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PARTIAL BREAKING OF EXTENDED SUPERSYMMETRY*

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Abstract

In this letter we use nonlinear realizations to examine the breaking of $N = 2$ extended supersymmetry to $N = 1$. We derive Lagrangians and transformations laws for the generalized $N = 2$ Akulov-Volkov goldstino field. We analyze the ghost states and show that they may be collected into $N = 1$ supermultiplets. We extend the transformation laws for the $N = 1$ chiral and vector multiplets to $N = 2$. Finally, we give the $N = 2$ generalization of an arbitrary $N = 1$ supersymmetric Lagrangian.

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Within the last few years, many models have been proposed in which $N = 1$ supersymmetry is spontaneously broken at some mass scale M_1 smaller than the Planck mass M_P. It is natural to ask whether such models can arise from a theory with extended supersymmetry, where the extra supersymmetries are first broken to $N = 1$ at some new mass scale $M_2 > M_1$. A model - independent way to investigate this question is through the use of nonlinear realizations [1]. In this letter, we use nonlinear realizations to show that any $N = 1$ supersymmetric theory (with chiral and vector superfields) may be obtained as the "low energy" limit of a corresponding $N = 2$ theory. Our method may be trivially extended to higher N as well.

This letter is organized as follows. We first derive transformation laws and Lagrangians for the generalized $N = 2$ Akulov-Volkov goldstino field. We find that the Akulov-Volkov Lagrangians contain ghost fields. This is, of course, expected from a general argument based on the supersymmetry algebra [2]. We construct the Fock space for the Akulov-Volkov fields, and show that the $N = 2$ ghost states may be collected into $N = 1$ supermultiplets. We then demonstrate how the transformation laws for the $N = 1$ chiral and vector multiplets may be extended to $N = 2$ with the help of the Akulov-Volkov fields. We conclude by giving the $N = 2$ generalization of an arbitrary $N = 1$ supersymmetric Lagrangian.

The formalism associated with nonlinear realizations is well known for the case of $N = 1$ supersymmetry [3-8]. The transformation law for the Goldstone spinor was first found by Akulov and Volkov [3]:

$$\delta_\xi \tilde{\lambda}_\alpha \quad = \quad \frac{1}{k}\xi_\alpha - ik\Big(\tilde{\lambda}\sigma^m\bar{\xi} - \xi\sigma^m\bar{\tilde{\lambda}}\Big)\partial_m\tilde{\lambda}_\alpha \ . \tag{1}$$

The coefficient $k^{-\frac{1}{2}}$ denotes the scale of supersymmetry breaking.* A somewhat simpler transformation law was used in Refs. [5, 6]:

$$\delta_\xi\lambda_\alpha \quad = \quad \frac{1}{k}\xi_\alpha - 2ik\Big(\lambda\sigma^m\bar{\xi}\Big)\partial_m\lambda_\alpha \ . \tag{2}$$

The fields λ and $\tilde{\lambda}$ are related as follows

$$\lambda_\alpha(x) \quad = \quad \tilde{\lambda}_\alpha(y) \ , \tag{3}$$

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* We use the conventions of Ref. [9].

where

$$y^m = x^m - ik^2\left(\tilde{\lambda}(y)\sigma^m \bar{\tilde{\lambda}}(y)\right) \quad . \tag{4}$$

This implicit definition may be rendered explicit by expanding (and inverting) Eq. (3) [5]:

$$\begin{aligned}
\lambda_\alpha &= \tilde{\lambda}_\alpha - i\Big[\, \tilde{v}^m - i\,\tilde{v}^n\,\partial_n\,\tilde{v}^m \\
&\quad - \tilde{v}^n\,\partial_n\,\tilde{v}^\ell\,\partial_\ell\,\tilde{v}^m - \frac{1}{2}\tilde{v}^n\,\tilde{v}^\ell\,\partial_n\partial_\ell\,\tilde{v}^m\,\Big]\,\partial_m\,\tilde{\lambda}_\alpha \\
&\quad - \frac{1}{2}\,\tilde{v}^m\,\tilde{v}^n\,\partial_m\partial_n\,\tilde{\lambda}_\alpha \\
\tilde{\lambda}_\alpha &= \lambda_\alpha + i\Big[\, v^m + ik^2 v^n\Big(\partial_n\lambda\sigma^m\,\bar{\lambda} - \lambda\sigma^m\partial_n\,\bar{\lambda}\Big) \\
&\quad + v^n\partial_n v^\ell\,\partial_\ell v^m + \frac{1}{2}\,v^n v^\ell\,\partial_n\partial_\ell v^m\Big]\,\partial_m\lambda_\alpha \\
&\quad - \frac{1}{2}\,v^n v^m\,\partial_n\,\partial_m\lambda_\alpha \quad ,
\end{aligned} \tag{5}$$

where $\tilde{v}^m = k^2\,\tilde{\lambda}\,\sigma^m\,\bar{\tilde{\lambda}}$ and $v^m = k^2\lambda\sigma^m\,\bar{\lambda}$. In Eq. (5), all fields are functions of x^m.

If λ_α and $\tilde{\lambda}_\alpha$ are promoted to $N = 1$ superfields Λ_α and $\tilde{\Lambda}_\alpha$, and ξ_α is replaced by ${\xi_\alpha}^{(2)}$, the transformation laws (1) and (2) realize the $N = 2$ supersymmetry algebra (without a central charge). The superfields Λ_α and $\tilde{\Lambda}_\alpha$ are the Goldstone superfields associated with breaking $N = 2$ to $N = 1$ supersymmetry. Because of the unbroken supersymmetry, the Goldstone spinors λ_α and $\tilde{\lambda}_\alpha$ are members of $N = 1$ supermultiplets.

An advantage of the transformation law (2) is that the $N = 1$ superfield Λ_α may be constrained to either be chiral $\bar{D}\Lambda = 0$ or antichiral $D\Lambda = 0$. To distinguish the two cases, we shall call the antichiral superfield X_α, $DX = 0$.

To derive the Lagrangian for A_α and X_α, we follow the method of Ref. [5]. We first construct the $N=2$ superfields associated with the transformation laws (1) and (2) [5,10]:

$$
\begin{aligned}
\tilde{\boldsymbol{A}}_\alpha\Big(x,\theta^{(1)},\bar{\theta}^{(1)},\theta^{(2)},\bar{\theta}^{(2)}\Big) &= exp\left[\delta_{\theta(2)}\right] \times \tilde{A}_\alpha\left(x,\theta^{(1)},\bar{\theta}^{(1)}\right) \\
\boldsymbol{A}_\alpha\Big(x,\theta^{(1)},\bar{\theta}^{(1)},\theta^{(2)},\bar{\theta}^{(2)}\Big) &= exp\left[\delta_{\theta(2)}\right] \times A_\alpha\left(x,\theta^{(1)},\bar{\theta}^{(1)}\right) \quad . \\
\boldsymbol{X}_\alpha\Big(x,\theta^{(1)},\bar{\theta}^{(1)},\theta^{(2)},\bar{\theta}^{(2)}\Big) &= exp\left[\delta_{\theta(2)}\right] \times X_\alpha\left(x,\theta^{(1)},\bar{\theta}^{(1)}\right)
\end{aligned}
\tag{6}
$$

These superfields may also be defined through constraints:

$$
\begin{aligned}
D_\beta{}^{(2)}\,\boldsymbol{A}_\alpha &= \frac{1}{k}\,\epsilon_{\alpha\beta} \qquad\qquad \bar{D}_{\dot{\alpha}}{}^{(1)}\,\boldsymbol{A}_\beta = 0 \\
\bar{D}^{\dot{\beta}\,(2)}\,\boldsymbol{A}_\alpha &= 2ik\,(\bar{\sigma}^m\,\boldsymbol{A})^{\dot{\beta}}\,\partial_m\,\boldsymbol{A}_\alpha \\
D_\beta{}^{(2)}\boldsymbol{X}_\alpha &= \frac{1}{k}\,\epsilon_{\alpha\beta} \qquad\qquad D_\alpha{}^{(1)}\,\boldsymbol{X}_\beta = 0 \\
\bar{D}^{\dot{\beta}\,(2)}\,\boldsymbol{X}_\alpha &= 2ik\,(\bar{\sigma}^m\,\boldsymbol{X})^{\dot{\beta}}\,\partial_m\boldsymbol{X}_\alpha \quad .
\end{aligned}
\tag{7}
$$

The constraints for $\tilde{\boldsymbol{A}}_\alpha$ may be worked out with the help of Eq. (5) [5]. The constraints (7) are consistent with the $N=2$ D-algebra [11],

$$
\begin{aligned}
\left\{D_\alpha{}^{(A)},\,\bar{D}_{\dot{\beta}}{}^{(B)}\right\} &= -2i\,\delta^{AB}\,\sigma^m_{\alpha\dot{\beta}}\,\partial_m \\
\left\{D_\alpha{}^{(A)},\,D_\beta{}^{(B)}\right\} = \left\{\bar{D}_{\dot{\alpha}}{}^{(A)},\,\bar{D}_{\dot{\beta}}{}^{(B)}\right\} &= 0 \quad .
\end{aligned}
\tag{8}
$$

From these superfields we can construct invariant Lagrangians for the Akulov-Volkov field:

$$-\frac{1}{2}k^2\int d^4xd^4\theta d^4\bar{\theta}\,\boldsymbol{\Lambda\Lambda\bar{\Lambda}\bar{\Lambda}} \;=\; \int d^4xd^2\theta^{(1)}d^2\bar{\theta}^{(1)}\Big[-\frac{1}{2k^2}-i\Lambda\sigma^m\partial_m\bar{\Lambda}+\ldots\Big] \quad (9a)$$

$$-\frac{1}{2}k^2\int d^4xd^4\theta d^4\bar{\theta}\,\boldsymbol{XX\bar{X}\bar{X}} \;=\; \int d^4xd^2\theta^{(1)}d^2\bar{\theta}^{(1)}\Big[-\frac{1}{2k^2}-iX\sigma^m\partial_m\bar{X}+\ldots\Big] \quad (9b)$$

$$-\frac{1}{2}k^2\int d^4xd^4\theta d^2\bar{\theta}^{(2)}\,\boldsymbol{\Lambda\Lambda\bar{X}\bar{X}}+h.c. \;=\; \int d^4xd^2\theta^{(1)}\Big[-\frac{1}{2k^2}-i\Lambda\sigma^m\,\partial_m\bar{X}+\ldots\Big]+h.c. \quad (9c)$$

The triple dots $(\ldots)$ stand for nonrenormalizable interactions, suppressed by powers of k. Note that the last Lagrangian is $N=1$ chiral. The integral over $d^2\theta^{(1)}d^2\bar{\theta}^{(1)}$ annihilates the constants, so there is no $N=1$ supersymmetry breaking for any of the three cases.

Before exhibiting the ghost structure, we first expand the $N=1$ superfields Λ_α and X_α in terms of component fields,

$$\begin{aligned}\Lambda_\alpha(y,\theta^{(1)}) &= \lambda_\alpha(y)+F_\alpha{}^\beta(y)\,\theta_\beta{}^{(1)}+\phi_\alpha(y)\,\theta^{(1)}\theta^{(1)}\\ \bar{X}_{\dot{\alpha}}(y,\theta^{(1)}) &= \bar{\chi}_{\dot{\alpha}}(y)+V_n(y)\,\theta^{\alpha\,(1)}\,\sigma^n_{\alpha\dot{\alpha}}+\bar{\psi}_{\dot{\alpha}}(y)\,\theta^{(1)}\theta^{(1)}\quad,\end{aligned} \quad (10)$$

where $y^m \;=\; x^m+i\theta^{(1)}\sigma^m\bar{\theta}^{(1)}$. The Lagrangians $(9a)$ - $(9c)$ become

$$\int d^4x\Big[-i\phi\sigma^m\partial_m\bar{\phi}-i\lambda\sigma^m\partial_m\Box\bar{\lambda}-\frac{1}{2}F^{*\,\dot{\alpha}}{}_{\dot{\beta}}\,\bar{\sigma}^{m\dot{\beta}\beta}\,\partial_m\partial_nF_\beta{}^{\alpha}\sigma^n_{\alpha\dot{\alpha}}+\ldots\Big] \quad (11a)$$

$$\int d^4x\Big[-i\psi\sigma^m\partial_m\bar{\psi}-i\chi\sigma^m\partial_m\Box\bar{\chi}-V^*_m\Big(\Box V^m-2\partial^m\partial\cdot V\Big)+\ldots\Big] \quad (11b)$$

$$\int d^4x\Big[-i\phi\sigma^m\partial_m\bar{\chi}-i\psi\sigma^m\partial_m\bar{\lambda}-\frac{i}{2}(\sigma^n\bar{\sigma}^m)_\alpha{}^\beta\,V_n\partial_mF_\beta{}^\alpha+\ldots\Big]+h.c. \quad (11c)$$

The first two Lagrangians have higher derivatives. Third is off-diagonal. All three contain ghosts.

The Lagrangians (11a) - (11c) are invariant under the following ($N = 1$) supersymmetry transformations:

$$\begin{aligned}
\delta_\xi \lambda_\alpha &= F_\alpha{}^\beta \xi_\beta \\
\delta_\xi F_\alpha{}^\beta &= 2\, \psi_\alpha \xi^\beta - 2i\, \partial_m \lambda_\alpha (\bar\xi \bar\sigma^m)^\beta \\
\delta_\xi \psi_\alpha &= -i\, \partial_m F_\alpha{}^\beta \sigma^m_{\beta\dot\beta}\, \bar\xi^{\dot\beta} \\
\delta_\xi \bar\chi_{\dot\alpha} &= \xi^\alpha \sigma^n_{\alpha\dot\alpha}\, V_n \\
\delta_\xi V^n &= -\, \xi \sigma^n \bar\phi \, + \, i\, \partial_m \bar\chi \bar\sigma^n \sigma^m \xi \\
\delta_\xi \bar\phi_{\dot\alpha} &= i (\bar\xi \bar\sigma^m \sigma^n)_{\dot\alpha}\, \partial_m V_n \quad .
\end{aligned} \tag{12}$$

These transformations also act on the states. They map physical states into physical states, and ghosts into ghosts. This is easiest to see for Lagrangian (11c). We shall examine it first.

To untangle the ghost structure, we canonically quantize the fields in (11c). The momentum conjugate to $F_\alpha{}^\beta$ is $-\frac{i}{2}(\sigma^n \bar\sigma^o)_\beta{}^\alpha\, V_n$, while $-i(\phi\sigma^o)_{\dot\alpha}$ and $-i(\psi\sigma^o)_{\dot\beta}$ are conjugate to $\bar\chi^{\dot\alpha}$ and $\bar\lambda^{\dot\beta}$. These lead to the following equal-time commutation relations:

$$\begin{aligned}
\Big[V^n(\boldsymbol{x},t),\; F_\alpha{}^\beta(\boldsymbol{y},t) \Big] &= -\,(\sigma^o \bar\sigma^n)_\alpha{}^\beta\, \delta^{(3)}(\boldsymbol{x}-\boldsymbol{y}) \\
\Big\{ \bar\chi_{\dot\alpha}(\boldsymbol{x},t),\; \phi_\alpha(\boldsymbol{y},t) \Big\} &= -\,\sigma^o_{\alpha\dot\alpha}\, \delta^{(3)}(\boldsymbol{x}-\boldsymbol{y}) \\
\Big\{ \bar\lambda_{\dot\alpha}(\boldsymbol{x},t),\; \psi_\alpha(\boldsymbol{y},t) \Big\} &= -\,\sigma^o_{\alpha\dot\alpha}\, \delta^{(3)}(\boldsymbol{x}-\boldsymbol{y}) \quad .
\end{aligned} \tag{13}$$

All other commutation relations vanish. As usual, the relations (13) can be extended to any two spacetime points,

$$
\begin{aligned}
\left[V^n(x),\ F_\alpha{}^\beta(y)\right] &= -\,(\sigma^m\bar\sigma^n)_\alpha{}^\beta\,\partial_m\,\Delta(x-y)\\
\left\{\bar\chi_{\dot\alpha}(x),\ \phi_\alpha(y)\right\} &= -\,\sigma^m_{\alpha\dot\alpha}\,\partial_m\,\Delta(x-y)\\
\left\{\bar\lambda_{\dot\alpha}(x),\ \psi_\alpha(y)\right\} &= -\,\sigma^m_{\alpha\dot\alpha}\,\partial_m\,\Delta(x-y)\ .
\end{aligned}
\tag{14}
$$

Here $\Delta(x)$ is the usual Lorentz-invariant commutator function normalized such that $(\partial/\partial x^o)\,\Delta(x) = \delta^{(3)}(\boldsymbol{x})$. The Fock space is constructed by expanding the fields in plane-wave solutions

$$
\begin{aligned}
F_\alpha{}^\beta(x) &= \int \frac{d^3\boldsymbol{p}}{(2\pi)^{3/2}}\left[G_\alpha{}^\beta(p)e^{ipx} + H^\dagger_{\dot\alpha}{}^\beta(p)e^{-ipx}\right]\\
V^n(x) &= \int \frac{d^3\boldsymbol{p}}{(2\pi)^{3/2}}\left[U^n(p)e^{ipx} + W^{n\dagger}(p)e^{-ipx}\right]\\
\phi_\alpha(x) &= \int \frac{d^3\boldsymbol{p}}{(2\pi)^{3/2}}\left[A_\alpha(p)e^{ipx} + B^\dagger_{\dot\alpha}(p)e^{-ipx}\right]\\
\chi_\alpha(x) &= \int \frac{d^3\boldsymbol{p}}{(2\pi)^{3/2}}\left[C_\alpha(p)e^{ipx} + D^\dagger_{\dot\alpha}(p)e^{-ipx}\right]\ ,
\end{aligned}
\tag{15}
$$

and similarly for λ_α and ψ_α. The operator relations

$$
\begin{aligned}
\left[W^{n\dagger}(p),\ g_\alpha{}^\beta(q)\right] = \left[U^n(p),\ h^\dagger_{\dot\alpha}{}^\beta(q)\right] &= \frac{1}{2}\,(\sigma^m\bar\sigma^n)_\alpha{}^\beta\,p_m\,E^{-1}\,\delta^{(3)}(\boldsymbol{p}-\boldsymbol{q})\\
\left\{A_\alpha(p),\ \bar C^\dagger_{\dot\alpha}(q)\right\} = \left\{B^\dagger_{\dot\alpha}(p),\ \bar D_{\dot\alpha}(q)\right\} &= \frac{1}{2}\,\sigma^m_{\alpha\dot\alpha}\,p_m\,E^{-1}\,\delta^{(3)}(\boldsymbol{p}-\boldsymbol{q})
\end{aligned}
\tag{16}
$$

allow us to recover the commutators (14). They also permit us to identify the daggered and undaggered quantities as creation and annihilation operators, respectively. From (16) we see that the Fock space is off-diagonal. The one-particle states have zero norm, and nonvanishing matrix elements with each other. To exhibit the ghost structure, we diagonalize the space of one-particle states. Without loss of generality, we consider positive helicity states moving along the $+z$ axis, $p^m = (E, 0, 0, E)$.

The following linear combinations

$$
\begin{aligned}
A^{(\pm)} &= \frac{1}{\sqrt{2E}}\,(A_1 \pm E\,C_1) \\
B^{(\pm)} &= \frac{1}{\sqrt{2E}}\,(B_1 \pm E\,D_1) \\
U_{\parallel}{}^{(\pm)} &= \frac{1}{\sqrt{2}}\,(U^o + U^3 \mp H_1{}^1) \\
U_{\perp}{}^{(\pm)} &= \frac{1}{\sqrt{2}}\,(U^o + iU^2 \mp H_1{}^2) \\
W_{\parallel}{}^{(\pm)} &= \frac{1}{\sqrt{2}}\,(W^o + W^3 \pm G_1{}^1) \\
W_{\perp}{}^{(\pm)} &= \frac{1}{\sqrt{2}}\,(W^1 - iW^2 \pm G_1{}^2)
\end{aligned}
\tag{17}
$$

diagonalize the one particle states. The superscripts $(\pm)$ denote positive and negative norm states, as may be seen from the following commutation relations:

$$
\begin{aligned}
\left[U_{\parallel}{}^{(\pm)}(p),\, U_{\parallel}{}^{(\pm)\dagger}(q)\right] &= \left[W_{\parallel}{}^{(\pm)}(p),\, W_{\parallel}{}^{(\pm)\dagger}(q)\right] = \pm\,\delta^{(3)}(\boldsymbol{p}-\boldsymbol{q}) \\
\left[U_{\perp}{}^{(\pm)}(p),\, U_{\perp}{}^{(\pm)\dagger}(q)\right] &= \left[W_{\perp}{}^{(\pm)}(p),\, W_{\perp}{}^{(\pm)\dagger}(q)\right] = \pm\,\delta^{(3)}(\boldsymbol{p}-\boldsymbol{q}) \\
\left\{A^{(\pm)}(p),\, A^{(\pm)\dagger}(q)\right\} &= \left\{B^{(\pm)}(p),\, B^{(\pm)\dagger}(q)\right\} = \pm\,\delta^{(3)}(\boldsymbol{p}-\boldsymbol{q}) \quad .
\end{aligned}
\tag{18}
$$

It is not hard to show that the supersymmetry transformations (12) preserve the norms of the states. They map physical states into physical states and ghosts into ghosts.The physical states and the ghosts form multiplets under the unbroken $N = 1$ supersymmetry algebra.

The Lagrangians with higher derivatives can be reduced to Lagrangians with first and second order derivatives through the introduction of auxiliary fields. This can be done in terms of component fields or in terms of $N = 1$ superfields.

The quadratic piece of the Lagrangian (9b) is reproduced with two extra superfields:

$$\int d^4x d^2\theta^{(1)} \left[-\frac{i}{2}\, \Gamma\, \sigma^m\, \partial_m \bar{X} \; + \; a\, \Gamma W \; + \; b\, WW \; + \; c\, \Gamma\Gamma \right] + h.c. \tag{19}$$

Here Γ_α and W_α are $N = 1$ auxiliary superfields, subject to the following constraints:

$$\begin{aligned} \bar{D}_{\dot{\alpha}}{}^{(1)}\, \Gamma_\beta \; &= \; \bar{D}_{\dot{\alpha}}{}^{(1)}\, W_\beta \; = \; 0 \\ \bar{D}^{(1)}\, \bar{W} \; &= \; D^{(1)}\, W \quad . \end{aligned} \tag{20}$$

The Lagrangian (19) gives rise to the following superfield equations of motion:

$$\begin{aligned} \bar{\sigma}^m\, \partial_m\, \Gamma \; &= \; 0 \\ -\frac{i}{2}(\sigma^m\, \partial_m\, \bar{X})_\alpha \; + \; a\, W_\alpha \; + 2c\, \Gamma_\alpha \; &= \; 0 \\ 2b\, D^{(1)}\, W \; + \; 2b^*\, \bar{D}^{(1)}\, \bar{W} \; + \; a\, D^{(1)}\, \Gamma \; + \; a^*\, \bar{D}^{(1)}\, \bar{\Gamma} \; &= \; 0 \quad . \end{aligned} \tag{21}$$

Eliminating W_α and Γ_α, and imposing the relation $a^2 = 8c\,\mathrm{Re}(b)$, we find the superfield equation for X,

$$\sigma^m_{\alpha\dot{\alpha}}\, \partial_m\, D^{(1)} D^{(1)}\, \bar{X}^{\dot{\alpha}} \; = \; 0 \quad . \tag{22}$$

In terms of component fields, Eq. (22) takes the following form:

$$\begin{aligned} \sigma^m\, \partial_m\, \bar{\psi} \; &= \; 0 \\ \sigma^m\, \partial_m \Box\, \bar{\chi} \; &= \; 0 \\ \Box\, V^n \; - \; 2\, \partial^n\, \partial \cdot V \; &= \; 0 \quad . \end{aligned} \tag{23}$$

These equations are the same as those which follow from Lagrangian (11b). Their ghost structure may be analyzed as before. In this case, the positive and negative norm (one-particle) states are not eigenstates of the Hamiltonian, so the norm is not preserved under supersymmetry transformations. Instead, one finds that the one-particle states may be grouped into two physical $N = 1$ supermultiplets, and one unphysical $N = 1$ dipole ghost supermultiplet [12]. This is similar to the ghost structure of conformal supergravity, first analyzed by Ferrara and Zumino [13].

The third Lagrangian, (11a), can be analyzed in terms of component fields, leading to analogous results. We have not, however, been able to find a formulation in terms of $N = 1$ superfields.

Having discussed the generalized Akulov-Volkov superfield, we are now ready to consider its coupling to $N = 1$ chiral and vector superfields. We start by extending the transformation law of an $N = 1$ chiral superfield to $N = 2$,

$$\delta_{\xi^{(2)}} \Phi = - 2ik \Lambda \sigma^m \bar{\xi}^{(2)} \partial_m \Phi \quad . \tag{24}$$

This transformation law preserves the $N = 1$ chirality constraint $\bar{D}_{\dot{\alpha}}{}^{(1)} \Phi = 0$. As before, it may be used to construct an $N = 2$ superfield $\boldsymbol{\Phi}$,

$$\boldsymbol{\Phi}\Big(x, \theta^{(1)}, \bar{\theta}^{(1)}, \theta^{(2)}, \bar{\theta}^{(2)}\Big) = exp\, [\delta_{\theta^{(2)}}] \times \Phi\Big(x,\ \theta^{(1)},\ \bar{\theta}^{(1)}\Big) \quad . \tag{25}$$

The superfield $\boldsymbol{\Phi}$ obeys the following constraints:

$$\bar{D}_{\dot{\alpha}}{}^{(1)} \boldsymbol{\Phi} = 0 \tag{26a}$$

$$\bar{D}^{\dot{\beta}\,(2)} \boldsymbol{\Phi} = 2ik\, (\bar{\sigma}^m \boldsymbol{\Lambda})^{\dot{\beta}}\, \partial_m \boldsymbol{\Phi} \quad . \tag{26b}$$

These constraints are consistent with the $N = 2$ D-algebra (8).

We proceed analogously for the $N = 1$ vector superfield V. Its $N = 2$ transformation law is given by

$$\delta_{\xi^{(2)}} V = -ik \left(\tilde{\Lambda} \sigma^m \bar{\xi} - \xi \sigma^m \bar{\tilde{\Lambda}}\right) \partial_m V \quad , \tag{27}$$

where $\tilde{\Lambda}_\alpha$ is expressed in terms of Λ_α as in Eq. (5). This preserves the $N = 1$ constraint $V = V^+$. The $N = 2$ superfield $\tilde{\boldsymbol{V}}$ obeys the following constraints:

$$\tilde{\boldsymbol{V}} = \tilde{\boldsymbol{V}}^+ \tag{28a}$$

$$D_\alpha{}^{(2)} \tilde{\boldsymbol{V}} = ik\, \sigma^m_{\alpha\dot{\alpha}}\, \bar{\tilde{\boldsymbol{\Lambda}}}{}^{\dot{\alpha}}\, \partial_m \tilde{\boldsymbol{V}} \quad . \tag{28b}$$

These constraints are also consistent with the $N = 2$ D-algebra.

To generalize the concept of a gauge transformation, we first consider the transformation law of a matter multiplet

$$\Phi\,' \;=\; exp\,(i\Sigma)\;\Phi \;. \tag{29}$$

The gauge parameter Σ must satisfy the same constraints as Φ. In analogy to $N = 1$, one might expect the transformation on Φ to be compensated by a transformation on $\tilde{V}$:

$$\tilde{V}' = \tilde{V} + i(\Sigma^{+} - \Sigma) \;\; . \tag{30}$$

However, this does not preserve the constraints (28). To circumvent this difficulty, we must discard (30), and find a function $V(\tilde{V}, \Lambda)$ whose constraints are also satisfied by Σ. There exists a general procedure for converting a superfield $\tilde{V}$, which satisfies a constraint of type (28*b*), into a new superfield V, a function of Λ_α and $\tilde{V}$, which satisfies a constraint of type (26*b*). One first performs the $N = 2$ chiral projection

$$V^o \;=\; -\frac{1}{4}k^2\; \bar{D}^{(2)}\bar{D}^{(2)}\;\bar{\Lambda}\bar{\Lambda}\;\tilde{V}\;, \tag{31}$$

and then constructs the superfield V:

$$V \;=\; V^o \;-k\,\Lambda^\alpha\, D_\alpha{}^{(2)}\; V^o \;-\; \frac{1}{4}k^2\;\Lambda\Lambda\, D^{(2)}D^{(2)}\; V^o \quad . \tag{32}$$

The superfield V obeys the constraint (26*b*). Its lowest component is the same as the lowest component of $\tilde{V}$. Equations (31) and (32) illustrate the general procedure for decomposing a chiral superfield into a standard form [5]. Armed with the superfield V, we can now construct a gauge transformation which preserves the $N = 2$ constraints. If we take

$$V \;\;\rightarrow\;\; V - i\;\Sigma \quad , \tag{33}$$

then

$$\Phi^{+}\;\,exp\left(\frac{1}{2}\,(V + V^{+})\right)\Phi \tag{34}$$

is both gauge invariant and $N = 2$ symmetric. For $\Lambda_\alpha = 0$, $\theta_\alpha{}^{(2)} = \bar{\theta}_{\dot{\alpha}}{}^{(2)} = 0$, these expressions reduce to the usual $N = 1$ gauge invariant expressions used in supersymmetric gauge theories.

The $N=2$ generalization of the gauge invariant $N=1$ superfield W_α is given by

$$\boldsymbol{W}_\alpha = -\frac{1}{8}\,\bar{D}^{(1)2}\,D_\alpha{}^{(1)}\,(\boldsymbol{V} + \boldsymbol{V}^+) \quad . \tag{35}$$

The $N=2$ superfield $\boldsymbol{W}_\alpha$ is both chiral and gauge invariant. The above construction can be immediately generalized to nonabelian gauge groups.

Having constructed the $N=2$ superfields corresponding to $N=1$ chiral and vector superfields, we now proceed to construct an $N=2$ Lagrangian which reduces to the appropriate $N=1$ Lagrangian when $\Lambda_\alpha = X_\alpha = 0$. We follow the general procedure given in Ref. [5]. This procedure takes advantage of the fact that the $\theta^{(2)}\theta^{(2)}\bar{\theta}^{(2)}\bar{\theta}^{(2)}$ components of $\boldsymbol{\Lambda\Lambda\bar{\Lambda}\bar{\Lambda}}$, $\boldsymbol{XX\bar{X}\bar{X}}$ and $\boldsymbol{\Lambda\Lambda\bar{X}\bar{X}}$ all contain a constant term. Therefore, these objects pick out the $\theta^{(2)} = \bar{\theta}^{(2)} = 0$ components of anything the multiply. This is just what we need to construct invariant Lagrangians with the correct low energy limits. Since we wish to preserve $N=1$ chirality properties, we use $\boldsymbol{\Lambda\Lambda\bar{X}\bar{X}}$ for $N=1$ F-terms. For simplicity, we also use it for $N=1$ D-terms. Thus an $N=2$ extension of the $N=1$ Lagrangian

$$\frac{1}{2}\int d^2\theta^{(1)}\,d^2\bar{\theta}^{(1)}\;\Phi^+\,exp\,(V)\,\Phi + \frac{1}{4}\int d^2\theta^{(1)}\,WW + \int d^2\theta^{(1)}\;f(\Phi) + h.c. \tag{36}$$

is given by

$$\begin{aligned}
&-\frac{1}{2}\,k^2\int d^4\theta\,d^2\bar{\theta}^{(2)}\;\boldsymbol{\Lambda\Lambda\bar{X}\bar{X}}\\
&+\frac{1}{2}\,k^4\int d^4\theta\,d^4\bar{\theta}\;\boldsymbol{\Lambda\Lambda\bar{X}\bar{X}}\;\boldsymbol{\Phi}^+\;exp\left(\frac{1}{2}(\boldsymbol{V}+\boldsymbol{V}^+)\right)\boldsymbol{\Phi}\\
&+\frac{1}{4}\,k^4\int d^4\theta\,d^2\bar{\theta}^{(2)}\;\boldsymbol{\Lambda\Lambda\bar{X}\bar{X}}\;\boldsymbol{WW}\\
&+k^4\int d^4\theta\,d^2\bar{\theta}^{(2)}\;\boldsymbol{\Lambda\Lambda\bar{X}\bar{X}}\;f(\boldsymbol{\Phi}) + h.c.
\end{aligned} \tag{37}$$

Equation (37) reduces to (36) when $\Lambda_\alpha = X_\alpha = 0$. It gives the low energy coupling of the $N=2$ goldstino to the $N=1$ effective theory. As discussed earlier, the Lagrangian (37) contains ghost fields. How many of these become gauge degrees of freedom when (37) is coupled to $N=2$ supergravity is currently under investigation.

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References

1. S. Weinberg, Phys. Rev. Lett. 18 (1967) 188; S. Coleman, J. Wess and B. Zumino, Phys. Rev. 177 (1969) 2239; C. Callan, S. Coleman, J. Wess and B. Zumino, Phys. Rev. 177 (1969) 2247; J. Wess, Springer Tracts in Modern Physics, Vol. 50 (1969) 132.
2. E. Witten, Nucl. Phys. B188 (1981) 513.
3. V. Akulov and D. Volkov, JETP Lett. 16 (1972) 438.
4. E. Ivanov and A. Kapustnikov, J. Phys. A11 (1978) 2375; J. Phys. G8 (1982) 167; T. Uematsu and C. Zachos, Nucl. Phys. B201 (1982) 250; S. Ferrara, L. Maiani and P. West, Z. Phys. C19 (1983) 267.
5. J. Wess, Karlsruhe preprint, Lopuszański lectures (1982); Karlsruhe preprint Dubrovnik lectures (1983); Karsruhe preprint, 7^{th} Workshop on Current Problems in High Energy Particle Theory, Bonn (1983).
6. S. Samuel and J. Wess, Nucl. Phys. B221 (1983) 153; Columbia preprints CU-TP-260, CU-TP-264.
7. S. Samuel, Columbia preprint CU-TP-265, 7^{th} Workshop on Current Problems in High Energy Particle Theory, Bonn (1983).
8. J. Wess and J. Bagger, **Supersymmetry and Supergravity**, (Princeton University Press, Princeton, 1983), Chapter 11.
9. Ibid., Appendix A.
10. Ibid., Chapter 4.
11. Ibid., Chapters 1, 4.
12. W. Heisenberg, Nucl. Phys. 4 (1957) 532; M. Froissart, Suppl. Nuovo. Cim. 14 (1959) 197.
13. S. Ferrara and B. Zumino, Nucl. Phys. B134 (1978) 301.