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A STUDY OF Z^0 DECAYS INTO LEPTON PAIRS AND QUARK PAIRS*

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ABSTRACT

We present a Monte-Carlo calculation of the decay $Z^0 \rightarrow q\bar{q}\ell^+\ell^-$ for all combinations of quarks and leptons in the three-generation world. We give distributions in lepton energy, dilepton invariant mass squared, and diquark invariant mass squared as well as the partial rates for each quark-lepton combination. These distributions can be used to distinguish between the above reaction and resonance production in Z^0 decays.

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I. INTRODUCTION

The discovery of the Z^o boson at the CERN SPS-collider[1,2] has confirmed the predictions of the standard Glashow-Salam-Weinberg[3] theory of electroweak interactions. The next major step in testing the standard theory will be the discovery of the Higgs boson, H^o. Precision measurements of the parameters of the standard theory such as the masses M_W and M_Z of the W and Z^o bosons respectively, and their widths, will also be important probes of the theoretical structure of gauge theories.

Furthermore, rare decays of the Z^o are expected to be a window into the physics in the TeV energy range as well as serving as a clean production mechanism of H^o via the reaction[4,5]

$$Z^o \rightarrow \ell^+ \ell^- H^o$$

if the mass of H^o is less than M_Z. More recently, exotic decays of Z^o such as those involving hypercolor hadrons or supersymmetric particles have also been considered. Most of those decays have branching ratios in the 10^{-5} to 10^{-7} range. With the anticipated production of about 10^4 Z^o events per day at LEP and about a tenth of this at SLC these are certainly important decays to seach for, and the Z^o will serve as a spectrometer for scalar particles.[4–6] This calls into attention the necessity of calculating the higher order effects such as radiative corrections[7] in the standard model. Obviously a complete understanding of these effects is essential to uncovering novel features of Z^o decays.

In this paper we study the four body decays of the Z^o given by

$$\begin{array}{l} Z^o \rightarrow \ell^+ \ell^- \; q \quad \bar{q} \\ \qquad\qquad\quad \lfloor\!\!\rightarrow \lfloor\!\!\rightarrow \textit{jets (or hadrons)} \end{array} \tag{1.1}$$

where $\ell = e, \mu, \tau$ and q denotes the quarks q = u, d, s, c, b, t. The case for t-quark is relevant only if its mass, m_t, is below $M_Z/2$. We are interested only in the inclusive rate where the quark and antiquark either form jets or simply hadronize. The special case where the $q\bar{q}$ form a meson such as π^o, η, η_c or any other mesons of heavy flavor can be estimated crudely using our results.

As an example we give a very primitive estimate of the branching ratio

$$Z^o \rightarrow e^+e^-\pi^o. \tag{1.2}$$

in our discussions.

The importance of studying reaction (1.1) is seen when one wishes to use the high Z^o production rate at LEP or SLC to search for resonance production in conjunction with a lepton pair.[5] Resonances such as the Higgs boson or hypercolor hadrons decay preferentially into heavy quark pairs, giving rise to final states similar to (1.1). We can easily estimate the width of the decay (1.1) by counting the order in gauge coupling to be e^6 and including the four-body phase-space factor of $1/768(\pi)^5$, as well as the logarithmic enhancement factor of $\ell n^2 \frac{M_Z^2}{4m_e^2}$ for the case $\ell = e$. Putting them together we obtain

$$\gamma\,(\,Z^o \rightarrow e^+e^-\,\bar{q}\,q\,)\;\approx\;\frac{e^6 M_Z}{768(2\pi)^5}\;\ell n^2\;\frac{M_Z^2}{4m_e^2}\;\simeq\;5\times 10^{-6}\quad GeV\,. \tag{1.3}$$

This is comparable[4,5] to the width of $Z^o \rightarrow e^+e^-H^o$ for a standard H^o of 47 GeV/c^2. Hence, we deem it necessary to study reaction (1.1) carefully if exotic rare decays of Z^o corresponding to a partial width $\sim 10^{-4}$ to 10^{-6} GeV are to be observed.

In Section II we present our calculations and state the couplings of gauge bosons to fermions in the standard model. This section also includes the results of our numerical calculations. Since the kinematics of the lepton pair are more easily accessible to experimentalists we focus on the energy spectra of $\ell^\pm$ as well as the lepton pair invariant mass distributions. We also present and discuss the usefulness of the quark pair invariant mass distributions in distinguishing reaction (1.1) from the production of a resonance along with a lepton pair. This distribution is accessible in a couple of ways. It could be obtained directly by detection of all the particles except the lepton pair, a very difficult task at hadronic machines. Alternatively, at an e^+e^- machine, the invariant mass squared of the quark pair is given by

$$M_{qq}^2 = (p_{\mathrm{beam1}} + p_{\mathrm{beam2}} - \ell^+ - \ell^-)^2$$

and, thus, can be obtained with measurement of the leptons. We have also calculated effects of experimental cuts on the various quantities studied here. The final section contains discussions of our results and conclusions.

II. THE CALCULATION OF REACTIONS (1.1)

The Feynman diagrams used to calculate reaction (1.1) are depicted in Fig. 1. Our notation is displayed there. Before showing in detail our calculation, we wish to emphasize the nature of the decay under study in order to outline the expected results. The reaction (1.1) is the first order electroweak correction to the decays

$$Z^o \rightarrow \ell^+ \ell^- \tag{2.1a}$$

and

$$Z^o \rightarrow q \bar{q} \quad . \tag{2.1b}$$

Since we consider Z^o decaying at rest, the intermediate Z^o in Fig. 1 is always off-shell. Consequently, this virtual state is suppressed and the case of an intermediate photon will dominate. The virtual photon (dominant) or Z^o will then most easily produce the lighter of the $q\bar{q}$ or $\ell^+\ell^-$ pair, having less energy available than the original Z^o. Therefore, the predominant scenario in this radiative correction process will be decay of the Z^o to the heavier of the quark or lepton pairs followed by emission of a virtual photon which, finally, produces the lighter pair of the four final state fermions. This situation is most relevant for the cases where the quark is much more massive than the lepton.

Thus, the energy of each of the heavier pair of fermions peaks towards it's maximum possible value, $\frac{M_Z}{2}$, while the much lighter fermions carry very little energy. Furthermore, one expects the very light pair to be produced approximately collinearly by a nearly on-shell photon while the opening angle between the heavy pair should be large. The invariant mass of the lighter pair must then peak towards zero while invariant mass of the heavier pair peaks towards M_Z^2. Our results are fully consistent with the above scenario in the relevant cases (e.g. for quark=b, lepton=electron).

The kinematics are not as clearcut for the cases for which the quark and lepton masses are not grossly different. Here, the fermion energy spectra, for instance, would be expected to exhibit a double peaking at the lower and upper energy limits. The heavier fermion should have somewhat stronger peaking at its upper energy limit corresponding to being produced by the decay of the original Z^o with the lighter pair having slightly enhanced production at the lower limit corresponding to preferential production by the virtual photon. The $q\,\bar{q}$ opening angle and that of the $\ell\,\bar{\ell}$ pair should also show this double peaking at 0^o and 180^o. We find this to be the case.

The matrix element for reaction (1.1) is given by

$$M = M_1 + M_2 + M_3 + M_4 + M_5 + M_6 + M_7 + M_8 \tag{2.2a}$$

with

$$M_1 = -\frac{iG^3}{(Q^2 - M_Z^2)[(k-q)^2 - m_q^2]}\Big\{\bar{u}\,(q)\, \not{\epsilon}_Z\, (a^q + b^q\gamma_5)\, (\not{q} - \not{k} + m_q)$$

$$[\gamma_\alpha(a^q + b^q\gamma_5)]v(\bar{q})\,\bar{u}\,(\ell^-)[\gamma^\alpha(a^\ell + b^\ell\gamma_5)]v(\ell^+)\Big\} \tag{2.2b}$$

$$M_2 = -\frac{ieQ_qG}{Q^2[\,(k-q)^2 - m_q^2]}\Big\{\bar{u}\,(q)\,\not{\epsilon}_Z\,(a^q + b^q\gamma_5)(\not{q} - \not{k} + m_q)\gamma_\alpha\, v(\bar{q})$$

$$\bar{u}\,(\ell^-)\,\gamma^\alpha\, v(\ell^+)\Big\} \tag{2.2c}$$

$$M_3 = -\frac{iG^3}{(Q^2 - M_Z^2)[(k-\bar{q})^2 - m_q^2]}\Big\{\bar{u}\,(\ell^-)\gamma^\alpha(a^\ell + b^\ell\gamma_5)v(\ell^+)$$

$$\bar{u}\,(q)\,\gamma_a\,(a^q + b^q\gamma_5)[\not{k} - \bar{\not{q}} + m_q]\;\not{\epsilon}_Z\,(a^q + b^q\gamma_5)\;v(\bar{q})\Big\} \tag{2.2d}$$

$$M_4 = -\frac{ieQ_qG}{Q^2[(k-\bar{q})^2 - m_q^2]}\Big\{\bar{u}\,(\ell^-)\gamma^\alpha v(\ell^+)\,\bar{u}\,(q)\gamma_\alpha[\not{k} - \bar{\not{q}} + m_q]$$

$$\not{\epsilon}_Z\,(a^q + b^q\gamma_5)\;v(\bar{q})\Big\} \tag{2.2e}$$

$$M_5 = -\frac{iG^3}{(Q'^2 - M_Z^2)[(k-l^-)^2 - m_\ell^2]}\Big\{\bar{u}\,(\ell^-)\;\not{\epsilon}_Z\,(a^\ell + b^\ell\gamma_5)[\not{\ell}^- - \not{k} + m_\ell]$$

$$\gamma_\alpha(a^\ell + b^\ell\gamma_5)\;v(\ell^+)\,\bar{u}\,(q)\Big[a^q\gamma^\alpha + b^q\gamma^\alpha\gamma_5 - \frac{Q'^\alpha}{M_Z^2}(a^q\,\not{Q}' + b^q\,\not{Q}'\gamma_5)\Big]v(\bar{\not{q}})\Big\} \tag{2.2f}$$

$$M_6 = -\frac{ieQ_qG}{Q'^2[(k-\ell^-)^2 - m_\ell^2]}\Big\{\bar{u}\,(\ell^-)\,\not{\epsilon}_Z\;(a^\ell + b^\ell\gamma_5)[\not{\ell}^- - \not{k} + m_\ell]\,\gamma_\alpha\; v(\ell^+)$$

$$\bar{u}\,(q)\gamma^\alpha v(\bar{q})\Big\} \tag{2.2g}$$

$$M_7 = -\frac{iG^3}{(Q'^2 - M_Z^2)[(k-\ell^+)^2 - m_\ell^2]}\Big\{\bar{u}\,(q)[a^q\gamma^\alpha + b^q\gamma^\alpha\gamma_5 - \frac{Q'^\alpha}{M_Z^2}(2m_q b^q)\gamma_5]$$

$$v(\bar{q})\,\bar{u}\,(\ell^-)\gamma_\alpha(a^\ell + b^\ell\gamma_5)v(\ell^+)\Big\} \tag{2.2h}$$

$$M_8 = -\frac{ieQ_qG}{Q'^2[(k-\ell^+)^2 - m_\ell^2]}\Big\{\bar{u}\,(q)\gamma^\alpha v(\bar{q})\,\bar{u}\,(\ell^-)\gamma_\alpha[\not{k} - \not{\ell}^+ + m_\ell]$$

$$\not{\epsilon}_Z\;(a^\ell + b_\ell\gamma_5)v(\ell^+)\Big\} \tag{2.2i}$$

$$\text{where}\quad G = g/cos\theta_W \tag{2.3}$$

$$\text{and}\quad Q = \ell^+ + \ell^- \tag{2.4a}$$

$$Q' = q + \bar{q}\quad . \tag{2.4b}$$

The Z^o couplings to fermion pairs are given by

$$a^{u,c,t} = -\frac{1}{4} + \frac{2}{3}sin^2\theta_W \tag{2.5a}$$

$$a^{d,s,b} = \frac{1}{4} - \frac{1}{3}sin^2\theta_W \tag{2.5b}$$

$$b^{u,c,t} = -b^{d,s,b} = \frac{1}{4} \tag{2.5c}$$

$$a^\ell = -\frac{1}{4}(cos2\theta_W - 2sin^2\theta_W) \tag{2.5d}$$

$$b^\ell = -\frac{1}{4}(cos2\theta_W + 2sin^2\theta_W) \tag{2.5e}$$

in the standard model. As seen in Eq. (2.2b-h) the matrix element peaks when the invariant mass squared of the virtual photon approaches that of $4m_e^2$. To obtain the partial widths we calculate $|M|^2$ and sum over all spins. The traces over the Dirac matrices were performed using the algebraic manipulation program Reduce II. The answer is too long to be presented here. The partial width in the Z^o rest frame is given by

$$d\Gamma = \frac{1}{2M_Z}\frac{d^3q}{2E_q}\frac{d^3\bar{q}}{dE_{\bar{q}}}\frac{d^3\ell^-}{2E_-}\frac{d^3\ell^+}{2E_+}\frac{1}{(2\pi)^8}\,\delta^4(P-q-\bar{q}-\ell^+-\ell^-)\frac{1}{3}\sum_{spins}|M|^2 \quad (2.6)$$

The four body phase-space integrations are done using the Monte-Carlo technique. We have employed importance sampling in our Monte-Carlo calculations so as to ensure rapid convergence. For all our calculations, we have used a quarter of a million integration points for each combination of quarks and leptons in the final state.

In Table I, we present the partial widths of the decays (1.1) for each quark-lepton combination. Two different values of $m_t = 30$ and 45 GeV/c^2 are used. For masses of the t-quark higher than this the kinematic suppression of this partial rate makes it undetectable. For the c- and b-quarks we have used the masses $m_c = 1.5$ and $m_b =$ 4.5 GeV/c^2. However, for the light quark masses, we have chosen $m_q = 0.3$ GeV/c^2 for all of them. This represents an average hadronic mass scale in the final states rather than the masses of the light quarks themselves. We have varied this scale from 100 MeV/c^2 to 1 GeV/c^2 and obtain at most 20%variation. This is in accordance with the expected slow log $(\frac{m_q}{M_Z})$ variation of these rates. Although the actual behavior of the width as a function of m_ℓ and m_q is complicated, a phenomenological parametrization of the form

$$\Gamma = A\, log^{\alpha}(\frac{M_Z}{2m_\ell}) \quad + \quad B \quad .$$

is quite accurate for a fixed m_q. The quantities A and B are functions of the quark masses.

Within the accuracy of our numerical calculation which is about 20%, the exponent α is found to be 1.75. From Table I we obtain the total inclusive width Γ_{incl} of $Z^o \rightarrow e^+e^- + hadrons$ to be

$$\Gamma_{incl} = 1.6 \times 10^{-3}\, GeV \quad (2.7)$$

The total width of the Z^o is given by [7]

$$\Gamma(Z^o \rightarrow \ hadrons) = \frac{3g^2 M_Z}{16\pi cos^2\theta_W} \left(1 - 2sin^2\theta_W + \frac{20}{9} sin^4\theta_W\right) \tag{2.8}$$
$$= 2.8 \ GeV$$

using $\frac{g^2}{8M_W^2} = 8.25 \times 10^{-6}$ GeV^2, $sin^2\theta_W = 0.22$, $M_W = 81$ GeV/c^2 and $M_Z = 95$ GeV/c^2. Then from Eq.(2.7) and (2.8) we get the branching ratio

$$\frac{\Gamma \ (Z^o \rightarrow e^+e^- \ + \ hadrons)}{\Gamma \ (Z^o + q\bar{q} \ + \ hadrons)} = 5.7 \times 10^{-4} \tag{2.9}$$

Since the process (1.1) is a radiative correction to the hadronic width the result of Eq. (2.9) is larger than simple power counting of α^2 of the hadronic width, most of the enhancement comes from the $\ell n^2 \frac{M_Z}{2m_e}$ factor. Furthermore, it is seen that the rare decay (1.1) represents about one percent of the $Z^o \rightarrow e^+e^-$ rate. Related to the process we are studying are the radiative decays of the Z^o

$$Z^o \rightarrow \ q\bar{q}\gamma \quad \text{and} \quad \ell^+\ell^-\gamma \tag{2.10}$$

with a "hard" photon detectable in the final state. It is well known[9] that reaction (2.10) has an infrared singularity and hence a suitable cut off ϵ on the photon energy must be imposed to get a finite rate for the three body decay involving a photon. As a check on our calculations we compare with the results of Ref. 9.

As an example this gives for the τ-lepton case

$$\frac{\Gamma \ (Z^o \rightarrow \tau^+\tau^- u \, \bar{u})}{\Gamma \ (Z^o \rightarrow u \, \bar{u} \, \gamma, \epsilon = 2m_\tau)} \simeq \ 0.01 \tag{2.11}$$

which is of the expected magnitude.

The four-body decay mode does not suffer from infrared divergences. These are tamed into ℓn m factors, where m is the mass of the lighter of the fermions in the final state. Consequently, as outlined at the beginning of this section, the energy spectra of the lighter fermions have peaking towards minimum energy. This peaking is very sharp for the cases where the quark and lepton masses are very different. In Figs. 2(a) and (b), we display the pure phase space lepton energy spectrum and invariant mass squared distribution. The overall normalization in Figs. 2 through 5 is arbitrary; however, the relative normalizations of the distributions are displayed correctly. In Figs. 3(a), (b), (c) we display the lepton energy spectrum, $\frac{d\Gamma}{dE_{\pm}}$, where E_+ and E_- are the energies of the ℓ^+ and ℓ^- of decay (1.1.), for the cases of $\ell = e, \mu$, and τ, respectively with the d-quarks. Figures 3(d), (e), and (f) contain the same information for the leptons with c-quark as do Figs. 3(g), (h), (i) for b-quark. Assuming CP invariance then the E_+ and E_- spectra are the same.

As expected, the electron energy spectra all exhibit very sharp peaking for small values of E_+ or E_-. A single peak at low energy in the muon spectrum only occurs for the b-quark case. It is less pronounced than is the electron energy peaking. The muon spectra exhibit double peaking at the upper and lower limits of $E_{\pm}$ for the d- and c-quark cases. The peaking at large $E_{\pm}$ is very slight for the charm quark case but this double peaking is accentuated in the d-case since the muon and quark masses are not too different. The situation is similiar for the tau lepton. Only for the top quark case, does the τ energy spectrum have a single peak at low $E_{\pm}$. For the c- and b-quarks, the spectrum has double peaking. The τ energy peak shifts to the high $E_{\pm}$ end for the d-quark case since, here, $m_\ell > m_q$. In this case, it is the lighter quark pair which is mainly being produced by the dominant virtual photon so the lepton energy spectra peak towards $M_Z/2$ while the quark energy spectra peak toward small $E_{\pm}$.

Another important quantity is the opening angle, θ_ℓ, of the lepton pair in the Z^o rest frame. For the decay of (1.1a) we find that θ_ℓ is small with a large fraction of the events concentrating near $cos\theta_\ell \approx 1$ for $\ell = e$. Since experimentally, for such a small opening angle, the lepton pair would appear as a single charged lepton we investigated the effects of imposing a cut of $\theta_\ell \geq 1$ mrad and $E_{\pm} \geq 1$ GeV on the decay rate. The results are presented in parentheses in Table I. Of course, the effect of this cut is largest for light quarks and $\ell = e$. It cuts out about one half to two-thirds of these events. For $\ell = \mu$ or τ the sensitivity to these cuts is less. In the cases of lepton and quark mass approximately equal, $\frac{d\Gamma}{dcos\theta_\ell}$ has peaks at both $\theta_\ell \sim 1$ and $\theta_\ell \sim -1$.

Next we examine the distributions in lepton invariant mass squared defined by

$$M_{\ell\ell}^2 \equiv (\ell^+ + \ell^-)^2 \quad .$$

This is given in Fig. 4 for the d-quark with τ-lepton and the c-quark with μ-lepton combinations. The sharp peaking seen in the distribution is significant for searches of Higgs boson or hypercolor hadrons. Compare for instance results of Ref. 5 and 10. Since the $M^2_{\ell\ell}$ distribution for

$$Z^o \rightarrow \ell^+\ell^- H^o$$

has a peak at high values[5] of $M^2_{\ell\ell}$, the higher order electroweak decay of (1.1) is not expected to be an important background. On the other hand, for searches of hyperpion or composite Higgs boson[5,10] with non-negligible $H^o\gamma Z$ coupling the distribution in $M^2_{\ell\ell}$ has the same peaking as seen in Fig. 4. In other words it will be difficult to distinguish between

$$Z^o \rightarrow \ell^+\ell^- + \textit{hyperpion}$$

$$\textit{or } Z^o \rightarrow \ell^+\ell^- + \textit{Composite Higgs Boson}$$

and the reaction (1.1) from the $M^2_{\ell\ell}$ distribution.

Additional information on the reaction (1.1) is contained in the distributions of the quark pairs. To the Born approximation, the quark energy, ω_q, gives the average energy of the jet containing that quark. Certainly hadronization of the quarks and quantum chromodynamics corrections are needed for an accurate description. However, the leading particles in a jet would reflect the characteristics of the quark. For $\ell = e$, the energy of the quark peaks sharply at $\frac{M_Z}{2}$; whereas for $\ell = \mu$ two peaks occur – one at each high and low values of ω_q. In this latter case as, discussed above, the muon and the d-quark are kinematically similar since they have approximately the same mass. Furthermore, they have similar Lorentz structure in their couplings to the Z^o and the photon. When we move to the τ-lepton, the ω_q distribution has only one peak at the low energy end. This is to be compared with the $E_\pm$ distributions [see Figs. 3 (c), (f), and (i)] which peak at the high energy end. This is complementary to the previous discussion on the lepton energy distribution.

Next we examine the invariant mass squared distribution of the quark pairs. We give the representative example of $q = b$ in Fig. 6 with $\ell = e$. For the b-quark, $M^2_{qq} \equiv (q + \bar{q})^2$ has a peak near the kinematic limit of M^2_Z. This behavior persists for both the muon and the tau lepton cases. This is to be contrasted with the situation of resonance production for which we expect M_{qq}^2 to have a sharp peak at the mass squared of the resonance if it decays predominantly into a pair of quarks. Such, for example, would be the case of a Higgs boson or a hyperpion. The invariant mass squared distribution of the lepton pair, $M_{\ell\ell}^2$, for the Z^o decay into a Higgs boson with a lepton pair peaks at high values of $M_{\ell\ell}^2$.[5] We have calculated the distribution in M_{qq}^2 for reaction (1.1) with a cut on the lepton pair mass of $M_{\ell\ell} > 20$ GeV. Thus, this is an appropriate calculation of the background to the Z^o decay into a Higgs boson. In Fig. 6, we show the distribution in M_{qq}^2 for the case of Z^o decay (1.1) into b-quarks and an electron pair with a cut of 20 GeV on the lepton pair invariant mass. We find the rate for this decay to be approximately 8.0×10^{-6} GeV. A standard model Higgs of mass 20 GeV produced via the decay $Z^o \rightarrow H^o e^+ e^-$ would decay predominantly into a $b\bar{b}$ quark pair producing the same final state as reaction (1.1) with a narrow peak in M_{qq}^2 at 400 GeV^2. We have previously calculated[5] the rate for Z^o decay into a 20 GeV mass Higgs with an electron pair to be 7.6×10^{-5} GeV. Thus, reaction (1.1), as a background to Higgs production, is down by an order of magnitude with the $M_{\ell\ell}^2$ distributions further distinguishing the two processes. The M_{qq}^2 distribution can be potentially useful in the search for resonances not near the Z^o mass. We also display in Fig. 5 the case of the d-quark with tau leptons which peaks at low values of M_{qq}^2 since $m_\tau > m_d$. The M_{qq}^2 distribution can be potentially useful in the search for resonances not near the Z^o mass.

III. CONCLUSIONS

We have calculated numerically the four body decay of the Z^o given in (1.1) for all combinations of quark and lepton pairs in the six quark and lepton world. We find the inclusive branching ratio to be

$$R_e \equiv \frac{\Gamma(Z^o \rightarrow e^+ e^- \ + \ hadrons)}{\Gamma(Z^o \rightarrow hadrons)} = 6.7 \times 10^{-4} \tag{3.1}$$

$$R_\mu = \frac{\Gamma(Z^o \rightarrow \mu^+ \mu^- \ + \ hadrons)}{\Gamma(Z^o \rightarrow \ hadrons)} = 1.4 \times 10^{-4} \tag{3.2}$$

and

$$R_\tau \equiv \frac{\Gamma(Z^o \rightarrow \tau^+ \tau^- \ + \ hadrons)}{\Gamma(Z^o \rightarrow \ hadrons)} = 6.4 \times 10^{-5} \tag{3.3}$$

We can also estimate the ratio of $Z^o \rightarrow e^+e^-\pi^0$ versus $Z^o \rightarrow e^+e^-q\,\bar{q}$ where $q = u$ or d quark.This ratio, which we call ρ, is crudely given by the ratio of the three-body to the four-body phase space divided by $(M_Z L)^2$. That is

$$\rho \simeq \frac{96\pi^2}{(M_Z L)^2} \tag{3.4}$$

where L is a characteristic length scale in the decay. It enters here due to the fact that the matrix elements in 3 and 4-body decays have different dimensions. If we choose L to be the confinement scale and take it to be 1.0 GeV^{-1}, then

$$\rho \simeq 0.1 \tag{3.5}$$

and hence we obtain the estimate

$$\frac{\Gamma(Z^o \rightarrow e^+e^-\pi^o)}{\Gamma(Z^o \rightarrow \ hadrons)} = 5.7 \times 10^{-5} \tag{3.6}$$

using (3.1). On the other hand, if we take L to be the Compton wave length of the π^o or F_π then

$$\rho \simeq 4.6 \tag{3.7}$$

and we get instead

$$\frac{\Gamma(Z^o \rightarrow e^+e^-\pi^o)}{\Gamma(Z^o \rightarrow \ hadrons)} \simeq 3.1 \times 10^{-3} \tag{3.8}$$

Our results reflect the uncertainty in the hadronic part of the calculation and is taken to be a rough order of magnitude estimate only.

We have given several energy spectra for different combinations of quarks and lepton species. In particular the distributions in $M^2_{\ell\ell}$ for light leptons always have a sharp peak at small values of $M^2_{\ell\ell}$ which is a characteristic of radiative processes. This is in contrast to the case of Z^o decay to a Higgs boson with a light lepton pair for which the $M^2_{\ell\ell}$ distribution peaks towards high values. On the other hand the M^2_{qq} distributions for reaction (1.1) with light leptons peak near M^2_Z. For resonance production and subsequent decay into quarks this distribution would peak at the mass of the resonance. These features can be used to distinguish between resonance production in Z^o decays and the four body radiative decay of reaction (1.1).

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TABLE I

The partial widths in GeV for the decays $Z^o \to \ell^+\ell^- q\bar{q}$. The flavor and masses in GeV/c^2 of the quarks used are given in the first column. Numbers in parentheses represent widths with cuts of lepton opening angles of > 1 mrad and energy greater than 1 GeV imposed.

ℓ / q	e	μ	τ
u, $m_u = 0.3$	7.5×10^{-4} (2.6×10^{-4})	1.5×10^{-4} (1.3×10^{-4})	8.1×10^{-5} (7.1×10^{-5})
d,s, $m_{d,s} = 0.3$	2.3×10^{-4} (1.1×10^{-4})	5.9×10^{-5} (4.5×10^{-5})	3.2×10^{-5} (2.7×10^{-5})
c, $m_c = 1.5$	5.1×10^{-4} (3.0×10^{-4})	1.1×10^{-4} (9×10^{-5})	2.8×10^{-5} (2.6×10^{-5})
b, $m_b = 4.5$	1.2×10^{-4} (7×10^{-5})	2.5×10^{-5} (2.2×10^{-5})	3.9×10^{-6} (3.7×10^{-6})
t, $m_t = 30$	4.5×10^{-5} (2.9×10^{-5})	7.4×10^{-6} (7.1×10^{-6})	8.7×10^{-7} (8.6×10^{-7})
t, $m_t = 42$	1.2×10^{-6} (9.9×10^{-7})	1.2×10^{-7} (1.1×10^{-7})	2.7×10^{-9} (2.7×10^{-9})

FIGURE CAPTIONS

1. Feynman diagrams for the decay (1.1).
2. Four-body phase space distributions. *a*) $d\Gamma/dE_{\pm}$ and *b*) $d\Gamma/dM^2_{\ell\ell}$.
3. The lepton energy differential distribution for several quark and lepton combinations.

   (a) d; e , (b) d; μ , (c) d; τ ,
   (d) c; e , (e) c; μ , (f) c; τ ,
   (g) b; e , (h) b; μ , (i) b; τ .

4. The differential distribution in dilepton invariant mass squared for the combinations (a) c;μ and (b) d;τ.
5. The differential distribution in diquark invariant mass squared for the combinations (a) b;e and (b) d;τ.
6. The differential distribution in diquark invariant mass squared for b-quarks and electrons with a cut in dilepton invariant mass of $M_{\ell\ell} > 20$ GeV.

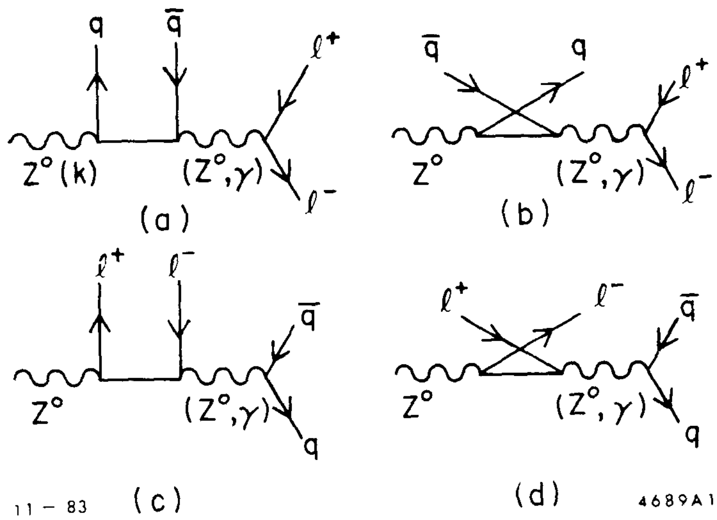
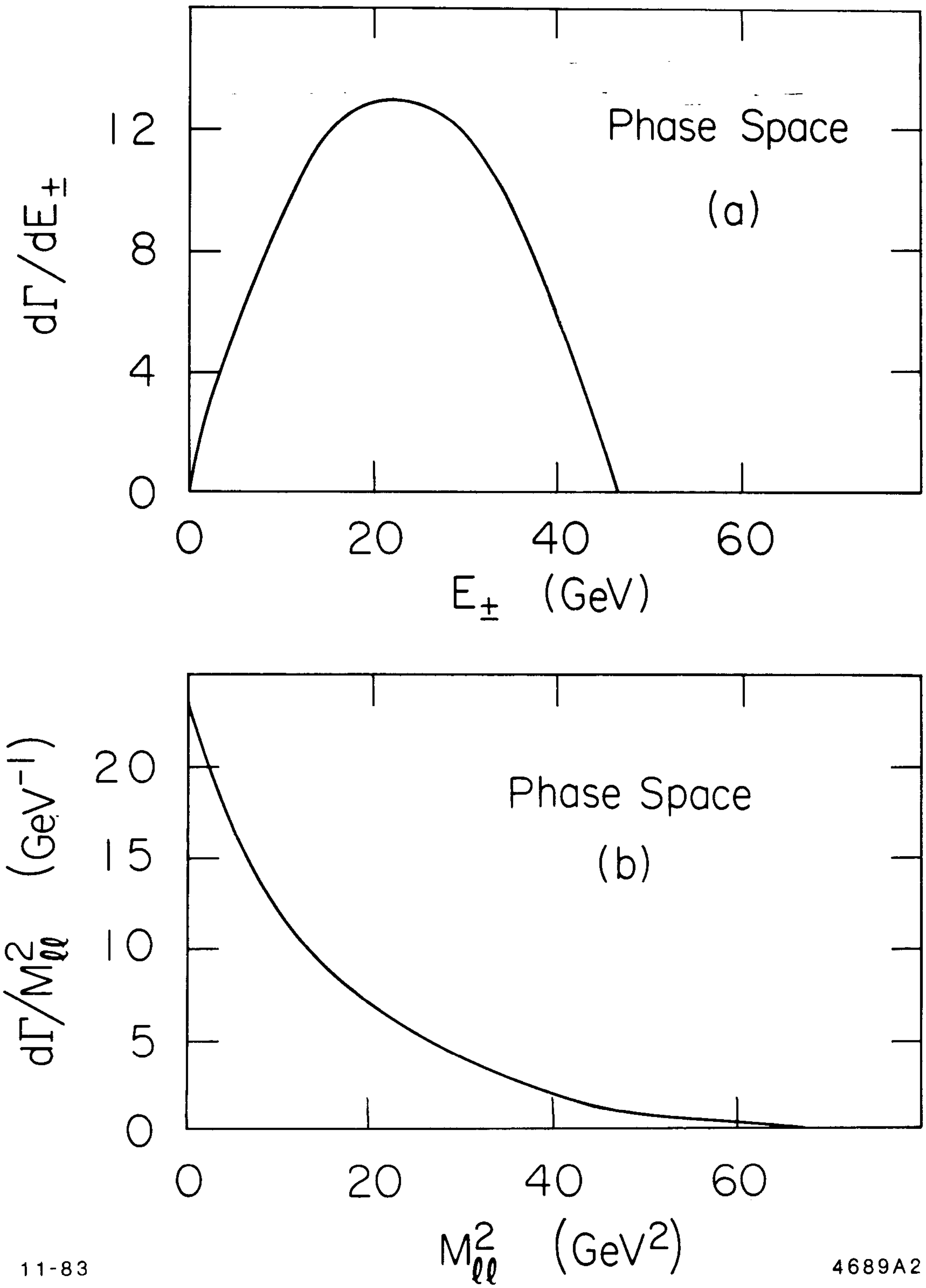


Fig. 1



Phase Space
(a)
$d\Gamma/dE_{\pm}$
0
4
8
12
0
20
40
60
$E_{\pm}$ (GeV)
Phase Space
(b)
$d\Gamma/M^2_{\ell\ell}$ (GeV^{-1})
0
5
10
15
20
0
20
40
60
$M^2_{\ell\ell}$ (GeV^2)
11-83
4689A2

Fig. 2

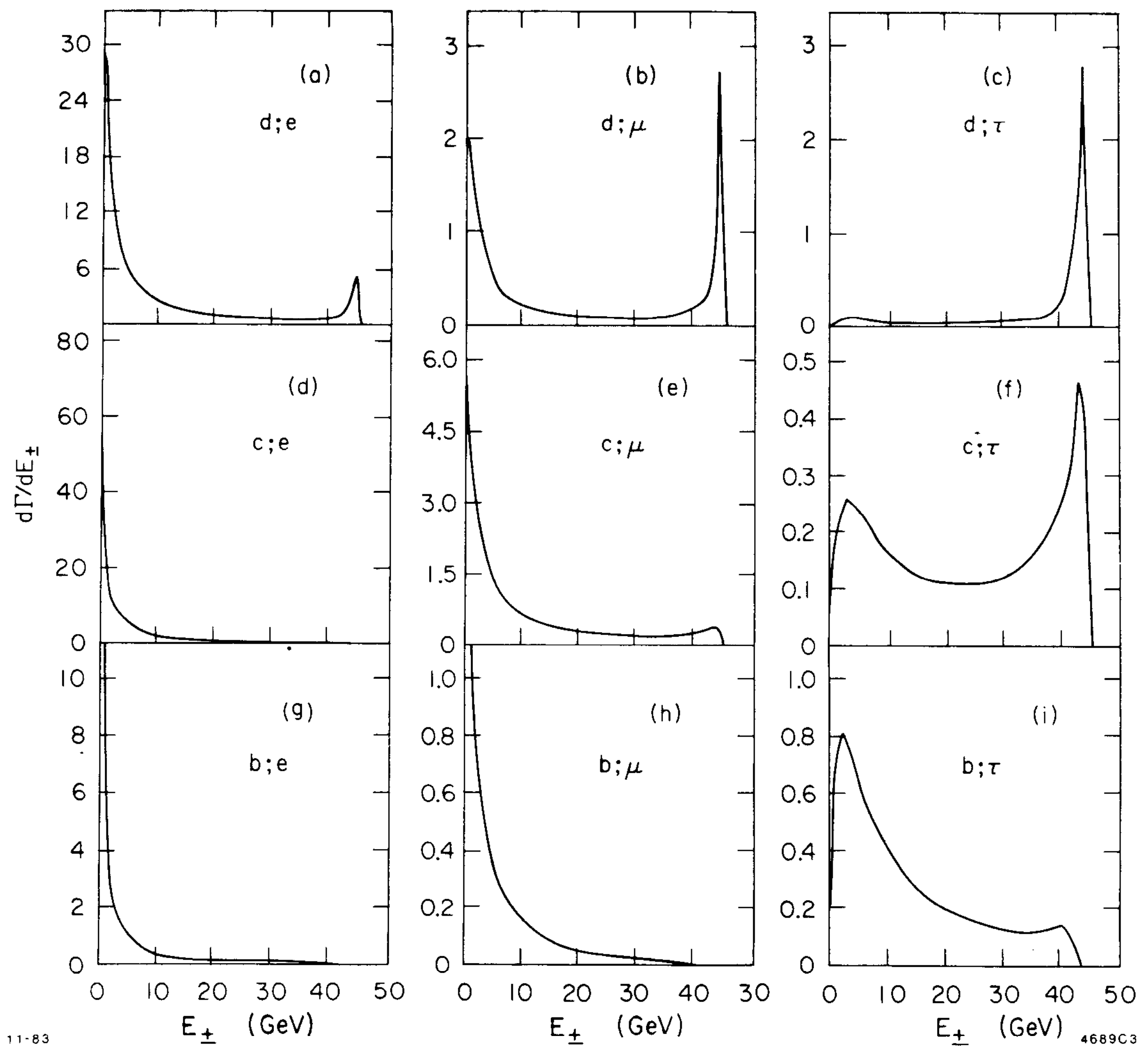
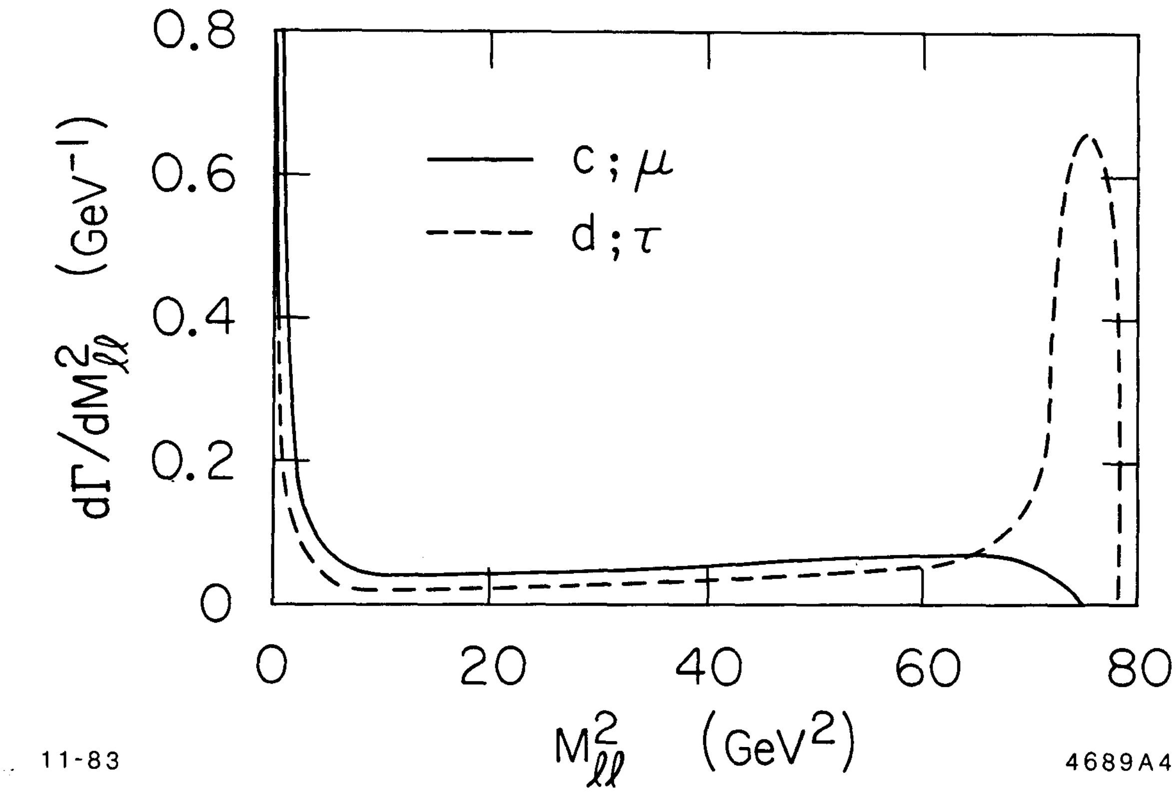
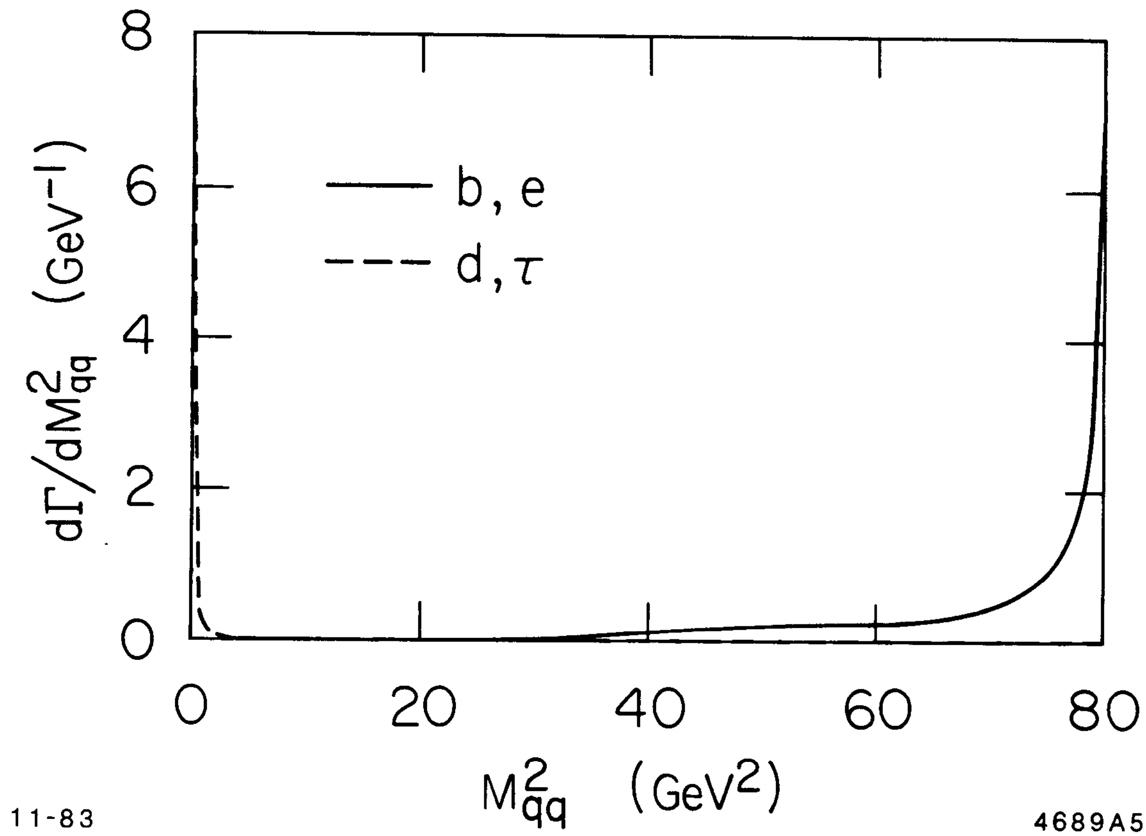


Fig. 3



0.8
0.6
0.4
0.2
0
$d\Gamma/dM_{\ell\ell}^2$ (GeV^{-1})
c ; μ
d ; τ
0
20
40
60
80
$M_{\ell\ell}^2$ (GeV^2)
11-83
4689A4

Fig. 4



8
6
4
2
0
0
20
40
60
80
$d\Gamma/dM^2_{qq}$ (GeV^{-1})
M^2_{qq} (GeV^2)
b, e
d, τ
11-83
4689A5

Fig. 5

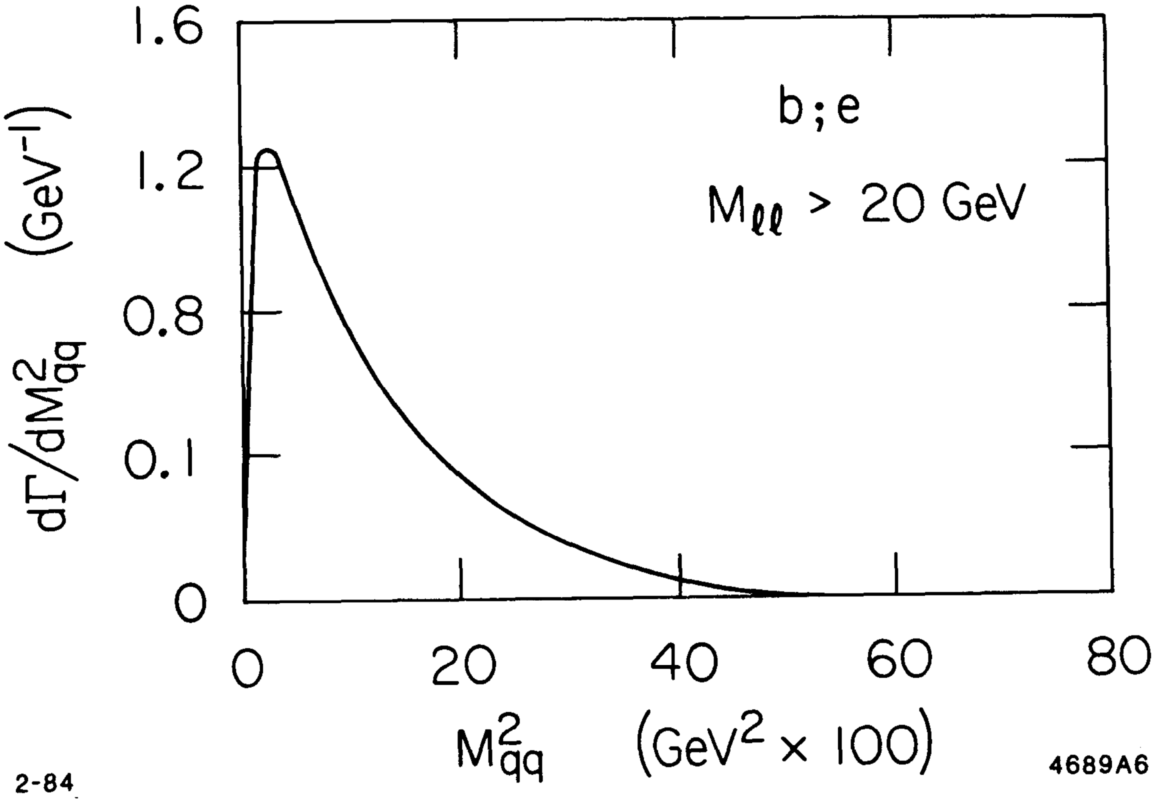

1.6
1.2
0.8
0.1
0
dΓ/dM$^2_{qq}$ (GeV^{-1})
b ; e
M$_{\ell\ell}$ > 20 GeV
0
20
40
60
80
M$^2_{qq}$ (GeV^2 × 100)
2-84
4689A6

Fig. 6