b}{10}\right)\Lambda_4^2 + \frac{c\Lambda_4}{2\sqrt{5}} + \alpha\eta_4^2 \\
\frac{1}{2}m_\Lambda^2 &= -\frac{\mu^2}{2} + \frac{3}{2}\left(a + \frac{13b}{10}\right)\Lambda_4^2 - \frac{3c\Lambda^4}{2\sqrt{5}} + \left(\alpha + \frac{4}{5}\beta\right)\eta_4^2 \\
\frac{1}{2}m_\eta^2 &= -\frac{\nu^2}{2} + \left(\alpha + \frac{4}{5}\beta\right)\Lambda_4^2 - \frac{2}{\sqrt{5}}\gamma\Lambda_4 + \frac{3}{2}\lambda\eta_4^2 \\
m_{\eta\Lambda}^2 &= 4\left(\alpha + \frac{4}{5}\beta\right)\eta_4\Lambda_4 - \frac{4}{\sqrt{5}}\gamma\eta_4 \quad .
\end{aligned}
\tag{26}
$$

If m_Σ^2 is negative this potential has a deep valley connecting the SU(4) symmetric saddle point to the global SU(3) $\times$ U(1) minimum. Even for small positive

m_Σ^2 such a valley exists, but separated from the SU(4) minimum by a small barrier. If, in some region of space, a fluctuation to a small nonzero Σ occurs then the classical equations of of motion which govern the evolution of $(\tilde{\Lambda}, \Sigma, \tilde{\eta})$ in that region are

$$\ddot{\tilde{\Lambda}} + 3H\dot{\tilde{\Lambda}} = -\frac{\partial V}{\partial \tilde{\Lambda}}$$
$$\ddot{\Sigma} + 3H\dot{\Sigma} = -\frac{\partial V}{\partial \Sigma} \tag{27}$$
$$\ddot{\tilde{\eta}} + 3H\dot{\tilde{\eta}} = -\frac{\partial V}{\partial \tilde{\eta}}$$

where

$$H^2 \simeq \frac{8\pi G}{3} V\Big(\Lambda_4;\, 0;\, \eta_4\Big) \quad . \tag{28}$$

We note that, once Σ is nonzero, the symmetry of the region is SU(3) $\times$ U(1) and it is inevitable that the state will be reached by following Eq. (27) will be the global SU(3) $\times$ U(1) minimum – the true vacuum of the theory – there can be no other intervening minimum.

Now we can study the potential near $\tilde{\Lambda} = \tilde{\eta} = \Sigma = 0$. In this region we can write

$$\frac{\partial V}{\partial \tilde{\Lambda}} = \tilde{\Lambda}\, m_\Lambda^2 + \tilde{\eta}\, m_{\Lambda\eta}^2 + \Sigma^2\Big[\Big(a + \frac{3b}{10}\Big)\Lambda_4 + \frac{c}{2\sqrt{5}}\Big] + \ \ldots$$
$$\frac{\partial V}{\partial \tilde{\eta}} = \tilde{\Lambda}\, m_{\Lambda\eta}^2 + \tilde{\eta}\, m_\eta^2 + 2\alpha\Sigma^2 + \ \ldots \tag{29}$$
$$\frac{\partial V}{\partial \Sigma} = \Sigma m_\Sigma^2 \ + \ \ldots \ .$$

The coefficients m_Λ^2, m_η^2 and $m_{\Lambda\eta}^2$ are all positive [this is the condition that $\Lambda = \Lambda_4$, $\eta = \eta_4$ is the SU(4) minimum]; but m_Σ^2 can be chosen to be negative.

Equations (29) then show that $\tilde{\Lambda}$ and $\tilde{\eta}$ grow as Σ^2 near the origin, and the evolution of the system is governed principally by the Σ evolution. Hence we will parameterize the evolution along the valley by

$$\begin{aligned} \Sigma(t) &= \sigma\,\epsilon(t) \\ \tilde{\Lambda} &= \sigma\,\epsilon^2\Big(\sum_{n=0}^{\infty} \ell_n\,\epsilon^n\Big) \\ \tilde{\eta} &= \sigma\,\epsilon^2\Big(\sum_{n=0}^{\infty} f_n\,\epsilon^n\Big) \end{aligned} \tag{30}$$

where σ is chosen so that $\epsilon = 1$ at the SU(3) $\times$ U(1) minimum of V.

If the potential is flat enough that a successful slow-roll-over can be achieved, then, as was the case for the single variable example discussed in the previous section, one can consistantly neglect the $\ddot{\Sigma}$, $\ddot{\Lambda}$ and $\ddot{\eta}$ terms in Eq. (27). By combining Eq. (27) with Eq. (30), one then has

$$\begin{aligned} \frac{1}{\sigma}\,\frac{\partial\,\tilde{\Lambda}}{\partial\epsilon} &= \frac{\partial V}{\partial\,\tilde{\Lambda}}\Big/\frac{\partial V}{\partial\Sigma} \\ \frac{1}{\sigma}\,\frac{\partial\,\tilde{\eta}}{\partial\epsilon} &= \frac{\partial V}{\partial\,\tilde{\eta}}\Big/\frac{\partial V}{\partial\Sigma} \quad . \end{aligned} \tag{31}$$

Equations (27), (30) and (31) then allow one to write

$$3H\Sigma = \sigma^2 \sum_{n=2}^{\infty} n b_n\,\epsilon^{n-1} \quad . \tag{32}$$

and solve for the b_n in terms of the couplings of the theory. The constraints (14) derived for the single variable problem can be translated into constraints on the b_n in Eq. (32) by the relationship

$$\begin{aligned} a_0 &= V(\Lambda_4, 0; \eta_4) - V(\Lambda_3, \Sigma_3; \eta_3) \quad , \\ a_n &= b_n\,\sigma^{4-n}\left(\frac{\sigma^2}{3H^2}\right); \qquad n \geq 2 \quad . \end{aligned} \tag{33}$$

We find

$$a_2 = \frac{m_\Sigma^2}{2}$$

$$a_3 = -\frac{2}{3}\sqrt{\frac{5}{3}}\left(\frac{3b\Lambda_4}{10} + \frac{c}{\sqrt{20}}\right)$$

$$4a_4 = a + \frac{7b}{6} + \frac{m_\eta^2\left[\left(a+\frac{3b}{10}\right)\Lambda_4 + \frac{c}{\sqrt{20}}\right]^2 + 16m_\Lambda^2\alpha^2\eta_4^2 + 8m_{\eta\Lambda}^2\eta_4\alpha\left[\left(a+\frac{3b}{10}\right)\Lambda_4 + \frac{c}{\sqrt{20}}\right]}{\left[2m_\Sigma^2 - m_\eta^2\right]\left[2m_\Sigma^2 - m_\Lambda^2\right] - m_{\eta\Lambda}^4} . \tag{34}$$

We have verified that the tuning of a_2, a_3 and a_4 is not in conflict with the zero temperature constraints. Also, since the reheating is controlled by $(d^2V/d\Sigma^2)|_{\Sigma=\sigma}$ and the coupling of the Σ field to fermions, we find it is not made small by these choices. Since a_5 and a_6 will also be independent functions of the original parameters they can also be tuned. [Nine parameters in V allow us to fix a_0, a_2 through a_6, the scale of σ, the zero temperature heirachy, and the choice of zero temperature global minimum.] Adding more Higgs multiplets gives a richer vacuum structure and more free parameters. Hence we believe that any Grand Unified theory with a sufficiently rich Higgs sector can, in principal, satisfy all the constraints. Clearly the result requires much parameter twiddling – it is highly artificial – however the same criticism can be applied to all other attempts to satisfy even a subset of the constraints A through F.

With all this fine tuning of parameters a word is in order about higher loop effects and sliding couplings. We take the point of view that for a problem such as this it is simplest to fix a renormalization definition once and for all, and then explicitly display all higher loop effects. Renormalization group "improvements" in a problem where many different physical scales enter can never sum all large logarithms and can be deceptive if naively applied. Thus we assume that all our Lagrangian parameters are defined in terms of quantities measurable (in principle) in the $T = 0$ vacuum at some momentum scale μ. The equations for the various a_n that are given in Eq. (34) are tree-level equations. At the one loop level they will acquire additional terms. The values of the original parameters which satisfy the one loop equations may be quite different from the values which satisfy the tree level equations; that is the problem of fine-tuning. However in general higher loop corrections do not make it impossible to satisfy constraints that could satisfied at tree level.

4. Comments and Conclusions

We have studied the possibility of a slow-rollover in a broken-symmetry to broken-symmetry transition. We chose to illustrate this possibility for a particular transition in a particular model but there is nothing special about this choice. Any grand-unified theory with a sufficient number of Higgs multiplets to get the zero-temperature physics right will probably also allow such transitions. Even in the context of the SU(5) model the choice of an SU(4) to SU(3) $\times$ U(1) transition is not special. [One could for example equally well study other possibilities, such as an SU(3) $\times$ U(1) to SU(3) $\times$ U(1) transition with the first minimum of the form

$$\Lambda \neq 0 \quad , \quad \Sigma = 0 \quad , \quad F = \begin{pmatrix} 0 \\ 0 \\ 0 \\ \rho \\ 0 \end{pmatrix}$$

and the second of the type previously discussed.]

We have presented a scenario for an inflationary universe history which we believe is at least as plausible as any other that has been discussed. We find that a high degree of fine tuning is necessary to produce all the required properties for the resulting universe. Lindé has argued that inflation is a "natural" phenomenon that will occur with almost any starting potential. Our analysis here shows that, in the context of inflation, the homogeneity of our present universe is still unexplained, except by unnatural fine-tuning. There is no longer a horizon problem, but the small range of values of $\delta\rho/\rho$ which are consistent with galaxy formation and the observed homogeneity of the $3°K$ background radiation is very difficult to achieve – at least if the present theory for the development of these fluctuations is correct. Since only very crude analyses of the origin of the fluctuations have yet been made, it is possible that further work on this problem will change the picture drastically.

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