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BREAKDOWN OF QCD FACTORIZATION THEOREMS FOR INCLUSIVE REACTIONS*

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ABSTRACT

Initial state interactions are shown to violate standard factorization for massive lepton pair production and hadron-induced hard-scattering inclusive reactions order-by-order in QCD perturbation theory. Initial and final state interactions lead to a number of new physical phenomena including $k_\perp$ fluctuations, color correlations, anomalous nuclear number dependence of inclusive cross sections, and induced hadron production in the central rapidity region.

1. Introduction

A basic tool of QCD phenomenology is the factorization ansatz, which allows inclusive cross sections at large momentum transfer to be computed directly in terms of the structure functions and fragmentation functions measured in deep inelastic lepton scattering. For example, the Drell-Yan cross section[1)] for massive lepton pair production in $\bar{p}$ nucleus collisions is written as

$$\frac{d\sigma}{dQ^2 dx_L}(\bar{p}A \to \ell\bar{\ell}X) = \sum_q e_q^2 \int_0^1 dx_a \int_0^1 dx_b \; G_{q/A}(x_a,Q) \; G_{\bar{q}/p}(x_b,Q) \; d\hat{\sigma} \qquad (1)$$

where

$$d\hat{\sigma} = \frac{1}{3}\frac{4\pi\alpha^2}{3Q^2}\,\delta(x_a x_b s - Q^2)\;\delta(x_L - x_a + x_b)\left[1 + \mathcal{O}\left(\frac{\alpha_s(Q^2)}{\pi}\right)\right]$$

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is the $q\bar{q} \rightarrow \ell\bar{\ell}$ subprocess cross section, and the distributions $G_{q/A}$ and $G_{\bar{q}/\bar{p}} = G_{q/p}$ are measured in deep inelastic lepton-nucleus and lepton-nucleon scattering. The factor of 1/3 results from assuming that the annihilating q and $\bar{q}$ have uncorrelated colors. Derivations[2] of this result in QCD perturbation theory are based on the observation that the logarithmic dependence of the distributions G(x,Q) arising from collinear gluon radiation is process independent and that, at least to leading twist (i.e., leading order in $1/Q^2$), all infrared divergences cancel. The distributions can also be related to the Fock state QCD wavefunctions for each hadron defined at equal time on the light-cone. In principle, one can compute the radiative corrections to the subprocess cross section $d\sigma$ as an expansion in powers of $\alpha_s(Q^2)$ (although one may need to sum certain large corrections to all orders). Given Eq. (1), one can predict the normalization, scaling and logarithmic evolution, and nuclear number dependence for the cross section. In particular, one predicts $d\sigma/dQ^2(pA \rightarrow \ell\bar{\ell}X) = A^1 d\sigma/dQ^2(\bar{p}N \rightarrow \ell\bar{\ell}X)$ in the region of large Q^2 where deep inelastic lepton scattering is additive in the nucleon number.

Although these results are familiar, they are nevertheless puzzling from a physical standpoint. Intuitively one expects that the incident $\bar{p}$ should be strongly affected by hadronic interactions upon passage through the nucleus—i.e., its Fock state structure should be profoundly disturbed by upstream elastic and inelastic collisions. One expects (A-dependent) modifications of the $k_\perp$ and x_L distributions of the quarks in the beam and target, accompanied by inelastic hadronic radiation. Certainly in the case of a macroscopic target, where one observes radiation losses, multiple scattering, and secondary beam production, Eq. (1) cannot be correct.

2. Initial State Interactions in QCD Perturbation Theory

In order to understand the above questions, we have recently investigated the effects of initial state (elastic and inelastic) interactions in the context of perturbative QCD.[3] The result to any order in α_s can be organized into the eikonal formalism when one uses time-ordered or light-cone perturbation theory and physical gauges, although the final results are gauge and frame independent. The leading initial state contributions at high energy due to gluon exchange between quarks of the interacting hadrons comes from the eikonal region of near-on-shell propagation. In the CM system the important region of exchanged gluon 3-momentum is finite transverse momentum $\ell_\perp$ (of the order of the inverse hadron size) and "wee" longitudinal momentum $\ell^z \leq \mathcal{O}(\ell_\perp^2/\sqrt{s})$. The contributions from the region $|\ell^z| >> \ell_\perp^2/\sqrt{s}$ are gauge dependent and factorize in the sense that they can be included in the structure functions defined in deep inelastic scattering when all contributions, including final state interactions, are taken into account.

The result of summing the elastic initial state interactions from the Glauber region is that the standard Fock state wavefunctions for the incident hadrons are multiplied (in impact space) by an unitarity eikonal phase:

$$\psi_A(x_a,\vec{z}_a^{\perp})\ \psi_B(x_b,\vec{z}_b^{\perp}) \Rightarrow \psi_A(x_a,\vec{z}_a^{\perp})\ \psi_B(x_b,\vec{z}_b^{\perp})\ U(\vec{z}_i^{\perp}) \qquad (2)$$

where the $\vec{z}_i^{\perp}$ are impact distances measured relative to the annihilation point. The phase U is a unitary color matrix constructed from a path ordered exponential of the eikonal potential

$$U(\vec{z}_i^{\perp}) = \mathscr{P}_\tau \exp\left\{-i\int_{-\infty}^{0} d\tau\ V(z_i^{\perp},\tau)\right\} \qquad (3)$$

integrated over the particles' trajectories up to the time of annihilation; in general, U can depend on log s. The (two-particle irreducible) eikonal potential includes all pairs of constituents, one taken from each of A and B:

$$V = \sum_{\substack{i\varepsilon A,\\ j\varepsilon B}} V_{ij}(\vec{z}_i^{\perp} - \vec{z}_j^{\perp} + \vec{\Delta}\tau) \qquad (4)$$

where $\vec{\Delta}$ is along the beam direction. Explicit evaluation of the lowest order eikonal potential is given in Ref. 3. We note that the trigluon coupling and cross graph diagrams (Figs. 1(h) and 1(i)) do not contribute to the order α_s^2 eikonal potential in physical gauges, whereas the reducible contribution of Fig. 1(g) is the iteration of 1(c) in the eikonal phase.

Since the eikonal phase varies with impact parameter, it obviously modifies the transverse momentum of the quarks. Thus the measured $\vec{Q}_\perp = \vec{k}_\perp^a + \vec{k}_\perp^b$ distribution of the lepton pair cannot be identified with the transverse momentum intrinsic to the Fock state wavefunctions. One expects that the fluctuations induced by initial state interactions would be increased in a nuclear target ($\delta\langle Q_\perp^2\rangle \propto A^{1/3}$). There is some empirical evidence[4)] for such an A-dependent smearing of the $Q_\perp$ distribution (see Fig. 2).

For the lepton pair cross section $d\sigma/dQ^2dx_L(A+B\to\ell\bar{\ell}X)$ (integrated over transverse momentum) the effect of the eikonal phase is to replace $d\hat{\sigma}$ by

$$U^{+}(\vec{z}_i^{\perp})\ d\hat{\sigma}\ U(\vec{z}_i^{\perp}) \qquad (5)$$

$d\hat{\sigma} = d\hat{\sigma}(qq\to\ell\ell)$ is the subprocess cross section; U^+ and U are to be evaluated at the same impact coordinates. For an Abelian theory this replacement has no effect on the cross section since U and $d\sigma$ commute and $U^+U=1$. Physically this corresponds to the fact that even though the transverse momentum of each quark is modified by the eikonal interactions, the total flux is conserved. In lowest order this can be seen explicitly since the square of the amplitude of Fig. 1(c) is cancelled by the interference of 1(g) with the Born amplitude 1(b).

However, in the case of non-Abelian gauge theories such as QCD, U and $d\sigma$ do not commute, since the eikonal phase is a color matrix. Thus initial state interactions involving color exchange can introduce color correlations, dramatically affecting the normalization of the Drell-Yan cross section. For example, consider the contribution of the eikonal potential in lowest order (two loop corrections). In an Abelian theory, the interference contribution $2R_e M_{1e}^{+} M_{1b}$ cancels the positive contribution of $M_{1c}^{+} M_{1c}$. In SU(n_c), the interference contribution is changed in sign and reduced in magnitude by a factor of $1/(n_c^2 - 1)$ (= 1/8 in QCD) relative to the Abelian result. Since the eikonal potential involves $\alpha_s(\ell_\perp^2)$ evaluated at low momentum transfer, we cannot reliably calculate the eikonal phase in perturbation theory. We can show, however, by unitarity, that the effect of the color correlation in the initial state is bounded: the standard Drell-Yan factor of $1/n_c$ could be enhanced by a factor as large as $n_c^2 = 9$ or reduced to zero. On the other hand, the final state eikonal interactions do not change the deep inelastic cross section.

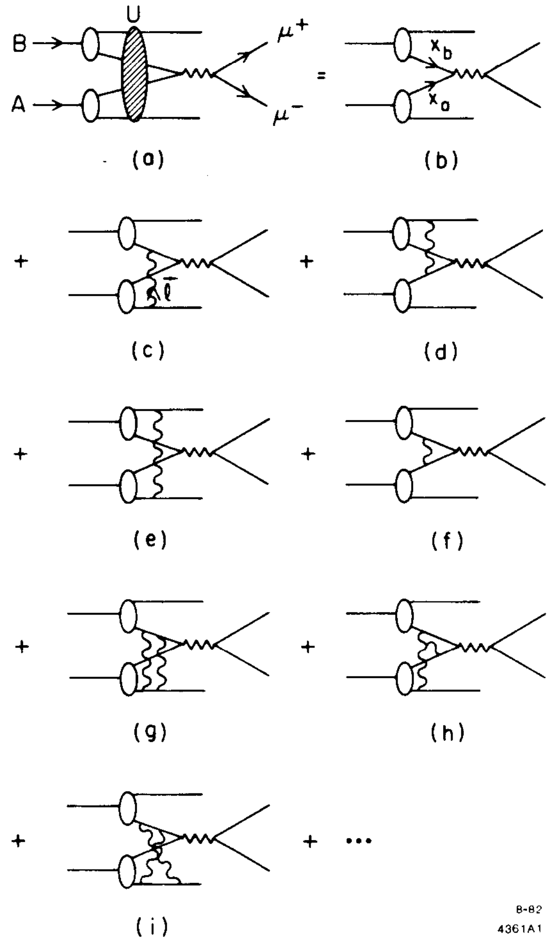

Fig. 1. Perturbative QCD expansion of the elastic eikonal initial state interaction for the Drell-Yan process.

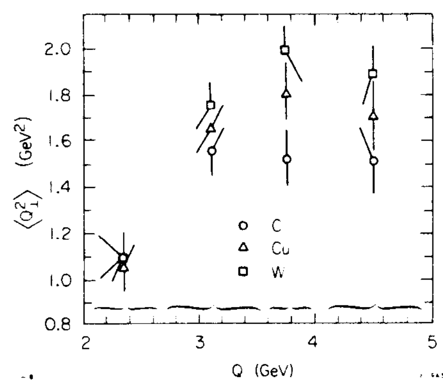

Fig. 2. Chicago-Illinois-Princeton data for the mean square transverse momentum of a muon pair produced in pion-nucleus collisions. (From Ref. 4).

Flavor-changing initial state interactions also affect the Drell-Yan cross section. As an extreme example, consider the Drell-Yan process for two mesons whose valence states are $\bar{s}d$ and $c\bar{u}$. If we neglect sea quarks, the process cannot occur without initial state interactions.[5] However, W-boson exchange between an active quark and spectator (as in Fig. 1(c)) leads to a finite Drell-Yan cross section at high energies. More generally, the Drell-Yan cross section is modified by any quantum number exchange components of the leading (Pomeron) trajectory for quark anti-quark scattering.

As emphasized by Mueller,[6] the color correlation effect in the Drell-Yan cross section is likely to be suppressed at asymptotic Q^2 by a quark (Sudakov) form factor $|S(Q^2)|^2$ once hard gluon radiative corrections are taken into account. The critical point is that once a color correlation appears (as in Fig. 3(a)) the usual cancellation of double logarithms between real (Fig. 3(b)) and virtual (Fig. 3(c)) gluon corrections to the hard-scattering cross section ($q\bar{q} \to \ell\bar{\ell}$) is spoiled. Assuming that the high energy behavior can be obtained by summing such logarithms to all orders, one finds that the actual correction to the Drell-Yan cross section due to color correlations is reduced by a Sudakov form factor

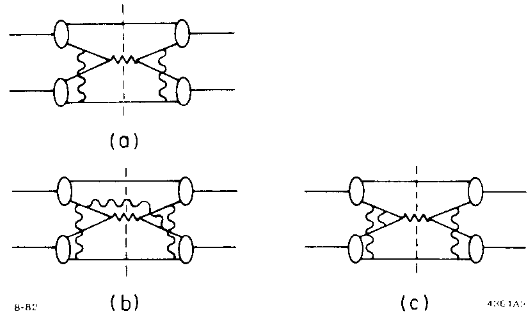

Fig. 3. Illustration of hard radiative corrections to the $q\bar{q} \to \ell\bar{\ell}$ subprocess after an initial state interaction.

$$|S(Q^2)|^2 \cong \exp\left\{\frac{-C_A}{\beta_0}\, \ell n\left(\frac{\ell n Q^2/\Lambda^2}{\ell n Q_0^2/\Lambda^2}\right) \ell n\, \frac{Q^2}{Q_0^2}\right\} \qquad (6)$$

where $C_A = n_c = 3$, $\beta_0 = 11\text{-}2/3\, n_f$, Λ is the QCD scale parameter, and Q_0 is the scale of the typical momentum transfer relevant to initial state corrections. For typical parameters ($\Lambda \sim 100$ MeV, $Q_0 \sim 400$ MeV), $|S(Q^2)|^2 \sim 1/3$ at $Q^2 = 10$ GeV^2 and ~1/10 at $Q^2 = 100$ GeV^2. The color correlation effect could thus make a significant effect on the normalization of QCD processes at current energy scales, and might be negligible only at very large Q^2. In addition the Q^2 dependence of $|S(Q^2)|^2$ introduces an important new source of scaling violation, quite different from that expected from conventional QCD structure function evolution.

3. Inelastic Initial State Effects

Central to the entire analysis outlined above is the fact that the longitudinal momenta of the active quarks are unchanged by initial state eikonal collisions. However, in gauge theories even the softest of collisions between high energy quarks can induce gluonic bremsstrahlung carrying off large fractions of the quarks' longitudinal momenta. Such radiation, if it could occur in an initial state collision before the $q\bar{q} \to \ell\bar{\ell}$ annihilation, would greatly soften the quark x distribution, destroying any semblance of factorization. In fact, there is insufficient time for hard collinear radiation to occur in a target of finite length if Q^2 is large enough. To see this, consider the diagrams of Fig. 4. Figure 4(a) corresponds to radiation induced by an initial active-quark spectator-quark collision, followed by near on-shell propagation and annihilation. In Fig. 4(b) the radiation is associated with the annihilation process. It is easy to check that these two process destructively interfere, provided that the longitudinal momentum transferred to the spectator quark of the target ($\ell_\perp^2/(x_b\sqrt{s})$ in the CM) is smaller than the resolution scale ($L^{-1}\ \sqrt{s}/M_A$) of the target wavefunction. The condition for no induced collinear radiation from the beam quark is then

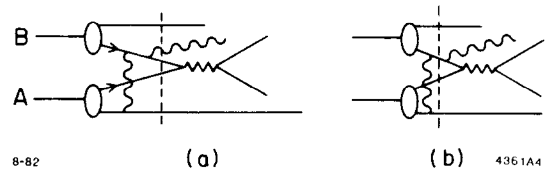

Fig. 4. Cancellation of induced collinear radiation. Figures (a) and (b) represent two different Glauber contributions of the same Feynman amplitude.

$$x_b\ s_{AB} \gg \langle \ell_\perp^2 \rangle\ L_A M_A \quad . \tag{7}$$

For a hadron interacting in a nuclear target, this is equivalent to the condition

$$Q^2 \gg x_a\ M_N L_N\ \langle \ell_\perp^2 \rangle\ A^{2/3} \tag{8}$$

where $L_N A^{1/3}$ is the nuclear target length. This condition is easily satisfied for a nucleon target, but significantly larger Q^2 is required to avoid induced collinear radiation in a heavy nucleus ($Q^2 > x_a$ 24 GeV^2 for uranium). It is also apparent from this condition that hadronic radiation is inevitable as the beam hadron passes through a macroscopic target. The condition (8) is a basic constraint for the validity of QCD formulae for hard scattering hadron-induced processes.

The absence of induced collinear radiation at sufficiently high energies is related to the concept[7] of the "formation zone" — the fact that the state of a system cannot be modified significantly (as in $q \to q+g$) in a time (in its rest system) less than its intrinsic scale. Thus at high energies collinear radiation occurs outside of the nuclear target; its effects can be factorized into structure functions (or in the case of final state processes, fragmentation distributions).

Initial state interactions create gluons in the central rapidity region (i.e., particles of limited longitudinal momentum in the CM) even when condition (8) is satisfied. An example of a contributing diagram is shown in Fig. 5. These inelastic interactions tend to destroy the coherence of the active q and $\bar{q}$ wavefunctions, and diminish the Drell-Yan cross section and quark momentum at large x_q and x_q. The effect is negligible provided that

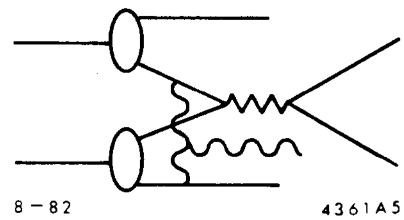

Fig. 5. Diagram leading to induced radiation in the central region.

$$\frac{Q^2}{x_a x_b} >> (M_N L_N)^2 \langle \ell_\perp^2 \rangle A^{2/3} \quad . \tag{9}$$

The central region multiplicity produced by these interactions should grow as the number of collisions in the nucleus ($\sim A^{1/3}$). Radiation at large transverse momentum is also not suppressed, according to condition (8); however, these effects are associated with the standard QCD radiative corrections to the hard-scattering process or the Sudakov form factor, as discussed above.

4. General Remarks Concerning Initial and Final State Interactions

A systematic approach to elastic and inelastic initial and final state interactions in QCD is discussed in Ref. 3. The net result is that the eikonal phase U is not only a matrix in color space, but in Fock space as well; it can create or destroy spectators in the central region as indicated in Fig. 6.

An intriguing and important feature of the eikonal interactions is their range. Since the essential scale of longitudinal momentum transfer in the CM is $\Delta\ell^z \sim \langle \ell_\perp^2 \rangle / \sqrt{s}$, the effective longitudinal range in this frame grows like $\sqrt{s}$, becoming much larger than the Lorentz-contracted lengths of the target or beam. Thus in $\bar{p}A \to \ell\bar{\ell}X$, the beam antiquark interacts with the entire nucleus even if the annihilation is at the front face. The initial-state color correlation thus has non-trivial A dependence. A large n_c analysis in QCD suggests, however, that any deviation from the A dependence of deep inelastic scattering is suppressed by a factor $A^{1/3}/n_c^2$; thus the nucleon cross section for massive lepton production could still be roughly additive in the nucleon number A.

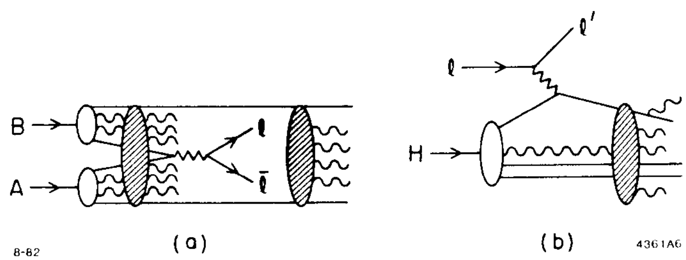

Fig. 6. Inelastic eikonal initial and final state corrections to lepton pair production and deep inelastic scattering.

It is clear that very high precision nuclear target measurements are necessary in order to analyze the initial and final state QCD effects. Such tests include measurements of the A-dependence of the lepton pair cross section, the pair transverse momentum $\langle Q_\perp^2 \rangle^{1/2}$, the associated central region production, and the depletion of the quark momentum distributions at large x. It is also important to investigate the transition between low and high Q^2 physics as set by condition (8). In the case of deep-inelastic scattering, the final state interactions of the outgoing quark as it passes through the nucleon target leads to analogous effects (see Fig. 6(b)), including associated central region radiation and k_T smearing.

The development of hadronic multiplicity in deep inelastic lepton scattering in the nucleus is particularly interesting, since one can study the influence of hadronic matter on quark jet propagation. As we have emphasized, formation of leading particles occurs outside the nuclear volume at high energies. The inelastic final state interactions correspond to cascading in the nucleus and demonstrate that, contrary to the usual assumptions made for the analysis of hadron-nucleus collisions, particle production in the target and central rapidity region cannot be directly correlated to the number of nucleons "wounded" by the beam. A model for the shape of the rapidity distribution based on "color cascading" is given in Ref. 8. We also note that interactions between quark or gluon constituents of a given hadron do not occur (to leading order in 1/s) during the transit through the target volume. Thus high energy interactions within nuclei are correctly described in terms of constituent quark and gluon propagation.

Although the discussion here has been limited primarily to the Drell-Yan process, virtually every inclusive hadronic target large momentum transfer test of QCD is affected by initial or final state interactions. The color correlation effects may be largest for processes such as $gg \to \chi$ which have strong color suppression in lowest order. [The usual color factor for these processes, 1/8, could be increased by a factor as large as 64.] The normalizations of the leading p_T^{-4} processes for high p_T hadron and photon production are also modified by initial state color correlations. In addition to the change in normalization of the leading twist subprocess cross section, hadron production at large transverse momentum in a nucleon or nuclear target collision is increased by the $k_\perp$ smearing effects of the initial and final state interactions.[9)] The multiple scattering series in a nucleus A leads to terms roughly of order A^1, $A^{4/3}/p_\perp^2$, etc. The coefficient of the A^α terms with $\alpha > 1$ can be large, since one is smearing a cross section that falls very rapidly with $p_\perp$.

Inevitably, the most strongly suppressed cross sections (such as $pA \to \bar{p}X$ and $pA \to K^- X$) receive the largest nuclear enhancement from quark and gluon scattering effects.

The nuclear initial and final state effects are, of course, enhanced in processes such as $A_1 A_2 \to HX$. Nuclear targets also enhance the effects of multiple scattering processes that lead to multiple jets in the final state. On the other hand, the part of the valence state of a hadron that consists of constituents at small transverse distances can pass through the target with no color or hadronic interactions. An application of this idea to diffractive dissociation processes in nuclei is discussed by G. Bertsch et al.[10]

More generally, any hard scattering inclusive process is accompanied by soft hadrons in the central rapidity region, which are the result of the initial state or final state interactions of the quark and gluon constituents. Even though the hard scattering cross section can be computed as if a single interaction occurs, the associated multiplicity distribution reflects the full scope of the actual QCD dynamics.

Finally let us briefly discuss the effects of initial and final state interactions on exclusive or semi-exclusive processes. Large momentum transfer exclusive reactions are controlled by the Fock states with the minimum number of constituents at transverse distances $b_\perp^2 \sim (1/Q^2)$.[11] The initial and final state collisions can probe transverse distances no smaller than the typical hadronic scale. Thus they cannot resolve the internal structure of the hadrons in exclusive reactions, and they do not couple to these color neutral objects. Formally, the initial and final state interactions cancel to leading order in $1/Q^2$ if one adds the contributions coming from all constituents of a color neutral hadron. This also implies that large momentum transfer quasi-elastic reactions such as $eA \to ep(A-1)$ and $\pi A \to \pi p(A-1)$ can occur deep inside a nuclear target without multiple scattering or bremsstrahlung in the target.[12] Color singlet cancellations also eliminate initial and final state interactions of hadrons interacting directly in hard scattering inclusive reactions.[13] For example, the "direct pion" has no initial state interactions in $\pi_D g \to qq$ (in $\pi p \to qqX$), and no final state interactions in $gq \to \pi_D q$ ($pp \to \pi X$). In the case of direct γ production $pA \to \gamma X$, the photon has no final state interactions, so only initial state interactions of the active q and g constituents are important. Similarly, the higher twist $F_L^\pi \sim 1/Q^2$ contribution to the meson structure function is unaffected by initial and final state interactions. Although their contributions are power law suppressed at large momentum transfer, the initial and final state interactions are expected to play an important role in exclusive processes at moderate kinematic values — possibly leading to non-trivial helicity and interference effects.[14]

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