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PROGRAM FOR A CONSTRUCTIVE PHYSICS *

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ABSTRACT

In this paper we outline a program for physics using as basic mathematics counting, binary arithmetic and randomness. We believe this approach makes sense because the discretization of physics has replaced arbitrary continuum standards of time, length and mass by *counting* the oscillations of an atomic clock, *counting* the number of wavelengths between two positions and *counting* the number of atoms in a macroscopic mass. These digital dimensional standards can be used to specify three dimensional constants: the limiting velocity c, the unit of action h, and either a reference mass (eg m_p) *or* a coupling constant (eg G related to the mass scale by $hc/(2\pi G m_p^2) \simeq 1.7 \times 10^{38}$). The objective of our program is to provide an algorithmic construction which allows us to relate this connection to specific laboratory paradigms. As in constructive mathematics, we hold that counting must be understood as the *practice* of mathematics in order to avoid redundancy. We allow no *completed* infinities and we aim to provide finite algorithms for the computation of any acceptable concept. We do not expect our formalism to be reducible in a mathematical sense to conventional physics. To succeed, our program must lead to an alternative quantitative description of accepted laboratory and experiential phenomena.

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1. BASIC IDEAS

The basic concepts in our program[1] are ***sequence***, ***event***, and ***velocity***. Each of these concepts is initially encountered in a digital context. Our objective is to give these concepts precision by means of a series of constructive, interrelated steps. Necessarily this requires us to loose some of the vague but creative richness contained in the standard experiential intuitive meanings associated with the historical practice of physics. Our basic sequence records unique events. Although these events are sequential, they are ***random*** in the sense that we cannot know which event occured; we can only explore the structure now by using the concept of equal prior probabilities to represent this state of ignorance. Our laboratory paradigm, that two events are connected to the sequential firing of two counters (or an equivalent natural phenomenon), allows retrodiction only via statistical arguments. Thus the universe we envisage has, conceptually, a "fixed past" and a "memory" of past events, but this past is only statistically accessible at the present. Further, even in principle, which event occurs next can only be predicted in the sense of relative frequencies, or ***probabilities***. Hence the future is always uncertain.

Our digital definition of "event" and connection between specific events is most easily understood in terms of the explicit construction of a "bit string universe" which we articulate in the Chapter 2. This construction leads to unique, ordered strings of the existence symbols "0" and "1", called ***bits***. The information content of the first 256 bits in any string is organized by an information-preserving mapping known as the ***combinatorial hierarchy***[2,3], which generates levels of rapidly increasing complexity characterized by the cardinals 3,10,137, $2^{127} + 136$ that Bastin[2] identifies with the scale constants of physics, and terminates at the fourth level. These first 256 bits are called the ***label***; the rest of the string is called the ***address***. We will see below that –after the early stages in the evolution of this universe – there will be many different addresses for each label, providing us with the concept of ***labeled ensembles***. Each time an event occurs the program adjoins a random bit to the end of each string in the universe; hence our "universal time sequence" is simply the (common) integral length of each bit string.

The way in which we recover a discrete version of the relative coordinates of special relativity and the Poincaré transformations is explained in Chapter 3. To connect two ordered events we concentrate on one label which is involved in the first, and after the bit string length has increased by b bits, is also involved in the second. Since, in general, there will be many different addresses involved, we take as our (dimensionless) definition of the ***velocity*** ν connecting these two events the average $<>$ over this ensemble $\nu = < N^1 - N^0 > / < N^1 + N^0 > = < N^1 - N^0 > /b$, where $N^1$is the number of 1's and N^0 the number of 0's in the address segments with $b = N^1 + N^0$ bits. Clearly all velocities lie in the interval $-1 \leq \nu \leq +1$. Modifying a construction due to Stein[4], we interpret this connection between events as a biased random walk of b steps with limiting velocity c, fixed step length ℓ, velocity of the most probable position of the peak $v = \nu c$ and mean position $d = \nu b\ell$; this interpretation implies that "1" represents a step in the positive direction and "0" a step in the other direction. Thanks to the limiting velocity, this allows us to derive a discrete version of the Lorentz transformation for intervals between events.

Associating a mass with each label (the mass ratios to be calculated later by the theory) and an invariant step length $\ell_0 = h/mc$ (the value of the step length in the zero velocity coordinate system) we can also define energy by $E = hc/\ell$ (our basic quantization condition) and momentum by $p^2c^2 = E^2 - m^2c^4$. Invoking our counter paradigm, we find that we need to introduce an ensemble of ensembles of differing momenta weighted with positive and negative coefficients in order to meet our quasi-local boundary conditions. This construction introduces both ***probability amplitudes*** whose squares are interpreted as predicted probabilities and a coherence length $\lambda = h/p$ in addition to the step length $\ell = hc/E$. We recover the DeBroglie relativistic wave mechanics for free particles as a continuum approximation to our theory with discrete corrections that –so far as we can see – are currently beyond the reach of experimental test. Then a double slit experiment or equivalent interference arrangement provides us with explicit ways to measure h.

From these definitions we develop in Chapter 4 a quantum scattering theory with asymptotic momentum conservation. The way in which we arrive at our version of quantum numbers and gravitational phenomena is explained in Chapter 5.

2. PROGRAM UNIVERSE

We start from very primitive finite mathematical structures (which we believe, but do not attempt to demonstrate here, can be grounded in the constructive mathematics of Bishop[5] and Martin-Löf[6]). "All" we need take from constructive mathematics are the symbols $0, 1, +_2, =$ with their usual significance [i.e., $0 +_2 0 = 0$; $0 +_2 1 = 1$; $1 +_2 0 = 1$; $1 +_2 1 = 0$], the "random" operator R which gives us 0 or 1 with equal probability, and ordered bit strings of the symbols 0 and 1. We take the symbols $0, 1, +_2$ to stand for primitive recursive functions. Then the expressions in [ ] above can, essentially, be seen as ***programs*** which give the information needed for their own evaluation[7]. By this strategy we aim at showing the expressions above to be self-explanatory vis-a-vis meaning; we do not have to embark on a reductionist strategy in order to justify the use of these expressions.

Our computer algorithm makes use of two processes which create new strings. For two ***ordered*** bit strings of length N, symbolized by $S_i = (..., x_i, ...)_N$ where $x_i \in [0,1]$, $i \in [1,N]]$, ***discrimination*** is defined by $D_N S_i S_j = (..., x_i +_2 x_j, ...)_N$. ***Complementation*** is defined by $\neg S_i = (..., x_i +_2 1, ...)_N = D_N S_i I_N$. where I_N is the ***antinull*** string containing N 1's, When neither of these operations succeeds in generating a string not already contained in the universe of bit strings we generate novelty by increasing the string length of all strings by appending a random bit randomly chosen for each string, at the growing end. This process is called *TICK* because it defines an irreversible sequence measured by bit length; note that it does not order happenings ***between*** ticks.

To get the program started (assuming an indefinitely extensible memory available, consistent with the mathematical practice of ***counting***), we assign the first string in the universe the value R (i.e. a random choice between 0 and 1) and the second again the value R, provided only it differs from the first. Since there is no way to produce further novelty at this level we enter the main program at *TICK*. From the universe containing SU unique strings we pick at random a string S_1 with prior probability $1/SU$ (expressing our ignorance of what else to do), pick another S_2 (testing that it is not the same) and discriminate them to produce a third, S_3. If it is not already in the universe , we adjoin it and proceed. If it is, we compute the complements of the first two strings, and adjoin them to the universe if they are not already there. When $D_N S_1 S_2 = S_3 = D_N \neg S_1 \neg S_2$ and all five strings are already in the universe, this failure to produce novelty causes the bit string universe to crank forward by one tick. We call this happening an *event.* Note that we have by this step abandoned the concept of ***simultaneity***, and not just "distant simultaneity" as is customary in special relativity.

In order to organize the information content of this universe into four levels of increasing complexity which preserve the information content of the lower levels in constructing the upper levels, we make use of the combinatorial hierarchy[2,3]. We first define a discriminately closed subset (DCsS) as a single non-null string or as that set of non-null strings which when any pair are discriminated yield another member of the set. If we start from linearly independent strings $a, b, c, ...$ (i.e. $a+b \neq 0, b+c \neq 0, c+a \neq 0, a+b+c \neq 0, ...$) we can clearly form the DCsS's $\{a\}, \{b\}, \{c\}, \{a, b, a+b\}, \{b, c, b+c\}, \{c, a, c+a\}, \{a, b, c, a+b, b+c, c+a, a+b+c\}$ and so on. Here we have used + for discrimination; since $a+a = 0$ the closure of the subsets is transparent. From j linearly independent strings we can obviously always form $2^j - 1$ DCsS's because this is the number of ways we can choose j distinct objects $1, 2, .., j$ at a time.

Starting from strings with two bits (N=2) we can form $2^2 - 1 = 3$ DCsS's, for example $\{(10)\}, \{(01)\}, \{(10), (01), (11)\}$. To preserve this information about discriminate closure we map these three sets by non-singular, linearly independent 2x2 matrices which have the members of these sets as eigenvectors. Rearranged as strings of four bits these form a basis for $2^3 - 1 = 7$ DCsS's. Mapping these by 4x4 matrices we get 7 strings of 16 bits which form a basis for $2^7 - 1 = 127$

DCsS's. We have now organized the information content of 137 strings into 3 levels of complexity. We can repeat the process once more to obtain $2^{127} - 1 \doteq 1.7 \times 10^{38}$ DCsS's composed of strings with 256 bits, but cannot go further because there are only 256x256 linearly independent matrices available to map them, which is many to few. We have in this way generated the critical numbers $137 \doteq hc/2\pi e^2$ and $1.7 \times 10^{38} \doteq hc/2\pi G m_p^2$ and a hierarchical structure which terminates at four levels of complexity.

Since the labeling capability of the combinatorial hierarchy scheme is exhausted using strings of 256 bits in length, but our program requires the string length to continue to grow, we reserve the first 256 bits in all strings as a ***label*** for an ensemble of strings, organized as discussed above. The remaining bits in each string in the ensemble are called the ***addresses***. From then on we assign strings to ensembles with the same first 256 bits, or labels, making new arrays when new labels turn up. Thereafter all that can happen are the discriminations and complementations which occur ***between*** ticks and that the number of members of each ensemble and the length of their addresses continue to grow.

3. TIME, SPACE AND PARTICLES

To connect events to coordinate systems we modify the random walk model pioneered by Stein[4]. We pick some event, which as we have seen will necessarily involve labels $L_1, L_2, L_3 = D_{256}L_1L_2, \neg L_1, \neg L_2$, later to be associated with a laboratory event such as the firing of a counter by a particle with quantum numbers L_1, as the origin of our coordinate system and then look for a second event involving also involving L_1 which occurs b ticks later. We now extract from the memory the address strings with label L_1 and form the address ensemble of string segments with the b bits which were added between the two events. We define the "velocity" connecting these two events by $\nu = < N^1 - N^0 > /(N^1 + N^0) = < N^1 - N^0 > /b$, where N^1 and N^0 are the number of 1's and 0's respectively in an address string in the ensemble and $<>$ is the ensemble average. We assume that the ensemble represents a biased random walk of b steps with a probability $p = < N^1(\nu, b) > /b = (1/2)(1 + \nu)$ of taking a step in the positive velocity direction defined by our two counters and a probability $q = 1-p = < N^0(\nu, b) > /b = (1/2)(1-\nu)$ of taking a step in the negative direction. The velocity of the peak, ν, is obviously bounded by $-1 \leq \nu \leq +1$ while the standard deviation from the peak is $\sigma(\nu, b) = (bpq)^{1/2} = (b/4)^{1/2}[1 - \nu^2]^{1/2}$. With this probabilistic interpretation, the random walk model, specified by the two parameters b, ν is equivalent to an ensemble of bit strings of length b, or to a binomial distribution specified by the same two parameters.

In order to convert this algorithm into a dimensional metric for our digital construction of space we assume not only that the limiting velocity is to be identified with the physical *limiting velocity* c but also that to each label we can associate a parameter m with the dimensions of *mass*. We then can introduce a constant with the dimensions of action called h and define an *invariant step length* h/mc. We take our random walk as describing the probability distribution for an event to occur, and the distance that the peak has moved, $\nu b\ell$, as only one point in the distribution. Thus, following Stein, we take our definition of the *distance* ξ between the two events to be $\xi - \nu b = \sigma(\nu, b) = (b/4)^{1/2}[1 - \nu^2]^{1/2}$. It might appear arbitrary that we have assigned our distance at one standard deviation *beyond* the peak rather than on the other side. However, we have at this stage no way to assign a significance to the sign of the velocity, and must choose this convention so that in the special case when $\nu = 0$, the distance $(b/4)^{1/2}$ is positive, because a negative distance in this case would be meaningless.

Accepting our definition of distance (so far in dimensionless form), our next step is to recognize that, although Program Universe defines all events in a unique way in a "coordinate system" which, when we turn to cosmology, will be identified with the frame at rest with respect to the 2.7^oK background radiation, we cannot (and probably never will be able to) define any laboratory procedure of sufficient precision to uncover directly the digital character of the bit strings. Hence we must content ourselves with constructing relative coordinates based on relative velocities. Thus, if we ask what is the distance interval ξ' in which the two events have zero relative velocity we must have that $\xi'_1 = \sigma(0, b) = (b/4)^{1/2} = (\xi - b\nu)/[1-(\nu)^2]^{1/2}$. If we define this coordinate system to have velocity $-\nu$ relative to the first, by reversing the argument we must have that $\xi = (\xi' + \nu b')/[1 - \nu^2]^{1/2}$, where we have made use of the fact that in this system the number of steps can be different and must be called b'. Now, by simple algebra, we can solve for b and b' to obtain $b = (\xi' + \nu b')/[1 - \nu^2]^{1/2}$; $b' = (\xi - \nu b)/[1 - \nu^2]^{1/2}$. In this way we claim to have derived a digital version of the Lorentz transformation for the interval between two events in our bit string universe, so far for a 1+1 Minkowski "space-time".

The next step is to note that at this stage the hierarchy construction has given us only four classes of labels, so we can have at most four different types of label L_3 occuring as the intermediate link in four events. Since we have by now a metric space, and the Lorentz transformation must apply to any interval connecting two events, we can from four events, in the general case, proceed immediately to the construction of our digital version of 3+1 Minkowski"space-time". Hence for us the hierarchy construction forces us to recognize that the basic space of description has to be three dimensional. Further, since the vertices in this basic tetrahedron are labeled, we will have a choice between two chiral alternatives, or in the language of chemistry, *stereo isomers*. Hence, once we can relate this to a basic asymmetry in the labels, or quantum numbers, related to scattering events (a much more complicated concept than the events so far considered) we can expect our interactions to have chiral properties, as has indeed been found experimentally. We find this basic argument for the three dimensionality of space a very satisfactory consequence of our approach. Once these details are worked out we must from now on assign three ensembles to each label, defining the components of a vector velocity $\vec{v}$ in the appropriate contexts.

We introduce our connection to physics by assuming that when we have two well separated counters of finite volume $\Delta x \Delta y \Delta z$ with a distance S between them greater than their spacial resolution which fire sequentially with a time interval T greater than their time resolution Δt that they define a velocity $v = S/T$ for some *object* which passed between them. Further we assume that the probability of the counters firing can be connected, statistically, to the events in the bit string universe. We now claim that the velocity c can be given laboratory significance and that we can take the Lorentz transformations derived above over into standard laboratory practice. That the uncertainty associated with the relativistic "wave packet spreading" will not trouble us directly is obvious, since it is scaled by Compton wavelength, and even the much larger Schrödinger spreading has never been measured experimentally. Further, since we develop below the usual connection between coordinate and momentum space via Fourier transformation and we have the usual Lorentz contraction of distances, the usual arguments for QED being tested down to $\approx 10^{-16} cm$ and the evidence for partons and quarks will survive in our theory in spite of our finite step lengths. However, in Chapter 5, we will see that this basic discreteness in our model will allow indirect confirmation in that it will enable us to understand the successful calculation of m_p/m_e achieved by Parker-Rhodes using a different starting point[8].

We now introduce dimensional units in the physical sense by identifying the random walk step length with the Compton wave length in the coordinate system in which two connected events have zero velocity and by postulating that the corresponding mass parameter is associated with one of our labels, which was the critical step taken by Stein. However, our treatment departs from his in that our basic counter paradigm compels us to see this length is Lorentz contracted in moving coordinate systems whereas he used it as a basic dimensional parameter. Our approach enables us to define relativistic energy and momentum for free particles correctly connected to the velocities we have already constructed by defining two new dimensional quantities through the basic relations $E^2 - p^2c^2 = m^2c^4$and $\vec{p} = m\vec{v}/[1-(v/c)^2]^{1/2}$. Then, since as already noted, our step length is $\ell = \ell_0[1-(v/c)^2]^{1/2} = (h/mc)[1-(v/c)^2]^{1/2}$ and our basic quantization is $\ell = hc/E$; we also have a second length $\lambda = h/p$ which will be discussed below. We now claim to have constructed a discrete version of classical relativistic particle kinematics for which the conventional continuum theory is a useful approximation.

4. CONSTRUCTING QUANTUM PARTICLES and SCATTERING THEORY

Returning to the bit string universe, all we have so far is that when two counters separated by a macroscopic space and time interval larger than the volumes and time resolutions of the counters have fired, some random walk connecting those two volumes has occured. We call the labeled ensemble connecting two events an *object*. But we do not know within those macroscopic volumes where this random walk started and ended. To meet this problem, we construct an ensemble of objects (which are themselves ensembles) all characterized by the same vector velocity $\vec{v}$ and the same label (or mass) chosen in such a way that, after k steps, each of length $l = (h/mc)[1-(v/c)^2]^{1/2}$, the peak of the random walk distribution will have moved a distance l in the direction of $\vec{v}$. We take as our unit of time the time to take one step, $\delta t = l/c$. Once "time" is understood in this digital sense the peak of each subensemble in this coherent ensemble has a velocity c/k. We call this *coherent* ensemble of ensembles a *free particle* of mass m, velocity $\vec{v}$, and momentum $\vec{p} = m\vec{v}/[1-(v/c)^2]^{1/2}$. We assume that the size of the counter Δz in this direction and in the plane perpendicular to this direction is so large that we can ignore end effects.

There is a second "velocity" associated with this ensemble of ensembles, namely that with which something moves at each step always in the direction $\vec{v}$. We call this the *phase velocity* $v_{ph} = kc$; hence $vv_{ph} = c^2$. Associated with each of the two velocities and the label (or mass) there is a characteristic length $\lambda_{ph} = l = hc/E; \lambda = kl = h/p$. Our next step is to show that these coherent ensembles of ensembles have *experimental* consequences that can be exemplified in the laboratory.

We now consider our coherent ensemble of ensembles specified by $\vec{v}$ and m incident on a "screen" perpendicular to $\vec{v}$ made of absorbers containing two holes (or slits in the two dimensional approximation in which the distances perpendicular to the line between the holes and to $\vec{v}$ are so large as not to produce appreciable end effects) a distance d apart. This is all well and good in the laboratory where we can established the meaning of absorbers by showing that they prevent counters from firing. In the bit string universe the absorbers can be thought of as containing so many events that their consequences are so diffuse as not to affect the progress of the experiment. Our coherent ensemble will pass through these two holes dividing into two subensembles without loosing its coherent properties. Thus we are led to the same conclusion as the wave theory when it is analyzed in this way[9] even though we have used a digital basis.

At some large distance D behind the screen we set up a counter array in a plane perpendicular to $\vec{v}$. We further assume that the source is a distance S on the other side of the array, and is equipped with a counter which fires when the particle leaves the source. Calling the time interval between when source and detector fire T, the velocity between source and detector is $v = (D+S)/T$. By making D and S large enough, and assuming that the source has a velocity spectrum which includes v, we can select in this way particles whose v is as precisely known as we like[9]. This step is necessary to insure that *all* elements in the coherent ensembles we consider have the same v to requisite precision; only such pairs of events will provide data for the experiment. Then, on a plane perpendicular to the center line of the slits at distance D, the coherent ensembles will have their maxima coincide, and hence counters be most likely to fire, at positions away from the center line given by $x_n = n\lambda D/d$, where d is the distance between the slits.

We now claim to have shown that our bit string universe contains something related to "deBroglie wave interference", and that by defining velocities and counting maxima under appropriate circumstances, we can measure h, which we are now justified in identifying with Planck's constant. We have also derived the deBroglie wave length and the relativistic phase wave length he introduced. Hence in the limit of negligible mass, we have the basic Einstein-Planck quantization condition $E = hc/\lambda_{ph}$ as well. The fact that energy is quantized is thus, for us, a direct consequence of our digitized step length.

Having constructed our ensembles of ensembles corresponding to a unique value of p we now note that these cannot be used to meet our basic counter boundary condition at $t = 0$, which confines the initial event to a finite space time volume. For this purpose we must make a superposition of these ensembles of ensembles for different values of p weighted by a function which must have both positive and negative values. Mathematically expressed this boundary condition is $\int_{-\infty}^{+\infty} dpf(p)\ \Sigma_{n=-N}^{+N}\delta(z+n\lambda) = \Theta(z-\Delta z)-\Theta(z+\Delta z)$. For the wave theory, ~~the~~ boundary condition[9] is $\int_{-\infty}^{+\infty} dpf(p)e^{ipz} = \Theta(z-\Delta z)-\Theta(z+\Delta z)$. Therefore by Fourier inversion $(1/2\pi)\int_{-\infty}^{+\infty} dze^{ip'z}\int_{-\infty}^{+\infty} dpf(p)e^{ipz} = \int_{-\infty}^{+\infty} dp\delta(p-p')f(p')$ and hence $f(p') = (1/2\pi p')[e^{ip'\Delta z} - e^{-ip'\Delta z}] = (i/\pi p')sin(p'\Delta z)$ But the *mathematical* operation of Fourier inversion can just as well be applied to the digital as to the wave boundary condition. Doing so, we recover the conventional result plus correction terms of order $(1/N)$. To extend our discussion to time dependent deBroglie waves we need only represent the bit string ensemble by $\delta(z+n\lambda - ct/\lambda_{ph}) = \delta[(pz+nh-Et)/h]$ We therefore claim to have *derived* wave mechanics as an *approximation* to our digital model in a form (laboratory boundary conditions based on counters of finite macroscopic size) which will serve for most of the practical applications of scattering theory. Further, we can now derive the Heisenberg uncertainty relations for continuum variables in the usual way. Thus we claim to have proved that we have constructed free particle quantum wave mechanics on a digital basis as an *approximate* theory. Finally, we see that the amplitude $f(p)$ must contain negative as well as positive values, and hence that we must take the *squares* of amplitudes, appropriately weighted to conserve flux, in order to make contact with the laboratory paradigm taken from physical optics and here extended to matter waves.

Since we now have standard relativistic particle wave mechanics for free particles, it would seem that we could now develop scattering theory in a conventional way. This true up to a point, but there is a critical conceptual difference. We have no Hamiltonian, so we *cannot* calculate scattering amplitudes as the matrix elements of such an operator between appropriate scattering states. This problem was met some time ago[10] by constructing a "Democritean scattering theory" starting from free particle wave functions and arriving at the standard Goldberger-Watson wave function[11] for N_A particles in and N_B particles out. The essential point is that the scattering amplitude then becomes a *kinematic* quantity describing any conceivable experiment of this type, including those which do not conserve flux. Then we are under the obligation of supplying *dynamical* equations for this amplitude which guarantee flux conservation, or in technical terms are unitary. Since this theory has been developed elsewhere[12–14] and its connection to the bit string universe discussed in more detail in Ref.1, we will be very brief here.

The basic process which drives the integral equations of the theory is given by a scattering amplitude describing the process in which two particles with masses m_1, m_2 and momenta $\vec{k}_1, \vec{k}_2$ coalesce to form a state of mass μ and momentum $\vec{k} = \vec{k}_1 + \vec{k}_2$ and then come apart with momenta $\vec{k}'_1, \vec{k}'_2$ conserving total momentum, i.e. $\vec{k}_1 + \vec{k}_2 = \vec{k}'_1 + \vec{k}'_2$. Although total momentum is conserved, the mass restriction to μ does not, in general, allow energy to be conserved in the intermediate state. Taking $\hbar = 1 = c$ as is conventional, in the zero momentum coordinate system where the relative coordinate between the two particles is r this scattering amplitude describes a bound state with binding energy $\kappa = m_1 + m_2 - \mu$ and wave function in configuration space(ignoring relativistic factors) proportional to $e^{-\kappa r}/r$; the particles scatter with a probability amplitude proportional to this wave function, so this model generates a "short range interaction". This, of course, simply represents the uncertainty principle in energy, which as Wick saw long ago[15] is the simplest way to understand the origin of short range forces in relativistic quantum mechanics. In momentum space the corresponding relativistic factor is $[\epsilon'_1 + \epsilon'_2 - \mu - i0^+]^{-1}$, where $\epsilon_i = [m_i^2 + k_i^2]^{1/2}$. The scattering is also proportional to the square of a *coupling constant* $g^2/\hbar c$. Putting this together, the basic scattering amplitude for the theory is

$$T(\vec{k}_1, \vec{k}_2; \vec{k}'_1, \vec{k}'_2) = g^2 f(\vec{k}_1, \vec{k}_2) f^*(\vec{k}'_1, \vec{k}'_2) \delta^3(\vec{k}_1 + \vec{k}_2 - \vec{k}'_l - \vec{k}'_2)/[\epsilon'_1 + \epsilon'_2 - \mu - i0^+] \tag{4.1}$$

Note that, as promised, the basic theory guarantees momentum conservation and hence allows us to measure mass ratios. The functions f are known as form factors and can be used to represent internal structure in the bound state wave function. In a minimal theory without internal structure they can be determined in terms of the masses by the requirements of relativistic invariance and unitarity or flux conservation[14].

To go from this scattering amplitude to the wave function in momentum space which in configuration space will represent a radially outgoing wave with asymptotic energy as well as momentum conservation we simply repeat the uncertainty principle argument and arrive at

$$\psi^+(\vec{k}_1,\vec{k}_2,\vec{k}'_1,k'_2) = \epsilon_1\epsilon_2\delta^3(\vec{k}_1-\vec{k}'_1)\delta^3(\vec{k}_2-\vec{k}'_2) - T/[\epsilon'_1+\epsilon'_2-\epsilon_1-\epsilon_2-i0^+] \quad (4.2)$$

The theory then goes on to sum sequences of scattering processes using relativistic Faddeev-Yakubovsky integral equations which by their structure guarantee flux conservation.

Our task now is to show that this theory can be derived from the bit string universe developed above. We start from the labels of the two initial states, restricted for simplicity to level 1 of the hierarchy and taken to be (10) and (01). Then the label content of the process just described in more conventional terms is simply $(10)+(01) \to (11) \to (01)+(10)$. As has been noted before[3] this simplest example of discrimination can be thought of, following Fermi and Yang[16], as a particle and an antiparticle forming a bound state which is to be thought of as a quantum. It also is the simplest example of an *event* in the bit string universe. If we think of particles and antiparticles as carrying a signed dichotomous quantum number, we see that this interpretation conserves the number of particles minus the number of antiparticles, which provides us with our first discrete (quantum number conserving) conservation law.

We have already seen that our experimental access to the bit string universe does not allow us to assign a unique integer to the event, and that we must use statistical arguments to connect events. We now extend this idea to connecting discriminations such as $D_2(10)(01) = (11)$ which occur *between* ticks but which are separated by several ticks. This gives us two vertices connected by a ***propagator*** and is our version of an elementary Feynman diagram for scattering. If the number of ticks is small enough, our experimental contact, which is asymptotic, cannot differentiate the elementary event from two discriminations separated by these ticks. So we extend our concept of event to a ***scattering amplitude*** which is the sum of these possibilities, appropriately weighted. Relying on the metric connection we have made (approximately via the wave theory) between energy and distance, we assume that this weighting factor will be inversely proportional to the distance, or in momentum space inversely proportional to the difference in energies and hence to $1/[\epsilon' - \epsilon]$.

This creates a new problem. We are summing over discrete rather than continuous energies; hence these energies could coincide creating an infinity which violates our basic finite philosophy. However, our minimum step is $\delta t = \ell/c$ and the quantization condition is $E = hc/\ell$. We see that the minimum energy step $\delta E = h/\delta t$ and hence that for a spread in energy δE and time δt we have that $\delta E \delta t \geq h$. We emphasize that this is *not* the Heisenberg uncertainty principle, which we have already seen comes in a conventional way from limitations on measurement due to finite counter size. Therefore we argue that the best way to meet the difficulty is to use $1/[E' - E - i\delta E]$ where we have used the imaginary factor i as the simplest way to insure that no infinity can occur. This argument for complex amplitudes can be reinforced by appeal to the scattering theory where they are needed to express flux conservation in the asymptotic region. We go over to the the continuum momentum space scattering theory in the limit $(i\delta E \rightarrow i0^+)$ consistent with our dropping of 1/N terms. If this argument is accepted for the scattering amplitude it carries over to the scattering wave function itself. We have now made the connection between conventional scattering theory and our construction, and can proceed to N particle scattering theory along the lines previously developed.

5. PARTICLE IDENTIFICATION,MASS SCALE and COSMOLOGY

We have now seen that our construction gives a complete *phenomenological* theory for relativistic N-particle scattering if we supply the masses and coupling constants from experiment. We took care in our original construction to show that the *label-address* schema was sufficient to construct the approximate theories of relativistic particle mechanics and relativistic quantum scattering theory *without* specifying the interpretation of the labels, a problem to which we now turn.

In our treatment of the scattering theory in the last section we made a start by showing how to interpret level 1 labels as referring to particle-antiparticle scattering as going through a composite quantum state. The same approach also can be applied to quantum-particle scattering, but we have not space here to develop the argument. The clue which led us to this interpretation, and which is discussed in earlier work[1,3], is that the mapping which leads from level 1 to level 2 necessarily leads to doubled descriptors in the new basis, for example $a = (1110), b = (1101), c = (1100)$. But in spite of the ambiguity in choice of representation (a problem still under study) any choice ends with the same maximal DCsS, namely $\{(0001), (0010), (0011), (1100), (1101), (1110), (1111)\}$. This suggested that, at least from level 2 on, the basis vectors of the hierarchy are *quanta*, interpreted as composites of particles and antipartices, but carrying additional quantum numbers such as charge, and that strings of the same length *outside* this representation of the hierarchy referred to particles or particulate systems. We try to make this guess more systematic in what follows.

In mapping level 2 to level 3, we first note that the three basis strings are of the form (11yz), which guarantee that the seven strings in the maximal DCsS are all of the form (wwyz). In contrast, the eight remaining possible non-null strings are of the form (wxyz) with $w \neq x$. Writing the mapping as $(ABCD, EFGH, IJKL, MNOP) \times (wxyz) = (Aw + Bx + Cy + Dz, Ew + Fx + Gy + Hz, Iw + Jx + Ky + Lz, Mw + Nx + Oy + Pz)$ we can see that the only way to have only the first seven strings as eigenvectors and exclude the other eight is to use mapping matrices of the form $(0100, 1000, IJKL, MNOP)$. Thus we again necessarily have a doubled descriptor, allowing us to continue our interpretation of quanta as composites of a particle-antiparticle pairs on to level 3. More detailed examination of the problem leads to the conclusion[1] that all of the eight remaining entries must be non-null in one or another of the seven mapping matrices. This suggests that up to level 3 all three level levels can be represented by a string of 16 bits, the first two referring to level 1, the next four to level 2, and the last 10 to level 3. The next problem is to interpret these sixteen slots as referring to dichotomous quantum numbers which will describe some of the basic elementary particle states. A tentative attempt to do this, which gives two component neutrinos at level 1, leptons, em quanta and weak vector bosons when this is combined with level 2, and $SU(3)$ baryons and mesons at level 3 has been given in Ref.1. Since we are no longer confident of the details of this scheme, we will not present it here.

Our final step is to use the evaluation of the charge on the electron as $e^2 = \hbar c/137$ to calculate the fundamental mass ratio between leptons and baryons by computing the mass of the electron from its electromagnetic interaction. Parker-Rhodes[8] started from a very different construction of space time and the combinatorial result; we provide here a modification of our previous discussion of this calculation[1,3]. Taking as our basic mass the baryon mass m_B (*because* of the connection to the gravitational constant G) the minimum distance we can assign to the diameter of this system in a rest system is h/m_Bc. We therefore scale the minimum distance distance we can assign to any happening measured from the center of this system by $r = (h/2m_Bc)y, 1 \leq y < \infty$. The charge in the lepton must separate by more than r into two lumps which by charge conservation we can write in terms of a dimensionless parameter x as ex and $e(1-x)$, where x is a statistical variable reflecting the fact that we have both charged and neutral leptons and baryons. Hence $< e^2/r >= (hc/2\pi \times 137) < x(1-x) > (2m_B/h) < 1/y >= m_lc^2$ and $m_B/m_l = 137\pi/ < x(1-x) >< 1/y >$.

Our basic identification of the statistics underlying our dynamics as due to random walks is now invoked to calculate the expectation value $<1/y>$. Since we have now established our space as necessarily three-dimensional, the discrete steps in y must each be weighted by (1/y) with three degrees of freedom. [Note that this is consistent with our previous use of the weighting factor $1/(E - E' - i0+)$ in momentum space.] Hence $< 1/y >= [\int_1^\infty (1/y)^4 dy/y^2]/[\int_1^\infty (1/y)^3 dy/y^2] = 4/5$. Since the charge must both separate and come together with a probability proportional to $x(1-x)$ at each vertex, the other weighting factor we require is $x^2(1-x)^2$. For one degree of freedom this would give $< x(1-x) >= [\int_0^1 x^3(1-x)^3 dx]/[\int_0^1 x^2(1-x)^2 dx] = 3/14$. Once the charge has separated into two lumps each with charge squared proportional to x^2 or $(1-x)^2$ respectively, we can then write a recursion relation[1,3,8] $K_n = [\int_0^1[x^3(1-x)^3+K_{n-1}x^2(1-x)^4]dx]/[\int_0^1 x^2(1-x)^2dx]$ and hence $K_n = 3/14 + (2/7)K_{n-1} = (3/14)\Sigma_{i=0}^{n-1}(2/7)^i$. Therefore, invoking again the three degrees of freedom, we must take $< x(1-x) >= K_3$ and we obtain the Parker-Rhodes result $m_B/m_l = 137\pi/[(3/14)[1+(2/7)+(2/7)^2](4/5)] = 1836.151497....$ Since the electron and proton are stable for at least 10^{31} years we identify this ratio with m_p/m_e in agreement with the experimental value 1836.1515 ± 0.0005, thus setting the basic mass ratio scale for the theory. Whether this mass ratio remains unchanged when we go on to level 4 and we must show how to calculate the masses of *unstable* baryons and bosons from our dynamical theory is under investigation.

As already noted, the absolute unit of mass in the theory must be approximately the proton mass because of our identification of $2^{127}+136$ with the inverse gravitational coupling constant. Since the calculation given above is a mass ratio, its success is independent of the absolute value of this unit. The corrections which take us from our single dimensional mass parameter m_B to the empirical value for the proton mass, given G (or equivalently to the empirical value for G, given m_p) and to the empirical value of the fine structure constant will have to come from level four of the theory, where we must also find a place for the equivalent of quarks and heavy leptons. Since we will then have 256 quantum numbers to play with, this will be challenging but not obviously impossible. Other problems, such as building up the electromagnetic field from our photons and the gravitational field from gravitons (we can obviously make the latter – so far as quantum numbers go – from leptons as spin 2 helicity states) is similar to that of any theory which starts from the weak coupling limit, and might even have advantages since we cannot encounter the infinities which plague conventional continuum approaches to this problem[17].

The reader immersed in special relativity may be troubled by the ticking universe, which provides a universal time and would seem to single out a particular coordinate system. We have been led to the construction which identified unique events with TICKs because we cannot allow our events to have a continuum limit in *points*; else we would get back to the agony of infinite energy at each point, which it has taken so much hard technical work for quantum field theory to deal with. *Our* "virtual" processes occur in the "void" as finite fluctuations which cannot be directly accessed by experiment. We claim this is a strength rather than a weakness. As to the special coordinate system, we claim to have shown that we can still define macroscopic velocities v to arbitrary precision, and derive the Lorentz transformation, thus recovering special relativity as a *macroscopic* approximation. As to the special coordinate system we claim that empirically there *is* such a coordinate system which defines $v = 0$ by the 2.7^oK background radiation. This is no more an embarrassment for us than for special relativity; the fact that it occurs so naturally in our theory we again count as a strength rather than a weakness. Although we have a special frame for *velocities*, our construction does not allow us to attach any significance to any particular choice for the origin of spacial coordinates. Any point will serve as the "center" of the universe, in agreement with the cosmological principle. Further, we have an event horizon defined at each address length N by the strings I_N and 0_Nwhich refer to to systems which have suffered no velocity-changing scatterings from the beginning of the construction; since they cannot be assigned any direction in 3-space, this horizon is isotropic.

Clearly we still have to show that we can get the particle physics right, and then go on to show that the big bang emerges from our initial generation operations. This is a problem for future research. We are encouraged by the fact that we have only one type of mass in the theory, and in that sense have no place for a difference between gravitational and inertial mass. Further, if we do indeed succeed in getting spin 2 gravitons in the weak coupling limit, we can hope to recover gravitational theory from that starting point, a problem already discussed by Weinberg[18]. As to the big bang itself, scattering events labeled by the full level 4 quantum number scheme can only start when the 256 bit hierarchy scheme closes off and we have $2^{256} - 1$ labels in the universe. If we can get our microphysics right, this is a reasonable estimate for the baryon number and lepton number of the universe.

Our final point is that by focusing on *velocity* rather than space and time as basic we believe we have the correct fundamental starting point for unifying macroscopic quasi-continuous measurement with a digital model, a point of view already stressed by S-matrix theorists. Further, our ticking universe allows us to fuse the special relativistic concept of *event* with the unique and indivisible events of quantum mechanics. Whatever else survives from this attempt to construct a digital model for the universe, we are convinced that this is the correct place to connect relativity with quantum mechanics in a fundamental way. We close by remarking that the cosmological implications of the model are not in obvious conflict with experience.

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