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A FINITE PARTICLE NUMBER APPROACH TO PHYSICS [1]

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Starting from a discrete, self-generating and self-organizing, recursive model and self-consistent interpretive rules we construct: the scale constants of physics ($3, 10, 137, 1.7\times10^{38}$); 3+1 Minkowski space with a discrete metric and the algebraic bound $\Delta\varepsilon\Delta\tau \geq 1$; the Einstein-deBroglie relation; algebraic "double slit" interference; a single time momentum space scattering theory connected to laboratory experience; an approximation to "wave functions;" "local" phase severence and hence *both* distant correlations *and* separability; baryon number, lepton number, charge and helicity; m_p/m_e; a cosmology not in disagreement with current observations.

I. INTRODUCTION

The claims made in the abstract will strike most physicists as incredible, as they do the author. They are the consequence of the convergence of a number of lines of research extending over three decades, some of which has been published (1,2,3), This contribution to the Symposium on Wave-Particle Dualism[2] is the first attempt to bring these strands together in a coherent fashion. A more detailed presentation by Amson, Bastin, Kilmister, Parker-Rhodes, Stein and this author is in preparation;[3] errors of omission and commission in this version are the sole responsibility of this author.

That quantum mechanics requires revision at a fundamental level, though often denied by workers in the vineyard of experimental and mathematical physics, is no news to the participants of this symposium celebrating Louis deBroglie's 90th birthday and his fundamental contribution to the new direction taken by physics in this century. The point of view developed here is that the discrete phenomena exemplified by quantum mechanics cannot be consistently

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incorporated within the continuum framework of Euclidean geometry and the Newtonian synthesis. Bohr believed that this haptic language was inescapable and tried to meet the problem by a "correspondence principle" and the profound idea of "complimentarity." Einstein and deBroglie challenged this approach as productive of paradox, or at least incompleteness, but for most working physicists the "Bohr-Einstein debate" left Bohr in possession of the field; they went on to more pragmatic concerns. The existence of the de Broglie Foundation and this Symposium is evidence that the issue is far from dead; indeed it has become a fruitful field for experimental study (4).

One way to see why the foundation of physics on continuum standards of mass, length and time with their implied "scale invariance" is inadequate is to follow the method of operational analysis advocated by Bridgman (5) applied to thought experiments in the tradition of Galileo and Einstein. For example we can consider the double slit experiment using deBroglie waves with particle detectors in the slits (6) as well as in the array where the statistical data showing both single and double slit interference accumulates. Since nonrelativistic quantum mechanics depends only on h and m, it is scale invariant, and it would seem that one could establish these effects at macroscopic dimensions and then extrapolate to arbitrarily short distances. However careful analysis shows (6) that if there is a smallest finite mass (usually assumed to be m_e) this extrapolation is frustrated.

This recent analysis is but another version of earlier studies. In discussing the measurability of the electromagnetic field, Bohr and Rosenfeld (7) made use of the fact that quantum electrodynamics rests on only two dimensional constants (h and c) and hence that their thought experiments incorporating the (nonrelativistic) uncertainty principle could be performed with macroscopic apparatus and then extrapolated to arbitrarily small distances. But they conclude their paper by noting that once there is a mass parameter in the theory, the analysis breaks down. This challenge to the operational justification for second quantization has never, in our opinion, been met; thus we distrust current quantum field theories for massive particles.

Of more immediate relevance to our current effort is the profound analysis by Wick (8) of Yukawa's meson theory (9). Since special relativity requires that a region of dimension r can only act coherently during a time interval Δt for $r < c\Delta t$ and by Heisenberg's uncertainty principle $\Delta t \approx \hbar/\Delta E$, if two systems within a region bounded by r communicate with a system of mass μ then if $\Delta E \geq \mu c^2$ (by special relativity again) such a mass can be present. Accepting Newton's third law, total momentum will be conserved but the relative momentum between the two systems becomes arbitrary except for that restriction and hence they will "scatter" if they approach each other closer than

$$r \leq c\Delta t \approx c\hbar/\Delta E \leq c\hbar/\mu c^2 = \hbar/\mu c$$

Extending the argument to shorter distance, if $r < \hbar/n\mu c$ there can be n "quanta" present and concept of "particle" as a point with mass breaks down. This is, of course, one of the sources of the infinities in conventional quantum field theory. Thus special relativity and quantum mechanics when coupled together prevent any consistent continuum description of short distances from being constructed.

Yet another example of the problem is provided by Dyson's analysis (10) of what must happen in *renormalized* quantum electrodynamics when more than 137 charged particle pairs are found within a radius $\hbar/2$ mc. Their electrostatic energy then exceeds the energy needed to create another pair and the problem looses precise definition. By extension, the same analysis applies to 1.7×10^{38} gravitating baryons of nucleonic mass (11) (they form a Laplacian "black hole"). Thus we already have within conventional physics a reason to take seriously a theory such as the one presented here which calculates the pure numbers 137 and 1.7×10^{38} in such a way as to associate them with these quantum phenomena.

We hope that these examples from within physics will motivate some of our colleagues to consider our claim that fundamental revision might be worth while. Mathematical objections to the continuum have, of course, a much longer history starting at least with the paradoxs of Zeno, as discussed by Aristotle, the adoption of a randomized version of Aristotle's least step by Epicurus, and continuing to the present day in the constructive mathematics of Brouwer, Bishop and Martin-Löf.

Another discipline that contributes to the development of our theory is computer science. Manthey (12) claims that concurrent asynchronous communicating digital systems whose communications are required to be synchronized *necessarily* have an uncertainty principle. He also shows that quantum systems can be viewed as concurrent processes described by finite algorithms. Of course any constructive mathematical theory must in principle be computable; this principle has been very helpful in our research.

For this author one important motivation for this research stems from a desire to provide a conceptual framework for physics that is broad enough to encompass all sciences. Given conservation laws, random background, and heritability of characteristics providing dynamic stability against that random background, neo-Darwinian evolution (for species defined by a statistical ensemble and not by a single genotype (13)) is inevitable. We will find that all these characteristics occur in the theory developed here. We will also see that although we can under appropriate circumstances talk about approximately separated systems (approximately because ultimately all our processes have a common origin) and hence in a sense can talk about "observers" our "measurement theory" does not allow them to obtrude in the way they do in von Neumann's quantum mechanics.

II. GENERATION AND CONSTRUCTION OF THE ALGEBRAIC MODEL

Although the precise form of the generating operation we adopt is not critical, provided it can start from the empty set $\emptyset$, we take for concreteness that due to Conway (14):

G: If L, R are two disjoint subsets of S, adjoin {L/R} to S.

This gives us (as we will see) the natural numbers 0,1,2,... which become (a) our ordering parameters, (b) the source of our "quantization," and (c) the tool we need for constructing "coherence." But since we do not know at any particular stage just what elements are in play using just this operation, we must define some process by which we can check whether an element generated by G is novel or not. This is done recursively as follows.

Specify a function $f: S\times S \to T$ (here T is some collection which includes S) with the property that there is a fixed subcollection Z of S and $f\,uv \in Z$ iff u and v are the same element. We also require (i) $f\,uu \in Z$, (ii) $f\,uv \in Z \Rightarrow f\,vu \in Z$, (iii) $f\,uv \in Z$ and $f\,vw \in Z \Rightarrow f\,uw \in Z$. If 0 is the unique element of some one element Z we can then prove[1] that

Theorem 1: Every f is equivalent to some g for which (i) $g\,uu = 0$, (ii) $g\,uv = g\,vu$, (iii) $g\,u(g\,vw) = g(g\,uv)\,w$.

Proof: (a) Define $\bar{f}\,uv = 0$ if $u = v$, $= f\,uv$ otherwise. Evidently $\bar{f} \equiv f$ and $\bar{f}\,uu = 0$. (b) Suppose the elements of S to have been ordered by any recursive process, and define $\bar{\bar{f}}\,uv = \min(\bar{f}\,uv, \bar{f}\,vu)$ where 0 is counted as the least element and 1,2,3,... are regarded as ordered in the usual way. Easily $\bar{\bar{f}} \equiv \bar{f} \equiv f$ and $\bar{\bar{f}}\,uu = 0$, $\bar{\bar{f}}\,uv = \bar{\bar{f}}\,vu$. (c) [Conway's trick] Define $g\,uv$ = the least element w such that $\bar{\bar{f}}\,w(g\,\bar{u}v) \neq 0$, $\bar{\bar{f}}\,w(g\,u\bar{v}) \neq 0$ for all $\bar{u}<u$, $\bar{v}<v$. Then, easily, $g \equiv \bar{\bar{f}} \equiv f$ and, since $g\,uv = 0$ iff u and v are the same element, we have $g\,uu = 0$. $g\,uv = g\,vu$, and also $g\,uv = 0$ and $g\,vw = 0$ implies that $g\,uw = 0$.

It remains to prove (iii) which, following Conway, is most easily done by explicit construction. From the definition $g\,00 = 0$, $g\,01 = 1 = g\,10$, so $g\,11$ cannot be 1 but it can be 0: therefore $g\,11 = 0$. $g\,02$ cannot be 1 or 0 but can be 2. Then $g12$ cannot be 2, 1, 0, so must be called 3. Next $g\,22 = 0$, and so on.

It is straightforward, if a little tedious to prove:

Theorem 2: If S is a closed system then $|S| = 2^n$ for integral n and there is an isomorphism $(S,g) \simeq (V_n,+_2)$ where V_n is the vector space of n dimensions over $\mathbb{Z}_2$.

One can then go on to discuss closure under this operation (discrimination), define a minimum labeling rule based on this property (15) and prove

Theorem 3: There is a unique complete hierarchy with more than two levels; it has successively completed levels of 3, 10, 137 and $2^{127} - 1 + 137 \approx 1.7 \times 10^{38}$ elements, beyond which further extension is impossible.

The proof of this theorem by Kilmister (15) is unpublished, but an earlier proof by Amson is given in the appendix to (2).

Since we will have need for explicit representations of the hierarchy later, we will use a more explicit construction. Thanks to Theorem 2, we are allowed to consider bit strings of length n $(x_1, \ldots, x_i, \ldots x_n)$ $x_i \epsilon 0, 1$ and for fixed n replace g by *discrimination* defined by $D_n xy = (\ldots x_i +_2 y_i \ldots)$ where $+_2$ is addition mod 2. We then define a "discriminately closed subset" (DCsS) as any set of nonnull strings such that discrimination applied to any two elements gives another element of the set, and we include also the singletons. For example, if we have two linearly independent strings a, b we have the three DCsS: $\{a\}, \{b\}, \{a,b,a+b\}$. If we had a third linearly independent element c, then we have in addition $\{c\}$, $\{b,c,b+c\}, \{c,a,c+a\}, \{a,b,c,a+b,b+c,c+a,a+b+c\}$ and so on. Clearly if we have j linearly independent strings then we can form $2^j - 1$ DCsS, since this is the number of ways we can take j distinct things 1, 2, ..., j at a time. Thus, starting with the strings (01) and (10) we have $\{(10)\}, \{(01)\}, \{(10),(01),(11)\}$. If we now ask for 2×2 matrices for which these sets are the only eigenvectors, which are linearly independent, and nonsingular so that they do not map onto zero, we can represent the information about discriminate closure contained in these $2^2 - 1 = 3$ DCsS at a level of greater complexity. Rearranged as strings of length 4, these matrices can then form a basis for constructing $2^3 - 1 = 7$ DCsS. In this case the maximal set with seven elements is (up to an arbitrary reordering of the bits in all the elements simultaneously) unique: $\{(1110),(1100),(1101),(0010),(0011),(0001),(1111)\}$. This can again be mapped (2) by 4×4 matrices, and a third level with $2^7 - 1 = 127$ DCsS constructed. These can again be mapped by 16×16 matrices (2) and these in turn used to construct a fourth level with $2^{127} - 1$ DCsS. But the process terminates here since there are only $(256)^2$ linearly independent matrices of "0's" and "1's" and these then cannot be used to map all the elements of level 4.

We see that the hierarchy labeling scheme is exhausted after $2^{127} + 136$ labels of length 256 have been assigned, and after generation and discrimination have filled up the remaining labels of length 256, all we can do is to start increasing the length of the strings. We call this additional part of the string, of length n, the "address;" that is, any string from now on will be of length $256 + n$ with the first 256 bits called the "label" and the n bits which follow called the "address."

III. CONSTRUCTION OF THE STATISTICAL MODEL

Since, because of discriminate closure, discrimination on the labels in our model will keep on throwing up the same values if we wait long enough, eventually we will come to have a large collection of strings all with the same label and with the same number of bits $b = N_0 + N_1$ of the address where N_0 and N_1 are the number of zeros and 1's in the address, however distributed. Our task is to find a well defined way to "break into" this, at first sight chaotic, situation using rules that can eventually be related to laboratory experience. We hope, ultimately, to derive these stability conditions from our basic operations; the rules we choose are, we claim, consistent with them but whether they are unique remains an open question.

The laboratory paradigm on which the rules are modeled comes from experience in high energy physics laboratories where "particles" are defined by the sequential firing of counters at fixed geometric positions, velocity defined by the ratio of the distance between counters to the time interval between firings, and their mass ratios measured by momentum-energy conservation. Our first step is to define the rational fraction $\omega b=(N_1-N_0)$. Clearly $-1\leq\omega\leq+1$, and each labeled address is characterized by $A_L(\omega,b)$. Since a large number of addresses carry the same label, we seek an elementary sampling procedure which will select out an ensemble $\mathbb{A}_L(\nu,b)$ for which $\nu=\langle\omega\rangle$. Consider three such samplings through which the label L_1 persists ordered by increasing bit number $b_1<b<b_1'$ and for which at the second sampling $b,\langle\omega\rangle=\nu_1$ while at the third $b_1',\langle\omega\rangle=\nu_1'$. Each sampling will involve additional sequences of labeled ensembles; the persistence of L_1 is insured by requiring that in the sequence we are discussing $DL_1L_i=L_j$, $DL_1L_2=L_3$, $DL_1L_i'=L_j'$. Consider now the second sequence characterized by label L_2 and ordered by $b_2<b<b_2'$, where the value b is required to be the same by requiring $D_bA_1A_2=A_3=D_bA_1'A_2'$; if the bit number b is not common, then no sampling can occur by our fundamental requirement on discrimination.

The label persistence we have now guaranteed is not sufficient to specify the connection between the values $\nu_1\nu_2$ and $\nu_1'\nu_2'$ before and after the sampling (i.e., ordered by $b_i<b<b_i'$). For this we assume that to each label L we can assign a unique positive value (which may be 0 for some labels), and define two functions $\varepsilon(\nu)$, $\rho(\nu)$ by $\varepsilon^2-\rho^2=\mu^2$, $\rho(\nu)=\nu\varepsilon(\nu)$. Then we require that for a sampling to take place the ensembles are connected by the requirement $\rho_1(\nu_1)+\rho_2(\nu_2)=\rho_1(\nu_1')+\rho_2(\nu_2')$.

In our last step we have clearly defined "momentum conservation" in a way that adumbers Einstein's modification of Newton's third law. We feel comfortable with this because the only obvious structure in our address strings other than $b=N_0+N_1$ is the difference defining a rational fraction bounded in absolute value by unity. We also have available the integer ordering parameters $b_1<b<b_1'$ and $b_2<b<b_2'$ which can serve as a "time ordering". The fact that we must rely on discrimination in constructing our information-preserving structure then provides "synchronization" because b must lie between both limits. What is missing is some construction that will connect these elements to a "metric space" in the geometrical sense. Here we are guided by Stein's insight (16) that an ensemble of bit strings of bit number b can be viewed as a "random walk" which has taken b steps.

With this model in mind, we think of the first sequence as starting when the bit length is $b_1^{(0)}$ and that the generation operation has added b bits on the end of this string. We now construct our ensemble which is sampled at the "intersection" of the two sequences using only these b bits, and define the "distance" from the reference position $\xi_1^{(0)}$ to the second sampling as $\xi_1(\nu_1,b)-\xi_1^{(0)}$ with $b=b_1-b_1^{(0)}$. Clearly in this interpretation the mean position of the distribution is at $\nu_1 b$. But the probability of a step in this direction is $\langle N_1/b\rangle=(1+\nu_1)/2$ while the probability of a step in the opposite direction $\langle N_0/b\rangle=(1-\nu_1)/2$. Thus the standard deviation $\sigma=\langle(N_1-N_0)^2\rangle^{\frac{1}{2}}=(b(1-\nu_1^2)/4)^{\frac{1}{2}}$. We see that even if $\nu_1=0$, there is a 50% chance that the sampling will give a position $\sqrt{b}/2$ or greater from the position of the mean. Hence we are lead to define our distance not by the peak of the distribution but by $\xi_1(\nu_1,b_1-b_1^{(0)})-\xi_1^{(0)}-\nu_1(b_1-b_1^{(0)})=(\sqrt{b}/2)\sqrt{1-\nu_1^2}$. But what we mean by the velocity parameter ν_1 is not yet clear. We specify it by assuming that for the second sequence $\nu_2=0$, and hence that $\xi_2(0,b_2-b_2^{(0)})=\sqrt{b}/2$. But the sign of the velocity is still arbitrary, since it comes from choosing N_1-N_0 rather than visa versa in our basic definition. Clearly we can resolve the ambiguity by treating ν as the <u>relative</u> velocity between the two ensembles. Thus we have derived $\sqrt{b}/2=(\xi_2-\xi_2^{(0)})=[\xi_1-\xi_1^{(0)}-\nu(b_1-b_1^{(0)}]/\sqrt{1-\nu^2}$; $\sqrt{b}/2=\xi_1-\xi_1^{(0)}=[\xi_2-\xi_2^{(0)}+\nu(b_2-b_2^{(0)})]/\sqrt{1-\nu^2}$ and hence also by elementary algebra that $b_1-b_1^{(0)}=[b_2-b_2^{(0)}+\nu(\xi_2-\xi_2^{(0)})]/\sqrt{1-\nu^2}$; $b_2-b_2^{(0)}=[b_1-b_1^{(0)}-\nu(\xi_1-\xi_1^{(0)})]/\sqrt{1-\nu^2}$. Thus our interpretation of the statistical distribution carrying a conserved label as the ordering parameter b increases leads directly, we claim, to the algebraic equivalent of the Poincaré transformation in a 1+1 "dimensional" Minkowski space, with the distinction that the "time" is represented by integers. We will see below that this integer character of our description cannot disappear.

So far we have not had to specify our "step length" in the random walks. From our basic definition if $\mu=0$ we necessarily have $\nu=\pm1$ (i.e., the bit string is all "1's" or all"0's" with <u>no</u> dispersion). But this will also be approximately true if ε is very large compared to μ. Thus we are forced to assume that our step length $\ell_\varepsilon=1/\varepsilon$. This as we will see below is in fact the Einstein-deBroglie relation. We thus have <u>derived</u> the "quantization of energy" directly from the discreteness of our construction. We are very pleased that this confirmation of deBroglie's basic insight that "particles," like photons, have an "internal" periodicity which lies at the heart of quantum mechanics should have been achieved in the year of his 90th birthday. Further, since our time steps are necessarily integral, $\Delta\tau\geq1$ while since also the number of steps ℓ_ε is integral $\Delta\varepsilon\geq1$. Thus, invoking the Lorentz invariance already established we have $\Delta\varepsilon\Delta\tau\geq1;\Delta\xi\Delta\rho\geq1$. We emphasize that these restrictions are <u>not</u> the "uncertainty principle," which we will discuss in due course, but simply an expression of the restrictions arising from the discreteness of our basic construction.

We now have "coordinate" pairs (ξ,b) where ξ can be any positive or negative number and b can be any positive or negative integer. We also have labeled "lines" L_1,L_2 which intersect at a position labeled by L_3. Since we have a finite metric coming from the integer structure, we can consider three labeled lines and three labeled intersections, and arrange the velocities such that the length $\xi_1-\xi_2$ and $\xi_2-\xi_3$ is greater than $\xi_1-\xi_3$. Then, since we have a metric, for common $b^{(0)}$ and three intersecting lines we have a triangle and Euclidean geometry in a plane. Further, if our velocity dependent transformation is to hold for arbitrary ν (bounded of course by ± 1), the coordinates perpendicular to the the lines must not transform. Since we have four levels to the hierarchy, we can extend this argument to three spacial dimensions by constructing a tetrahedron, but the fact that there are only four types of labels we can distinguish at this point in the construction prevents us from going further. Thus we claim that our model requires us to use as our basic space the 3+1 dimensional Minkowski space with coordinates that have (except for the integer character of the "time" parameter) the usual Poincaré transformations. Since $\underline{\nu}$ is now a 3-vector, the definition of ρ as a 3-vector is clearly $\underline{\rho}=\underline{\nu}\varepsilon$, from which the 3+1 momentum space follows.

We have now constructed ensembles of labeled binary distributions (LBD's) characterized by a statistical parameter $\underline{\nu}$ which can be accessed by statistical samplings in a space defined by the coordinates of the samplings containing a discrete internal periodicity $\ell_\varepsilon=1/\varepsilon$. From these we construct an array of LBD's in three dimensions all having the common directed $\underline{\nu}$ in a single direction synchronized to a common $b^{(0)}$ in such a way that at that time all the maxima along this direction are separated by the step length ℓ_ε. After some number of steps η the maxima will move one step length. Since each step takes a time $1/\varepsilon$, the velocity $\nu=\ell_\varepsilon/\eta(1/\varepsilon)=1/\eta\leq 1$. But we can also define a second velocity, that in which each step is taken in the direction $\underline{\nu}$ which is $\nu_\varepsilon=\eta\ell_\varepsilon/(1/\varepsilon)=\nu\geq 1$; clearly $\nu\nu_\varepsilon=1$. Were we dealing with a wave phenomenon we would call ν the group velocity and ν_ε the phase velocity; we emphasize that our construction is discrete and algebraic and should not be allowed to carry this continuum implication. But we now can define a coherence length referring to this synchronization namely $\ell=\eta\ell_\varepsilon=1/\nu\varepsilon=1/\rho$. If we now consider a second synchronized array with a different label and $\underline{\nu}^1$ synchronized to the same $b^{(0)}$, the most probable positions for elementary sampling events to exhibit this periodicity will be separated by this coherence length ℓ. Clearly ℓ plays for us the same role as the "deBroglie wavelength" in a wave theory.

We now consider a situation in which there are an enormous number of LBD's in a plane perpendicular to $\underline{\nu}$ so densely distributed that the probability of the original ensemble of LBD's which will encounter a large number of sampling events in this region, emerging on the other side of this plane is negligible, except for two "slits" a distance $d\ell_\varepsilon$ apart, where they are absent. If we go some large distance S behind this screen in the direction $\underline{\nu}$ and consider a plane perpendicular to $\underline{\nu}$ and a distance x in this plane away from the center line the most probable positions for a sampling taken using some other array to occur will be where the paths from the slits differ by an integral number of coherence lengths $n\ell$. Thus by the usual geometric construction $\ell = xd/nS$, and we claim to have derived "double slit interference" from our discrete model. Note that even when x,d and S are "macroscopic" in the sense that they are all very large compared to our metric length ℓ_ε, we can always find an arrangement in which this discrete structure is exhibited, thus relating macroscopic dimensions to the "microscopic" parameter ℓ.

It might appear that this coherent superposition is a rather special case, but in fact it is an idealization of actual laboratory practice which sets up time scales by using counts in detectors, as we will see in the next section. Thus we can use this "basis" as the starting point for constructing a mathematical description of actual experiments. We will therefore call this coherent ensemble of labeled binary distributions identified by a single label L (and hence value of μ) and a statistical parameter $\underline{\nu}$ an "elementary quantum particle." Note that the old problem of whether the "particle" goes through one or the other hole is resolved by us by saying that the particle is not a single entity but a discrete coherent statistical ensemble which can be divided spacially into two subensembles without losing either its coherence or its uniqueness.

IV. SCATTERING THEORY AND MEASUREMENT

It is now time for us to show that our constructions are, at least sketchily, directly related to the practice of physics and laboratory experience. Physics is usually taught in terms of basic macroscopic standards of Mass, Length, and Time against which laboratory standards are calibrated, thus allowing pure number ratios to be used in calculations. This was appropriate in a continuum theory which used Euclidean geometry as its mathematical paradigm. But in practice the standard meter is now defined by counting the number of wavelengths of light emitted by a well defined atomic source, the second is defined by counting the number of oscillations of an atomic clock, and the standard kilogram can be defined, if one likes, by counting the number of hydrogen atoms in the volume which would balance it. Thus in practice the standards of physics are already "absolute" pure numbers, and our only task is to show how our integers can be related to these standards. In practice the continuum has already disappeared from physics at the level of the fundamental dimensional units and been replaced by "counting."

The basic device we need for our discussion is a "counter." In practice this can be quite a complicated gadget, but it is assumed that the chain of events which leads to the firing of a counter starts from a single elementary scattering event. All we need do is to identify this initial event with our "elementary sampling event"; we can then leave the discussion of the accuracy to which this event can be localized in the laboratory in the competent hands of the experimental physicist. We also assume him equipped with "rods" and "clocks" calibrated against the primary standards, and "sources" of particles which produce sequential firings of counters separated by macroscopic distances, and by time intervals measured on his clock, thus giving measured velocities. We anticipate from our construction that no matter what source he uses there will be some maximum velocity that is never exceeded, which is borne out in practice, and which we call c in dimensional units. Thus his velocity is related to our statistical parameter by $\underline{v}=\underline{\nu}c$.

The particle physicist cannot measure "mass" directly by a balance, but instead uses the particles from some standard source and measures mass ratios by using relativistic momentum-energy conservation. Particle identification in practice can be quite complicated, but usually starts from the fact that electron and proton carry the same absolute value of electric charge, and the fact that this charge is also the quantized unit carried by other particles. Since we will identify this parameter within our discrete construction in the next section, we can follow his practice, identify his mass ratios μ with our μ and the mass in our theory as $m=\mu m_0$ where m_0 is some appropriately identified standard. To fix our third dimensional constant we note that we now have $E=\mu m_0 c^2 \varepsilon$, $\underline{p}=\mu m_0 c\underline{\rho}$ and hence can relate our microscopic length ℓ to macroscopic standards by introducing a universal constant h of dimensions ML^2T^{-1} and taking $\ell=\lambda=h/p$. Thus we also have the fundamental Einstein-deBroglie relation $E=hc/\lambda$. The final step is then to set up the double slit experiment and relate the macroscopic distances d,S,x to this by $\lambda=xd/nS$ where n is the number of the interference maxima counted away from the center line and the distances are measured using the laboratory standard. Thus, as claimed, we have reduced our basic measurements to the counting of integers. Now that we have made these laboratory identifications, we have no more freedom in making contact between the mathematical structure and laboratory experience (except, as we will see in the next section, some still unresolved questions of how we assign quantum numbers at level 4, similar to the problems now being faced by theorists trying for "grand unification" and "supergravity").

Now that we have identified particle sources and counters, we can conduct scattering experiments and measure the number of particles scattered into a specified solid angle relative to the flux in the incident beam. The cross section so defined is dependent only on particle identification and momentum conservation, so does not directly involve us in specifically quantum effects. For ease of contact with conventional theory we will treat momentum as a continuous variable $\underline{k}$ and $\varepsilon=(\mu^2+k^2)^{\frac{1}{2}}$, although we will have occasion in the next section to return to the discrete picture for a fundamental calculation. Then we can describe any scattering experiment in terms of a set of basis states $\phi_k(\underline{k}^1)_\infty=\varepsilon_k\delta^3(\underline{k}-\underline{k}^1)$ which are orthogonal and complete in the sense that $\int d^3k/\varepsilon_k)\phi_k(\underline{k}_1)\phi_k^*(\underline{k}_2)=\int(d^3k/\varepsilon_k)\phi_{k_1}(\underline{k})\phi_{k_2}{}^*(\underline{k})=\varepsilon\delta^3(\underline{k}_1-\underline{k}_2)$. Thus we are able to describe any energy-momentum conserving process as a transition which takes us from some product of these states representing the intial degrees of freedom. What is needed is a dynamical theory describing such processes.

We have seen that, although we can assume momentum precisely conserved in each elementary sampling event (or scattering process), that the uncertainty principle in energy prevents a precise specification of the energy in such processes. Thus what we need is to sum up all momentum conserving processes allowed by the combination of initial and final labels we consider in such a way that the uncertainty principle is respected but only energy conserving processes are described by the boundary states. For simplicity we consider the system with zero total momentum so that the invariant four momentum $P=\varepsilon_1+\varepsilon_2=\varepsilon_1'+\varepsilon_2'$ for the boundary states. Then

$$\psi^{\pm} = \delta^3\left(\underline{k}_1+\underline{k}_2-\underline{k}_1'-\underline{k}_2'\right)\left[\varepsilon_1\varepsilon_2\delta^3(\underline{k}_1-\underline{k}_1') - \frac{g(\underline{k}_1\underline{k}_2)g(k_1'k_2')}{\varepsilon_1'+\varepsilon_2'-P\mp i\Delta\varepsilon}\right]$$

This gives a high probability of energy conservation when the boundary states are far enough separated (differ by large values of n) but respects the uncertainty principle. The appearance of the imaginary i is forced on us by the fact that if we used a real linear function we could hit an actual zero in the denominator and produce an infinity that would be incompatible with our philosophy. We could try some nonlinear real structure, but this would introduce aribtrary and unwarranted structure into our statistical space. Thus the introduction of the complex field into our theory comes about basically because we insist on linearity in the basic scattering processes.

This argument does not give us anything more than the singularity and does not define the residue, which is also true in conventional scattering theory. The form we select comes from our zero range or single time scattering theory (17,18) and is chosen to correspond to a particle and quantum forming a "bound state" with the same mass and quantum numbers as the particle, and a particle and anti-particle forming a "bound state" with the mass of the quantum (19,20). This, as we will see more clearly when we discuss quantum numbers below, is precisely the structure of our elementary sampling events. Thus we have arrived at a scattering theory in momentum space which, it is claimed, can be used to obtain the usual results of relativistic quantum mechanics. The further development of this theory, which amounts to summing up all scattering processes consistent with the uncertainty principle and leads to Faddeev and Faddeev-Yakubovsky type integral equations, will be found in the series of papers to which we have just referred.

We now have a full momentum space description of the scattering process and a dynamics which allows us to compute cross sections and make experimental predictions. To connect this with the usual s-matrix formalism for quantum mechanics, all we need do is to make a Fourier transform of our momentum space wave functions to configuration space. This is a smoothing out of our discrete theory, and hence an approximation whose limitations we must keep in mind. In particular, we saw that our initial specification of an elementary sampling event involved a chain of three samplings. In a conventional wave function only the coordinates of the event itself appear, and the coordinates which in classical Hamilton Jacobi theory would represent the constants of the motion have disappeared. This is clearly a mistake, as was first pointed out by Phipps (21). We see that our current development forces us to the same point of view, so that our configuration space wave function becomes (22) $\Psi = \exp\left(-i\sum_k \underline{P}_k \cdot \underline{Q}_k\right)\psi_{sch}(\underline{q},t)$ where ψ_{sch} is the conventional Schroedinger wave funtion and $\underline{P}_k$, $\underline{Q}_k$ are the "constants of the motion"; for us these represent the first in the chain of three samplings (which usually occurs somewhater in the collimatior defining that degree of freedom as it enters the scattering region) needed to define the event at $\underline{q}$. Note that this factor does not affect the predictions of probabilities based on $|\Psi|^2$ nor the distant correlations built into ψ_{sch}. However, our approach tells us that the event at $\underline{q}$ (which now is being described only statistically) cannot be specified until a third sampling has taken place for each degree of freedom. Until this has happened, Ψ represents a prediction and not an "element of reality" in the classical sense. In particular it will contain distant correlations which can be checked experimentally by making local measurements which complete the sampling chains, in our paradigm by the firing of a counter. Until all degrees of freedom are accounted for, there will still be correlations; once all relevant counters have fired, the process is completed and joins the fixed past. Of course these firings are themselves the start of new scattering chains. We see that for us there is no overall "collapse of the wave function," only a local severance of phase chains. When all degrees of freedom are accounted for we are describing separated systems. Thus the question of whether or not two systems are separated for us boils down to whether we can account for all relevant degrees of freedom with or without distant correlations. If we make a mistake in this, we may find unexpected correlations in the laboratory; all this tells us is that we have left out one or more relevant degrees of freedom and should start our analysis over using a more articulated system. Thus we claim that our theory leads naturally to a view of measurement that fits both quantum mechanical and classical experience without generating paradoxes.

V. QUANTUM NUMBERS; m_p/m_e; COSMOLOGY

Our next step is to make a specific interpretation of the bits in our labels, calculate the basic mass ratio to which all others are to be referred, and discuss the cosmological structure implied by our theory. In our earlier presentation (2) it was noted that the 16-bit strings at level 3 suggested identification of these bits with the conserved quantum numbers of baryon number, lepton number, charge and spin, and that if we could assign a space-time direction sense to (01) and (10) dichotomous pairs, we could start to interpret discrimination as a Feynman diagram vertex. Now that we have succeeded in constructing addresses for our labels and have showed that $\langle N_1 - N_0 \rangle$ does indeed define the sense of the velocity in 3+1 Minkowski space-time, we can make this connection precise.

Consider again our elementary sampling event $DS_1S_2=S_3=DS_1'S_2'$ where all strings have the common length 256+b, $DL_1L_2=L_3$ in both cases while A_1A_2 have velocities $\underline{v}_1\underline{v}_2$ referring to two samplings before the event (i.e., $b_1,b_2<b$) and $A_1'A_2'$ have velocities $\underline{v}_1'\underline{v}_2'$ referring to two samplings after ($b_1',b_2'>b$) the event. We see that interchanging "0's" and "1's" in all the address strings reverses all directions, and hence corresponds to the parity operation, while the "direction" of time referes to the $\pm i\Delta\varepsilon$ in the propagator for a scattering process described in the last section. Thus, as in conventional theory, "time reversal" corresponds to complex conjugation. Consider first the labels for level 1. We have three basic processes: (10)+(01)→(11)→(10)'+(01)', where we have used an obvious shorthand for the addresses, (11)+(10)→(01)→(11)'+(10)', and (01)+(11)→(10)→(01)'+(11)'. At this level we cannot say whether this quantum number refers to charge, or baryon number or lepton number or helicity for spin 1/2. For concreteness think of it as charge. Then interpret the string (xy) with x=1, y=0 as + charge, x=0, y=1 as - charge and x=1, y=1 as a "bound state" of two particles with positive and negative charge which, under macroscopic examination, behaves as an uncharged particle. Then the three basic processes are seen, externally, to conserve charge.

The "internal" structure requires discussion. Consider the second case in which we have + charge externally and - charge internally. But internally, in an elementary process we cannot assign the sign of time flow meaningfully, since it is connected to only two and not three processes, and these are "vertices" and not sampling events. The obvious convention to adopt is that at a "vertex" all lines are "incoming" or "outgoing," and that a negative particle moving backward is the same as a positive particle moving forward. Then the apparent contradiction with charge conservation in this process is removed. In fact, this suffices to establish the usual Feynman rule, and can be extended to all our dichotomous quantum numbers as we see below. Thus we claim to have replaced the heuristic argument in (2) by a rigorous definition justified by the construction of "space-time" achieved above.

A second point requires care. In the addresses there is a symmetry between the interchange of "0's" and "1's" which we exploited in our derivation of the Lorentz transformation. But this symmetry does not apply to the labels; it would take (11) into (00) which is not allowed because the latter is the null string and has an absolute significance. However, we can make an interchange (xy)→(yx) which interchanges + and - charge (or baryon and antibaryon or lepton and antilepton) in our dichotomous notation. Clearly this corresponds to "charge conjugation" in the conventional theory. Spin or helicity cannot be discussed in quite the same way; we defer it to the discussion of level 3.

Our construction of level 2 above was based on a matrix mapping. If we note, following Vanzani (23), that the minimal number of amplitudes needed to describe N particle scattering is $2^{N-1}-1$, we can reinterpret level 2 in terms of seven processes generated by four particles. If we interpret out 4-bit string as $(a\bar{a}+-)$ where a and $\bar{a}$ stand for particle and antiparticle, we have two choices depending on whether the particle has positive or negative charge. Both lead to the unique 7 strings of level 2 as we see:

<u>a positive</u>	<u>b negative</u>			
a+(1010)	b−(1001)	(a+,$\bar{a}$0),(b−,$\bar{b}$0) = q−(1101)	(q−,q0)	(0001)
$\bar{a}$−(0101)	$\bar{b}$+(0110)	(a0,$\bar{a}$0),(b−,$\bar{b}$+) = q0(1100)	(q−,q+)	(0011)
a0(1011)	b0(1000)	($\bar{a}$−,a0),($\bar{b}$+,b0) = q+(1110)	(q0,q+)	(0010)
$\bar{a}$0(0111)	$\bar{b}$0(0100)		(q+,q0,q−)	

Clearly if we think of the first case as the charge structure of $p\bar{p}n\bar{n}$, q+q0q− correspond to π+π0π− and the four additional quanta to ρ+ρ0ρ− and ω0. The alternative choice would be $e\bar{e}\nu\bar{\nu}$. In both cases we have the basic charge structure of the mesons that describe phenomena in the few hundred MeV region—nuclear forces for the first and electron-positron scattering in the second, where the ρ is indeed the prominent resonant structure.

This interpretation holds up when we go to level 3 and interpret the 8 basic particles we need to form the 7 string basis by adding the left and right helicity states to baryon number, lepton number and charge according to the schema

	B	$\bar{B}$	B+	B−	BL	BR	$\bar{B}$L	$\bar{B}$R	
pL	(1	0	1	0	1	0	0	0	00000000)
pR	(1	0	1	0	0	1	0	0	00000000)
$\bar{p}$L	(0	1	0	1	0	0	1	0	00000000)
$\bar{p}$R	(0	1	0	1	0	0	0	1	00000000)

		ℓ	$\bar{ℓ}$	ℓ+	ℓ−	ℓL	ℓR	$\bar{ℓ}$L	$\bar{ℓ}$R
eL	(00000000	1	0	0	1	1	0	0	0)
eR	(00000000	1	0	0	1	0	1	0	0)
$\bar{e}$L	(00000000	0	1	1	0	0	0	1	0)
$\bar{e}$R	(00000000	0	1	1	0	0	0	0	1)

Here we must use care since a Lorentz transformation can, for finite mass particles, change the sign of <u>v</u> without affecting the label, and hence take an L state into an R state when referred to an <u>external</u> coordinate system. Consequently this set contains only four basic particles ($p\bar{p}e\bar{e}$) and we must add to them $n,\bar{n},\nu,\bar{\nu}$ as already foreshadowed in our discussion of level 2. We do not have space here to go into details of how this is done and how we arrive at our version of the CPT theorem. We note that by putting together a particle and an anti-particle (aLaR$\bar{a}$L$\bar{a}$R) we get vector quanta VL(1010), V_0(1111), VR(0101) and a singlet quantum (0000). Since we have reason to believe from the zero range scattering theory (18) that it may be convenient to take a Wheeler-Feynman view of "photons" and give them a mass below the current threshold of detection, in that case the singlet quantum becomes the "coulomb" photon in the radiation gauge. We do not have to commit ourselves at this point to whether or not the neutrino has a mass as the question is still open experimentally. If it does not then we should use a "two component" theory which in our notation will be ν:(0000000010001000) $\bar{\nu}$:(00000000010000001). The decision will only be made after we have faced the dynamics of level 4, where we must achieve weak-electromagnetic unification, calculation of the Cabbibo angle, quarks, gluons, and heavy leptons. Since we obviously have an 8+8 structure at level 3, this might come about through some version of Harari's $0(8)\times 0(8)$ "rishons"; we leave this as a question for future research.

Although we are not ready to discuss dynamical calculations at level 4 where, as we have already seen, the basic structure is tetrahedral, the four corners being labeled by the four levels of the hierarchy and connected to external lines, and hence among other processes to the decay of unstable particles such as the neutron, we can already calculate the basic mass ratio m_p/m_e using only level 3 concepts. Since the calculation has already been published (2,3) starting from slightly different but closely related points of view, and the algebraic details are unchanged, we only summarize the argument here, using the current context. The idea is to calculate the electron mass-energy from its electrostatic energy, an idea that antedates quantum mechanics. If we start from an electron at rest with its level 3 label, it can "open up" via the uncertainty principle fluctuations into two vertices labeled by level 1 and level 2 labels and close again with the electron label forming a closed tetrahedral structure, two of whose corners have the same label and which is therefore not chiral. Since we have seen that charge is conserved, one of these corners will carry xe and the other (1-x)e where x is a variable we estimate statistically and lies between 0 and 1 (inclusive). Assuming that these are separated by a distance r, again a statistical variable, we then have that $m_ec^2=e^2\langle x(1-x)\rangle\langle 1/r\rangle$. Since the only other stable (or at least stable enough for the purposes of this calculation, since decays only come in at level 4) particles are the proton and antiproton, the energy which, in this static calculation, defines our shortest discrete step length is $2m_pc^2$, we have that $\langle 1/r\rangle=2m_pc/h\langle 1/y\rangle$ where y is a discrete random variable ranging from 1 to some large number N which for practical purposes can be taken to infinity. Hence $m_ec^2=(2m_pce^2/h)\langle x(1-x)\rangle\langle 1/y\rangle$.

Since the "coulomb photon" through which the process starts and ends is only one of the 137 "quanta" available to us at level 3, the a priori probability of the process is 1/137, which we identify with $e^2/\hbar c$. After the first step, we keep on until the process eventually closes back on the electron. Since each step will increase the separation of the two (statistically defined) lumps of charge and the steps have three degrees of freedom (either the three dimensions of the space or the three degrees of freedom of the three levels of the hierarchy used in this context) we find that the weighting factor $P(1/y)=1/y^2$ and hence that $\langle 1/y\rangle = \int_1^\infty dy/y^6 \,/\, \int_1^\infty dy/y^5 = 4/5$. Since the probability of the separation occurring is proportional to $x(1-x)$ and the process must close, the weighting factor for the charge is $(x(1-x))^2$. For one degree of freedom this gives $K_1 = \langle x(1-x)\rangle_1 = \int_0^1 x^3(1-x)^3 dx \,/\, \int_0^1 x^2(1-x)^2 dx = 3/14$. Once the charge has separated the effective charge is x^2 or $(1-x)^2$, so we can write the recursion relation

$$K_n = \frac{\int_0^1 x^3(1-x)^3 + K_{n-1}x^2(1-x)^4}{\int_0^1 x^2(1-x)^2 dx} = \frac{\int_0^1 x^3(1-x)^3 + K_{n-1}x^4(1-x)^2}{\int_0^1 x^2(1-x)^2 dx}$$

$$= \frac{3}{14} + \frac{2}{7} K_{n-1} = \frac{3}{14}\sum_{j=0}^{n-1}\left(\frac{2}{7}\right)^j$$

Putting all this together we find that

$$m_p/m_e = \frac{137\pi}{\langle x(1-x)\rangle\langle 1/y\rangle} = \frac{137\pi}{3/14(1+2/7+4/49)4/5}$$

$$= 1836.151497 \;....$$

in remarkable agreement with the empirical value of 1836.15152(70). This completes the chain of discrete rules and calculations which allows us to connect our numerical theory with laboratory experience.

It is worth noting that our linear theory forces us (as we saw in the discussion of the Einstein-deBroglie relation) to take as our basic step length $\lambda_E = hc/E$ rather than the $\hbar/p$ that seems more natural in a wave theory. This has the consequence that we expect the discrete aspects of our theory to come into prominence in the nuclear force problem at distances of $h/2m_p c = 0.63$ fm, rather than the usual estimate from the Wick-Yukawa mechanism of $\hbar/m_\pi c = 1.4$ fm. Actually this is all right since our zero range theory gives us (19,20) the usual one pion exchange "interaction" at these distances. But as is well known, though not always admitted, the conventional theories of the nuclear force based on hadrons break down completely at "distances" of $\hbar/2m_\pi c = 0.7$ fm, which is still outside our absolute limit. Current theoretical efforts based on quantum chromodynamics try to meet this with a phenomenological "quark bag" whose radius in some versions is ~1 fm, which may already be in trouble with what we "know" empirically about the nuclear force from nuclear physics. Thus, if our fundamental theory is to be approximated phenomenologically, we are comfortably and unequivocally on the side of the "small bag" theorists in this controversy.

To calculate the central mass values of unstable particles will have to be deferred, as already noted, until we can make a firm assignment of quantum numbers at level 4. At that point we will also have to calculate the correction to 1/137 arising from weak-electromagnetic unification, and a host of other problems. Clearly the theory will at that level face stringent empirical tests, and in spite of this promising beginning, could still fail. But there is one problem we must anticipate already when it comes to gravitation. Using the Dyson argument to relate the gravitational coupling constant to the maximum number of gravitating massive particles we can define within their own reduced compton wave length, we find a rest mass $m_G=[(\hbar c/G)/(2^{127}-1+137)]^{\frac{1}{2}}\approx 936$ MeV/c^2 rather than the proton mass $m_p \approx 938$ MeV/c^2. This of course does not upset our m_p/m_e calculation since even if m_G should be taken as the fundamental unit of mass, it will cancel out in the ratio. But it does raise the question of why the proton is not the fundamental unit. Perhaps it is in fact unstable, and the difference we encounter here is due to an electromagnetic correction which we should ultimately be unable to compute. The question is open for us at present, but we incline to the view that neither is the fundamental unit. We think it would be best to take the mass unit as the rest mass of the universe, which brings us to cosmology.

Returning to our three fundamental processes, we have seen that G and D and our minimum labeling rule first bring us to 1.7×10^{38} labels which we have now identified with "quanta," then fill up the remaining labels of the $2^{256}-1$ non-null possibilities in the 256-bit string with labels we have been able to identify with "particles" (although so far only a few of them with laboratory definitions), and then go on to add "addresses" of increasing length n. Once this starts we have scattering processes which, because of our conservation laws, will settle down to mainly electrons, protons, neutrinos and photons, and if we get our particle physics right, to nuclei, stars, galaxies, planetary systems, biological evolution, social evolution, and "intelligent" technological societies which can begin to understand these processes. Thus our cosmology, up to a point, is fairly conventional. But it has constraints. Clearly the lepton number and baryon number of the universe cannot exceed, for us, $(1/2)(2^{256}-1)$ and must lie between that upper limit and $2^{127}-1+137$. This is not in conflict with current estimates of this number. We must ultimately be able not only to compute this number with precision, but also the finite energy of the universe. Thus we will eventually face stringent empirical tests coming from cosmological "observations."

A second aspect of our model deserves mention. Because for us the maximum discrete address length at any time, n, refers back to our initial generating operation, the "time since the big bang" is indeed for us an <u>absolute</u> time which (outside of being discrete and statistical) is conceptually analogous to the absolute time of Newton. Further, the address strings of length n_{max} which contain all "1's" or all "0's" provide an "event horizon" which keeps on growing. Unless the rest energy associated with our "big bang" turns out to be unexpectedly large, we anticipate an "open" universe. We find this encouraging, particularly in the light of Dyson's recent analysis entitled "Time Without End" (24) which suggests that the evolution of complex communicating systems can continue forever in such an environment.

VI. CONCLUSION

We claim in this article to have shown that three well defined recursive processes supplemented by self-consistent rules of interpretation provide a self-generating, self-organizing system into which the current practice of physics fits in a natural, and we hope compelling, way. Thus we claim to provide an "exoskeleton" for our understanding of the universe, based on the conventional dimensional units of physics, — mass, length and time — all of which have "derived." Yet we find it remarkable that only these three dimensional units have sufficed for three centuries to form the basis of physics. Our model is clearly richer than this. Our definitions show how the informational content associated with physics can be accessed, but there could be other stable structures with implications that could be tested in the laboratory. For instance, "naked charm" and other quark flavors have been demonstrated indirectly by means of particle experiments. But the quarks and heavy leptons only scratch the surface of the $2^{127}-1+137$ possibilities we necessarily have in our theory. Perhaps the three "colors" of quantum chromodynamics are a selection from a much richer spectrum which (though ultimately discrete from our point of view) might add a new "dimension" that goes beyond mass, length and time. Although our theory might seem to set rather rigid boundaries to what has been called "physics," it could at the same time open up unexpected possibilities for research. Only the uncertain future can decide.

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VII. FOOTNOTES

1. Work supported by the Department of Energy, contract DE-AC03-76SF00515.
2. A preliminary version of this paper prepared before the symposium is available as SLAC-PUB-2906 (April 1982).
3. The work summarized here will be discussed in detail in a paper with the tentative title "A Reconstruction of Quantum Mechanics using a Discrete, Recursive Model" intended for submission to Physical Review D.
4. This contention is presented in detail in T. Bastin, *A Combinatorial Basis of the Physics of the Quantum* (in preparation).

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