SLAC-PUB-2964
August 1982
(T/E)

ENERGY EIGENVALUES AND STRING TENSION*

IN THE SCHWINGER MODEL

Belal E. Baaquie§
Stanford Linear Accelerator Center
Stanford University, Stanford, California 94305

ABSTRACT

We exactly evaluate the energy eigenvalues of the Schwinger model, and find that due to a nontrivial cancellation the fermion eigenvalues are eliminated. The eigenspectrum of the interacting theory reduces to that of a free massive boson field. We then exactly evaluate the energy of the fermion-string-antifermion state and the string tension. We discuss the relation of the string tension to the Wilson criterion of confinement.

Submitted to Physical Review D

---

*Work supported by the Department of Energy, Contract DE-AC03-76SF00515.
§Permanent address: Modern Consultants, 146/A New Baily Road, Dacca, Bangledesh.

I. INTRODUCTION

The Schwinger model is QED in one space and one time dimension. It has been studied extensively[1] and is now being studied numerically.[2] Exact results[3] provide a useful check to computer calculations, and in this paper we derive some exact results that are amenable to numerical studies.

Although a number of authors[1] have shown that the gauge field picks up a mass due to its interaction with the fermions, there has been, within the author's knowledge, no calculation for the energy eigenspectrum of the interacting theory using the path integral and showing the cancellation of fermion eigenenergies.

This paper is essentially a continuation of Ref. 3, and we will use the notation and results of Ref. 3 extensively. We evaluate the eigenenergies in Section II. In Section III we calculate the energy of the fermion-string-antifermion state and lastly, in Section IV we discuss the string tension and the Wilson criterion for confinement.

II. ENERGY EIGENVALUES

Let H be the Hamiltonian operator for Schwinger QED, E_n the eigenenergies and $|\Phi_n\rangle$ the orthonormal eigenfunctions. That is ,

$$\langle\Phi_n|H|\Phi_m\rangle = E_n \delta_{nm} \tag{2.1}$$

The fermion eigenfunctions using the anticommuting variables has been discussed in Ref. 4.

Consider the 'partition function' for finite time ,

$$Z(\tau) = \mathrm{Tr}\left(e^{-\tau H}\right) \qquad (2.2)$$

$$= \sum_n e^{-\tau E_n} \qquad (2.3)$$

where the sum is over all the eigenenergies. The time τ plays the role of inverse temperature and $Z(\tau)$ can be evaluated using finite temperature methods.[5]

To evaluate $Z(\tau)$, we consider the finite time action for Schwinger QED in two-dimensional Euclidean space, with the boson (fermion) variables being periodic (antiperiodic) with period τ. In effect the field theory is defined on a cylinder of infinite length.

The finite-time Euclidean action is defined using the two component spinors $\bar{\psi},\psi$ and the gauge field A_μ, and we have for coupling constant g,

$$S = -\frac{1}{4g^2} \int_0^\tau dt \int dx \left(\partial_\mu A_\nu - \partial_\nu A_\mu\right)^2 + \int_0^\tau dt \int dx \; \bar{\psi}\gamma_\mu \left(\partial_\mu + iA_\mu\right)\psi \qquad (2.4)$$

$$\equiv S_B + S_F + S_I \quad . \qquad (2.5)$$

Note:

$$A_\mu(t,x) = A_\mu(t+\tau,x) \qquad (2.6)$$

$$\psi(t,x) = -\psi(t+\tau,x) \qquad (2.7)$$

$$\bar{\psi}(t,x) = -\bar{\psi}(t+\tau,x) \qquad (2.7')$$

and

$$\left\{\gamma_\mu,\gamma_\nu\right\} = 2\delta_{\mu\nu} \quad , \qquad \int dx \equiv \int_{-\infty}^{+\infty} dx \quad . \qquad (2.8)$$

We have the path integral for the partition function

$$Z(\tau) = \prod_{t=0}^{\tau} \prod_{x\mu} \int dA_\mu(t,x)\, d\bar{\psi}(t,x)\, d\psi(t,x) \exp\left\{S\right\} . \qquad (2.9)$$

We repeat the calculation of Ref. 3 to perform the fermion integration. To do so, make the change of variables[3] ($\varepsilon_{01} = -\varepsilon_{10} = 1$)

$$A_\mu = \varepsilon_{\mu\nu}\, \partial_\nu s + \partial_\mu \phi \qquad (2.10)$$

where s and ϕ are pseudoscalar and scalar fields respectively. Note s is gauge-invariant. The measure of the gauge field transforms as

$$\prod_{t=0}^{\tau} \prod_{x\mu} \int dA_\mu(t,x) = \det\left(\partial^2\right) \prod_{t=0}^{\tau} \prod_{x} \int ds(t,x)\, d\phi(t,x) \quad , \qquad (2.11)$$

where

$$\partial^2 = \partial_\mu\, \partial_\mu$$

The determinant of ∂^2 is defined on functions periodic in t with period τ.

For the action, we have[3]

$$S = -\frac{1}{2g^2} \int_0^\tau dt \int dx \left(\partial^2 s\right)^2 + \int_0^\tau dt \int dx \left(\bar{\psi}\gamma_\mu \partial\psi_\mu + \bar{\psi}\gamma_\mu \gamma_5 \psi \partial_\mu s\right) \qquad (2.12)$$

where we have decoupled ϕ from the fermions by performing a gauge-transformation and have used

$$\gamma_5 = \gamma_0\gamma_1 \quad , \quad \gamma_\mu\gamma_5 = i\varepsilon_{\mu\nu}\, \gamma_\nu \quad . \qquad (2.13)$$

To perform the fermion path integral, we need the finite-time fermion propagator given by

$$G_\tau(t,x) = \frac{1}{\not\partial} \sum_{n=-\infty}^{+\infty} e^{in\pi} \delta(t-n\tau)\,\delta(x) \qquad (2.14)$$

$$= \sum_{n=-\infty}^{+\infty} e^{in\pi} G(t-n\tau,x) \quad , \qquad (2.15)$$

where

$$G(t,x) = \frac{1}{2\pi} \frac{t\gamma_0 + x\gamma_1}{x^2+t^2} \quad . \qquad (2.16)$$

Hence, for $t^2 + x^2 = a^2 \to 0$, we have

$$G_\tau(t,x) = G(t,x) + \mathcal{O}(a) \quad , \qquad (2.17)$$

which shows that the short distance behavior of G_τ is the same as the $\tau = \infty$ case.

Note in Ref. 3, the axial vector current coupling $\bar\psi\gamma_\mu\gamma_5\psi\,\partial_\mu S$ was regularized and the fermion path integral was then performed using the axial anomaly. The only property of the fermion propagator which entered in the axial anomaly was its zero-distance behavior, which is unchanged for finite time as in (2.17). Hence we have, as in Ref. 3,

$$\exp\{S'\} \equiv \prod_{t=0}^{\tau} \prod_{x} \int d\bar\psi\, d\psi\, \exp\left(S_F + S_I\right) \qquad (2.18)$$

$$= \exp\left\{-\frac{1}{2\pi} \int_0^\tau dtdx\left(\partial_\mu s\right)^2\right\} \prod_{tx} \int d\bar\psi\, d\psi\, \exp\left\{\bar\psi\not\partial\psi\right\} \qquad (2.19)$$

$$= \exp\left\{-\frac{1}{2\pi} \int_0^\tau dtdx\left(\partial_\mu s\right)^2\right\} \det{}_F(\not\partial) \quad . \qquad (2.20)$$

Note that the fermion determinant $\det_F(\not{\partial})$ has to be evaluated over the functions antiperiodic in time with period τ.

To perform the boson integration, we drop the redundant integration over ϕ (which is equivalent to choosing the Landau gauge), and have, for $m^2 = g^2/\pi$,

$$Z(\tau) = \left(\det_F \not{\partial}\right) \left(\det \partial^2\right) \prod_{t=0}^{\tau} \prod_x \int ds(t,x)$$

$$\times \exp\left\{- \frac{1}{2g^2} \int_0^\tau dt\, dx \left[\left(\partial^2 s\right)^2 + m^2(\partial s)^2\right]\right\} \quad (2.21)$$

$$= \frac{\left(\det \partial^2\right)\left(\det_F \not{\partial}\right)}{\sqrt{\det\left[\partial^2\left(\partial^2 + m^2\right)\right]}} \quad (2.22)$$

$$= \frac{\det_F(\not{\partial})\sqrt{\det \partial^2}}{\sqrt{\det\left(\partial^2 + m^2\right)}} \quad . \quad (2.23)$$

From standard results[5] we have for $d = 1+1$,

$$\sqrt{\det\left(\partial^2 + m^2\right)} = \prod_p \operatorname{sh}\left(\frac{\tau}{2}\sqrt{p^2 + m^2}\right) , \quad (2.24)$$

$$\det_F \not{\partial} = \prod_p \operatorname{ch}^2\left(\frac{|p|\tau}{2}\right) \quad . \quad (2.25)$$

To have well-defined products over the momentum p, let $x \in [0,N]$ with periodic boundary conditions. Then $p = (2\pi/N)\ell$, $\ell = 0, 1, \ldots N-1$.

We now show that in the product $\sqrt{\det\partial^2} \cdot \det_F(\not{\partial})$, there is a cancellation which removes the fermion eigenenergies. Note that

$$\sqrt{\det \partial^2} \cdot \det_F \not\partial = \prod_p \mathrm{sh}\left(\frac{\tau}{2} |p|\right) \mathrm{ch}^2\left(\frac{\tau}{2} |p|\right) \tag{2.26}$$

$$= \exp\left\{ \sum_p \left[\ln\left(1 - e^{-\tau|p|}\right) + 2\ln\left(1 + e^{-\tau|p|}\right)\right] + \frac{3\tau}{2} \sum_p |p| \right\} . \tag{2.27}$$

We have for the first two terms in (2.27), for $N \to \infty$, using Ref. 6

$$N \int_{-\infty}^{+\infty} \frac{dp}{2\pi} \left[\ln\left(1 - e^{-\tau|p|}\right) + 2\ln\left(1 + e^{-\tau|p|}\right)\right] = \frac{N}{\pi\tau} \left[-\frac{\pi^2}{6} + 2\,\frac{\pi^2}{12}\right] = 0 \tag{2.28}$$

and hence

$$\sqrt{\det \partial^2} \cdot \det_F \not\partial = \exp\left\{ \frac{3\tau}{2} \sum_p |p| \right\} \tag{2.29}$$

where the term linear in the exponential is simply the zero-point energy. Note the fermion determinant with its negative energy solutions has cancelled and the fermions have been eliminated from the eigenspectrum.

We therefore have from (2.23) and (2.29)

$$Z(\tau) = \frac{\exp\left\{ \frac{3}{2}\, \tau \sum_p |p| \right\}}{\sqrt{\det\left(\partial^2 + m^2\right)}} \tag{2.30}$$

$$= \exp\left\{ -\frac{\tau}{2} \sum_p \left(\sqrt{p^2 + m^2} - 3|p|\right) - \sum_p \ln\left(1 - \exp\left\{-\tau\sqrt{p^2 + m^2}\right\}\right)\right\} \tag{2.31}$$

Dropping the vacuum energy, we obtain

$$Z(\tau) = \prod_p \sum_{n_p=0}^{\infty} \exp\left\{ - \tau \sum_p n_p \sqrt{p^2 + m^2} \right\} \tag{2.32}$$

Comparing (2.32) and (2.3) we find that the eigenenergies are

$$E[n] = \sum_p n_p \sqrt{p^2 + m^2} \quad , \quad n_p = 0, 1, \ldots \quad . \tag{2.33}$$

We see that the eigenspectrum is simply the equally spaced energy levels of the free massive boson field. The integers n_p denote the number of particles n_p with momentum p that are in the system. These massive excitations of the field are the bound states[1] of the fermion-antifermion pair interacting via the gauge field. The eigenfunctions of the interacting theory are $|\Phi_n\rangle \equiv \otimes_p |n_p\rangle$ with eigenenergy $\sum_p n_p \sqrt{p^2 + m^2}$, and form a complete basis for the Hilbert space of states.

The fermion-like negative eigenenergies have been eliminated from the spectrum due to its interaction with the gauge-gield, and the cancellation in (2.28) reflects this. The absence of fermionic energy levels is the first indication that the fermions are confined.

III. THE MESON STATE

By the meson state, we mean the fermion-string-antifermion gauge invariant state. Before calculating the energetics of the meson state, we briefly review the definition of energy for a field theory.

Recall from quantum mechanics the energy of an unnormalized state is

$$E = \frac{\langle\Phi|H|\Phi\rangle}{\langle\Phi|\Phi\rangle} \tag{3.1}$$

where H is the Hamiltonian. To use the above definition for a quantum field, a suitable limiting procedure has to be used since the states are generally not normalizable. Also, since we are using the path integral, we derive the Hamiltonian from the action functional. Let $|\chi_\varepsilon\rangle$ be a state close to $|\Phi\rangle$ such that

$$\lim_{\varepsilon \to 0} |\chi_\varepsilon\rangle = |\Phi\rangle \quad . \tag{3.2}$$

Then

$$E = \lim_{\varepsilon \to 0} \frac{\langle\chi|H|\Phi\rangle}{\langle\chi|\Phi\rangle} \tag{3.3}$$

$$= \lim_{\varepsilon \to 0} \left(\lim_{t \to 0} - \frac{\partial}{\partial t} \frac{\langle\chi|e^{-tH}|\Phi\rangle}{\langle\chi|\Phi\rangle} \right) \quad . \tag{3.4}$$

The amplitude $\langle\chi|\exp\{-tH\}|\Phi\rangle$ can be evaluated using the path integral, and hence the energy can be obtained using (3.4).

We are interested in the energy of the gauge-invariant fermion-antifermion state separated by distance L. In the Schrodinger representation

$$|\Phi\rangle = \bar{\psi}_{(0L)} \exp\left\{ i \int_0^L A_1(0,x)\, dx \right\} \psi_{(00)} |\Omega\rangle \tag{3.5}$$

where $|\Omega\rangle$ is the vacuum state (see Fig. 1). The state $|\chi\rangle$ is obtained by infinitesimally displacing the fermion and antifermion in $|\Phi\rangle$. For notational simplicity, we will treat $|\chi\rangle$ as identical to $|\Phi\rangle$ and introduce ε at the end. Let

$$Q = \langle\Phi|e^{-tH}|\Phi\rangle \tag{3.6}$$

Then in the Heisenberg representation, we have

$$Q = \langle\Omega|\bar{\psi}_{(t0)} \exp\left\{ -i \int_0^L A_1(t,x)\, dx \right\} \psi_{(tL)} \; \bar{\psi}_{(0L)}$$

$$\times \exp\left\{ i \int_0^L A_1(0,x)\, dx \right\} \psi_{(00)} |\Omega\rangle \tag{3.7}$$

where the Heisenberg operators are appropriately time-ordered for $t > 0$. We can represent Q as a path integral using the infinite-time action and have[7] (see Fig. 2):

$$Q = \frac{1}{Z}\left\langle \bar{\psi}_{(t0)} \exp\left\{- i \int_0^L A_1(t,x)\, dx\right\} \psi_{(tL)}\, \bar{\psi}_{(0L)} \times \exp\left\{i \int_0^L A_1(0,x)\, dx\right\} \psi_{(00)}\, e^S \right\rangle \tag{3.8}$$

The action in (3.8) is the infinite-time action obtained from (2.12) and Z is also the infinite-time partition function; the bracket denotes integration over all boson and fermion field variables, and $\bar{\psi}$, ψ and A_μ in (3.8) are the field variables.

Introducing the regularizer ε and performing the path integral[3] gives

$$Q_\varepsilon = \langle \chi | e^{-tH} | \Phi \rangle \tag{3.9}$$

$$= \frac{1}{2\pi^2}\, \frac{1}{t^2 + \varepsilon^2}\, \exp\left\{ R(t,L) + P(t) \right\} \tag{3.10}$$

where

$$R(r,L) = -\frac{g^2}{2}\int \frac{d^2p}{(2\pi)^2}\, \frac{\left|1 - \exp\{ip_0 t\}\right|^2 \left|1 - \exp\{ip_1 L\}\right|^2}{p_1^2 (p^2 + m^2)} \tag{3.11}$$

$$P(t) = 2g^2 \int \frac{d^2p}{(2\pi)^2}\, \frac{\left|1 - \exp\{ip_0 t\}\right|^2}{p^2 (p^2 + m^2)} \tag{3.12}$$

$$= 2\left[\gamma + \ln\left(\frac{mt}{2}\right) + K_0(mt)\right] \tag{3.12'}$$

and γ is Euler's constant. The term R(t,L) comes from the gauge-field variables, and the term P(t) from a combination of the gauge-field and fermion variables. NOte for $L \to 0$, $R(t,L) \to 0$ and we recover the result given in Ref. 3.

We have the following exact properties of $R(t,L)$ and $P(t)$:

$$R(0,L) = 0 \quad , \qquad P(0) = 0 \quad , \tag{3.13}$$

$$-\left.\frac{\partial R(t,L)}{\partial t}\right|_{t=0} = \frac{g^2 L}{2} \quad , \qquad \left.\frac{\partial P(t)}{\partial t}\right|_{t=0} = 0 \quad . \tag{3.14}$$

Using (3.4) for the energy, and taking the limit of $\varepsilon \to 0$ gives

$$E = -\left.\frac{\partial R(t,L)}{\partial t}\right|_{t=0} = \frac{g^2 L}{2} \quad . \tag{3.15}$$

The expression for energy is exact and is a gauge-invariant quantity. We see that energy increases linearly with distance, and the string tension, i.e., the energy per unit length, is $g^2/2$.

It is obvious that the state $|\Phi\rangle$ is not an eigenstate, given its complicated time dependence. To understand how the state $|\Phi\rangle$ is constructed, we expand it in the energy eigenfunction basis. Let

$$|\Phi\rangle = \prod_p \sum_{n_p} C_{n_p} |n_p\rangle \quad . \tag{3.16}$$

Then,

$$Q = \langle\Phi|e^{-tH}|\Phi\rangle$$

$$= \sum_{\{n\}} |C_n|^2 \exp\left\{-tE_n\right\} \quad . \tag{3.17}$$

From (3.10) we have for $mL >> 1$ $(\alpha = g^2 L/2m)$,

$$Q = \frac{e^{2\gamma}}{8\pi^2} \exp\left\{ -\alpha + \alpha e^{-mt} + 1 - mtK_1(mt) + 2K_0(mt) + \mathcal{O}\left(e^{-mL}\right)\right\} \quad (3.18)$$

$$\simeq \frac{e^{2\gamma}}{8\pi^2} \exp\left\{ -\alpha + \alpha e^{-mt}\right\} \quad . \quad (3.19)$$

In obtaining (3.19) we have kept only the leading order terms in mL, and in effect have disregarded eigenfunctions with nonzero momentum p. We call this the static approximation, and a detailed study shows that this approximation has the leading order effects.

Ignoring the overall constant, we have

$$Q \simeq e^{-\alpha} \sum_{n=0}^{\infty} \left(\frac{\alpha^n}{n!}\right) e^{-nmt} \quad . \quad (3.20)$$

Hence, from (3.17) and (3.20),

$$E_n = nm \quad (3.21)$$

$$|C_n|^2 = e^{-\alpha}\left(\frac{\alpha^n}{n!}\right) \quad , \quad \sum_n |C_n|^2 = 1 \quad . \quad (3.22)$$

As expected, only the static eigenstates $|n_{p=0}\rangle \equiv |n\rangle$ with energy nm contribute to $|\Phi\rangle$; note $|\Phi\rangle$ is a Poisson distribution in $|n\rangle$ and

$$|\Phi\rangle \simeq \sum_{n=0}^{\infty} C_n |n\rangle \quad (3.23)$$

Hence

$$E = \langle\Phi|H|\Phi\rangle \quad (3.24)$$

$$= m \sum_n n|C_n|^2 = m\alpha \quad (3.25)$$

$$= \frac{g^2 L}{2} \quad (3.26)$$

Note the static approximation gives the correct expression for the energy. Using the properties of the Poisson distribution, we have for the energy dispersion

$$\Delta E = \langle \Phi | \hat{H}^2 | \Phi \rangle - E^2 \tag{3.27}$$

$$= m^2 \sum_n n^2 |C_n|^2 - E^2 \tag{3.28}$$

$$= mE \quad . \tag{3.29}$$

The dispersion in the energy is large and is proportional to the energy of the state. Hence in any numerical calculation it would be difficult to separate out the energy of the state from the background statistical fluctuations. We can see the origin of this large fluctuation by expanding the $|C_n|^2$ about its maxima, and assuming all the coefficients are real we obtain, up to a normalization constant

$$|\Phi\rangle \simeq \sum_{n=0}^{\infty} \exp\left\{-\frac{1}{4\alpha}(n-\alpha)^2\right\} |n\rangle \quad . \tag{3.30}$$

We see that $|\Phi\rangle$ is peaked at the state $|N\rangle$ with integer $N = \alpha$, and has a spread of 2α which gives rise to the large dispersion for E.

Identifying the eigenstates $|n\rangle$ as n pairs of fermion-antifermion bound states, we have the interpretation of (3.30) that the excited meson state is, with the largest amplitude, a state of $N \sim mL$ pairs. That is, for large L the string 'breaks' instantaneously into a number of pairs proportional to the length of the string. See Fig. 3.

Hence, if we view the state $|\Phi\rangle$ as a case where the quarks (fermions) are well-separated and attempt to see the single quark, we will end up observing the bound-state pairs and not the isolated fermion. Once the fermions constituting the meson are separated by a distance larger than m^{-1}, pair production takes place. Hence we conclude that the fermions are permanently confined within a distance of m^{-1}, and no isolated single fermion can be seen for separations much larger than m^{-1}.

This view of string break-up has been discussed by other authors[8] and our calculation provides a quantitative basis for this.

IV. STRING TENSION AND CONFINEMENT

We define the string tension μ as the change in the energy of the string when the length is varied. That is

$$\mu = \frac{\partial E}{\partial L} \tag{4.1}$$

where E is the energy and L the length of the string. For the meson state $|\Phi\rangle$ we have from (3.15)

$$\mu = \frac{g^2}{2} \quad . \tag{4.2}$$

We now discuss the connection of μ with the Wilson loop integral. Consider a square contour of length L and width t and with enclosed area Γ (see Fig. 4). Then, using the results of Ref. 3 we have [using (2.10) and Stokes theorem],

$$W = \left\langle \exp \left\{ i \oint_{\mathscr{C}} A_\mu \, d\ell_\mu \right\} \right\rangle \tag{4.3}$$

$$= \left\langle \exp \left\{ i \int_{\Gamma} \partial^2 s \right\} \right\rangle \tag{4.3'}$$

$$W = \exp\left\{R(t,L) + R(L,t)\right\} \quad , \tag{4.4}$$

where R(t,L) is given by (3.11).

For large loops and large time, we have

$$W = \exp\left\{-\frac{g^2}{2m}(t+L) + \mathcal{O}\left(e^{-mt}, e^{-mL}\right)\right\} \quad . \tag{4.5}$$

We see that W does not have exp {- area} Wilson behavior,[7] and the string tension μ cannot be extracted as the coefficient of the area term. The reason for this is the string 'breaks' into $(t+L)$-pairs and gives the exp {- perimenter} behavior for the loop; in other words, fermion pair creation removes the Wilson behavior for the loop.

The Wilson loop in the absence of the fermions can be obtained by setting m=0 in R(t,L), and we obtain the exact result

$$W = \exp\left\{-\frac{g^2}{2} tL\right\} \tag{4.6}$$

$$= \exp\left\{-\mu tL\right\} \quad , \tag{4.7}$$

and the string tension is the coefficient of the area term. Hence μ can be obtained by studying the large gauge field loops in the absence of fermions, and the introduction of fermions does not spoil the result, at least in Schwinger QED. Note that in the presence of the fermions, for small loops, i.e., $t,L \ll 1$, we again have

$$W \cong \exp\left\{-\mu tL + \mathcal{O}\left(t^2, L^2\right)\right\} \quad . \tag{4.8}$$

We hence have the following picture for confinement in Schwinger QED. The small gauge-field loops show the Wilson exp {- area} behavior. As the loops are made larger and larger, due to the Wilson behavior of the pure gauge loops large energy is required to make these loops, and instead the system produces pairs of fermion-antifermion, breaking the string and giving the exp {- perimeter} behavior.[8] The string breaks, i.e., crosses over from the area to perimeter behaviour at the characteristic length scale of the system, which for Schwinger QED is m^{-1}. See Fig. 5.

In summary, the behavior of the large gauge field loops in the pure gauge theory determine whether or not there is confinement of the fundamental fermions, and the Wilson criterion is appropriate. However, in the presence of the fermions, confinement can be equally studied by looking at the energy of the fermion-string-antifermion state, which involves looking at the short time behaviour of the system. Both these approaches can be used for evaluating the string tension and give the same result in Schwinger QED.

ACKNOWLEDGMENTS

I thank Professor Sidney Drell and SLAC for their hospitality. I also thank Professor Abdus Salam for hospitality at the International Center for Theoretical Physics, where a part of this work was done.

This work was supported by the Department of Energy under contract DE-AC-3-76SF00515.

REFERENCES

1. J. Schwinger, Phys. Rev. 128, 2425 (1962);
L. S. Brown, Nuovo Cimento 29, 617 (1963);
J. Lowenstein and J. Swieca, Annals of Physics 68, 72 (1971);
Y. Frishman, Lecture Notes in Physics 32, (1975).

2. E. Marinari, G. Parisi and C. Rebbi, Nuclear Physics B190, 734 (1981);
T. Burlit and P. Corvi, work in progress (1982), Edinburgh, U.K.

3. B. E. Baaquie, ICTP preprint IC/80/123 (to be published in Journal of Physics G).

4. B. E. Baaquie, ICTP preprint IC/82/79.

5. D. J. Gross, R. D. Pisarski and L. G. Yaffe, RMP 53, Vol.1, 43 (1981).

6. T. S. Gradshteyn and I. J. Ryzhik, Table of Integrals, Academic Press (New York), 1975.

7. K. G. Wilson, Phys. Rev. D10, 2455 (1974).
K. G. Wilson, Erice Lecture Notes (1973).

8. A. Casher, J. Kogut and L. Susskind, Phys. Rev. D10, 732 (1974).

FIGURE CAPTIONS

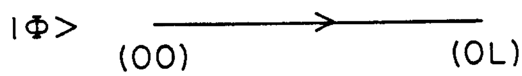
Fig. 1. Meson state.

Fig. 2. Time evolution of the meson state.

Fig. 3. Eigenfunction expansion of the meson state.

Fig. 4. The Wilson loop.

Fig. 5. Breaking of the string.



8 – 82 4359A1

Fig. 1

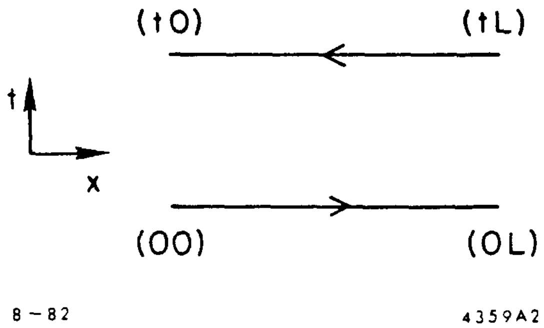


Fig. 2

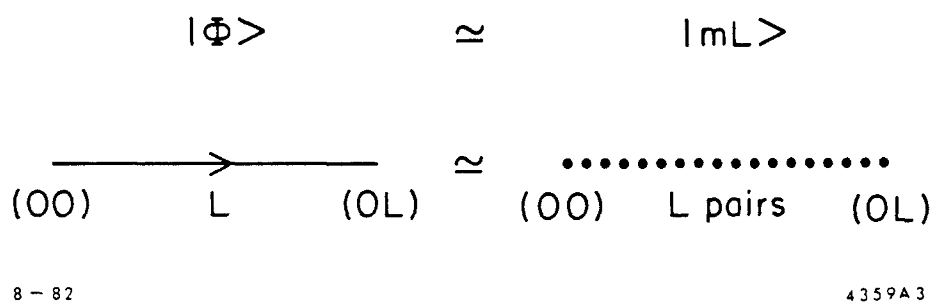


Fig. 3

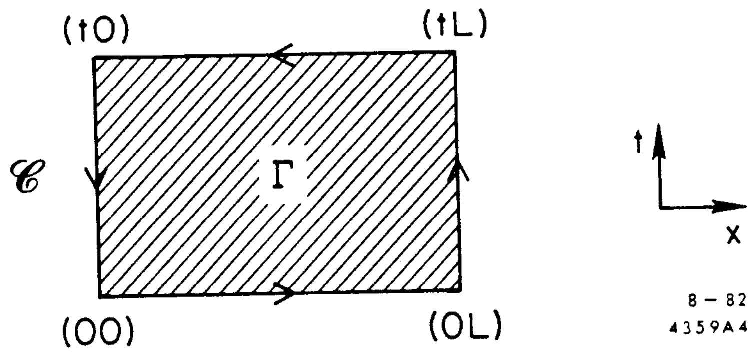


Fig. 4

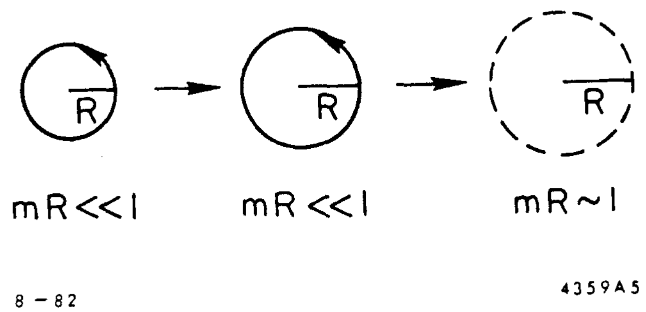


Fig. 5